THE PHYSICS OF FOAM

- Boulder School for Condensed Matter and Materials Physics

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   - Formation
   - Microscopics
2. Structure
   - Experiment
   - Simulation
3. Stability
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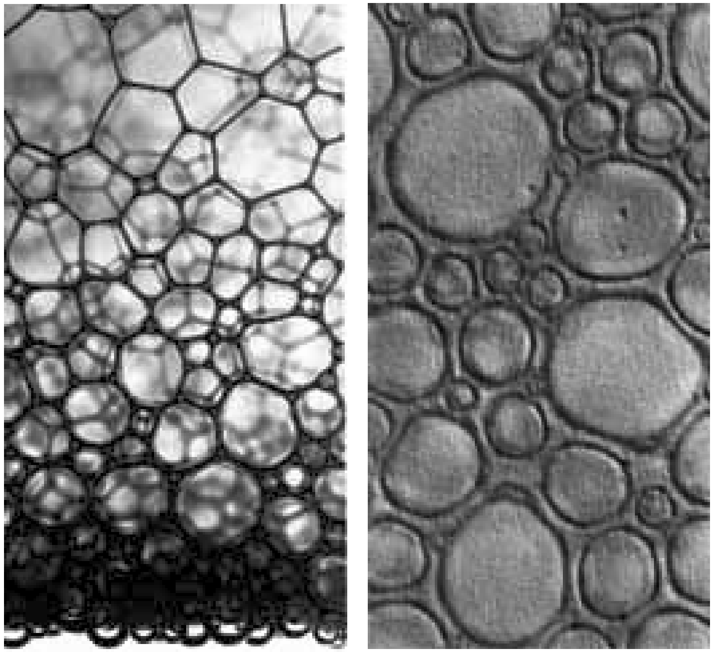
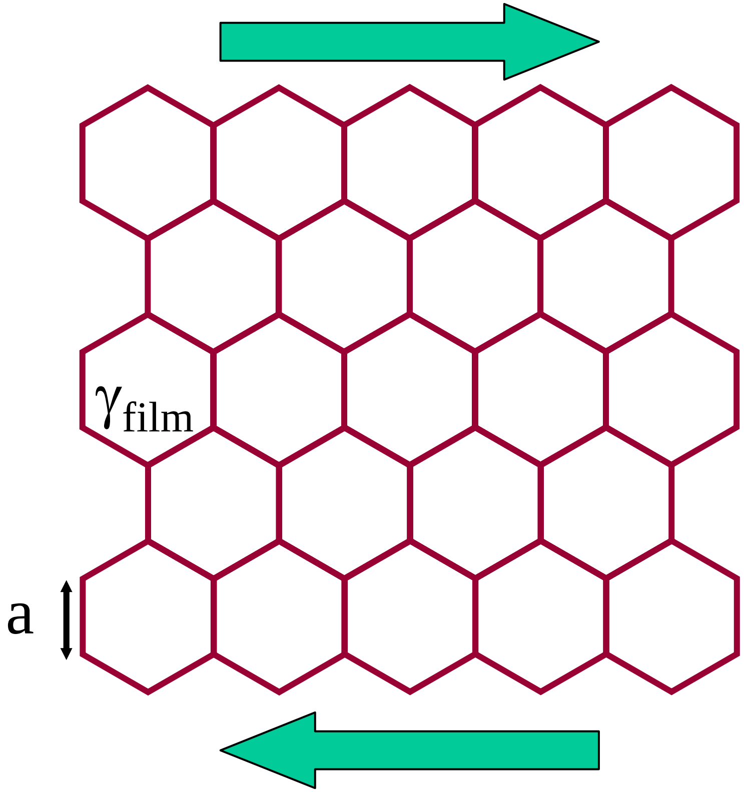
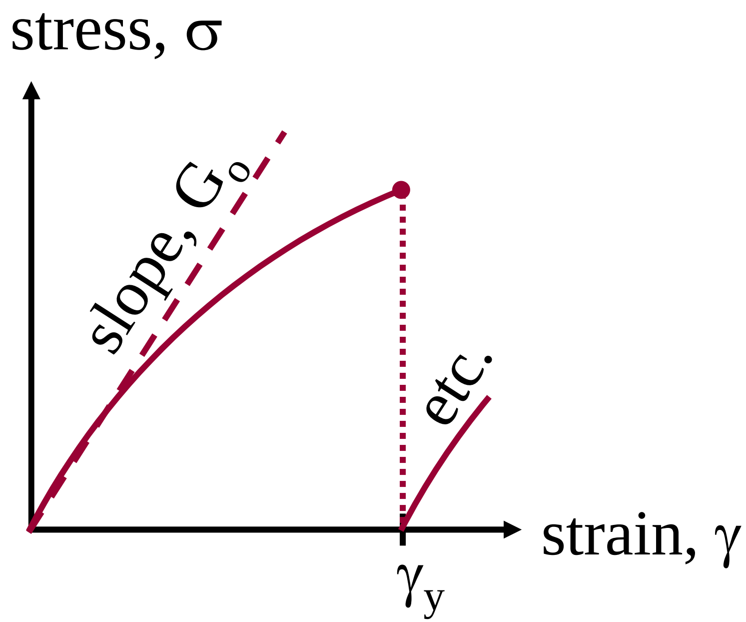
   - Drainage
4. Rheology
   - Linear response
   - Rearrangement & flow



Princen-Prud'homme model

- shear a 2D honeycomb, always respecting Plateau's rules
  - deformation is not affine: vertices must rotate to maintain 120-120-120

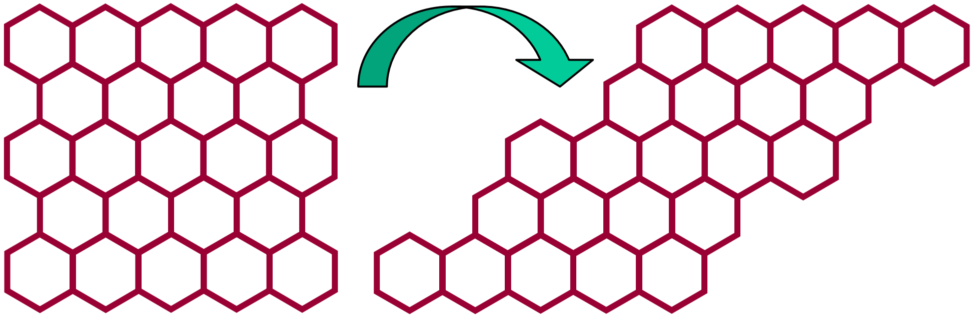




shear modulus: $G_o = \gamma_{film}/\mathrm{Sqrt}[3]a$
yield strain: $\gamma_y = 1.2$

Yes, but…

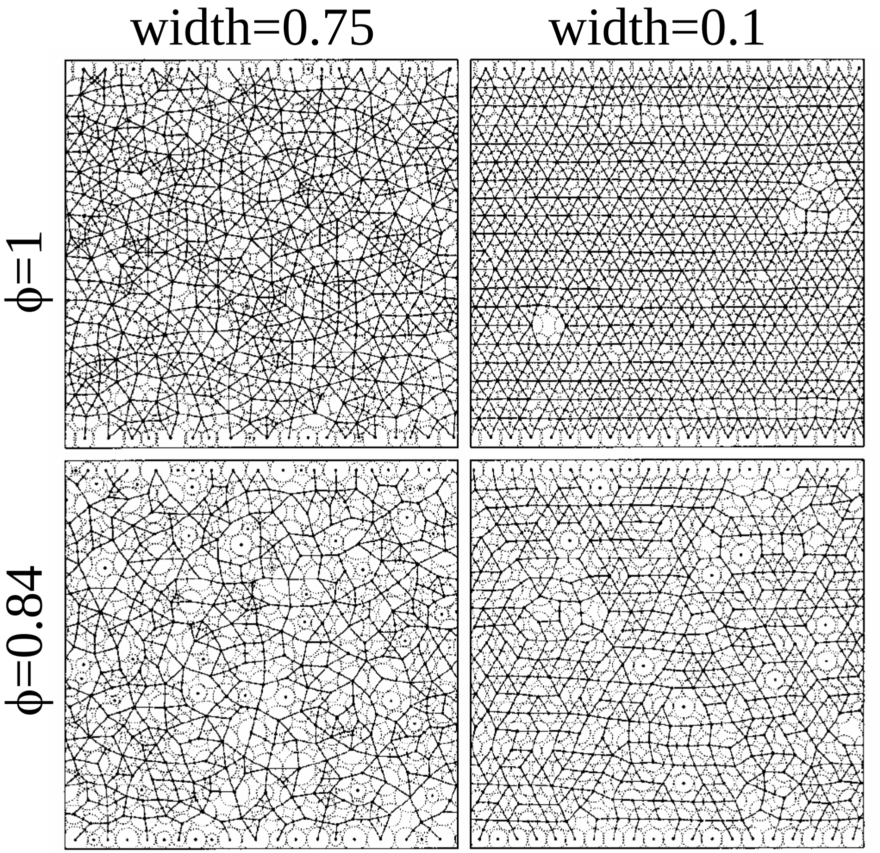
- Princen-Prud’homme is for 2D periodic static dry foam
  - dimensionality?
    - generally expect G ~ Laplace pressure (surface tension/bubble size)
  - wetness?
    - shear modulus must vanish for wet foams
  - dissipation?
    - nonzero strainrate or oscillation frequency?
  - disorder?
    - smaller rearrangement size (not system)?
    - smaller yield strain?

