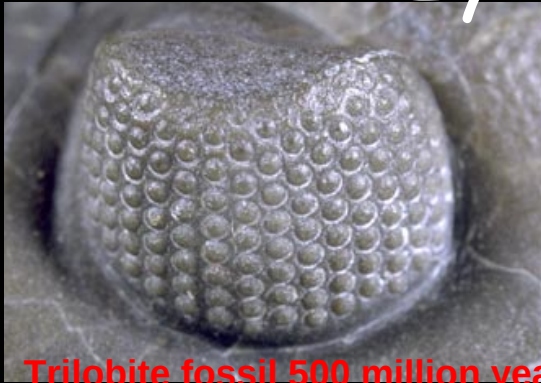


Eyes everywhere...



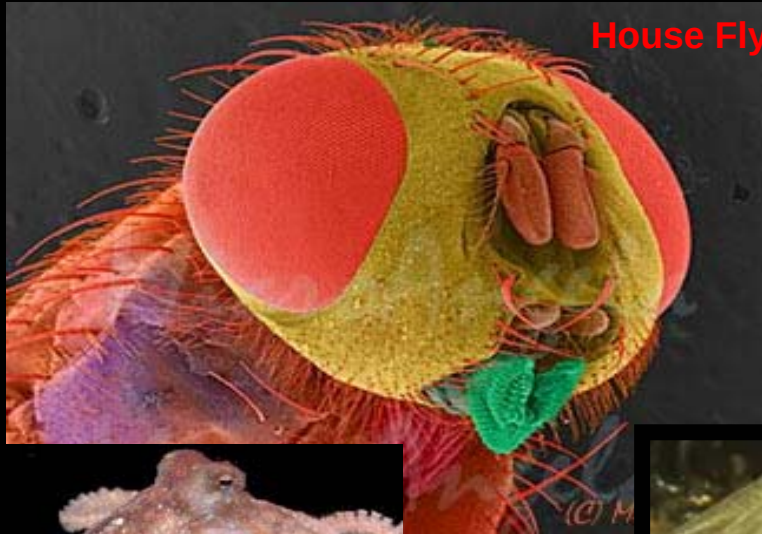
Trilobite fossil 500 million years



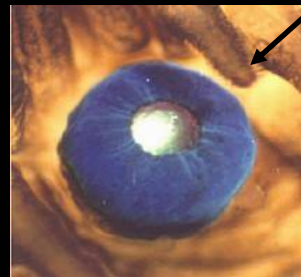
Copepod's 2 lens telescope



Scallop



House Fly



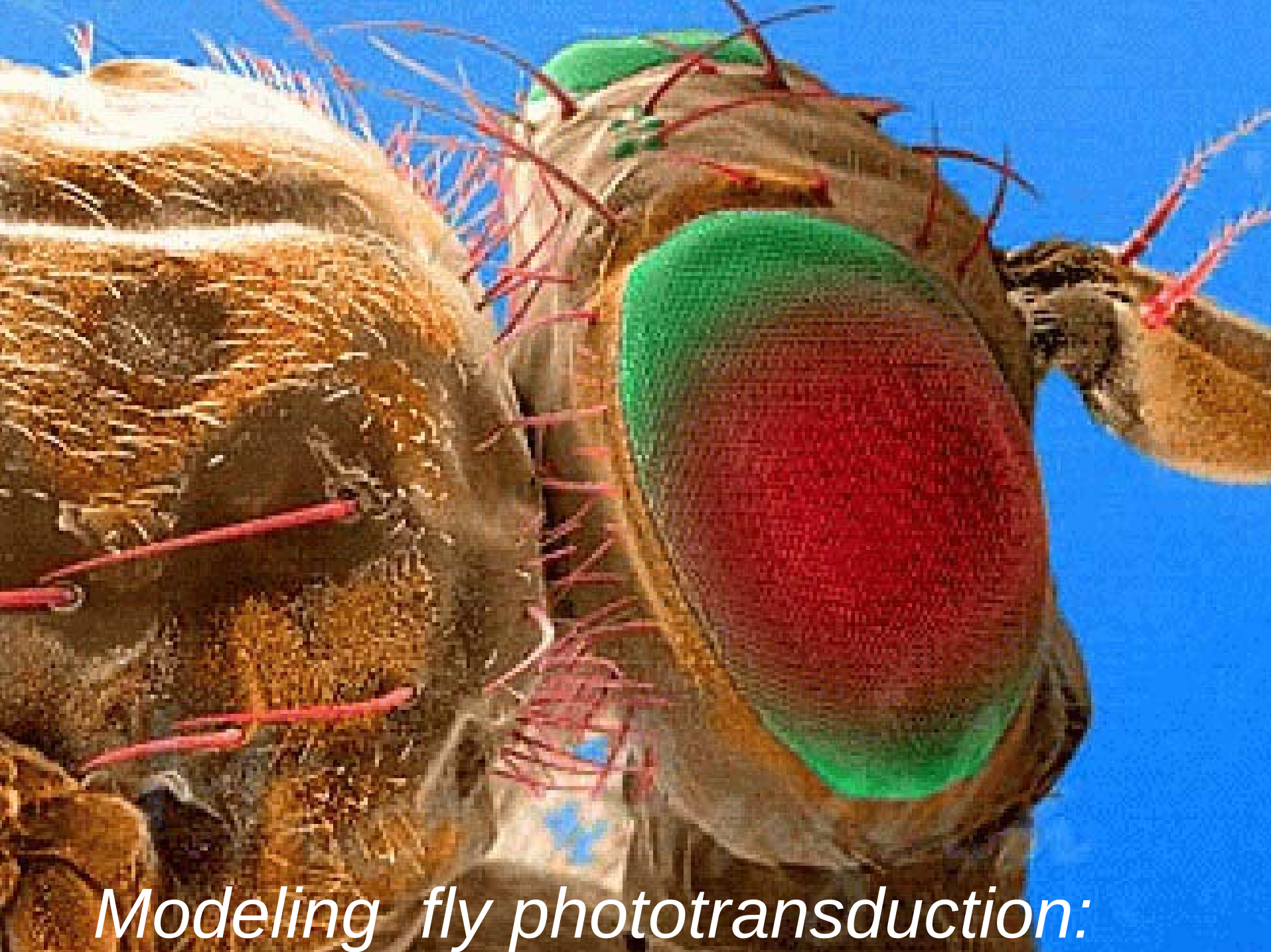
Octopus



Cuttlefish



Black ant



Modeling fly phototransduction:

Limits of modeling?

Comparative systems biology?

- vertebrate phototransduction (rods, cones)
- insect phototransduction
- olfaction, taste, etc...

Fly photo-transduction

Outline:

- About the phenomenon
- Molecular mechanism
- Phenomenological Model
- Predictions and comparisons with experiment.

