THE PHYSICS OF FOAM

- **Boulder School for Condensed Matter and Materials Physics**

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- **Drainage**

4. Rheology
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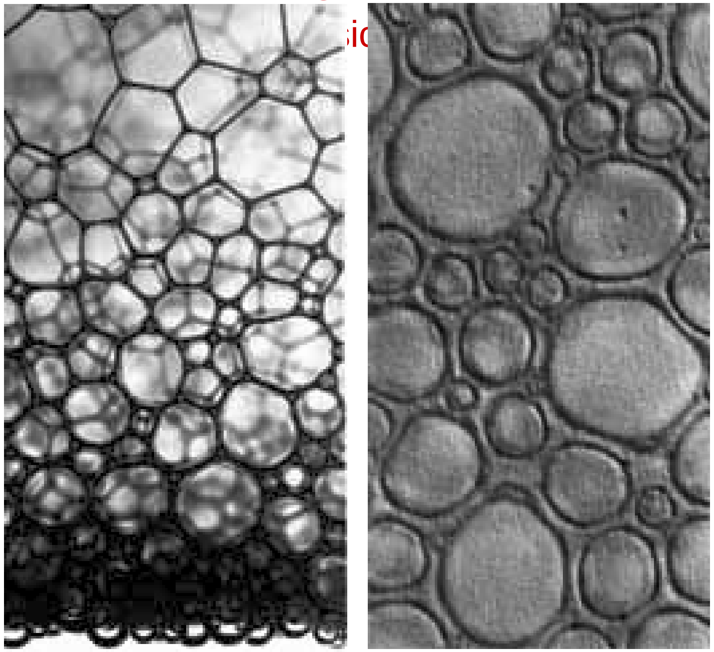
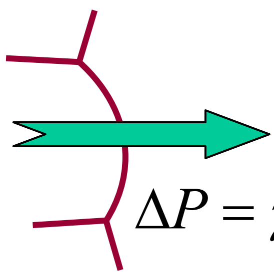
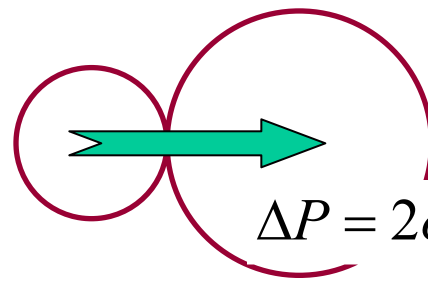


Gas diffusion

- bubble volumes can change by the diffusion of gas across films
  - gas flux goes from high to low pressure bubbles, as set by Laplace's law
    {generally, from smaller to larger bubbles}

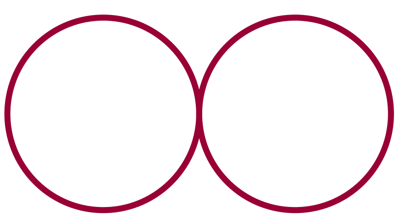


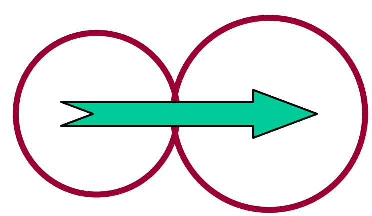
$$\Delta P = \gamma_{film} / r_{film}$$



$$\Delta P = 2\sigma\left(1/r_1 - 1/r_2\right)$$

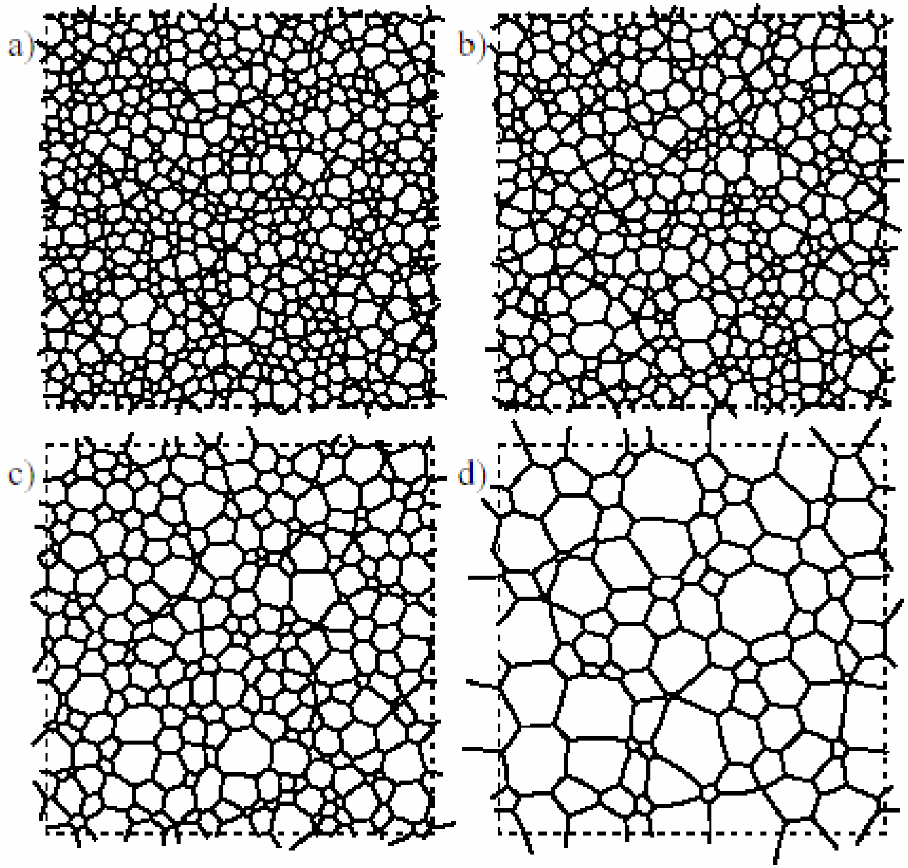
  - monodisperse foams are unstable: fluctuations are magnified…





Coarsening

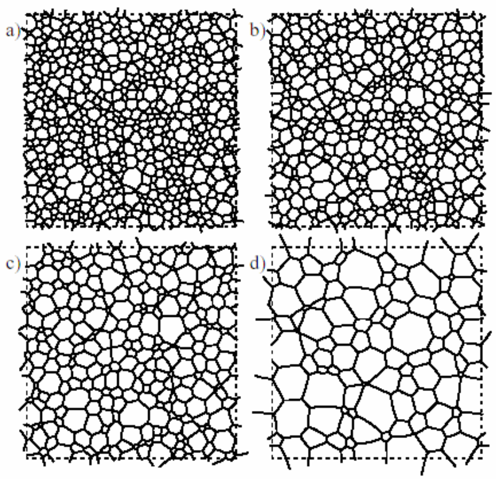
- small bubbles shrink…large bubbles grow…the texture ***coarsens***

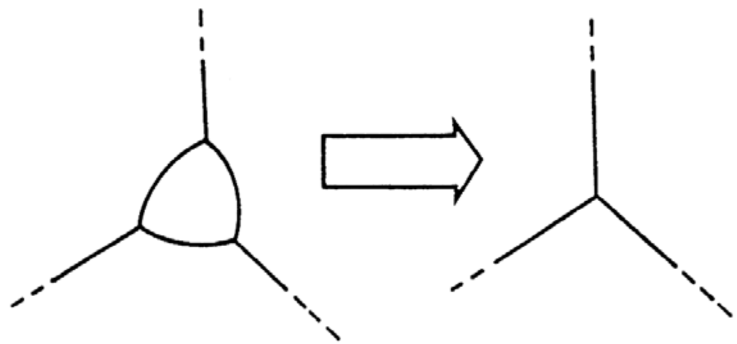


  - interfacial area decreases with time (driven by surface tension)
  - similar behavior in other phase-separating systems
    - eg called Ostwald ripening for grain growth in metal alloys

Coarsening alters the topology

- number of bubbles decreases as small bubbles evaporate



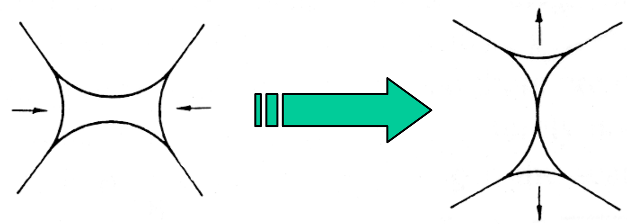


  – this is called at "T2" process

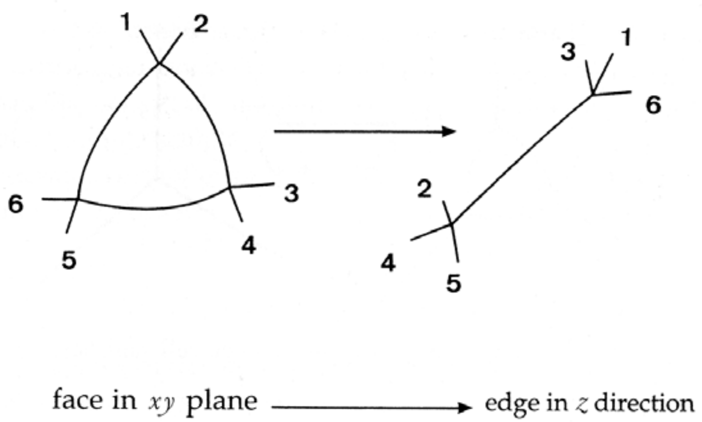
    {topology change of the second kind}