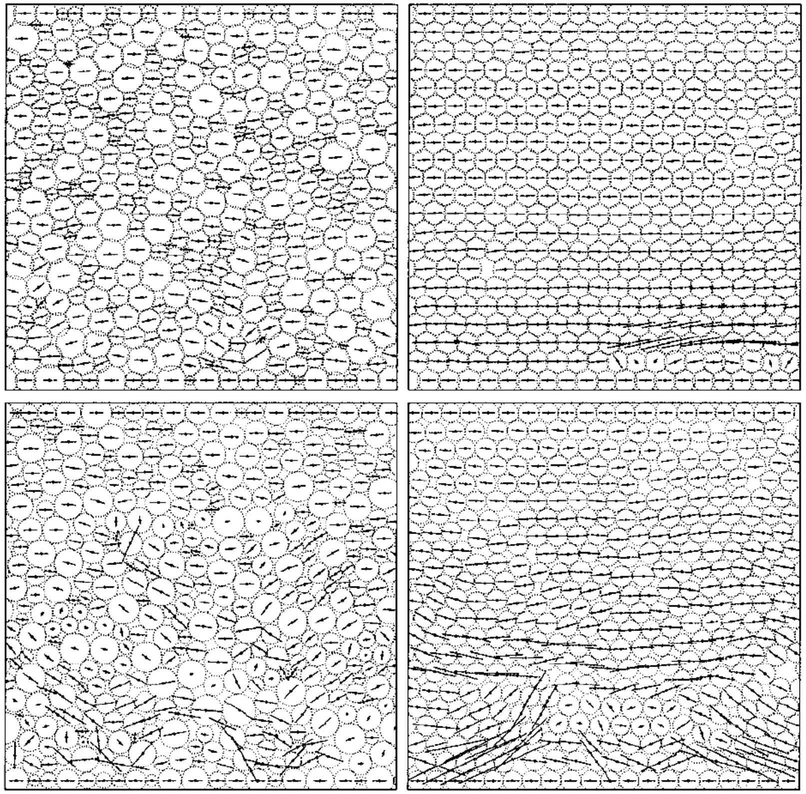


small-strain non-affine motion

(DJD)

- bubble motion is up & down as well as more & less
  - compare bonds and displacements (normalized to affine expectation)
  - trends vs polydisersity and wetness





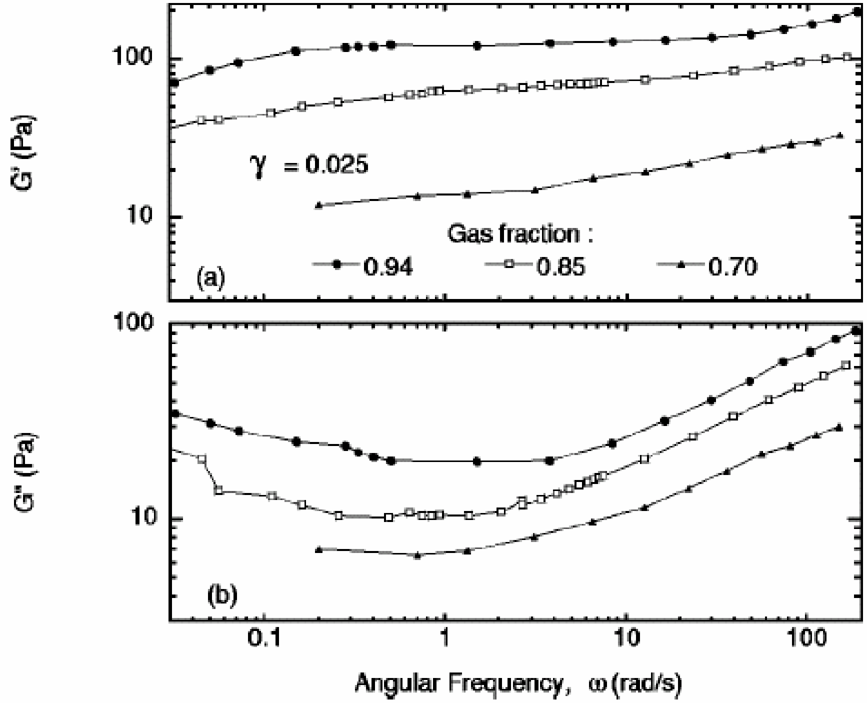
Frequency dependence, $G^*(\omega)$

(A. Saint-Jalmes & DJD)

- Simplest possibility: Kelvin solid

  $G^*(\omega) = G_0 + i\eta\omega$ {i.e. $G'(\omega) = G_0$ and $G''(\omega) = \eta\omega$}

- But typical data looks much different:
  - G'(ω) isn't flat, and increases at high ω…
  - G"(ω) doesn't vanish at low ω…