Compound eye of the fly

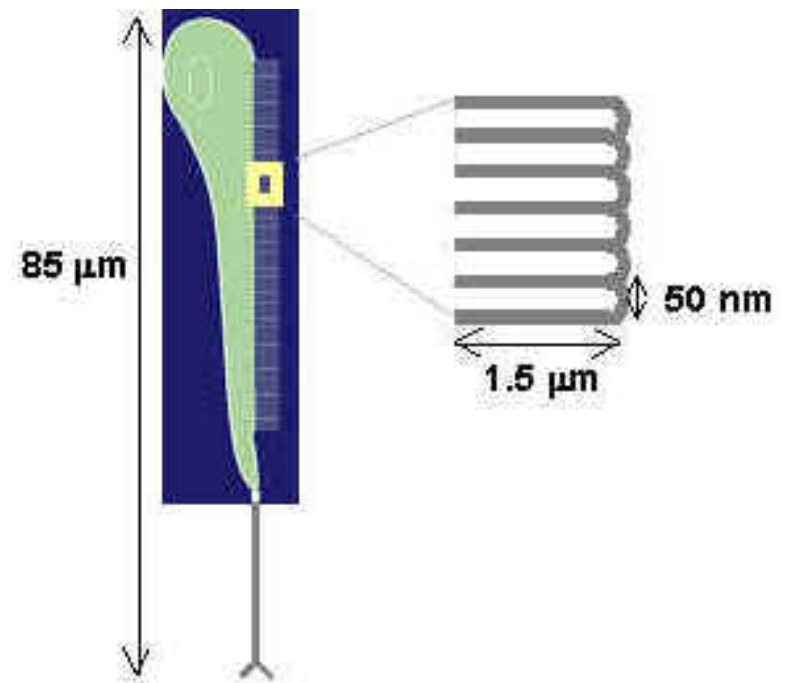
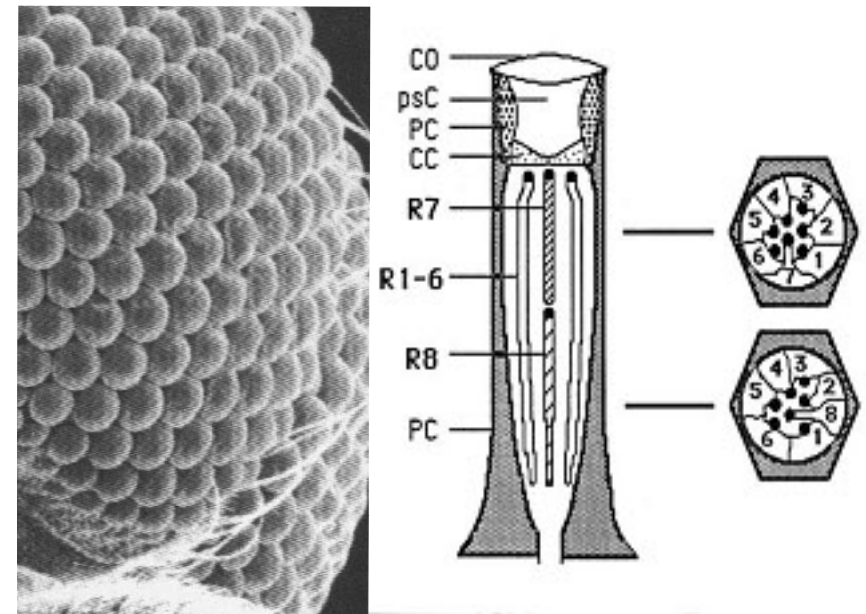
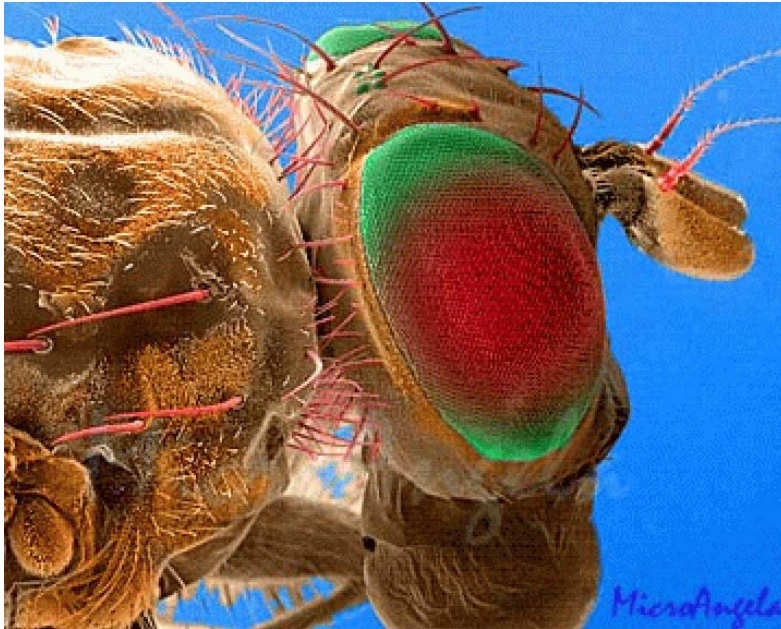
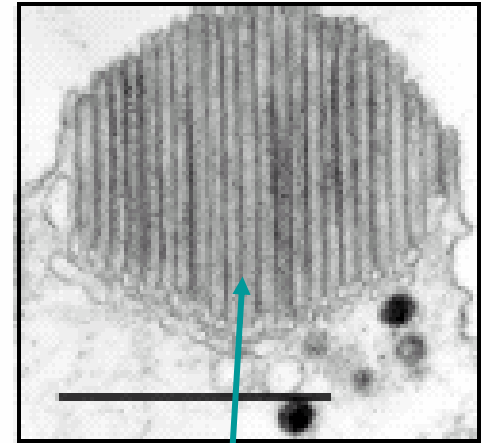