Other topology changes

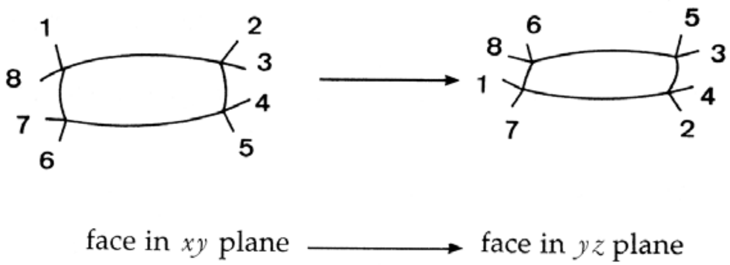
- in 2D, neighbor switching happens only one way:
  - the so-called "T1" process



- in 3D, there is more than one type of neighbor-switching process:

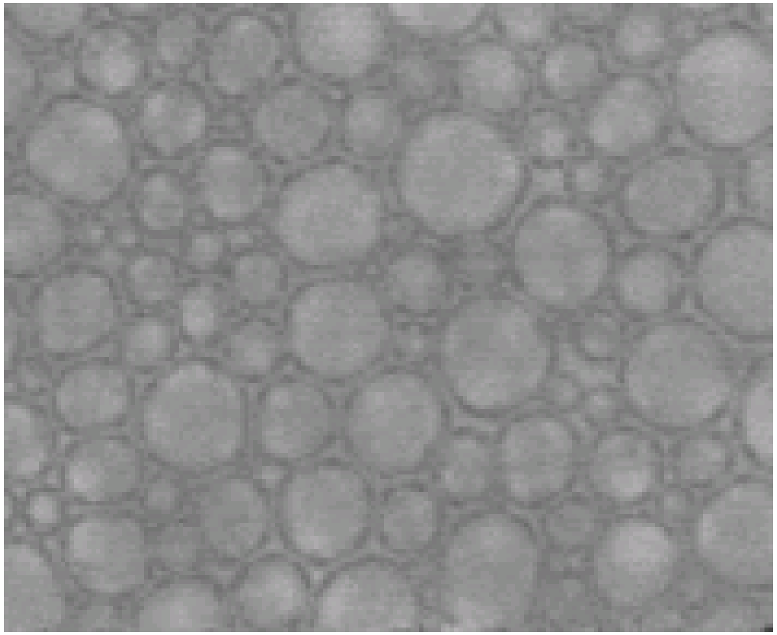


the "quad-flip" is most prevalent



Rearrangement dynamics

- these events can be sudden / avalanche-like:

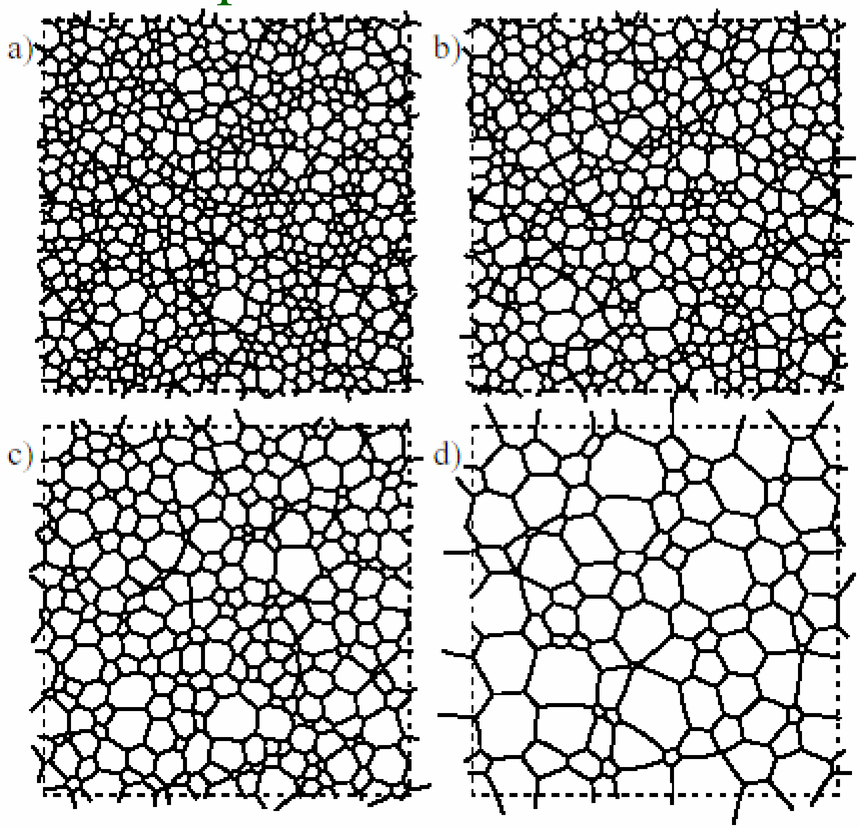


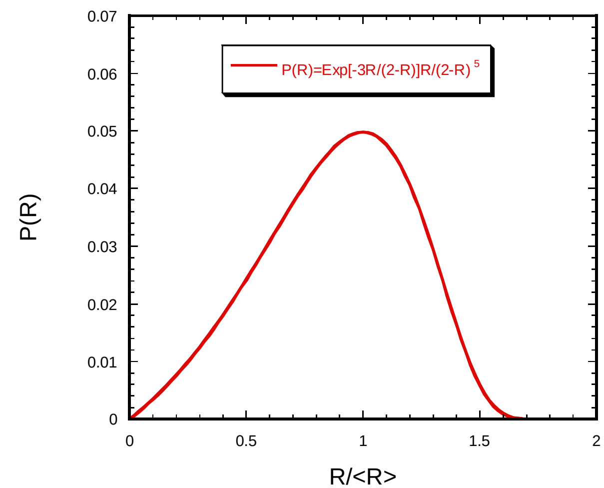
- surface of a bulk foam
- 30 μm diameter bubbles

  – similar rearrangements occur during flow… (next time)

self-similarity

- bubble-size distribution scales with the average
  - p(R,t) = F(R/<R(t)>) where all t-dependence is in <R(t)>
  - arbitrary initial distribution evolves to this distribution
  - time sequence looks like an increase in magnification





this property makes it simple
to compute the rate of coarsening…

Rate of coarsening I.

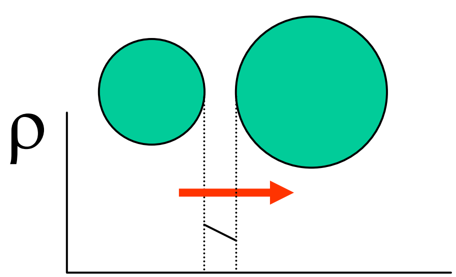
- The bubbles in a foam are polydisperse
  - smaller bubbles have higher pressures (Laplace)

$$P_{inside}=P_{outside}+\gamma/r$$

  - concentration of disolved gas is therefore higher just outside smaller bubbles (Henry)

$$\bullet\ \rho=\text{const}\ \text{x}\ P_{inside}$$

  - hence there is a diffusive flux of disolved gas down the concentration gradient from smaller to larger bubbles (Fick)



Rate of coarsening II.

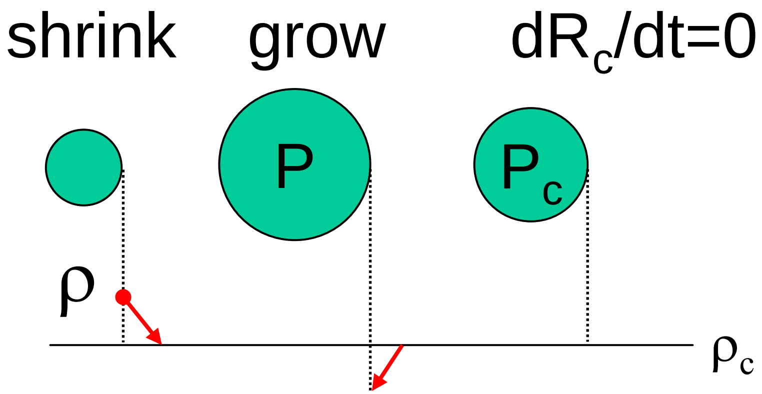
- mean-field argument: $dV/dt \sim -A\,(P-P_c)$

  V = average bubble volume, A = average bubble area

  {$dV/dt = A\, dR/dt$, so $dR/dt \sim -(P-P_c)$ in any dimension}

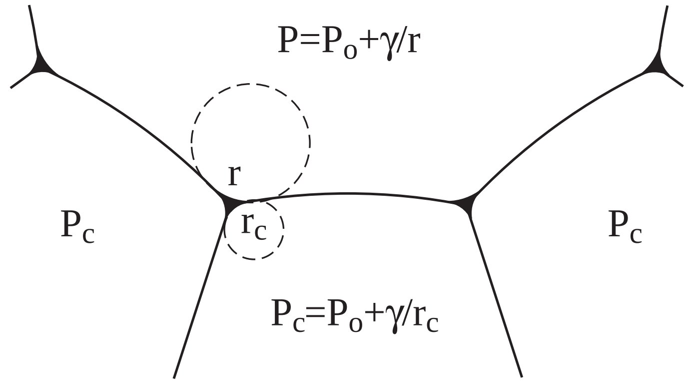
  {proportionality constant scales as diffusivity x solubility / film thickness}

$(P-P_c)$ = pressure difference of average bubble with neighboring 'crossover' bubbles that neither grow nor shrink:



Rate of coarsening III.