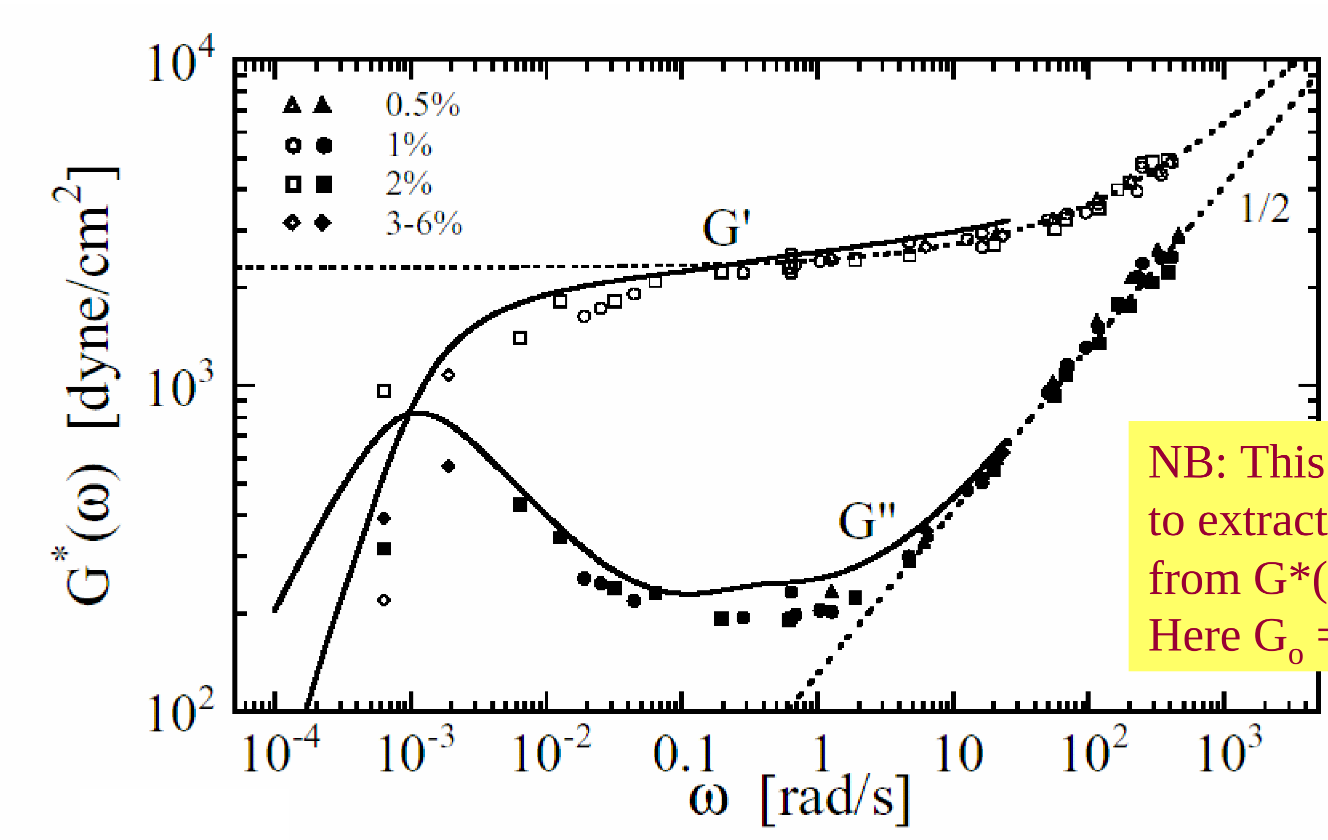
High-ω rheology

(A.D. Gopal & DJD)

- Due to non-affine motion, another term dominates:

$$G^*(\omega) = G_0(1 + \mathrm{Sqrt}[i\omega/\omega_c])^{**}$$

shown by dotted curve for ε=0.08 foam:

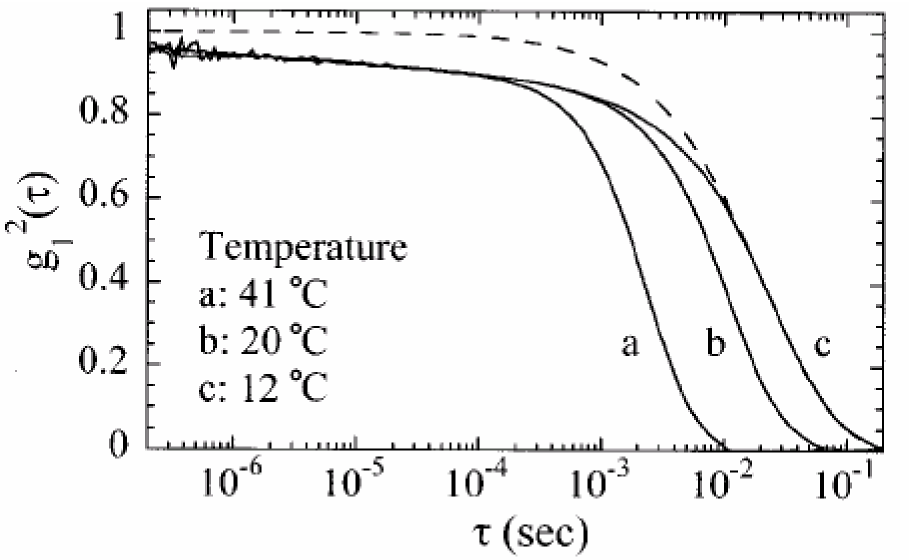


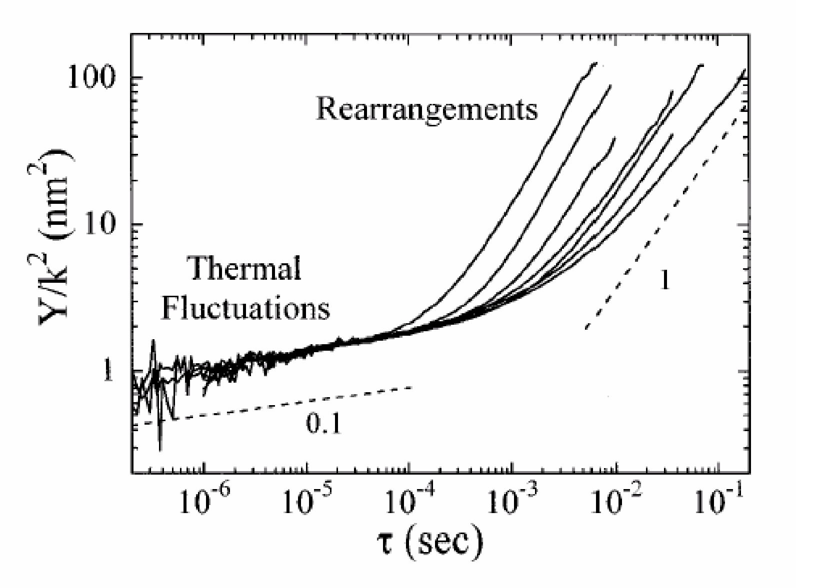
NB: This gives a robust way to extract the shear modulus from G*(ω) vs ω data.
Here G_o = 2300 dyne/cm²

[**seen and explained by Liu, Ramaswamy, Mason, Gang, Weitz for compressed emulsion using DWS microrheology]

Short-τ DWS

- long-τ: rearrangements give exponential decay
- short-τ: thermal interface fluctuations (Y=<Δr^2(τ)>)



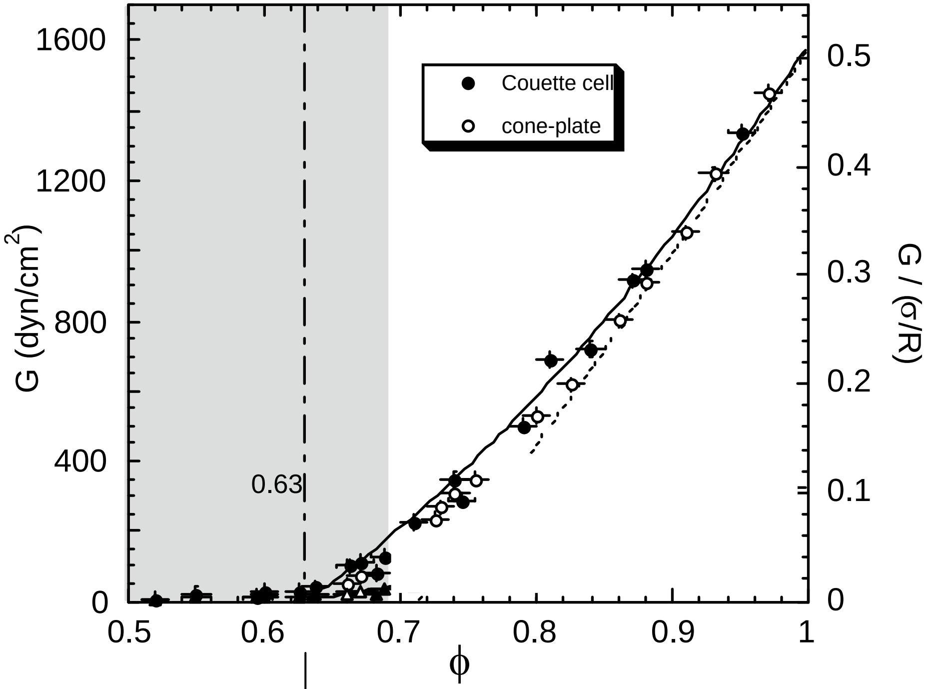


  - amplitude of fluctuations: $\delta = 13 \pm 3$ angstroms
  - microrheology: $G_o \approx \dfrac{k_B T}{R\delta^2} \approx 1000 \pm 300 \quad \text{dyne/cm}^2$

Unjamming vs gas fraction

(A. Saint-Jalmes & DJD)

- Data for shear modulus:
  - symbols: polydisperse foam
  - solid curve: monodisperse emulsion (Mason & Weitz)
  - dashed curve: polydisperse emulsion (Princen & Kiss)