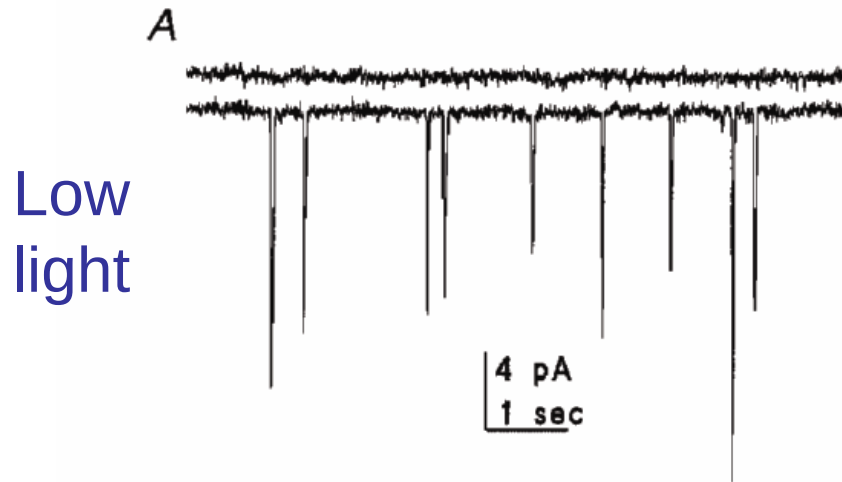
Diagram illustrating the structure and function of a rod cell, showing the SMC (Sensory Motor Cell) and Rhabdomere.

The diagram shows a cross-section of the rod cell, with the SMC (Sensory Motor Cell) and Rhabdomere labeled. The Rhabdomere is shown with microvilli (labeled Microvillus) and rhodopsin molecules (labeled Rhodopsin). The rhodopsin molecules are shown as red lightning bolts labeled $h\nu$ and Rhodopsin.

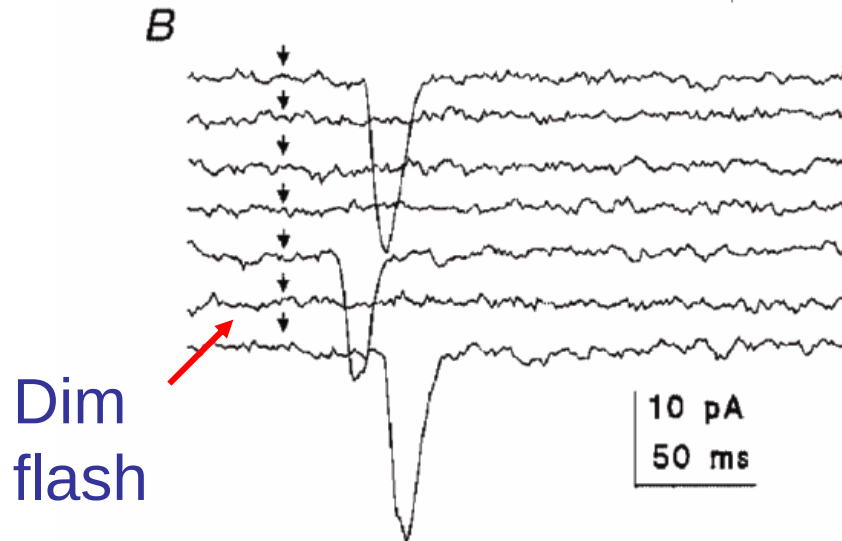
The diagram also shows the flow of Na^+ and Ca^{2+} ions through the rhabdomere membrane. The length of the rhabdomere is indicated as $1.5 \mu\text{m}$ and the width as 50 nm .