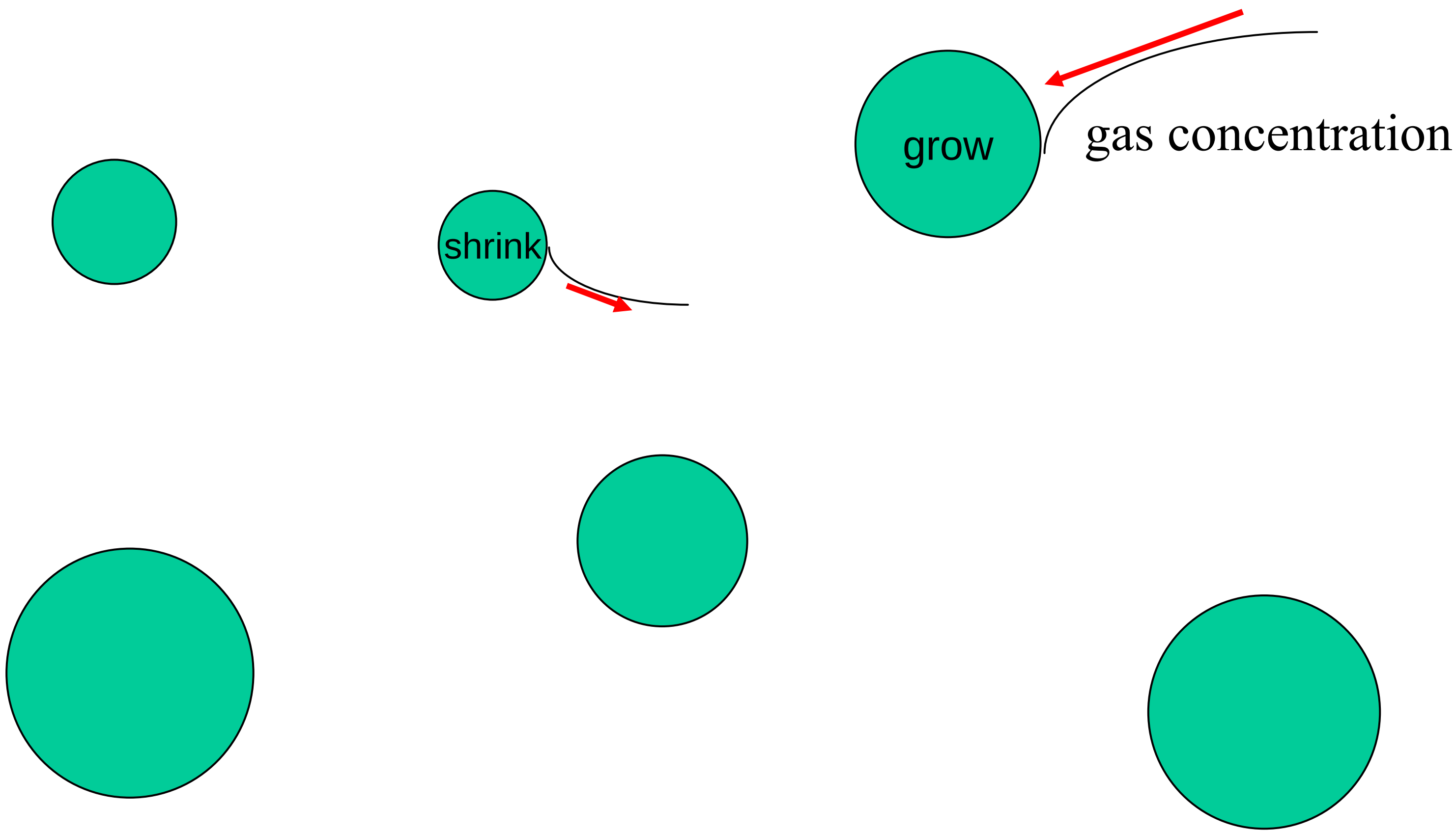
- $(P_c - P) = (\gamma/r_c - \gamma/r)$, difference of Plateau border curvatures



- two steps to connect to bubble size:
  - self-similarity of the bubble-size distribution implies that R is *exactly* proportional to R_c
  - $\varepsilon \sim (r/R)^2 = (r_c/R_c)^2$
- Altogether: $dR/dt \sim (P_c - P) = (\gamma/r_c - \gamma/r) \sim 1/(\mathrm{Sqrt}[\varepsilon]R)$
  - therefore, $R \sim t^{1/2}$ {in both 2D and 3D}

Lifshitz & Slyozov (1961)

- considered coarsening of metal alloys
  - droplets separated by a distance >> droplet size
  - full distribution size distribution $f(R,t)$, with $\langle R(t)\rangle \sim t^{1/3}$



von Neumann's law for 2D dry foams

- sum rule for change in tangent angles going around an n-sided bubble with arclengths l_i and radii r_i is $$2\pi = \sum_{i=1}^{n} \frac{l_i}{r_i} + n \cdot \pi/3$$

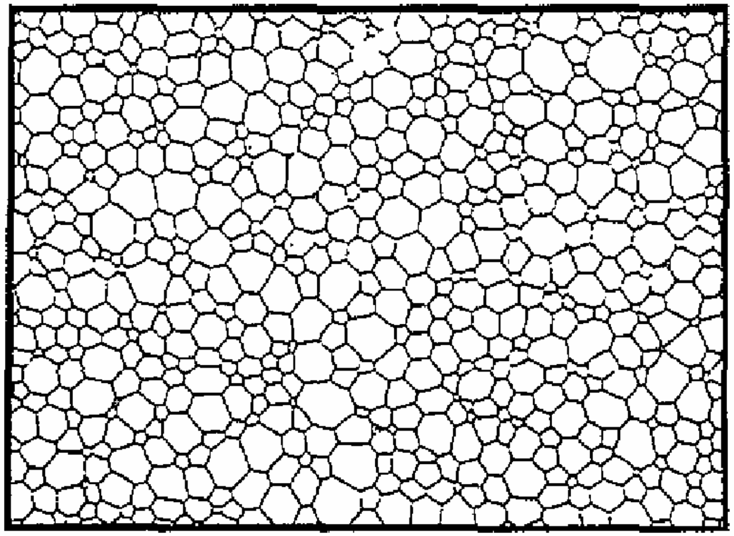
- flux across each arc scales as l_i / r_i

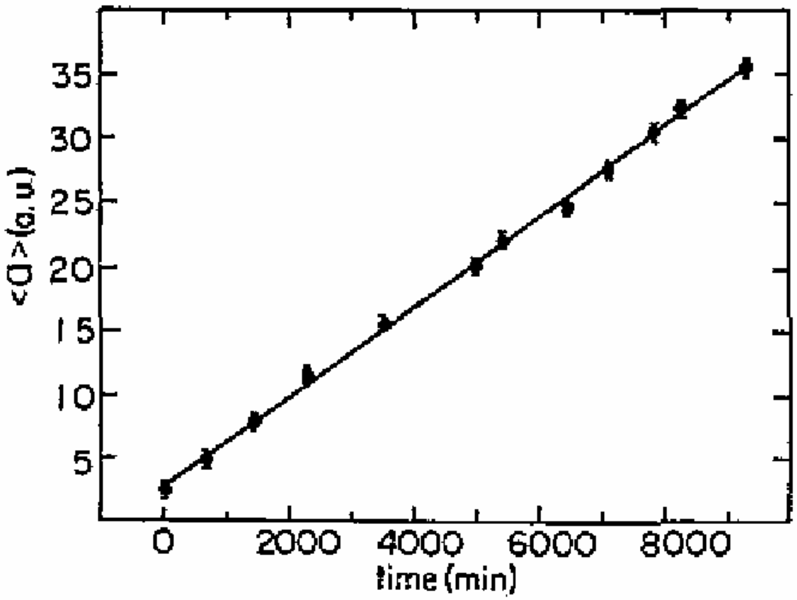
- rate of change of area thus scales as $$\frac{dA_n}{dt} \sim \left(-\sum_{i=1}^{n} \frac{l_i}{r_i} \right) \sim (n-6)$$
  - the crossover bubble is six-sided
  - the average bubble area grows as A~t {consistent with $r \sim t^{1/2}$}

