THE PHYSICS OF FOAM

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3. Stability
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- Drainage

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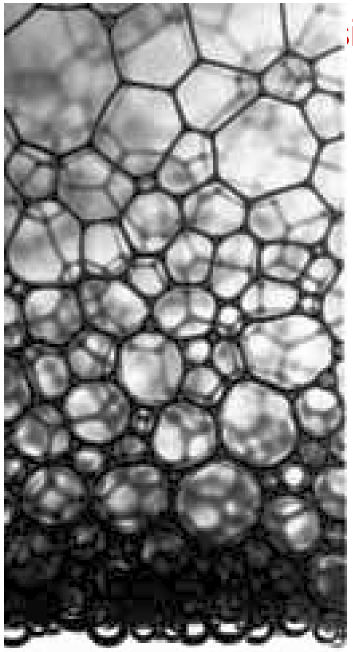
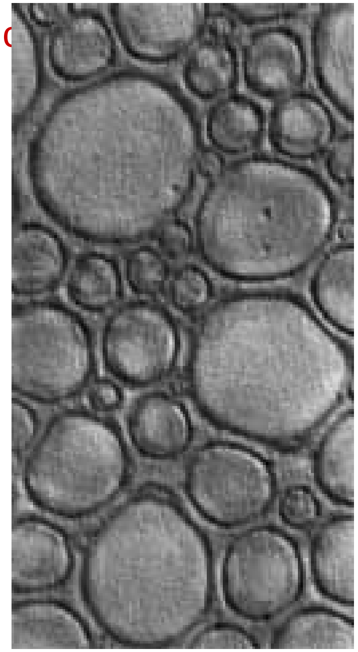
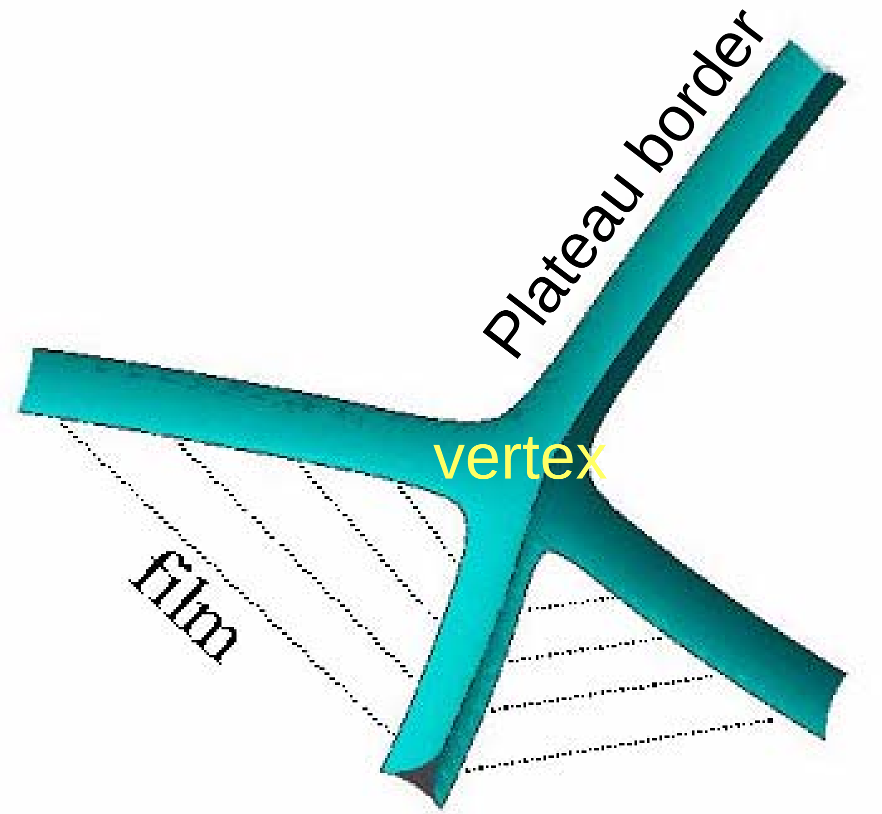
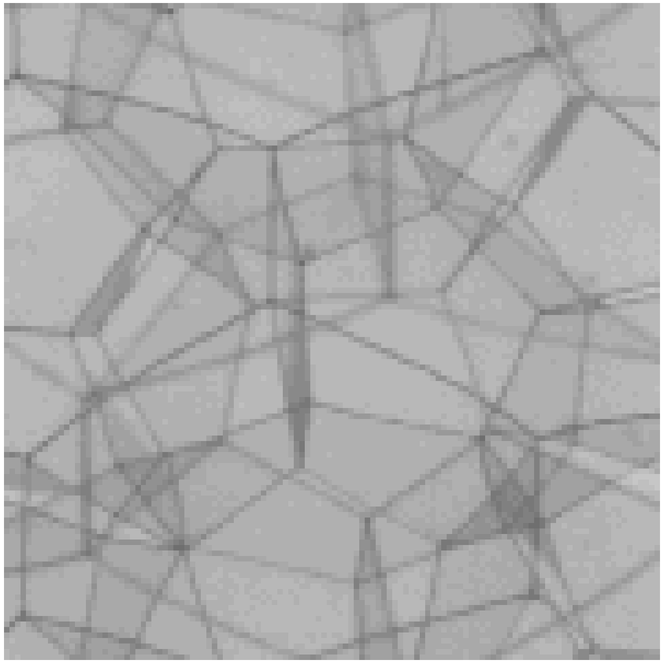




local structure recap

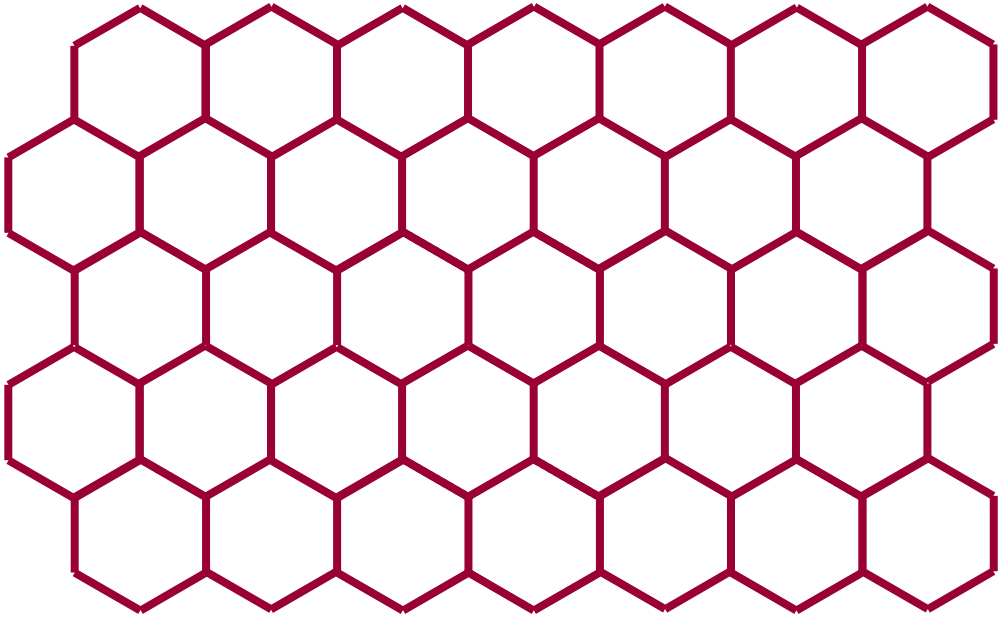
- liquid fraction ε ~ [(border radius r) / (bubble radius R)]^2
- Plateau's rules for mechanical equilibrium:
  - (1) films have constant curvature & intersect three at a time at 120^o
  - (2) Plateau borders intersect four at a time at $\cos^{-1}(1/3)=109.47^o$