Single photon response in Drosophila: a Quantum Bump



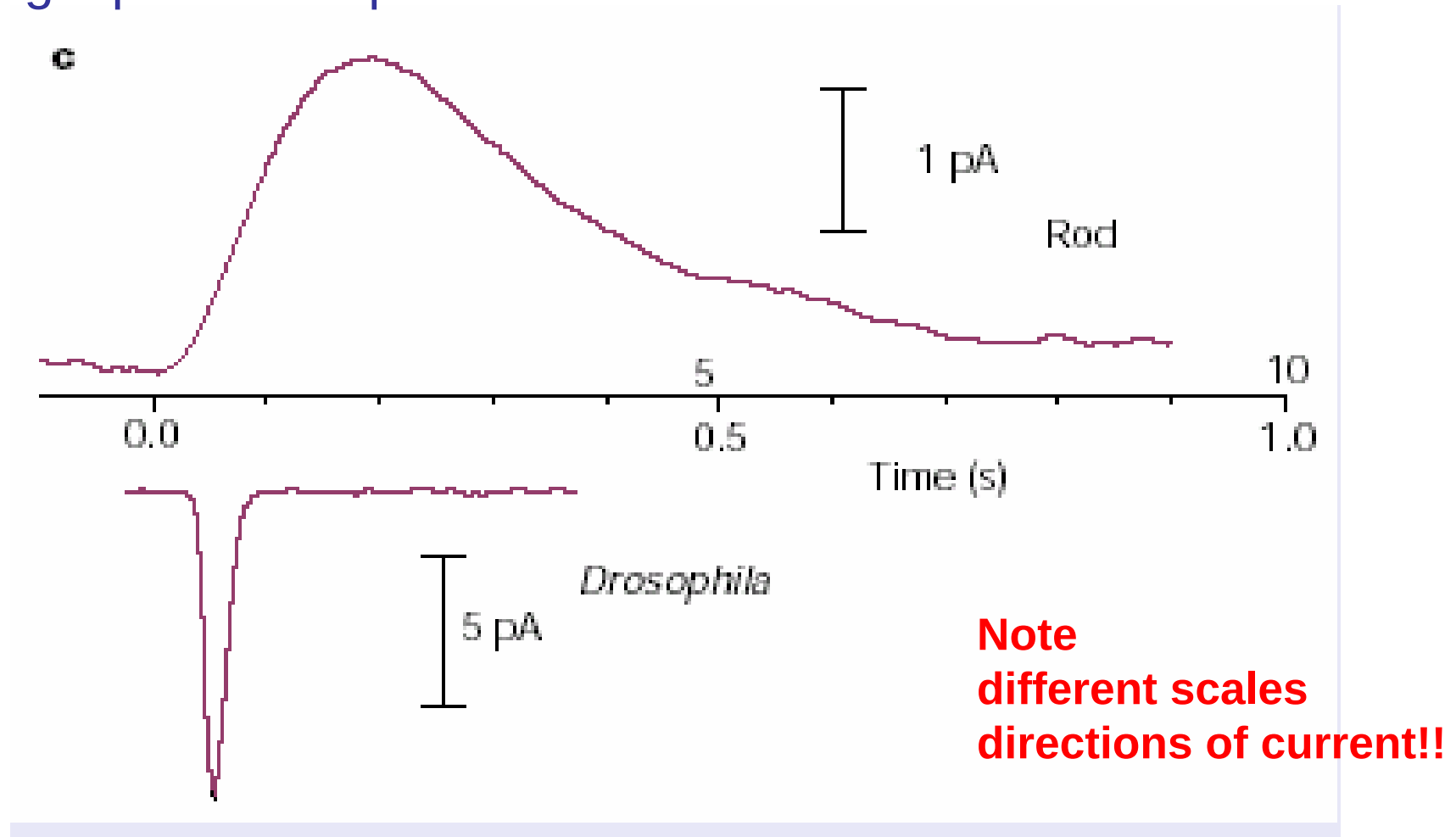
*“All-or-none”
response*



Henderson and Hardie,
J.Physiol. (2000) 524, 179