- cannot be carried into 3D, but approximations have been proposed
  - $RdR/dt \sim (F-F_o)$ with $F_o \sim 14$ {Glazier}
  - $RdR/dt \sim F^{1/2}-F_o^{1/2}$ {Hilgenfeldt}

experiments, 2D

- soap bubbles squashed between glass plates:





- Gillette Foamy, from multiple light scattering

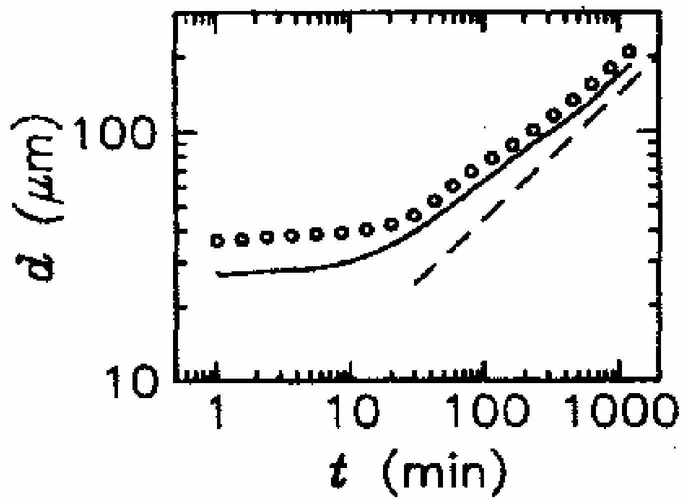


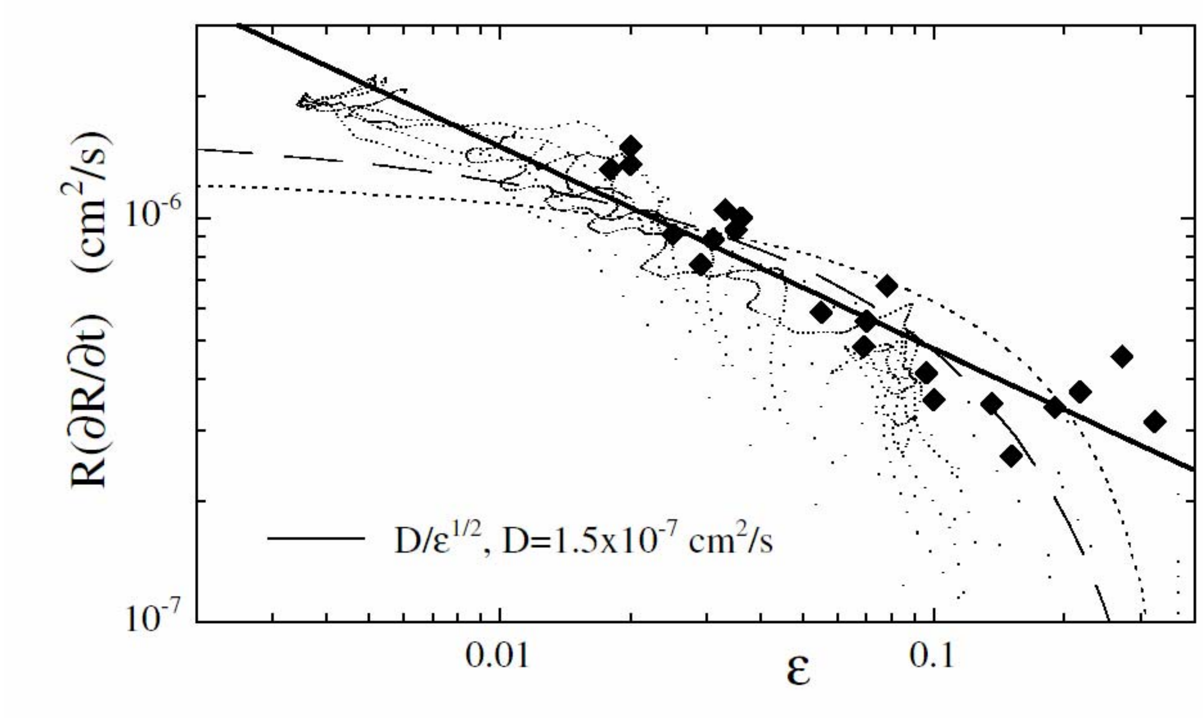
FIG. 3. Average bubble diameter vs foam age as determined from static (——) and dynamic (o) multiple light-scattering measurements. The dashed line has slope $\frac{1}{2}$ and shows that the growth of d is nearly consistent with the scaling prediction for densely packed bubbles.

experiments, 3D

- custom made foams of uniform liquid fraction (large symbols)
- a single foam sample that is draining and coarsening (small dots)

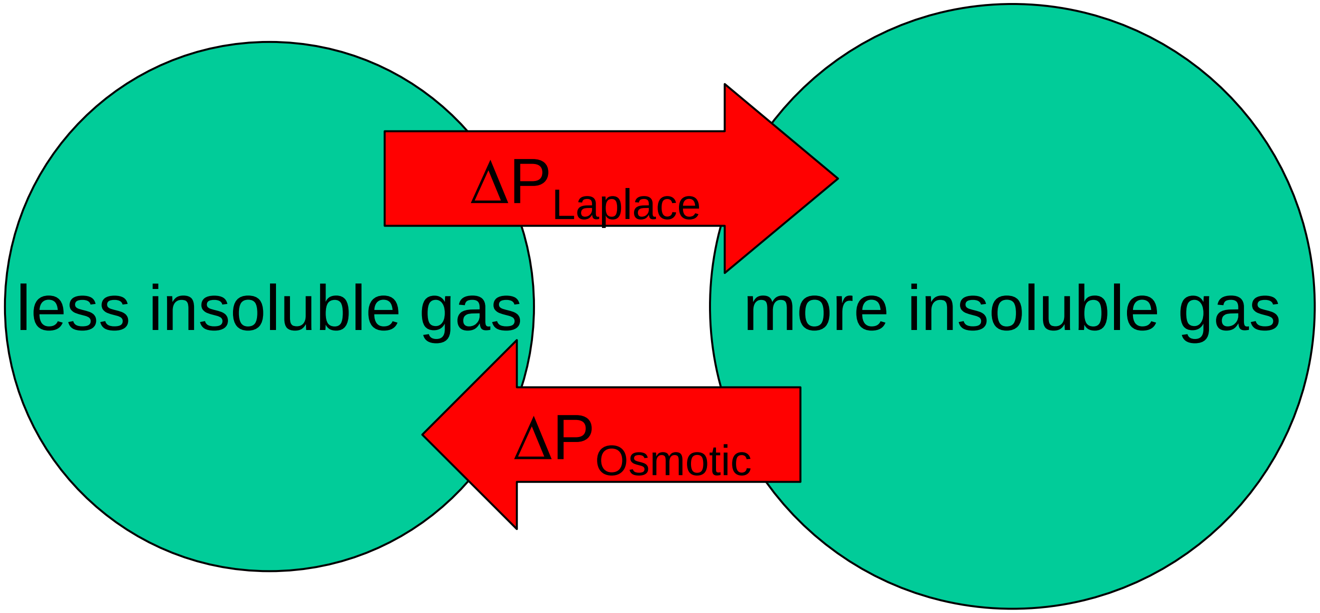
- liquid-fraction dependence: $dR/dt \sim 1/(\mathrm{Sqrt}[\varepsilon]R)$

  {cf competing arguments where liquid-filled Plateau borders completely block the flux of gas: $dR/dt \sim (1-\mathrm{Sqrt}[\varepsilon/0.44])^2$ (dash)}



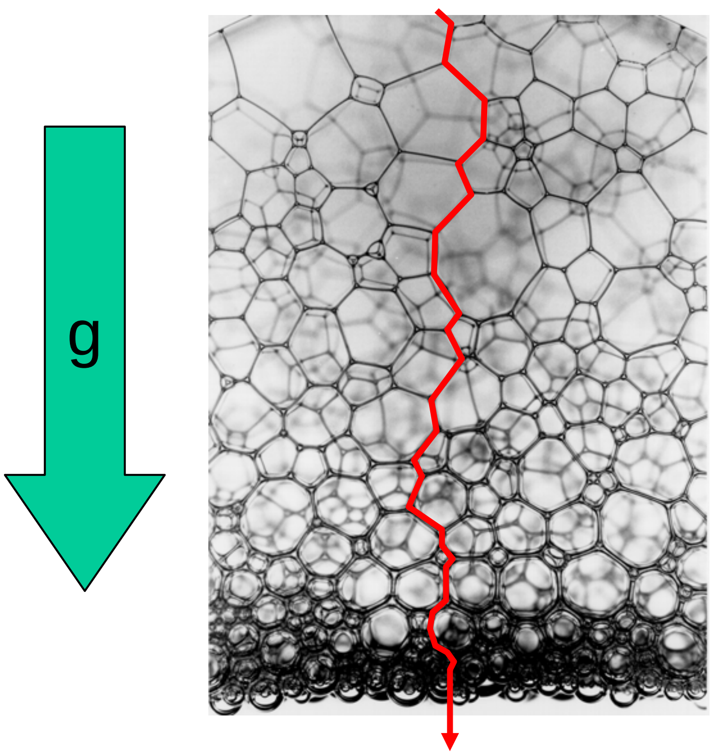
Coarsening can't be stopped

- but it can be slowed down:
  - make the bubbles monodisperse
  - choose gas with low solubility and low diffusivity in water
  - add trace amount of "insoluble" gas
    - works great for liquid-liquid foams (ie emulsions)
    - composition difference & osmotic pressure develop that oppose Laplace