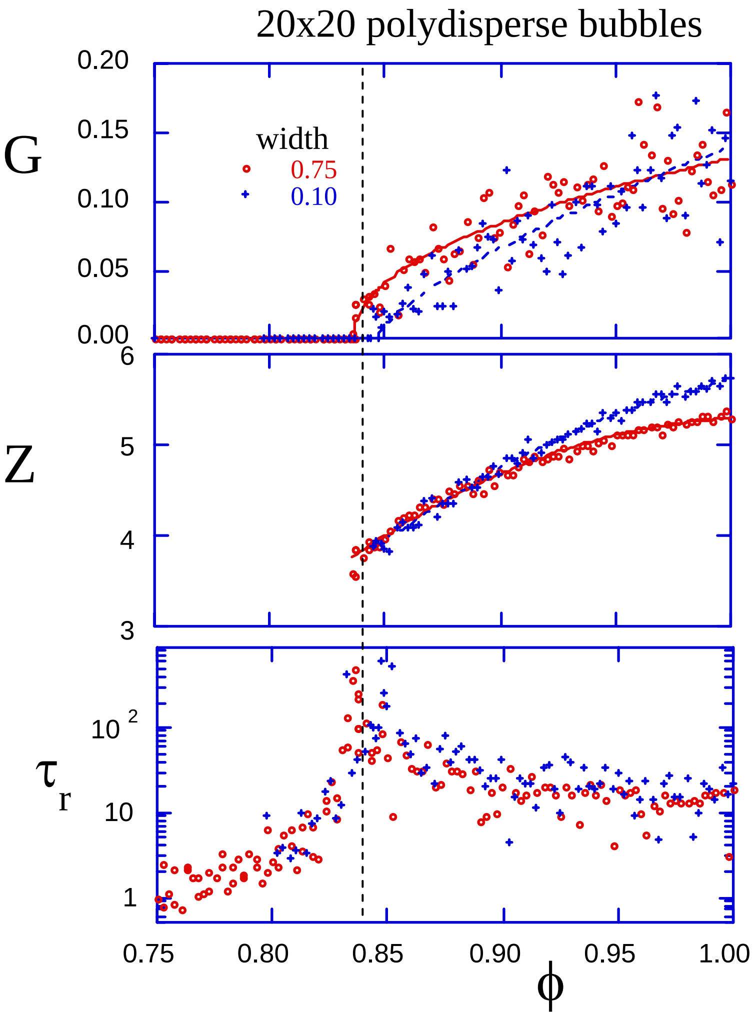
unjammed | jammed

polydispersity makes little or no difference!

Behavior near the transition

(DJD)

- simulation of 2D bubble model



shear modulus

coordination number

stress-relaxation time

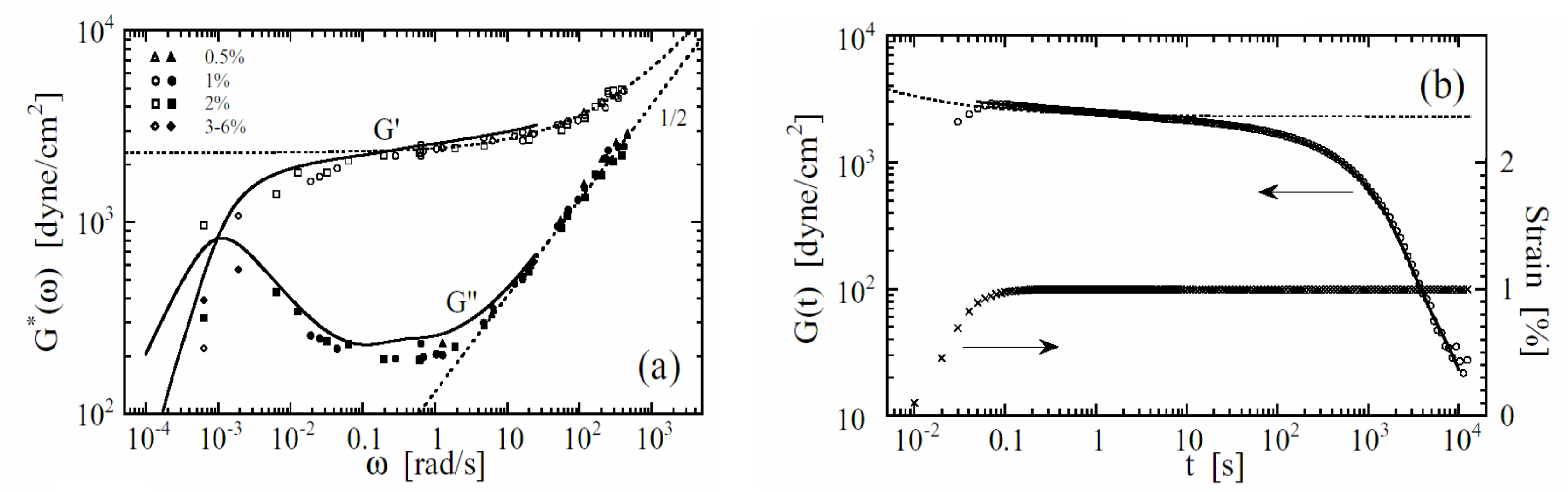
better "point J" statistics by O'Hern, Langer, Liu, Nagel

τ_r appears to diverge at ϕ_{rcp} ~0.84

Unjamming vs time

(A.D. Gopal & DJD)

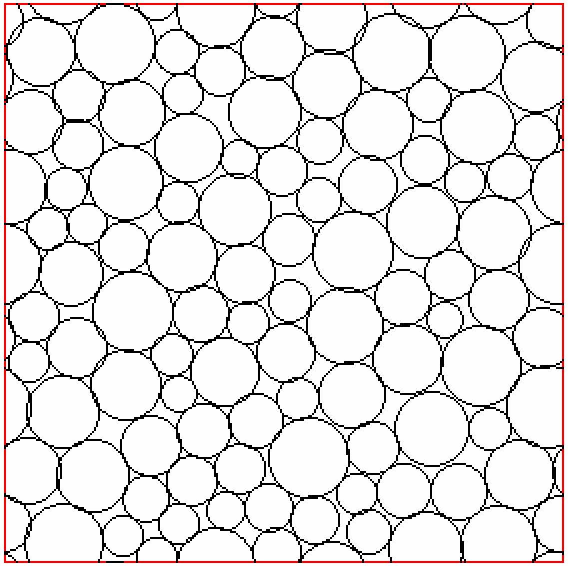
- elasticity vanishes at long times…
  - stress relaxes as the bubbles coarsen
  - time scale is set by foam age
    - ie how long for size distribution to change
    - not set by the time between coarsening-induced rearrangements (20s)



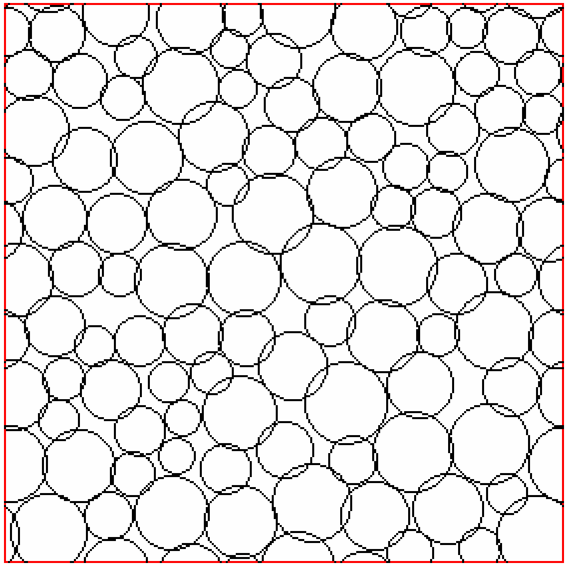
- Even though this is not a thermally activated mechanism like diffusion or reptation, the rheology is linear
  - $G^*(\omega)$ and G(t) date are indeed related by Fourier transform