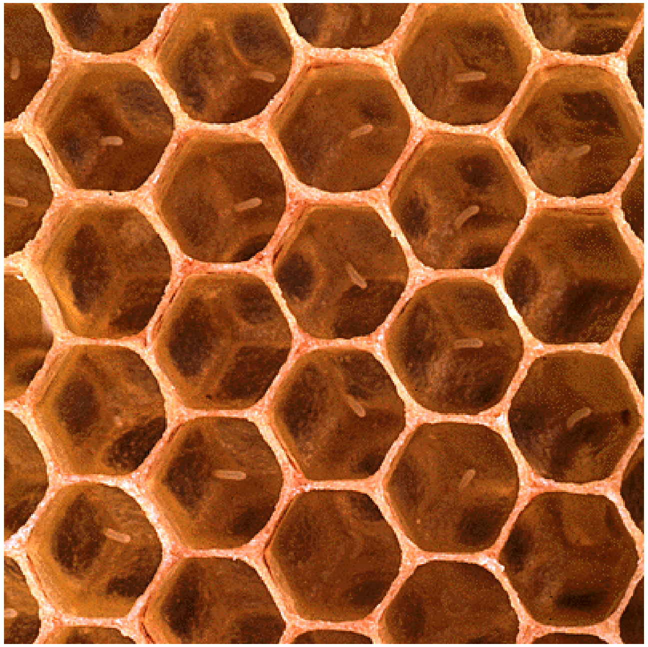




Periodic foams, 2D

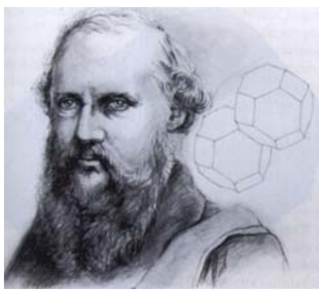
- the simplest structure to satisfy Plateau's rules is a honeycomb
  - seems obvious, but only proved in 2001 by T.C. Hales to be the partitioning of 2D space into cells of equal area with the minimum perimeter

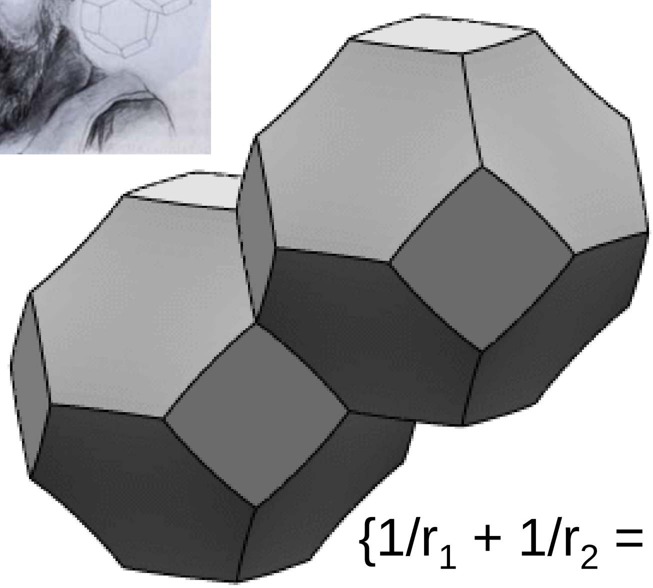




Periodic foams, 3D

- it's not possible to satisfy Plateau's rules with regular solids having flat faces & straight Plateau borders
  - Kelvin foam: like Wigner-Seitz cell for BCC lattice, but with films and Plateau borders curved to satisfy Plateau
    - tetrakaidecahedron (14 sided): 6 quadrilaterals + 8 hexagons





$\{1/r_1 + 1/r_2 = 0\}$

Honeycomb for a 4D bee?

- Bees build a 2D foam that minimizes perimeter/cell

- What foam structure minimizes area at unit cell volume?
  - values for Wigner-Seitz cells curved according to Plateau
    - SC {1x1x1}: 6
    - FCC: 5.34539
    - BCC/Kelvin: 5.30628

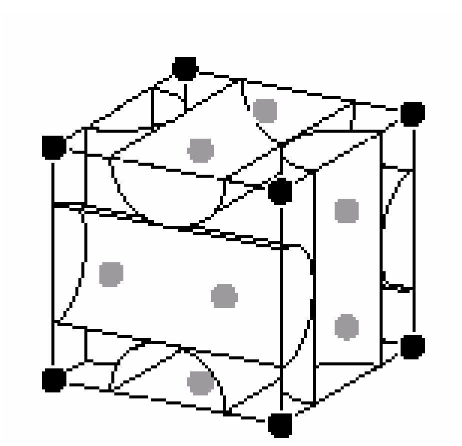
    {sphere: $(36\pi)^{1/3} = 4.83598$}

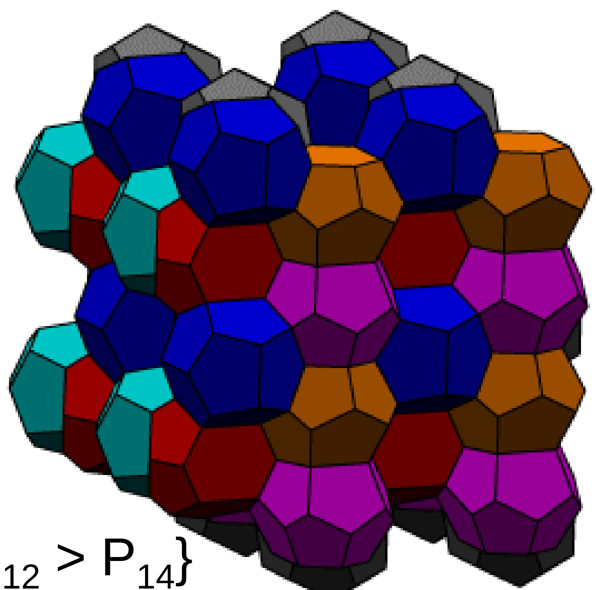
Long believed to be the optimal 3D periodic foam

A15/Weaire-Phelan foam

- BCC/Kelvin: 5.30628
- A15/Weaire-Phelan: 5.28834
  - constructed from *two* different cell types
    - tetrakaidecahedron: 12 pentagons and 2 opposing hexagons
      - these stack into three sets of orthogonal columns
    - dodecahedron: 12 pentagons
      - these fit into interstices between columns

A new champion! (0.3% improvement)