Drainage intro

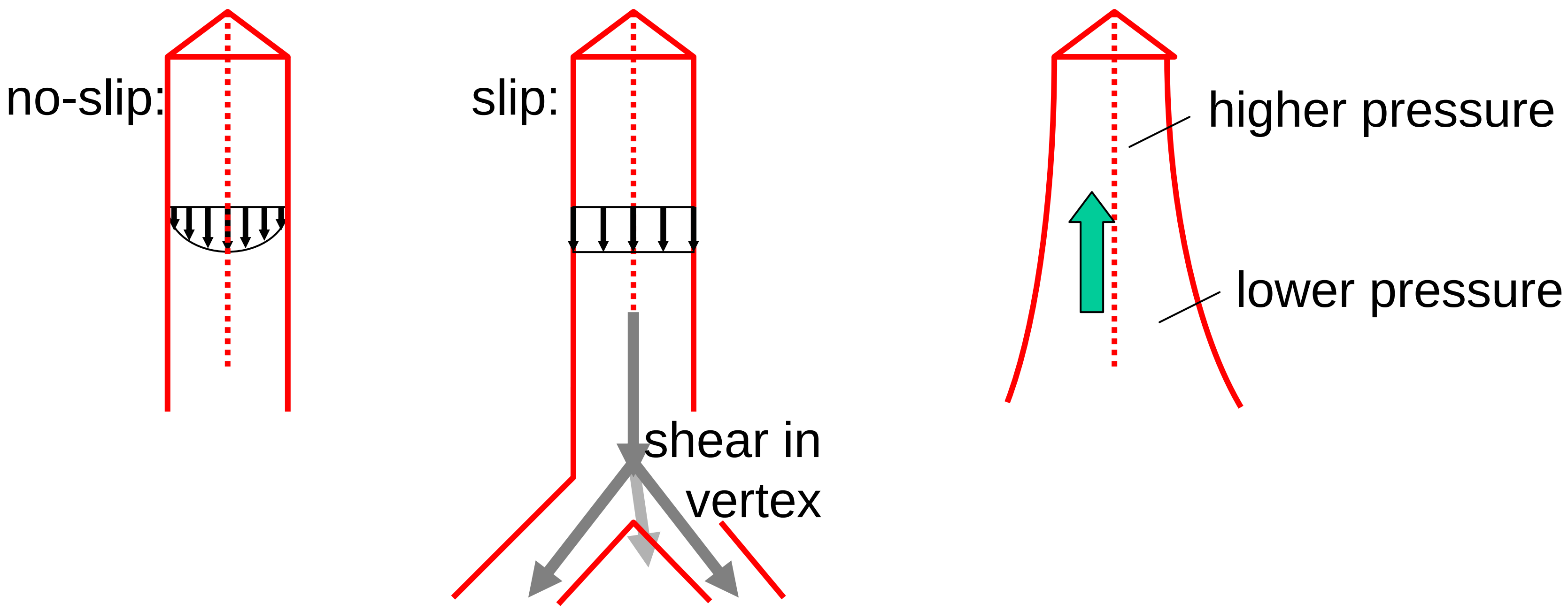
- Under influence of earth's gravity, the liquid drains downwards in between the bubbles - primarily through the Plateau borders
    - some debate about role of films in liquid transport…
  - unlike coarsening, this mechanism can be turned off (microgravity)
  - drainage and/or evaporation are often a prelude to film rupture



different from ordinary porous medium: the pore (i.e. Plateau borders) shrink as drainage proceeds: $\varepsilon \sim (r/R)2$

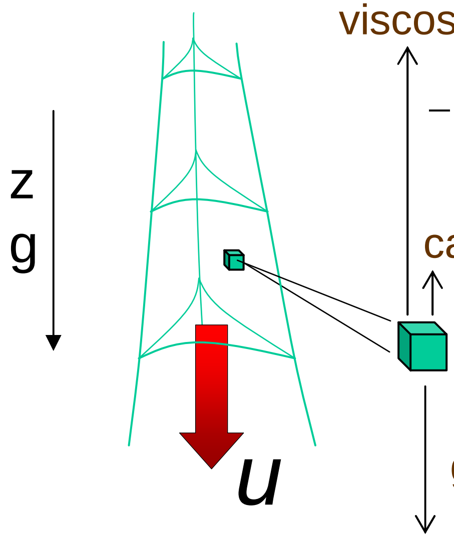
Forces?

- drainage is driven by gravity, but opposed by two other forces
  - viscous dissipation
    - if the monolayer are rigid:
      - no-slip boundary, so Shear Flow in Plateau borders
    - if the monolayers are mobile:
      - slip boundary, so Plug Flow in Plateau borders and shear flow only in vertices
  - capillarity



Liquid flow speed, u?

- estimate ΔE/time in volume r^2L for all three three forces:



viscosity: $$-\eta\left(\frac{u}{r}\right)^2 \times \begin{cases} r^2 L & \text{rigid films}^* \text{ (shear in PB's)} \\ r^3 (L/R) & \text{mobile films}^{**} \text{ (shear in vertices)} \end{cases}$$

capillarity: $$-\left(\frac{d}{dz}\left[P_o - \frac{\gamma}{r}\right]R\right) r^2 u$$

gravity: $$+\rho\left(r^2 L\right) g u$$

- use $r \sim \varepsilon^{1/2} R$ and require $\Sigma(\Delta E/\text{time})=0$:

$$u = u_o\left(1 - \sqrt{\frac{\varepsilon_c}{\varepsilon}\frac{\xi}{\varepsilon}\frac{\partial\varepsilon}{\partial z}}\right) \times \begin{cases} \varepsilon & \text{shear in PB's}^* \\ \varepsilon^{1/2} & \text{shear in vertices}^{**} \end{cases}$$

$$\left\{ u_o \approx \rho g R^2/\eta \ \text{(characteristic flow speed)}, \quad \xi \approx \gamma/\rho g R \ \text{(capillary rise length)}, \quad \varepsilon_c \approx 0.36 \right\}$$

Drainage Equation: PDE for ε(z,t)

- continuity equation for liquid conservation:

$$0=\frac{\partial\varepsilon}{\partial t}+\frac{\partial(u\varepsilon)}{\partial z}+\frac{u\varepsilon}{A}\frac{\mathrm{d}A}{\mathrm{d}z}$$

$\varepsilon(z,t)=$ liquid volume fraction

$u(z,t)=$ liquid flow speed (previous slide)

$A(z)=$ cross-sectional area of container

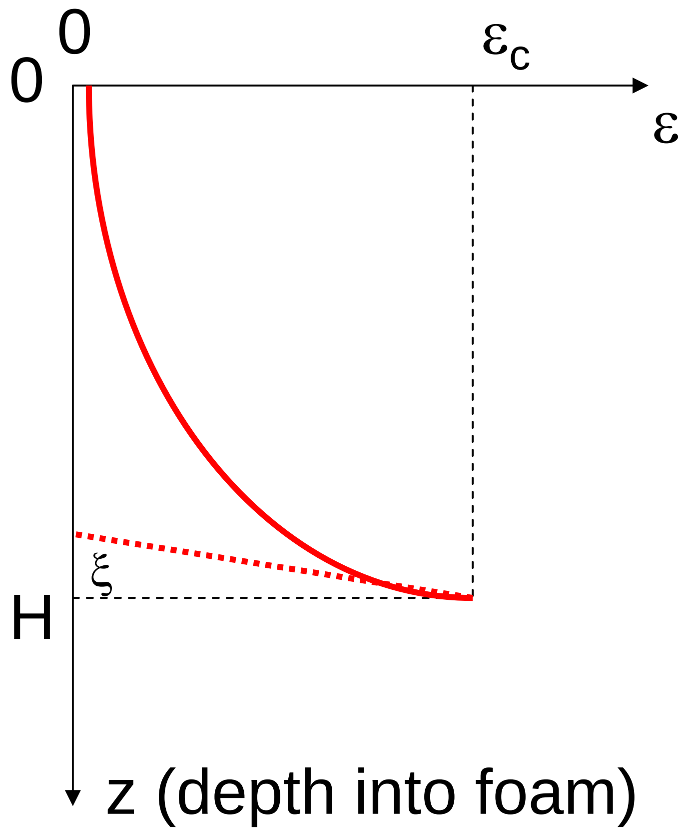
- boundary conditions:

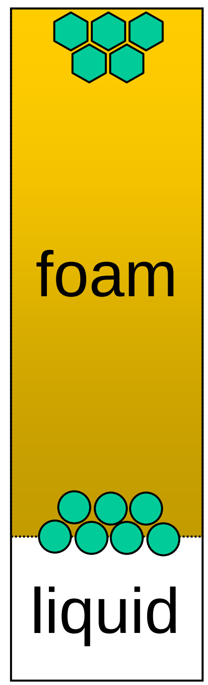
$$\varepsilon\big|_{boundary}=\varepsilon_c \text{ (flow at bottom), or } \left.\frac{\varepsilon}{\partial\varepsilon/\partial z}\right|_{boundary}=\xi\sqrt{\frac{\varepsilon_c}{\varepsilon}} \text{ (no flow at top or bottom)}$$

Equilibrium capillary profile

- u=0 everywhere: gravity balanced by capillarity

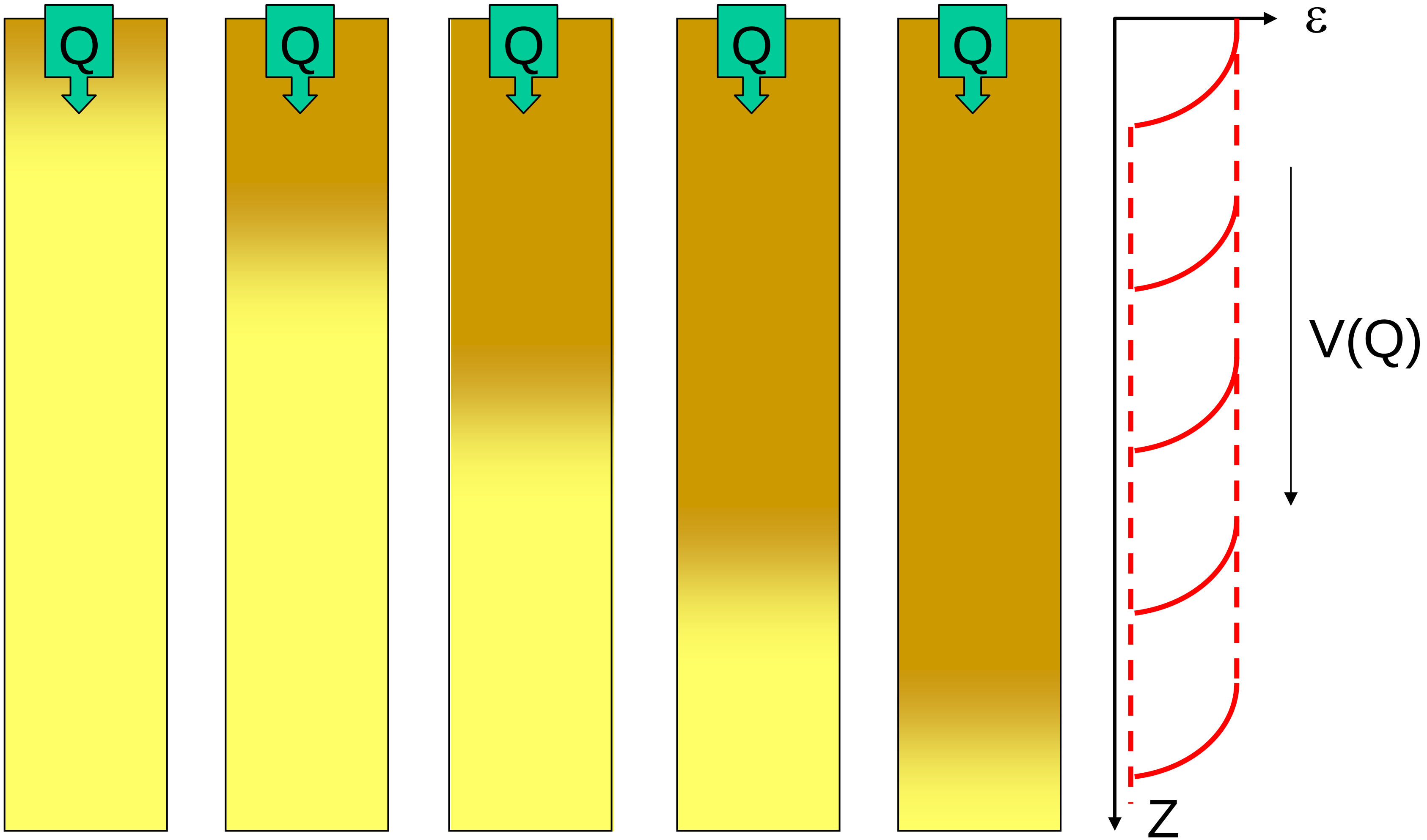
$$\varepsilon(z) = \varepsilon_c / [1-(H-z)/2\xi]^2, \ \text{H} = \text{column height}$$





Forced-drainage

- pour liquid onto foam column at constant rate Q
  - wetness front propagates at constant speed & shape (solitary wave)

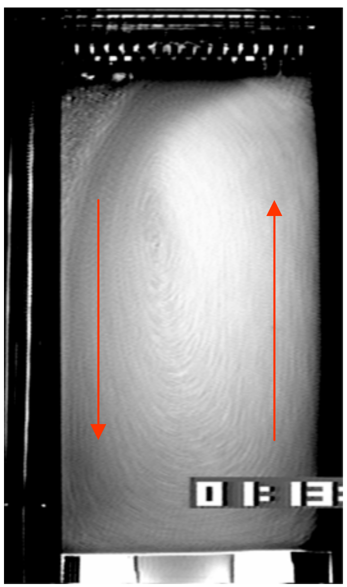


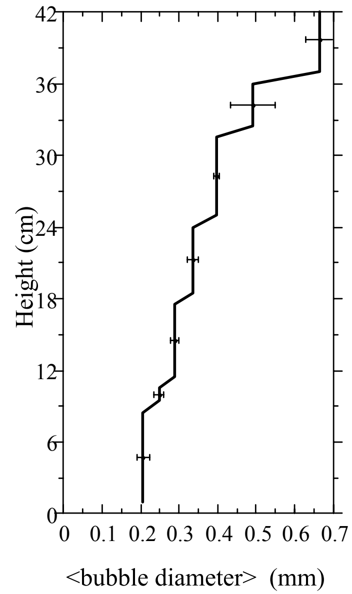
Convection & size segregation

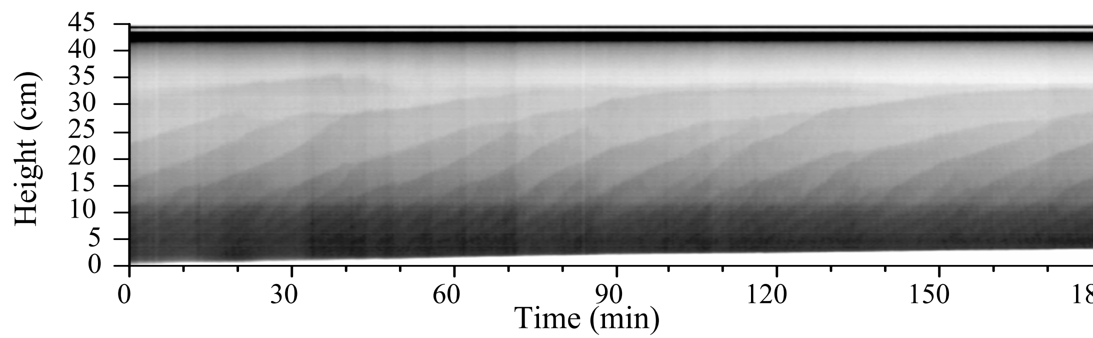
- but don't pour too hard!