Unjamming vs shear

- make bubbles rearrange & explore packing configurations
  - slow shear:
    - sudden avalanche-like rearrangements of a few bubbles at a time
  - fast shear:
    - rearrangements merge together into continuous smooth flow



slow & jerky

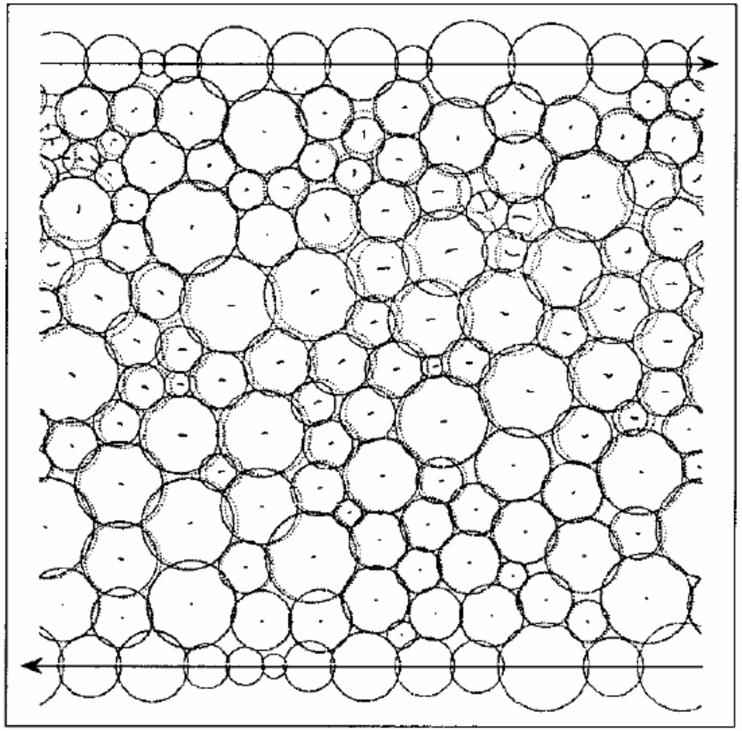


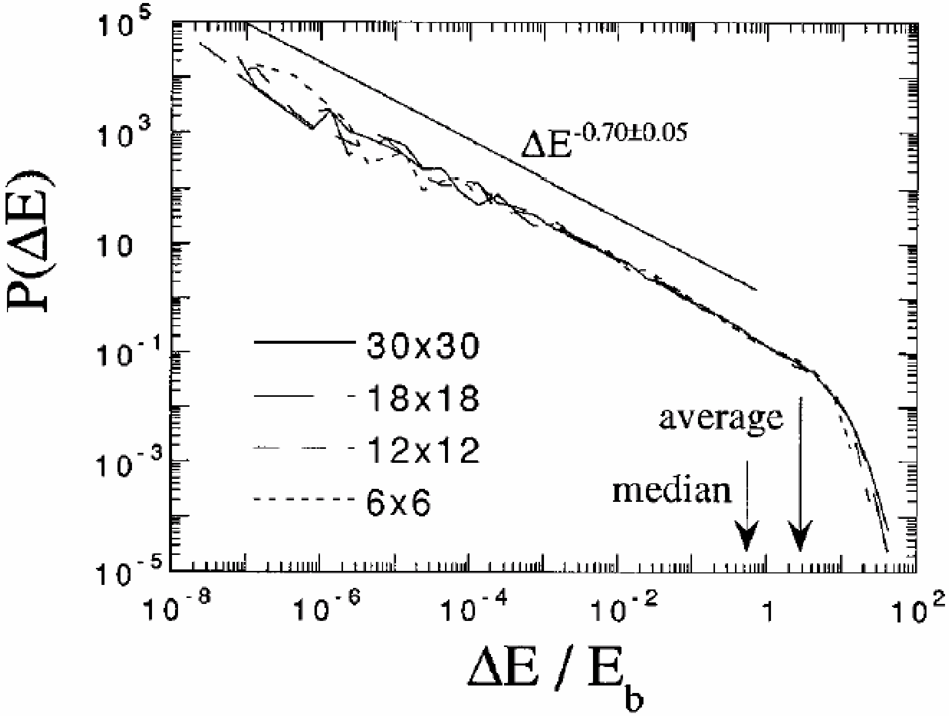
fast & smooth

Rearrangement sizes

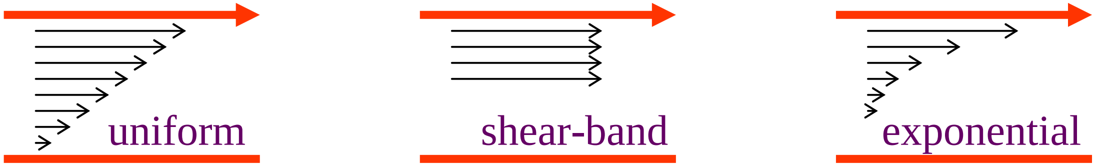
- even the largest are only a few bubbles across
  - picture of a very large event
  - distribution of energy drops (before-after) has a cutoff

    {but it moves out on approach to ϕ_c "point J"}

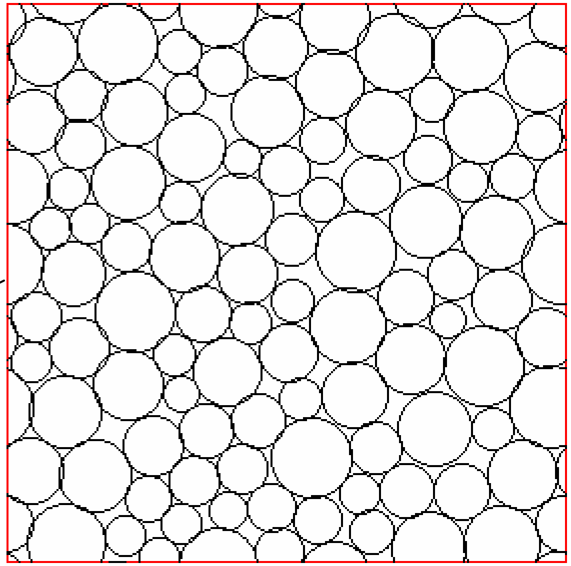




NB: shear deformation is uniform



- UCLA:
  - direct observation of free surface in Couette cell
  - viscosity and G*(ω) are indep. of sample thickness & cell geometry
  - DWS gives expected decay time in transmission and backscattering
  - viscous fingering morphology
- Hohler / Cohen-Addad lab (Marne-la-Vallee):
  - multiple light scattering and rheology
- Dennin lab (UC Irvine):
  - 2D bubble rafts and lipid monolayers
- Weitz lab (Exxon/Penn/Harvard):
  - Rheology of emulsions
- Computer simulations:
  - bubble model (DJD-Langer-Liu-Nagel)
  - Surface evolver (Kraynik)
  - 2D (Weaire)
  - vertex model (Kawasaki)



important scales

notation:

τ_{oq} : time between coarsening - induced rearrangments

τ_{d} : duration of rearrangement events

γ_y : yield strain

$\dot{\gamma}_c = \gamma_y / \tau_{\mathrm{oq}}$: whether coarsening - or shear - induced rearrangements dominate

$\dot{\gamma}_m = \gamma_y / \tau_{\mathrm{d}}$: whether rearrangements are smooth or avalanche - like

values for Foamy:

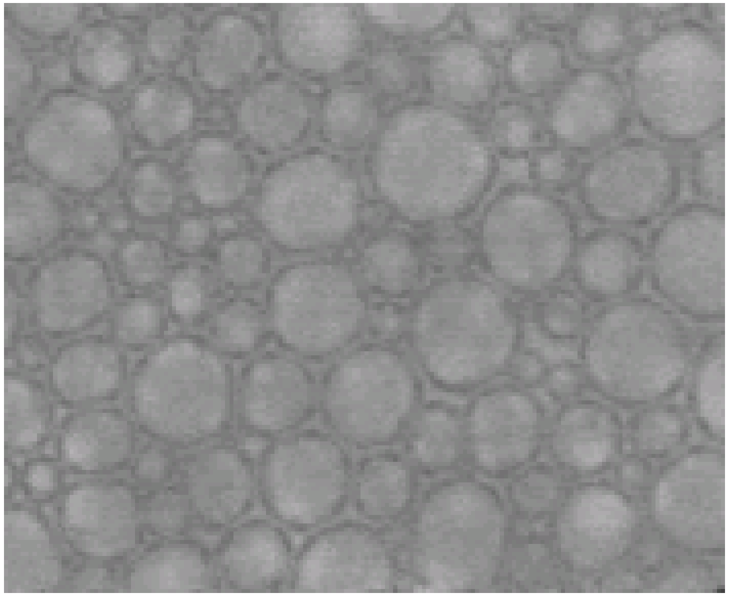
$\tau_{\mathrm{oq}} = 20s$

$\tau_{\mathrm{d}} = 0.1s$

$\gamma_y = 0.05$

$\dot{\gamma}_c = 0.003/s$

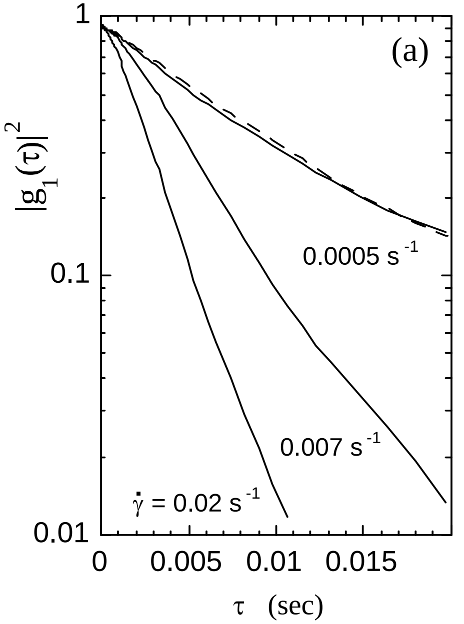
$\dot{\gamma}_m = 0.5/s$



bubble motion via DWS

(A.D. Gopal & DJD)

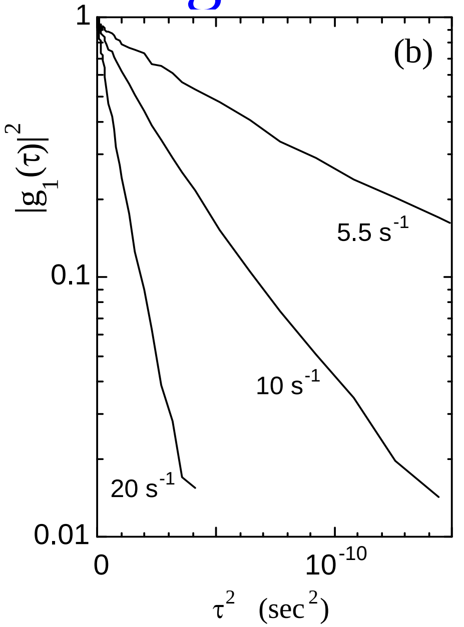
low shear



$$g_1(\tau) \approx \exp\left[-6\left(\frac{L}{l^*}\right)^2 \frac{\tau}{\tau_o}\right]$$

τ_o = time between rearrangements at each scattering site

high shear



$$g_1(\tau) \approx \exp\left[-6\left(\frac{L}{l^*}\right)^2 \left(\frac{\tau}{\tau_s}\right)^2\right]$$

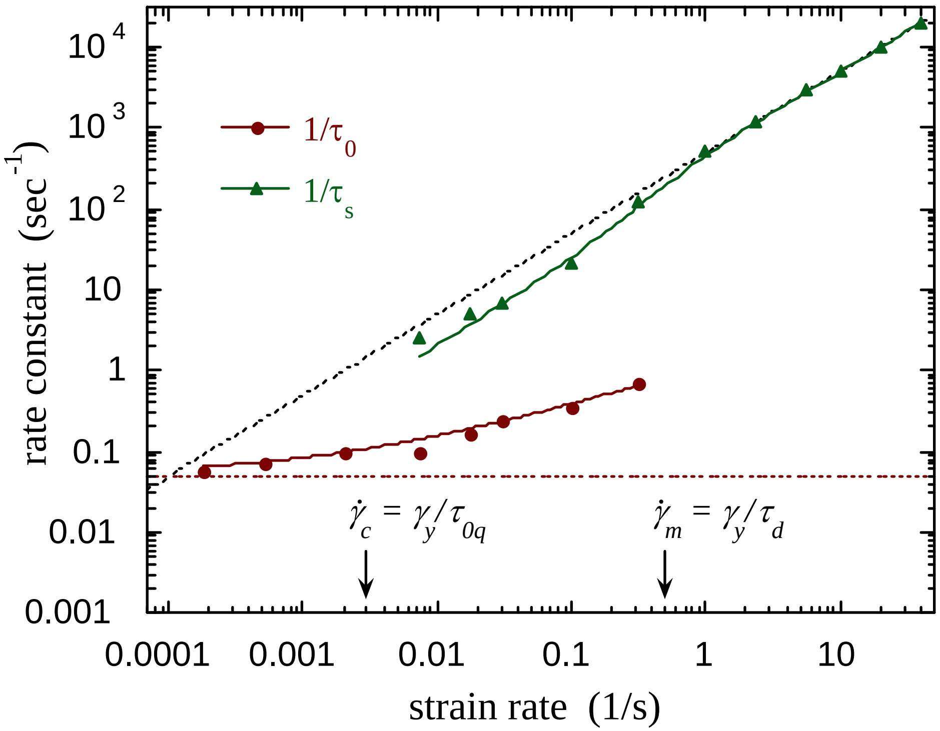
$$\tau_s = \frac{\sqrt{30}}{kl^{*}\dot{\gamma}}$$

time for adjacent scattering sites to convect apart by λ

DWS times vs strainrate

(A.D. Gopal & DJD)

- behavior changes at expected strainrate scales



discrete rearrangements
jammed/solid-like

laminar bubble motion
unjammed/liquid-like

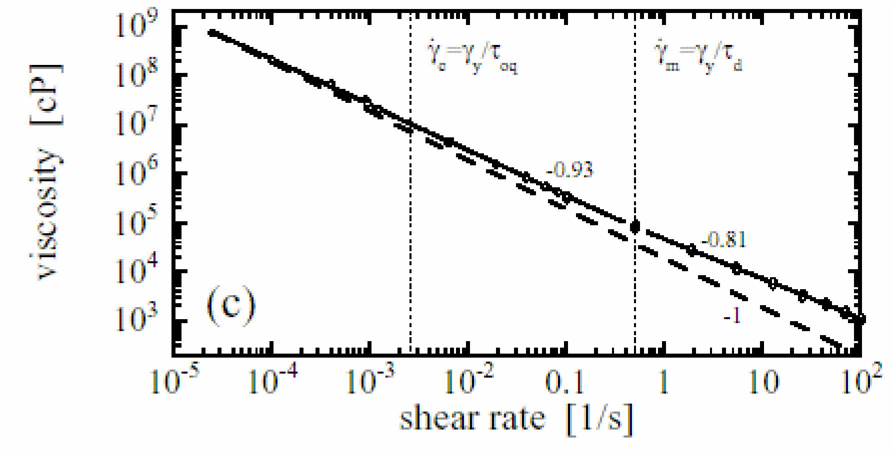
Rheological signature?