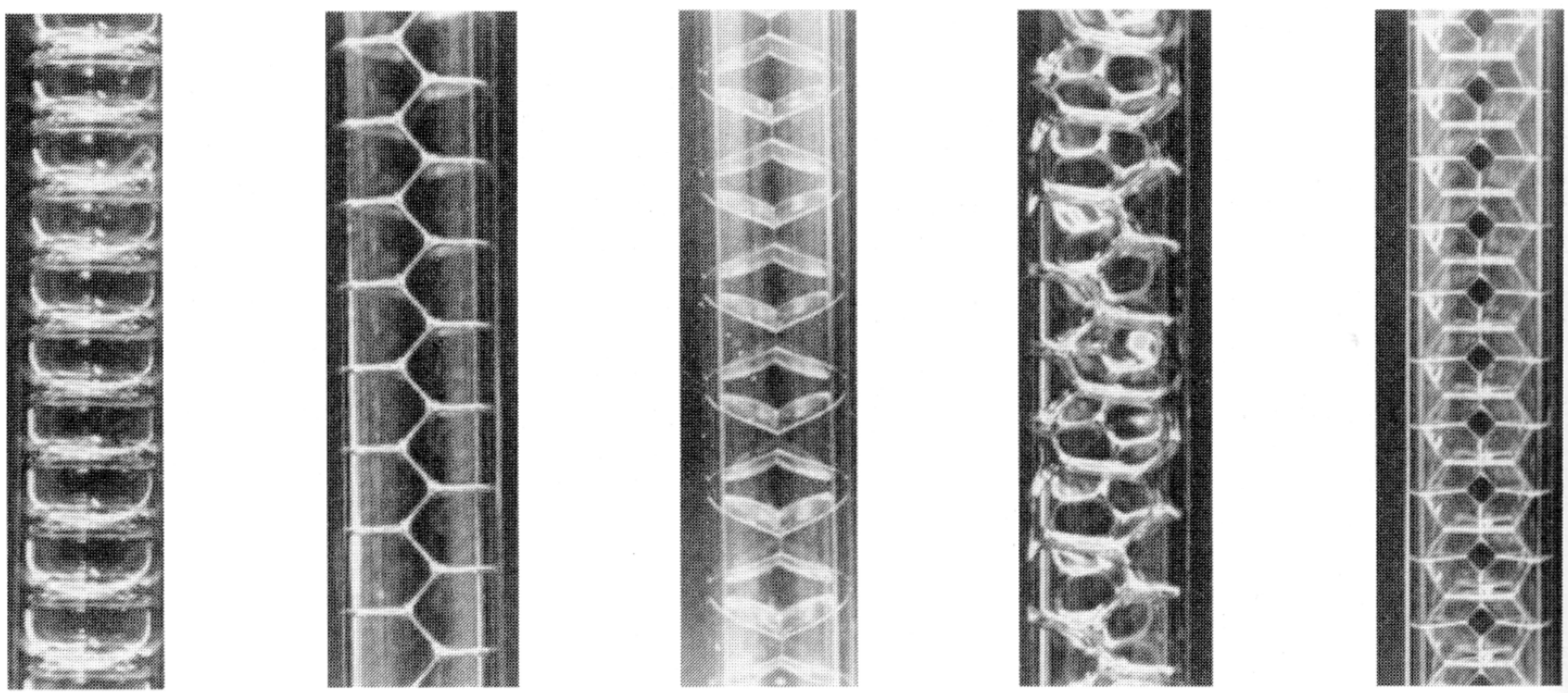




$\{P_{12} > P_{14}\}$

Bubbles in a tube

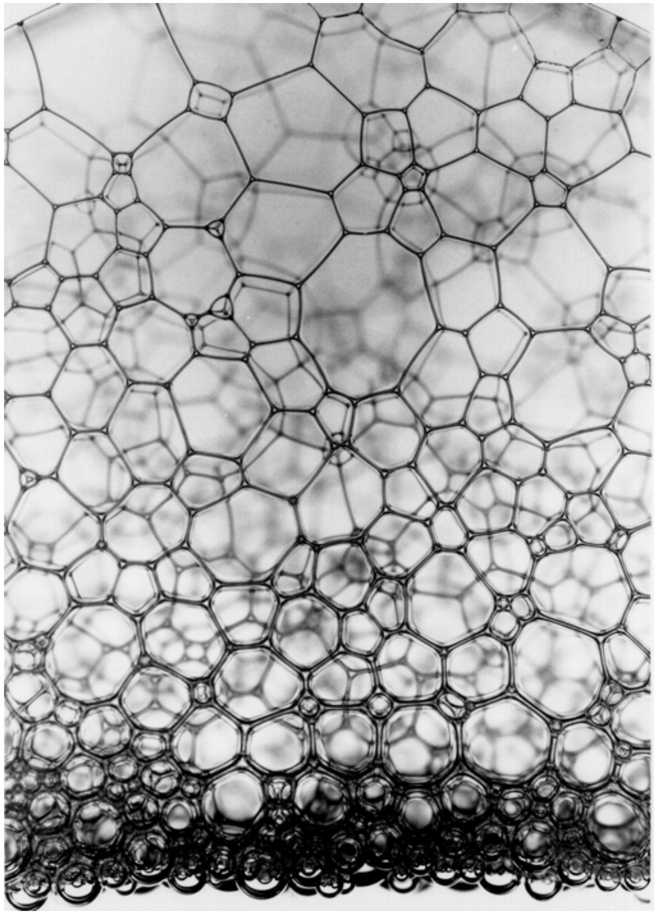
- other ordered structures can readily be produced by blowing monodisperse bubbles into a tall tube:

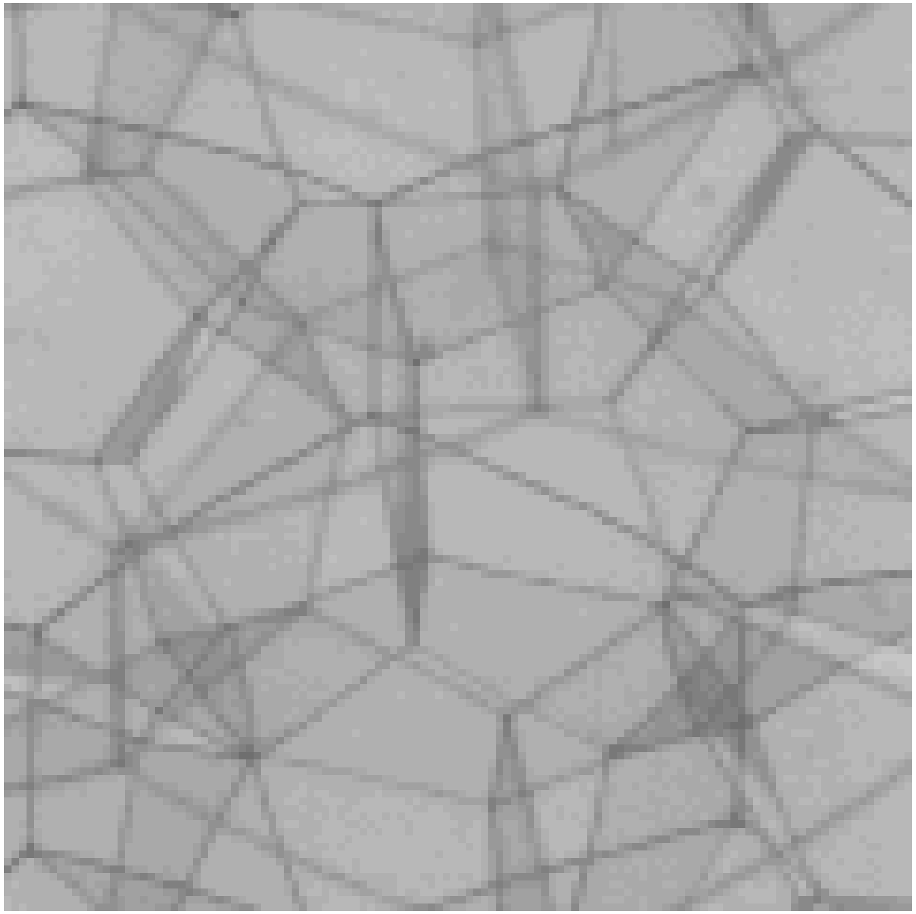


bamboo   (2,1,1)   (2,2,0)   (3,2,1)   (4,2,2)