$Q>>Q_m$
convection & size segregation

$Q>Q_m$
convection

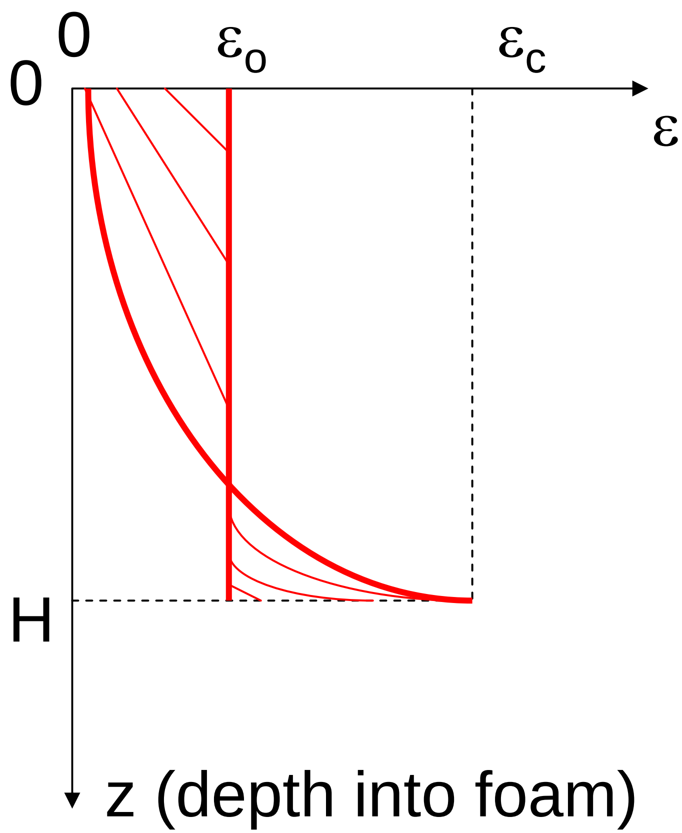


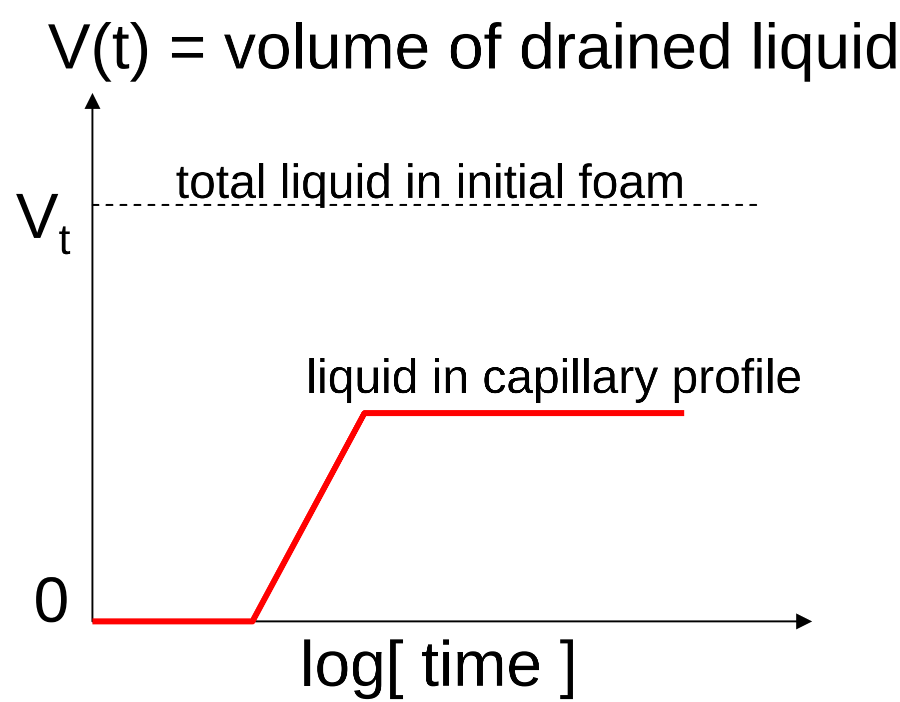




Free-drainage in straight column

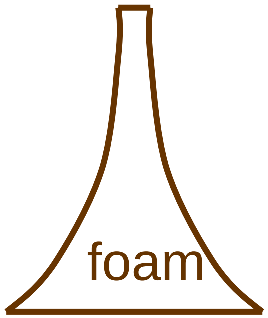
- no analytic solution is known!
  - initially, becomes dry/wet at top/bottom; ε=constant in interior
  - leakage begins when $\varepsilon \to \varepsilon_c$ at bottom
  - eventually, rolls over to equilibrium capillary profile





Free-drainage in Eiffel Tower

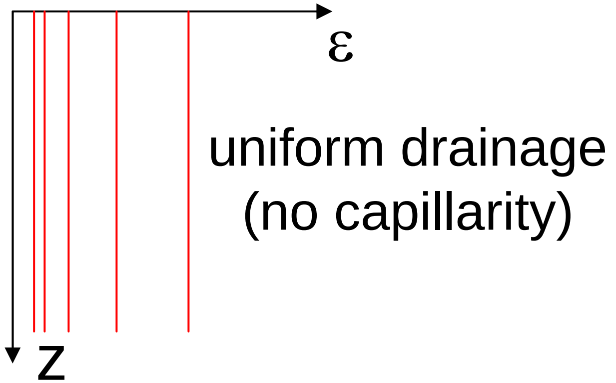
- exponentially-flaring shape: $A(z) \sim Exp[z/z_o]$



- simple analytic solution (ignoring boundary conditions)

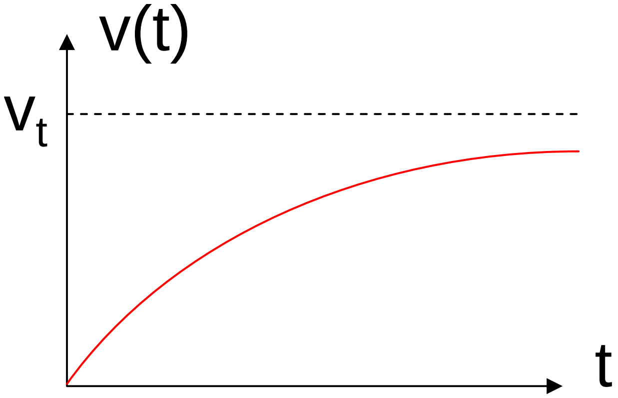
liquid fraction:

$$\varepsilon(z,t) = \varepsilon_o \Big/ (1 + t/t_o)^{1/m}$$
$$\text{with } t_o = z_o \Big/ \left(u_o m \varepsilon_o^{\ m}\right)$$



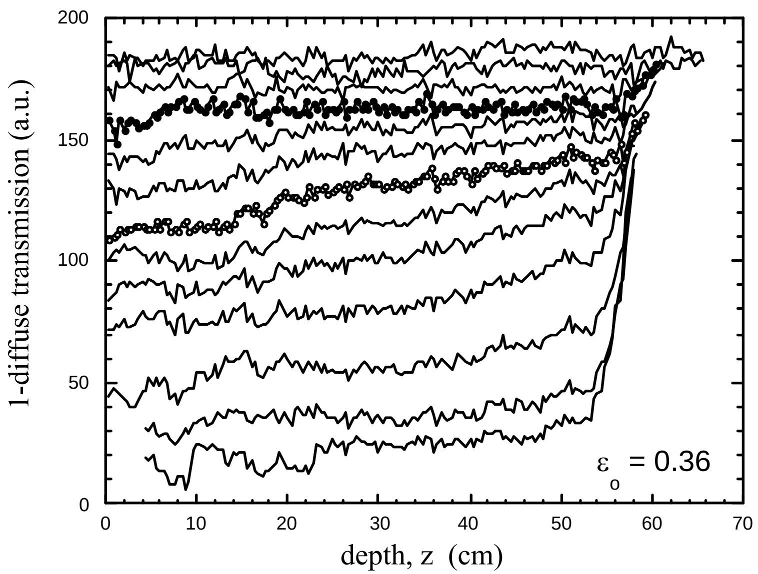
vol. of drained liquid:

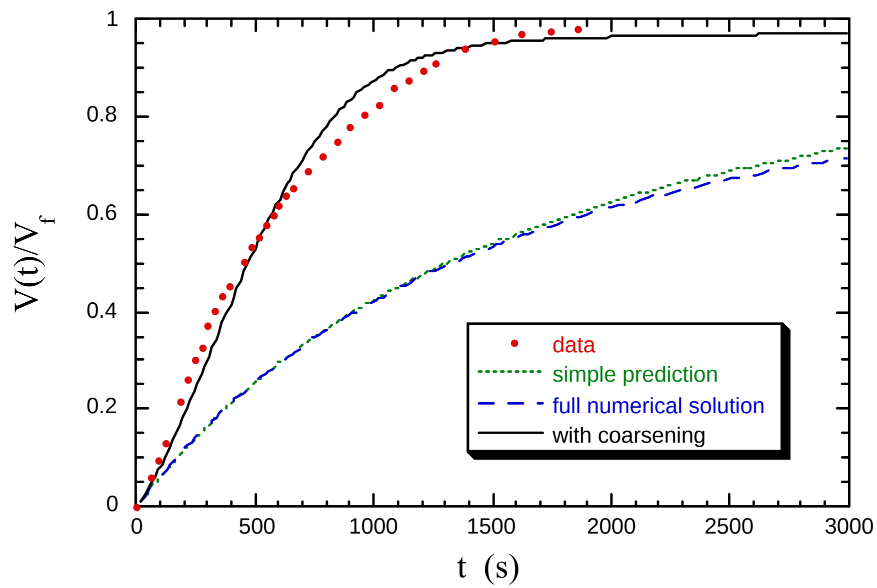
$$V(t)/V_t = 1 - 1\Big/(1 + t/t_o)^{1/m}$$