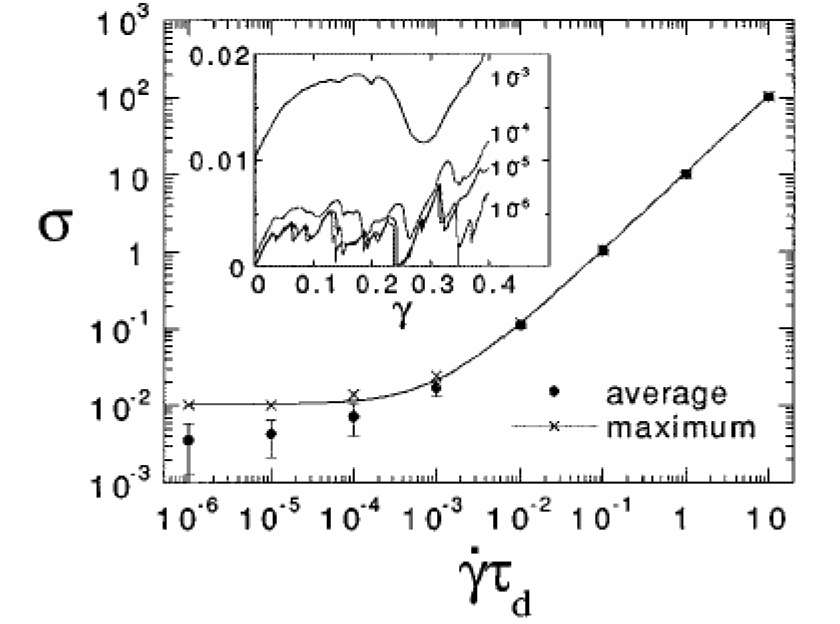
(A.D. Gopal & DJD)

- simplest expectation is Bingham plastic:

$$\sigma = \sigma_y \left(1 + \dot{\gamma}\tau_d / \gamma_y\right) \text{ and hence } \eta = \sigma_y \left(1/\dot{\gamma} + 1/\dot{\gamma}_m\right)$$

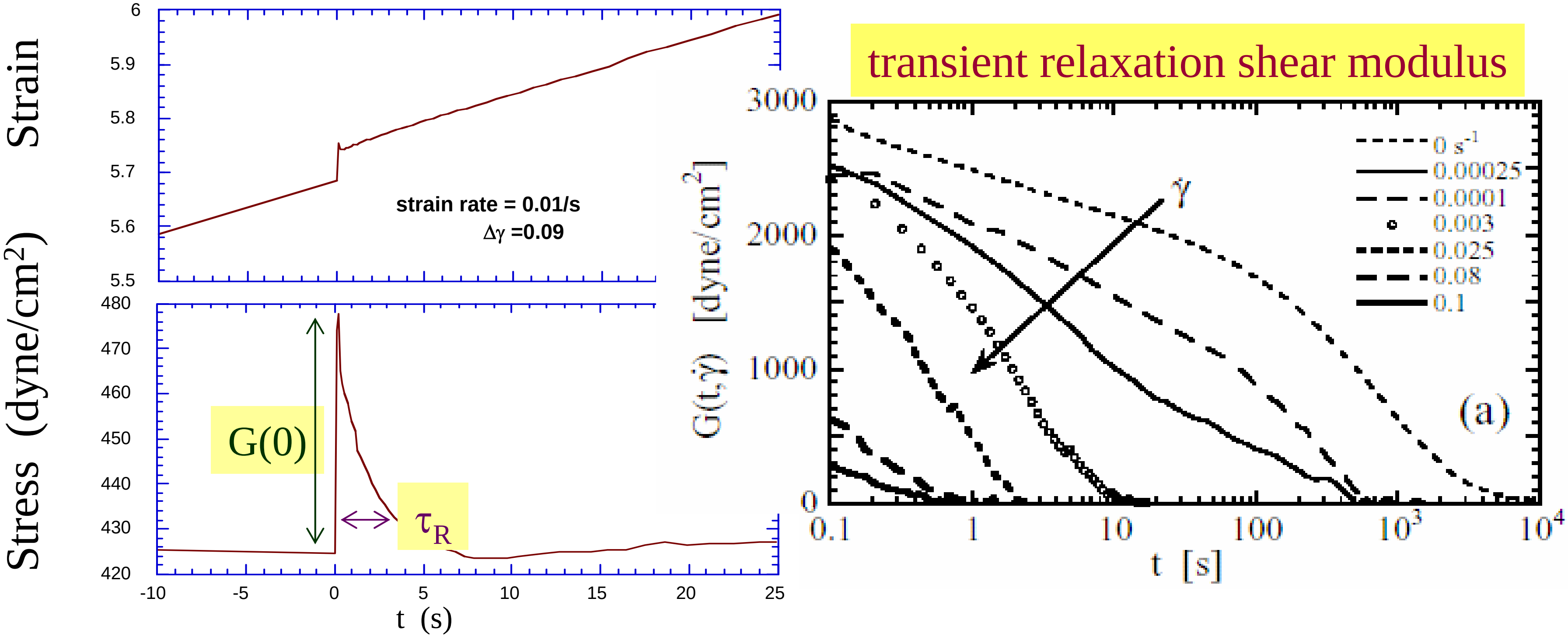
  - but $1/\dot{\gamma}$ isn’t seen in either data or bubble-model:
  - no real signature of $\dot{\gamma}_m$ in data





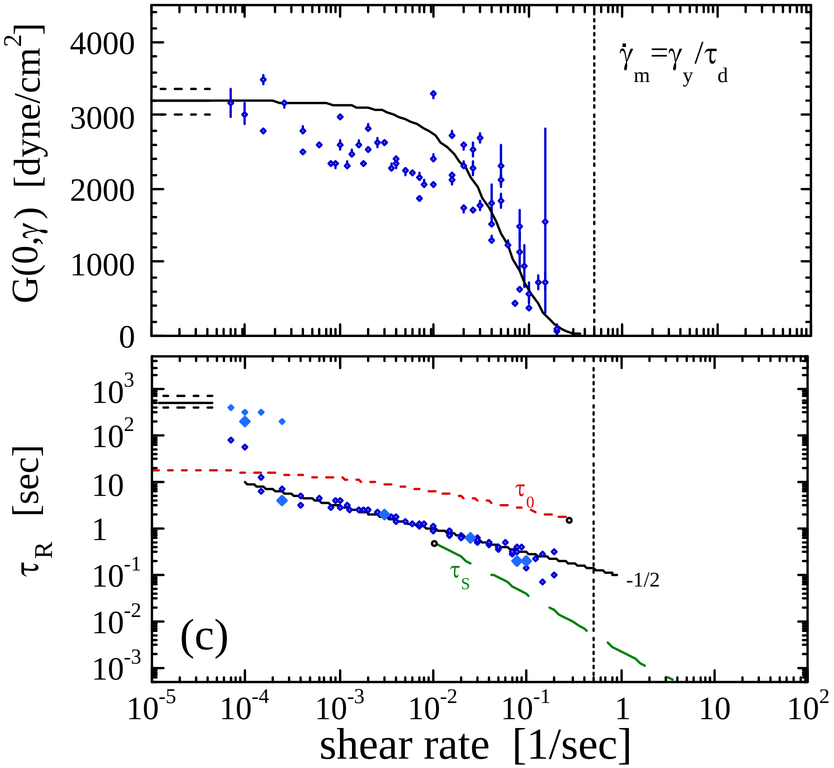
Superimpose step-strain!

- Stress jump and relaxation time both measure elasticity



bubble motion vs rheology

- All measures of elasticity vanish at same point where rearrangements merge together into continuous flow
    - “unjamming” shear rate = yield strain / event duration



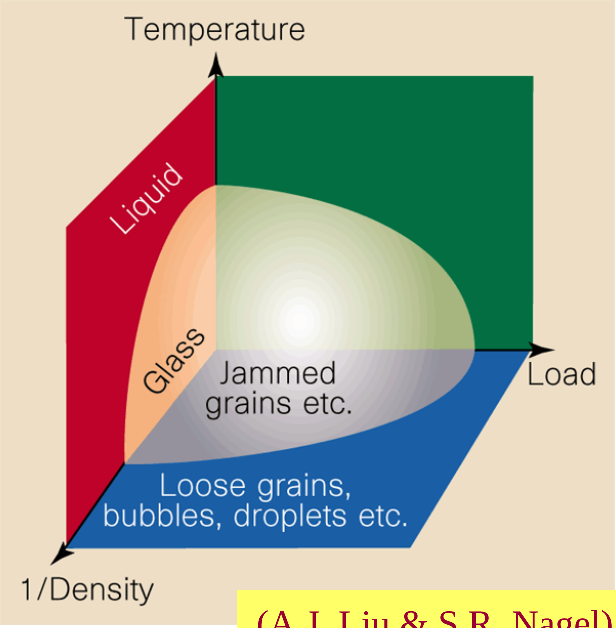
This completes the connection between bubble-scale and macroscopic foam behavior

Jamming

- We’ve now seen three ways to unjam a foam
    - ie for bubbles to rearrange and explore configuration space

  – vs liquid fraction (gas bubble packing)
  – vs time (as foam coarsens)
  – vs shear

- Trajectories in the phase diagram?
  – does shear play role of temperature?