Random structures

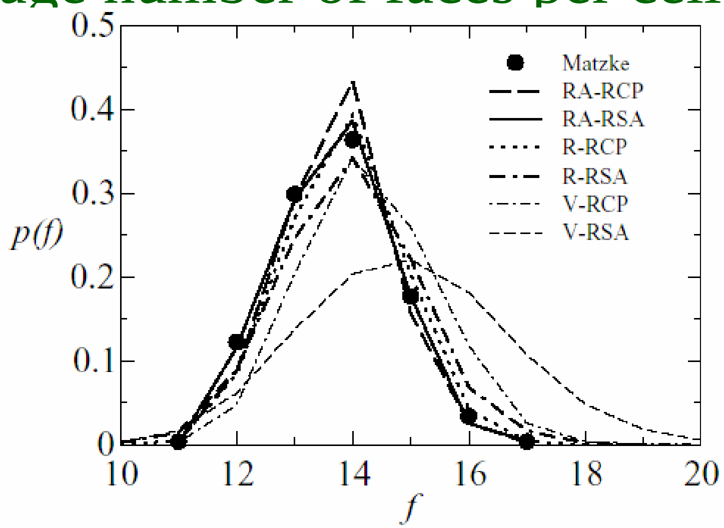
- bulk foams are naturally polydisperse and disordered!
  - (we'll see later that ordered foams are unstable)





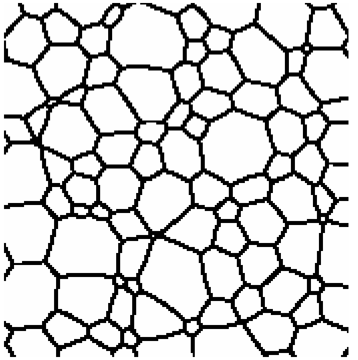
A tedious experiment

- Matzke (J. Botany 1946) constructed random monodisperse foams by individually blowing ~10^3 bubbles and placing them into a container by hand
  - most abundant cell: 13-hedron
    - 1 quadrilateral, 10 pentagons, 2 hexagons
    - Matzke didn’t find even a single Kelvin tetrakaidecadedron!
  - almost all faces were 4, 5, or 6 sided
  - average number of faces per cell <f>=13.70



easier for 2D foams

- bubbles squashed between glass plates
- bubbles floating at an air/water interface (“raft”)
- domains of phase-separated lipid monolayers



- distribution of edges per bubble, p(n)
  - average number of edges per bubble: $\langle n \rangle = \Sigma[n\ p(n)] = 6$
  - second moment, $\mu_2 = \Sigma[(n-6)^2 p(n)] = 1.4$
    - hexagons are common, but there is considerable width
- neighbor correlations
  - average number of edges of neighbors to n-sided bubbles, m(n)
  - Aboav law: $m(n) = 5 + 8/n$
    - combined with Lewis “law” ($A_n \sim n+n_o$, which actually doesn’t work so well) big bubbles are surrounded by small bubbles and vice-versa

Topology

- Euler equation for total # of cells, faces, edges, vertices:

$$N_F - N_E + N_V = 1 \quad (2D)$$

$$-N_C + N_F - N_E + N_V = 1 \quad (3D)$$

- Combine with Plateau in 2D

$N_V = 2/3\ N_E$, so large $N_F = N_E - N_V = 1/3\ N_E$

hence $\langle n \rangle = 2\ N_E / N_F = 6$

  - as observed

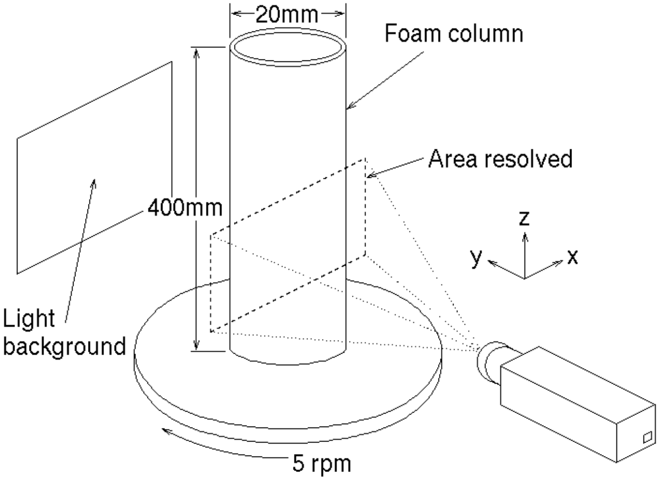
- Combine with Plateau in 3D

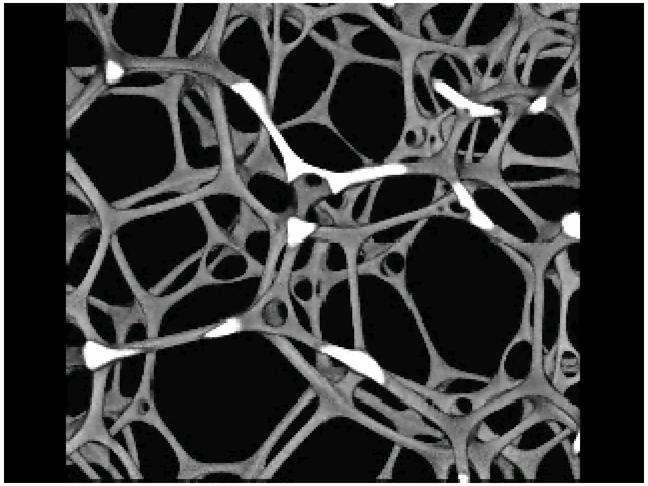
$\langle f \rangle = 12/(6 - \langle n \rangle)$

  - Matzke result $\langle f \rangle = 13.70$ implies $\langle n \rangle = 5.12$

Imaging methods

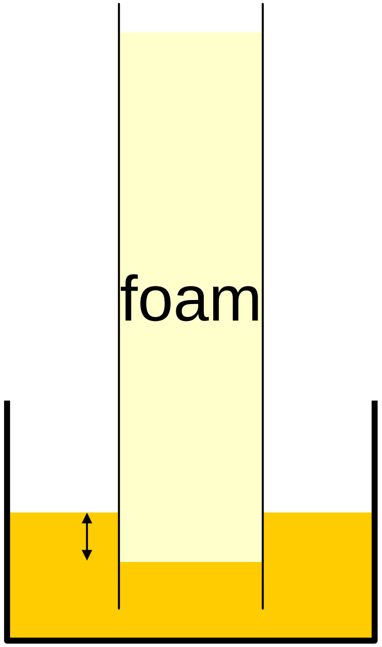
- ordinary microscopy / photography (eg Matzke)
  - good only for very dry foams a few bubbles across
- large numerical-aperture lens
  - image one 2D slice at a time, but same defect as (1)
- confocal microscopy – reject scattered light
  - *slightly* wetter foams / larger samples
- medical (MRI, tomography)
  - slow





Other structural probes

- Moving fiber probe
  - drive optical fiber through a bulk foam: reflection spikes indicate proximity of a film; gives ~cell-size distribution
    - doesn’t pop the bubbles!
- Electrical conductance
  - conductivity is proportional to liquid fraction
    - independent of bubble size!
- Archimedes – depth of submerged foam
  - deduce liquid fraction