Eiffel Tower - data

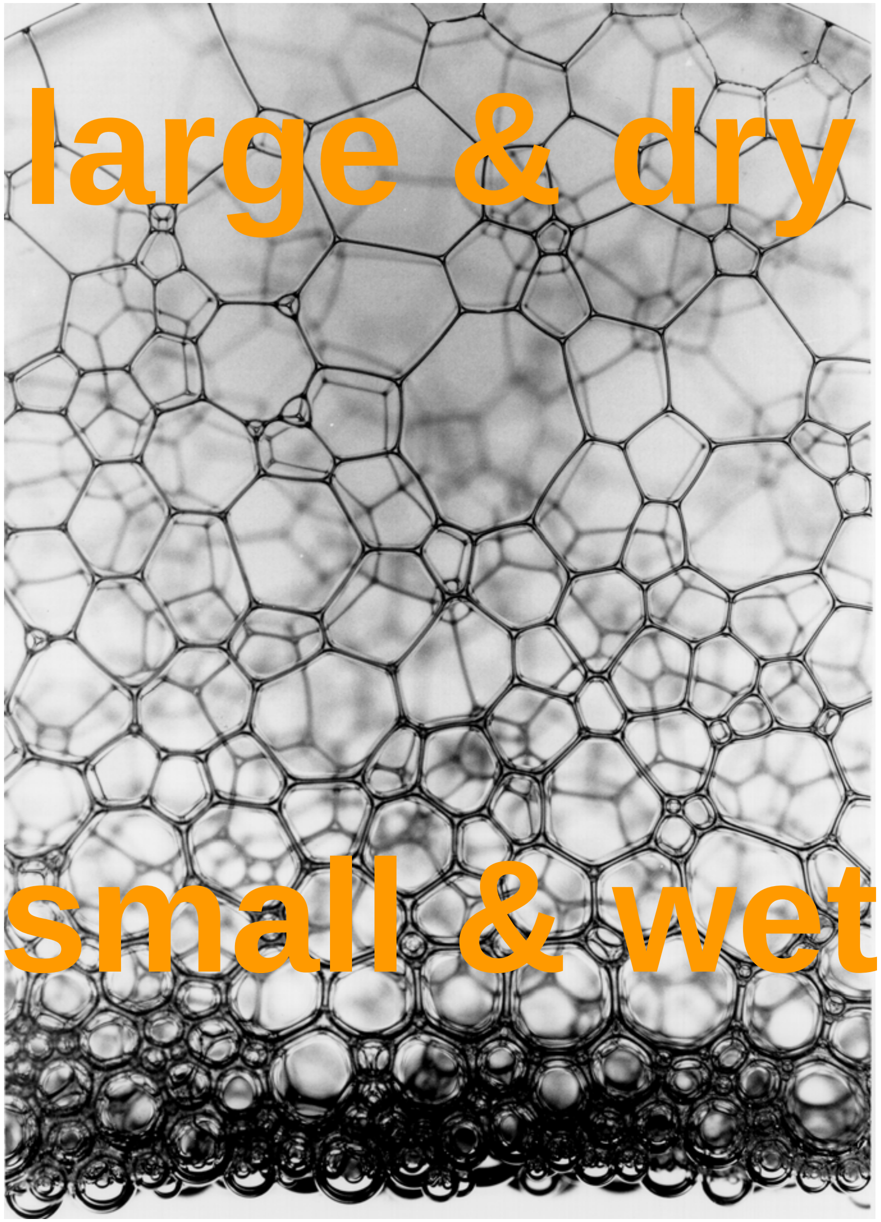
- uniform drying (no ε-gradients, until late times)
- but much faster than predicted
  - capillarity in BC's slows down leakage
  - *must* be due to effects of coarsening…





Drainage-coarsening connection

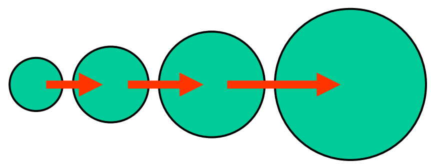
- vicious cycle:
  - dry foams coarsen faster…
  - large bubbles drain faster…
  - etc.



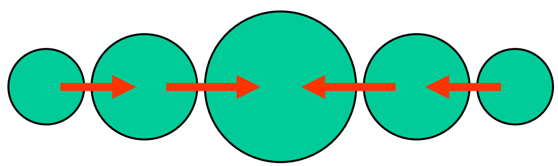
- to model this effect:
  - combine with RdR/dt=1/Sqrt[ε]
  - add one more ingredient…

Coarsening Equation

- Previous treatments assume spatial homogeneity, which isn’t the case for freely draining foams
    - gradient causes net gas transport



    - curvature contributes to bubble growth

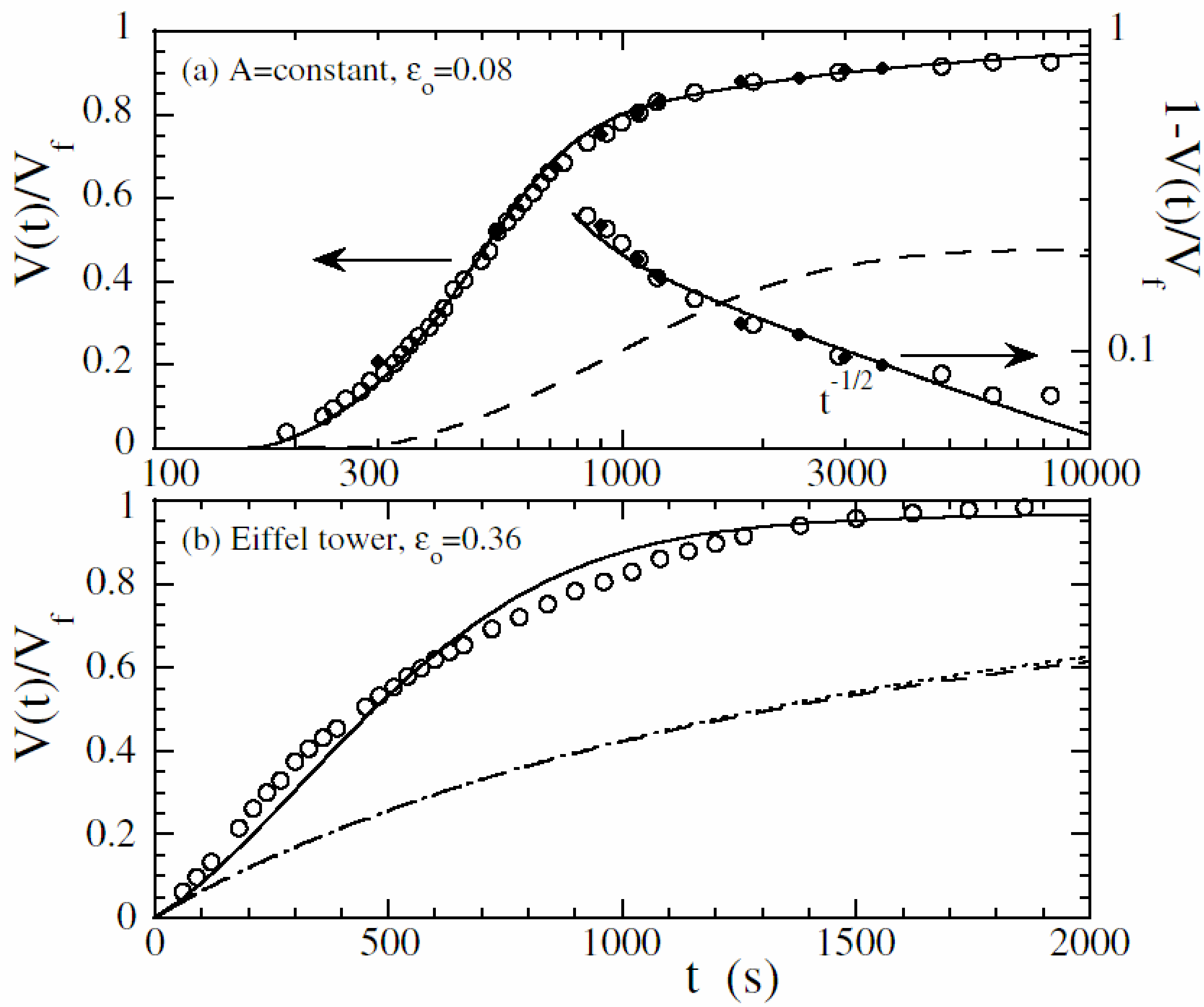


- The full ***coarsening equation*** must thus be of the form $\partial R/\partial t = D[\ X + (R^2/\alpha)\partial^2 X/\partial z^2\ ]$:

$$\frac{\partial R}{\partial t} = D\left[\frac{1}{\sqrt{\varepsilon R}} + \frac{R^2}{\alpha}\frac{\partial^2}{\partial z^2}\left(\frac{1}{\sqrt{\varepsilon R}}\right)\right]$$

Compare with data

- simultaneously capture straight and flaring columns:



Next time…

- Foam rheology
  - linear response (small-amplitude deformation)
  - bubble rearrangements and large-deformation flow