(A.J. Liu & S.R. Nagel)

Three effective temperatures

(I.K. Ono, C.S O'Hern, DJD, S.A. Langer, A.J. Liu, S.R. Nagel)

- Compressibility & pressure fluctuations

$$\kappa_S{}^{-1} = \frac{A}{T}\left\langle (p-\langle p\rangle)^2 \right\rangle$$

- Viscosity & shear stress fluctuations

$$\eta = \frac{A}{T}\int_0^\infty dt \left\langle \sigma_{xy}(t)\sigma_{xy}(0) \right\rangle_c$$

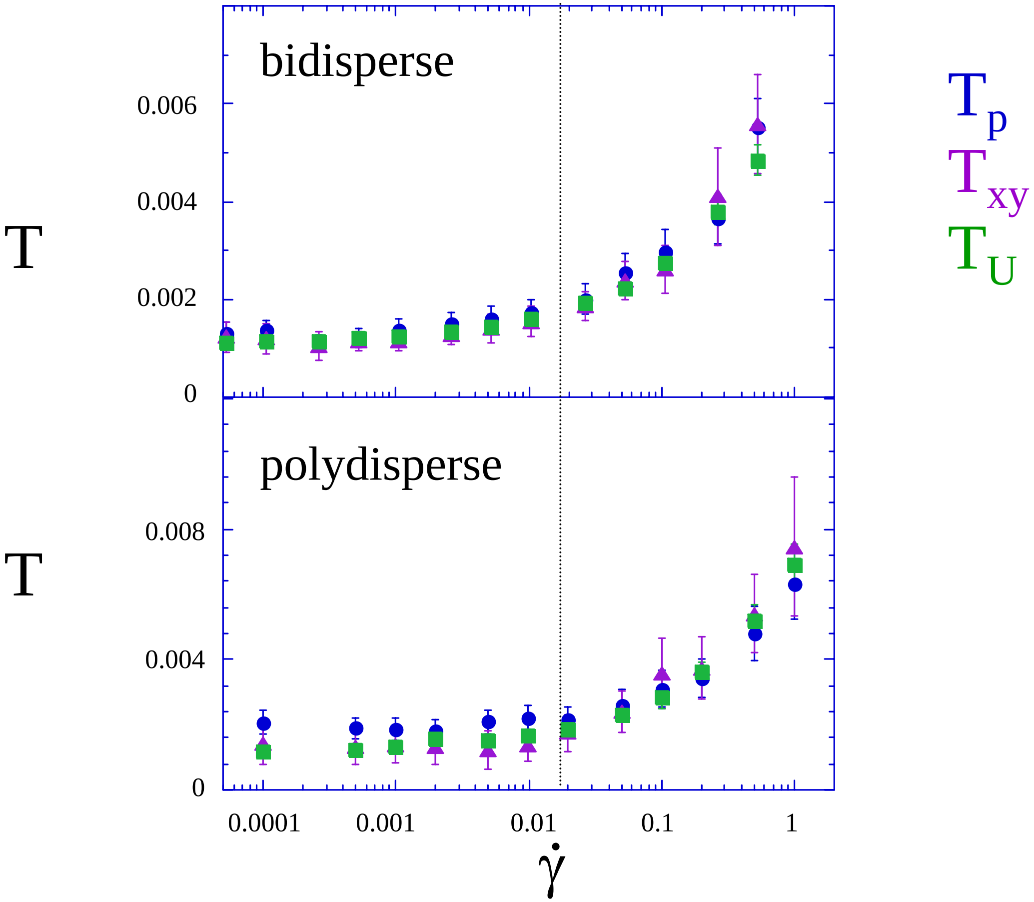
- Heat capacity & energy fluctuations

$$\frac{\partial \langle U\rangle}{\partial T} = \frac{1}{T^2}\left\langle (U-\langle U\rangle)^2 \right\rangle$$

  - T's all reduce to $(dS/dU)^{-1}$ for equilibrated thermal systems
  - What is their value & shear rate dependence; are they equal?
    - Difficult to measure, so resort to simulation…

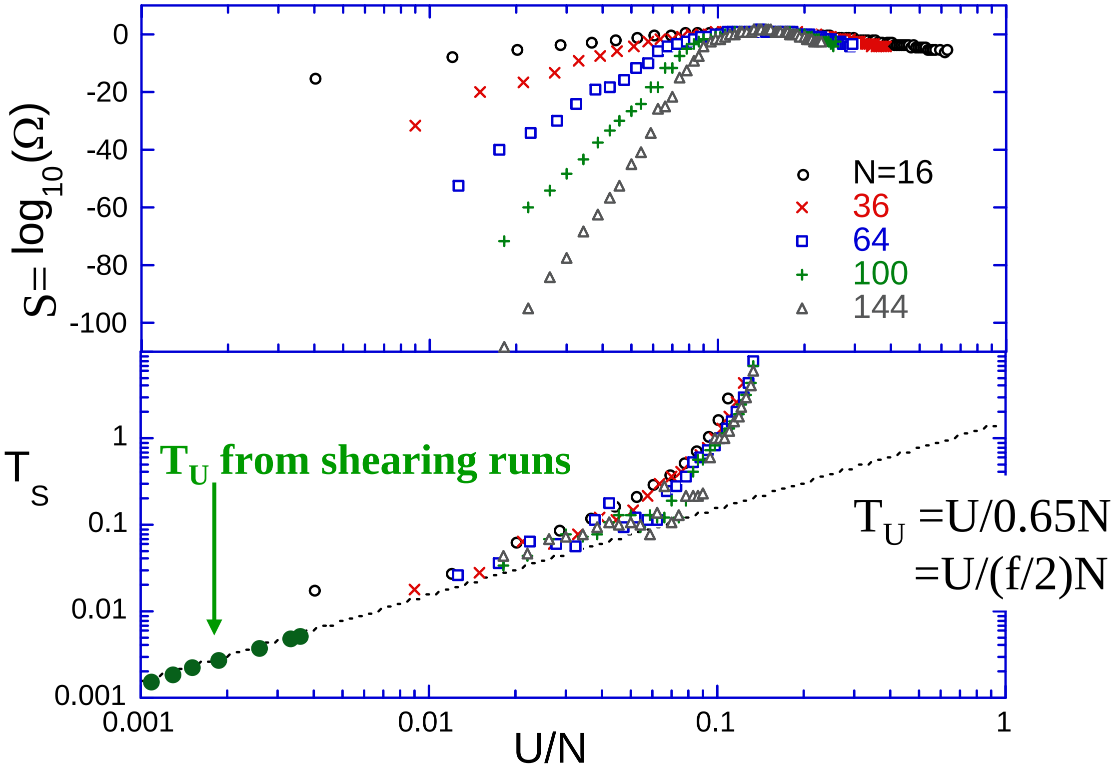
The three T_{eff}'s agree!

- N=400 bubbles in 2D box at ϕ=0.90 area fraction
  - T_{eff} approaches constant at zero strain rate
  - T_{eff} increases very slowly with strain rate



…also agree with $T=(dS/dU)^{-1}$

- Monte-Carlo results for $\Omega(U)$, the probability for a randomly constructed configuration to have energy U



T_U from shearing runs

$T_U = U/0.65N$
$= U/(f/2)N$

For this system, the effective temperature has all the attributes of a true statistical mechanical temperature.

Mini-conclusions

- ***Statistical Mechanics works for certain driven athermal systems, unmodified but for an effective temperature***
  - When does stat-mech succeed & what sets the value of T_{eff}?
    - Elemental fluidized bed (R.P. Ojha, DJD)
      - upflow of gas: many fast degrees of freedom, constant-temperature reservoir
    - Uniformly sheared foam (I.K. Ono, C.S O'Hern, DJD, S.A. Langer, A.J. Liu, S.R. Nagel)
      - neighboring bubbles: many configurations with the same topology
    - In general
      - Perhaps want fluctuations to dominate the dissipation of injected energy?
  - When does stat-mech fail & what to do then?
    - eg anisotropic velocity fluctuations in sheared sand
    - eg $P(v) \sim Exp[-v^{3/2}]$ in shaken sand
    - eg flocking in self-propelled particles

THE END.

- Thank you for your interest in foam!