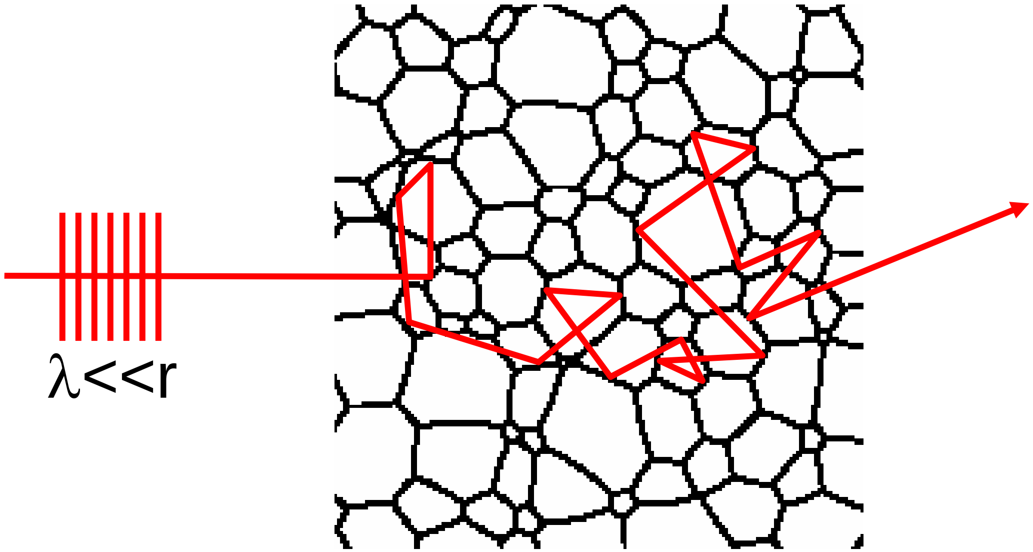
Photon diffusion

- 3D foams are white / opaque

  clear foams do not exist!

  – photons reflect & refract from gas/liquid interfaces

  – multiple scattering events amount to a random walk (diffusion!)



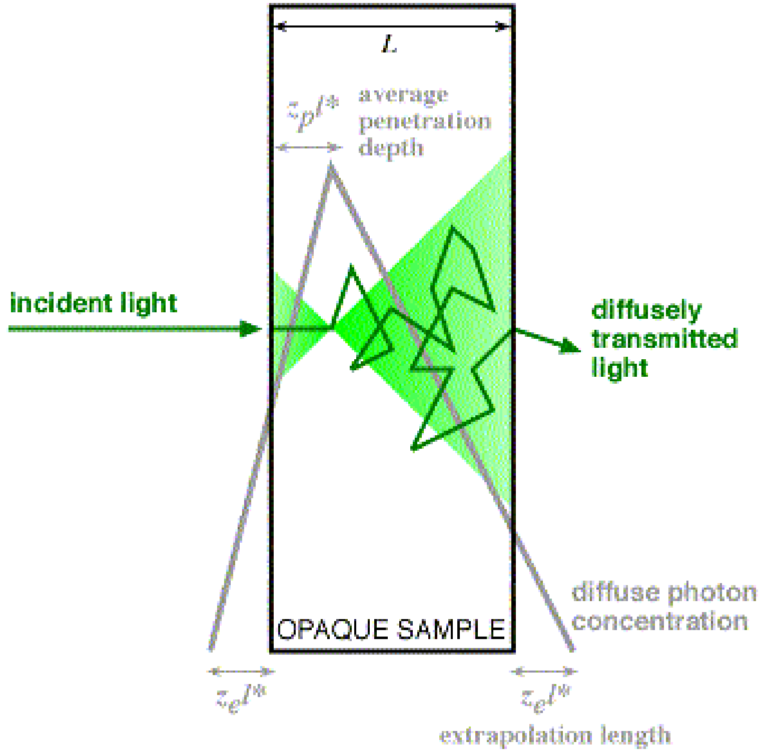
- while this limits optical imaging methods, it can also be exploited as a probe of foam structure & dynamics…

Transmission probability

- how much light gets through a sample of thickness L?
  - *ballistic* transmission is set by scattering length

    $T_b = \mathrm{Exp}[-L/l_s]$ (vanishingly small: 10^{-5} or less)
  - *diffuse* transmission is set by transport mean free path ($D = cl^*/3$)

    $T_d = (z_p + z_e)/(L/l^* + 2z_e) \sim l^*/L$ (easily detectable: 0.01 – 0.1)



Foam optics

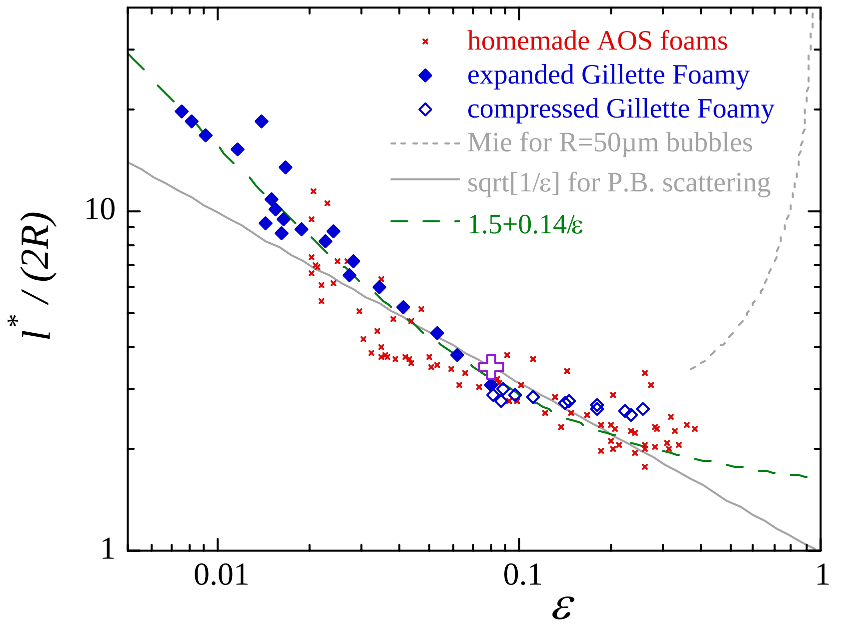
- Plateau borders are the primary source of scattering
  - recall liquid fraction ε ~ (border radius r / bubble radius R)^2
  - estimate the photon transport mean free path from their number density and geometrical cross section:

$$l^* = \frac{1}{\rho\sigma^*} \sim \frac{1}{\left(1/R^3\right)\left(rR\right)} \sim \frac{R}{\sqrt{\varepsilon}}$$

FAST & NON-INVASIVE PROBE:

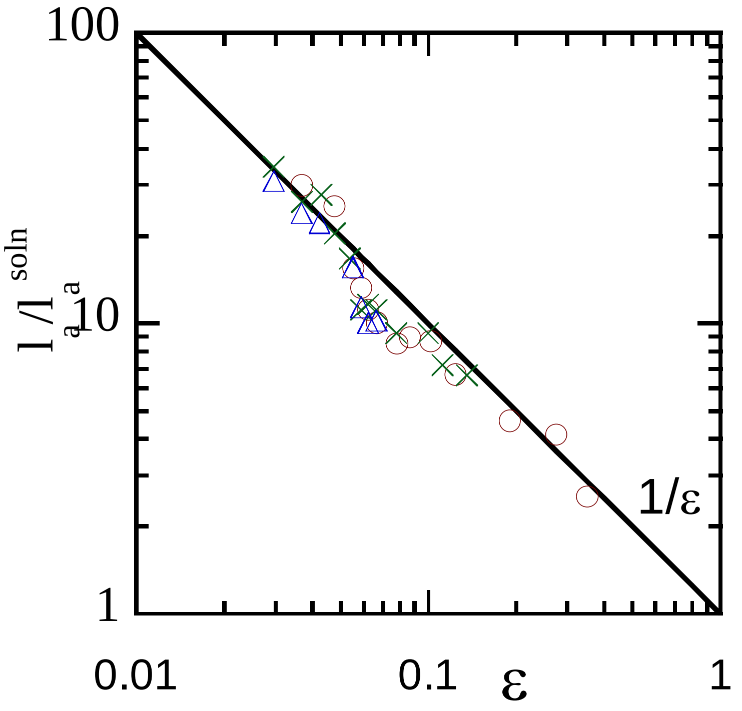
*diffuse transmission gives l**

l gives bubble size or liquid fraction*



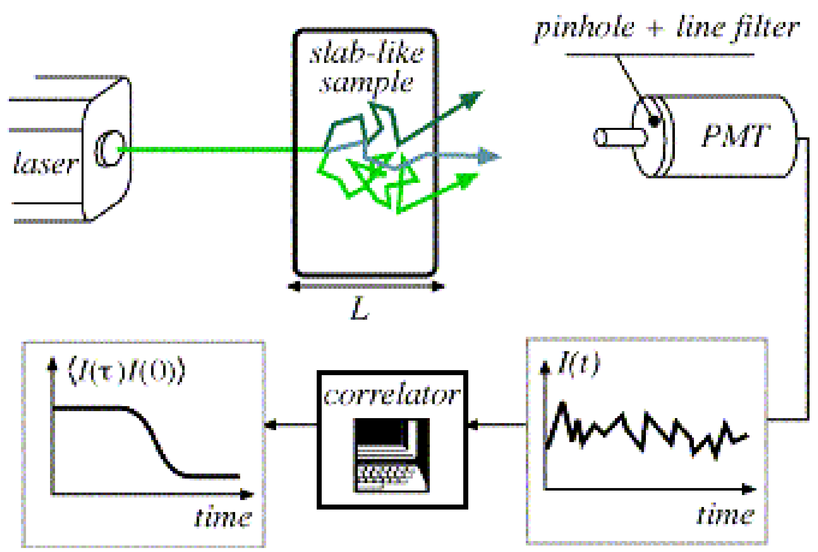
How random is the walk?

- the foam absorbs more light than expected based on the volume fraction of liquid $\{l_a/l_a^{soln} = 1/\varepsilon\}$
  - Plateau borders act like a random network of optical fibers
    - effect vanishes for very wet foams: Plateau border length vanishes
    - effect vanishes for very dry foams: photons exit at vertices



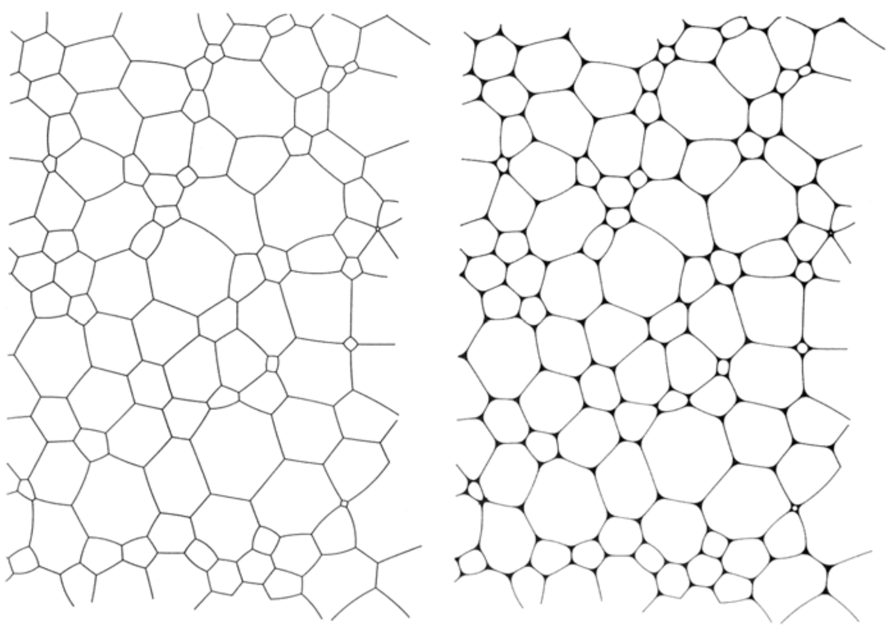
Diffusing-wave spectroscopy

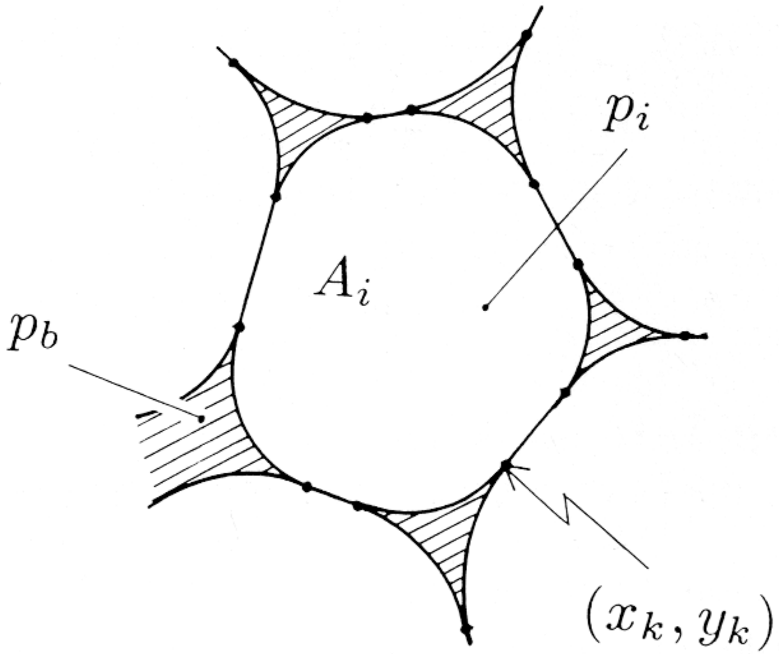
- Form a speckle pattern at plane of detector
- As scattering sites move, the speckle pattern fluctuates
  - for maximum intensity variation: detection spot = speckle size
  - measure <I(0)I(t)> to deduce nature & rate of motion



Simulation of structure

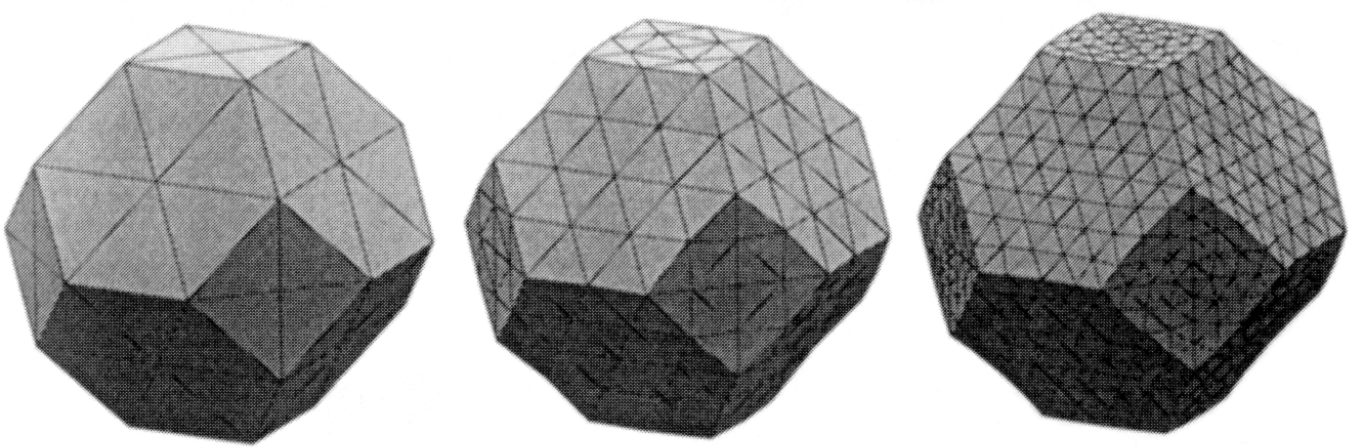
- in 2D the elements are all circular arcs (wet or dry)
  - otherwise the gas pressure wouldn't be constant across the cell
- adjust endpoints and curvature, while maintaining constant area, until Plateau is satisfied everywhere
  - iteratively or all at once





Surface Evolver

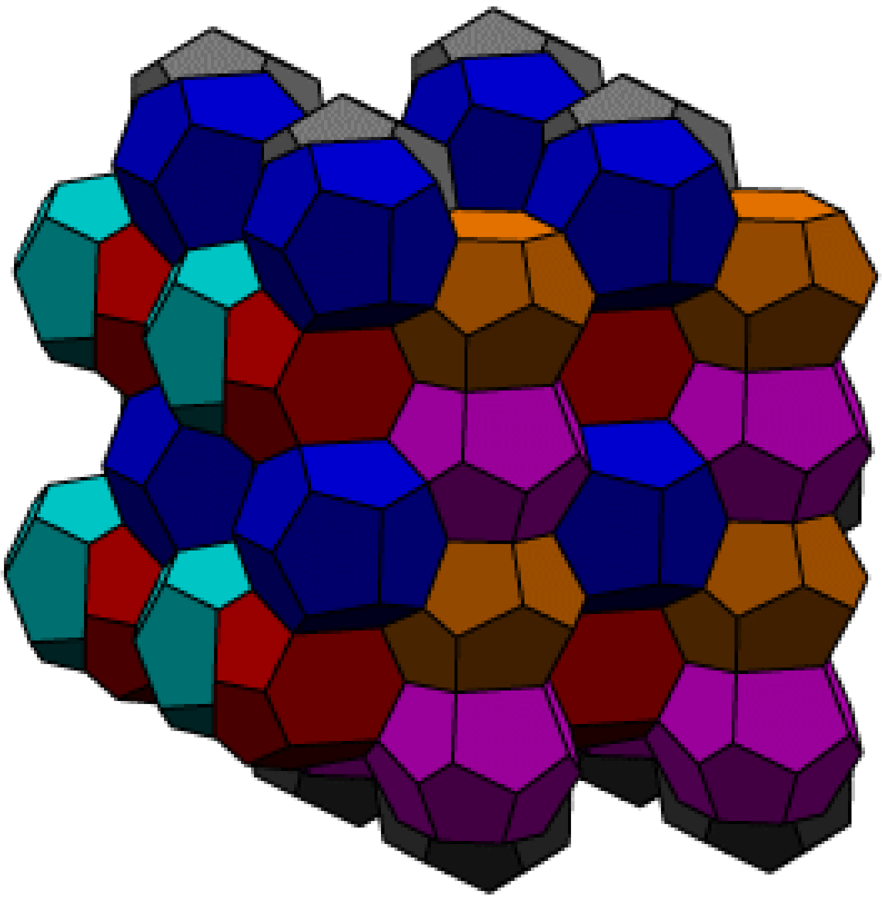
- in 3D it's much harder…
  - films have constant curvature but are not spherical
  - Plateau borders have arbitrary shape
- The "Surface Evolver" program by Ken Brakke minimizes film area at fixed topology
  - approximate surfaces by flat triangular plaquets
    - eg successive refinement of Kelvin cell:

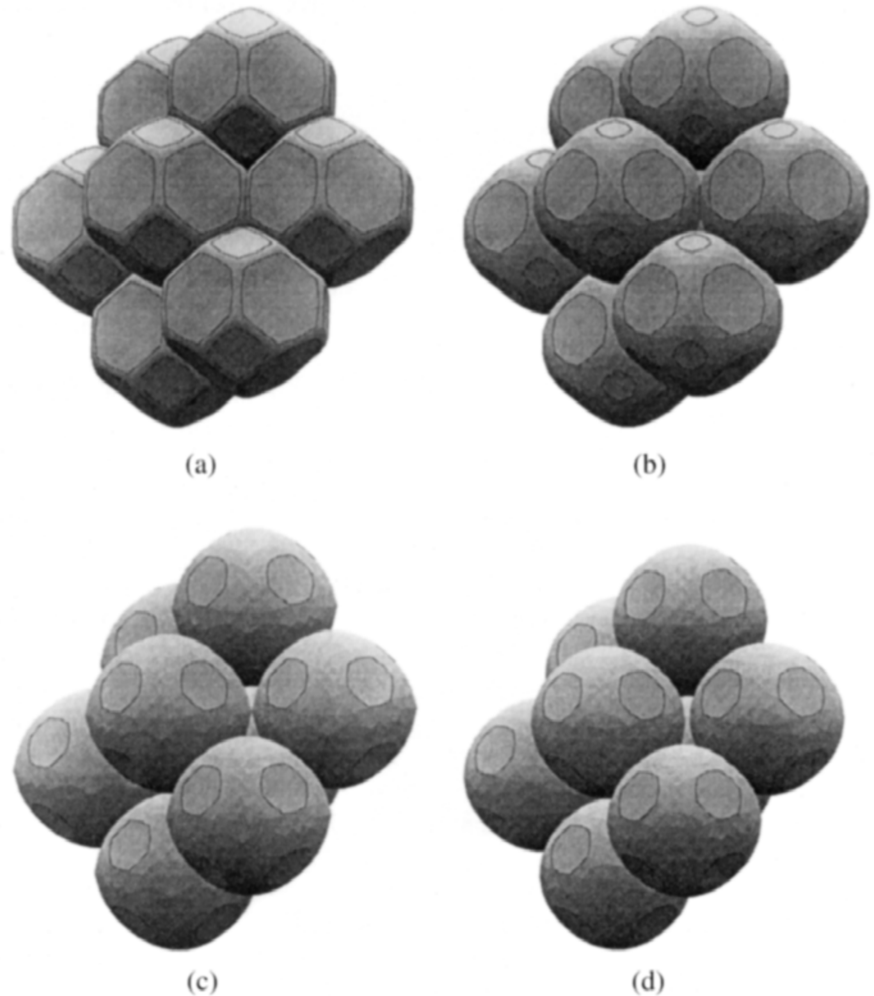


(a) R0 (b) R1 (c) R2

Surface Evolver – uses

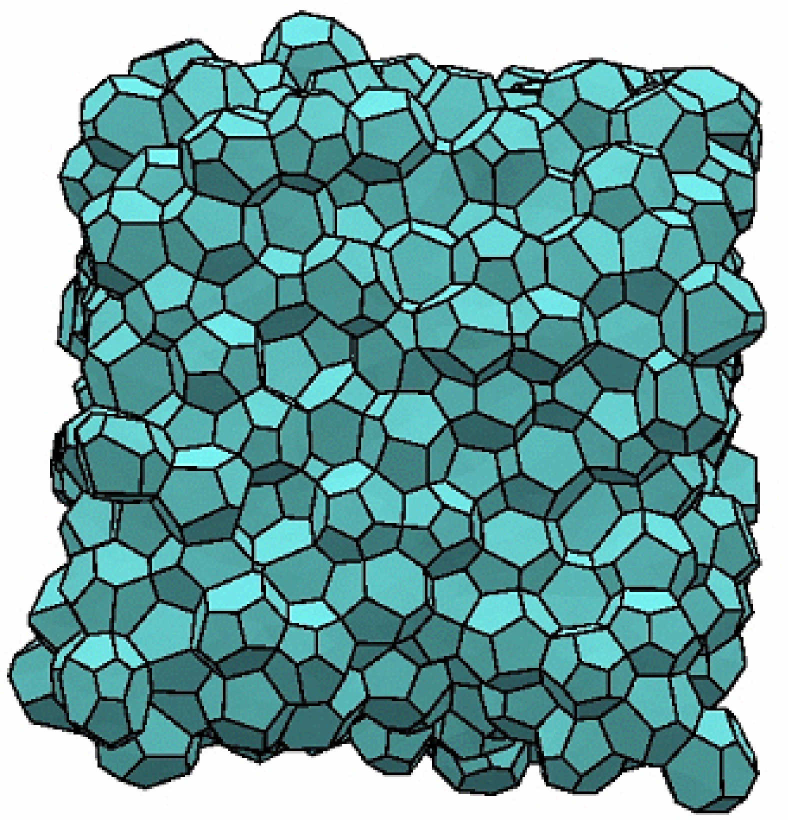
- discovery of A15/Weaire-Phelan foam
- wet Kelvin foams

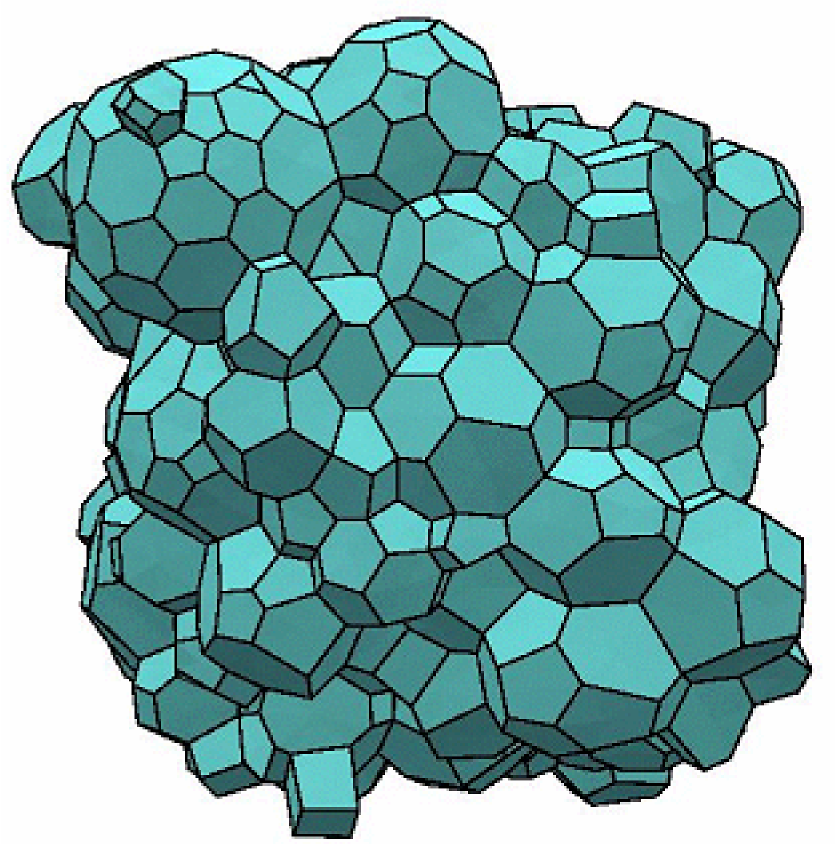




Surface Evolver – uses

- random monodisperse foams
- polydisperse foams



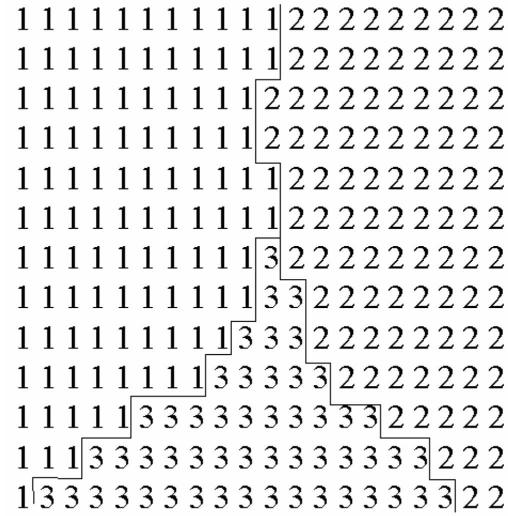


Surface Evolver – uses

- Apply shear to any of the above
- Reconstruct full structure from partial tomographic data
  - eg finding films and volumes knowing only Plateau borders
- In general: statistics of dry foams in static equilibrium
- Drawbacks
  - fixed topology (must be reset by hand during equilibration / flow / evolution)
  - progressively slower for wetter foams
  - no true dynamics (film-level dissipation mechanisms cannot be included)

Q-state Potts model

- each lattice site has a spin, with a value that depends on the cell to which it belongs; eg:

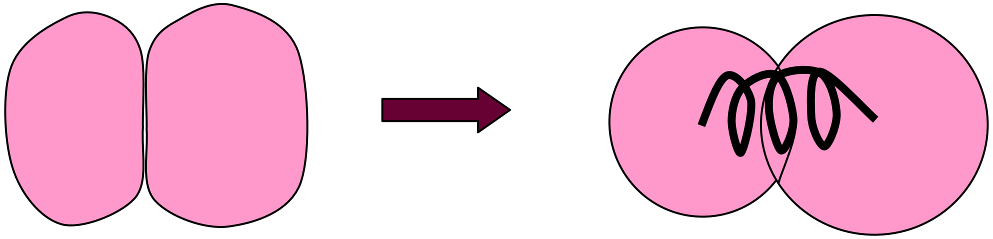


  - energy penalty for neighbors of different spin
  - flip spins at interface by Monte-Carlo
    - minimizes interfacial area, like Surface Evolver, but slower
    - avoids the issue of setting topology by hand
    - but no true dynamics

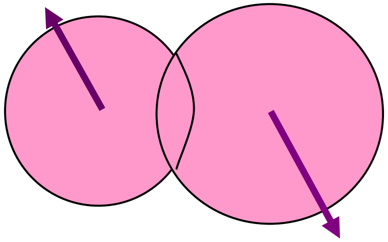
Bubble model

- Consider bubbles, not films, as the structural elements
  - ignore shape degrees of freedom
  - move bubbles according to pairwise interactions:
    1. Spring force for overlapping bubbles (strictly repulsive)

       {exact in 2D, good approximation in 3D}

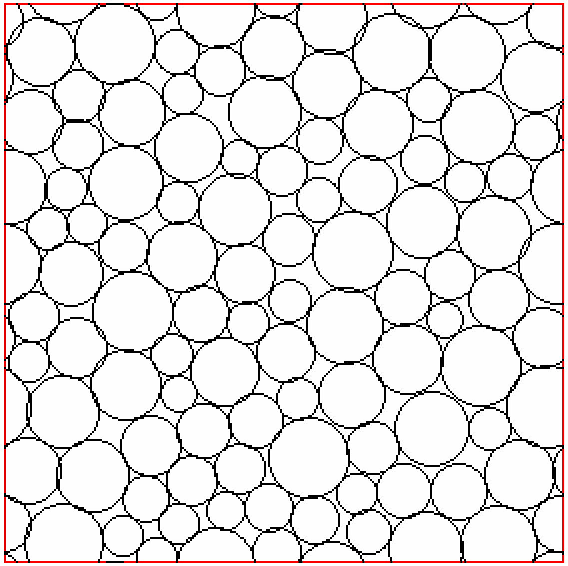


    2. Dynamic friction for neighboring bubbles (~velocity difference)



Bubble model – uses

- rough caricature of essential microscopic physics
  - exact for wet foam limit of close-packed spheres
- no need to keep track of topology by hand
- true dynamics, and computationally cheap
  - not useful for topology statistics
  - good for evolution and flow



Next time…

{not yet ready foam rheology}

- evolution of aqueous foams
  - coarsening, in response to gas diffusion
  - drainage, in response to gravity