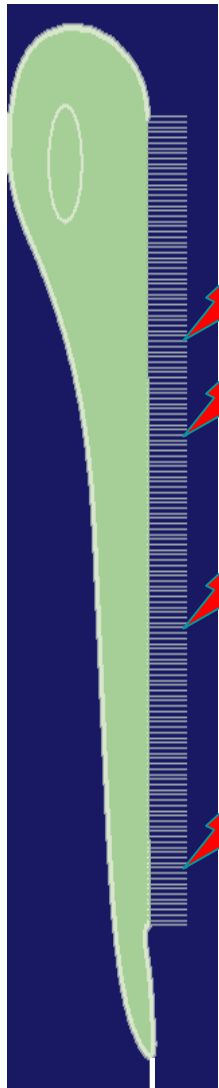
Comparison of a fly with a toad.

Single photon response:

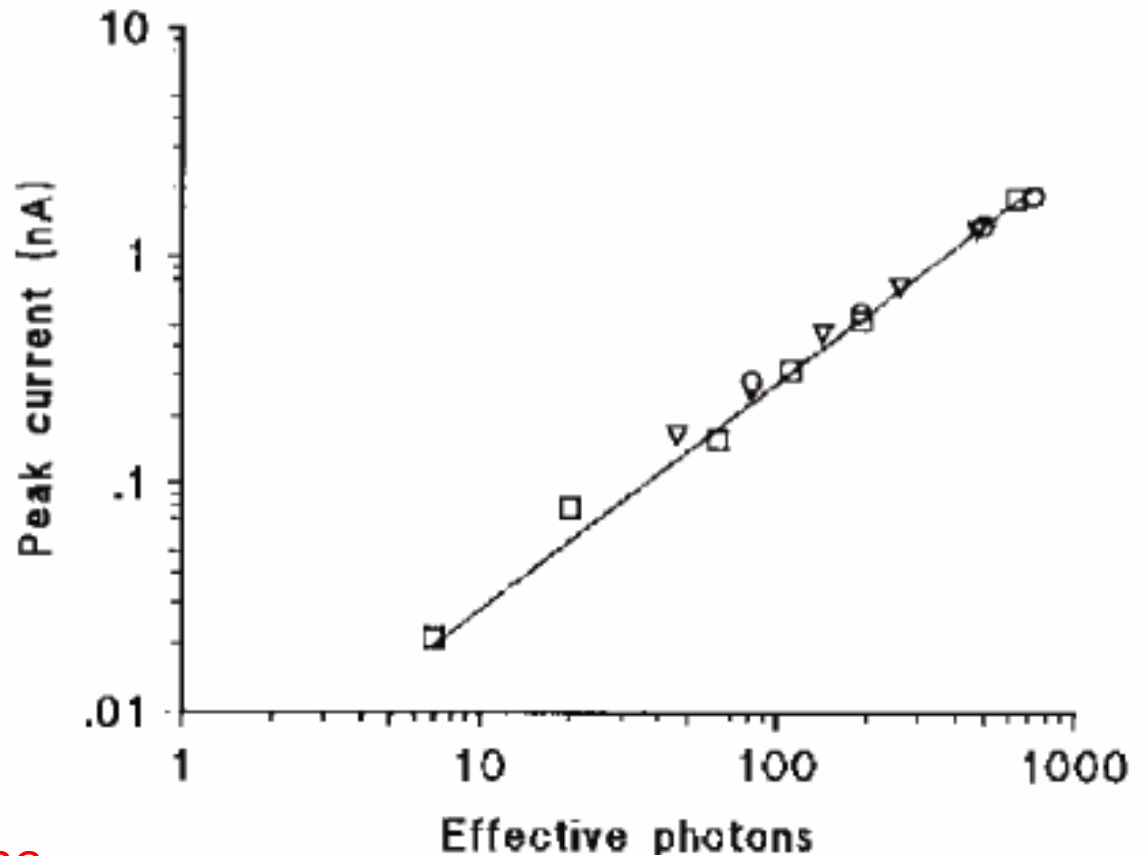


From Hardie and Raghu, Nature 413, (2001)

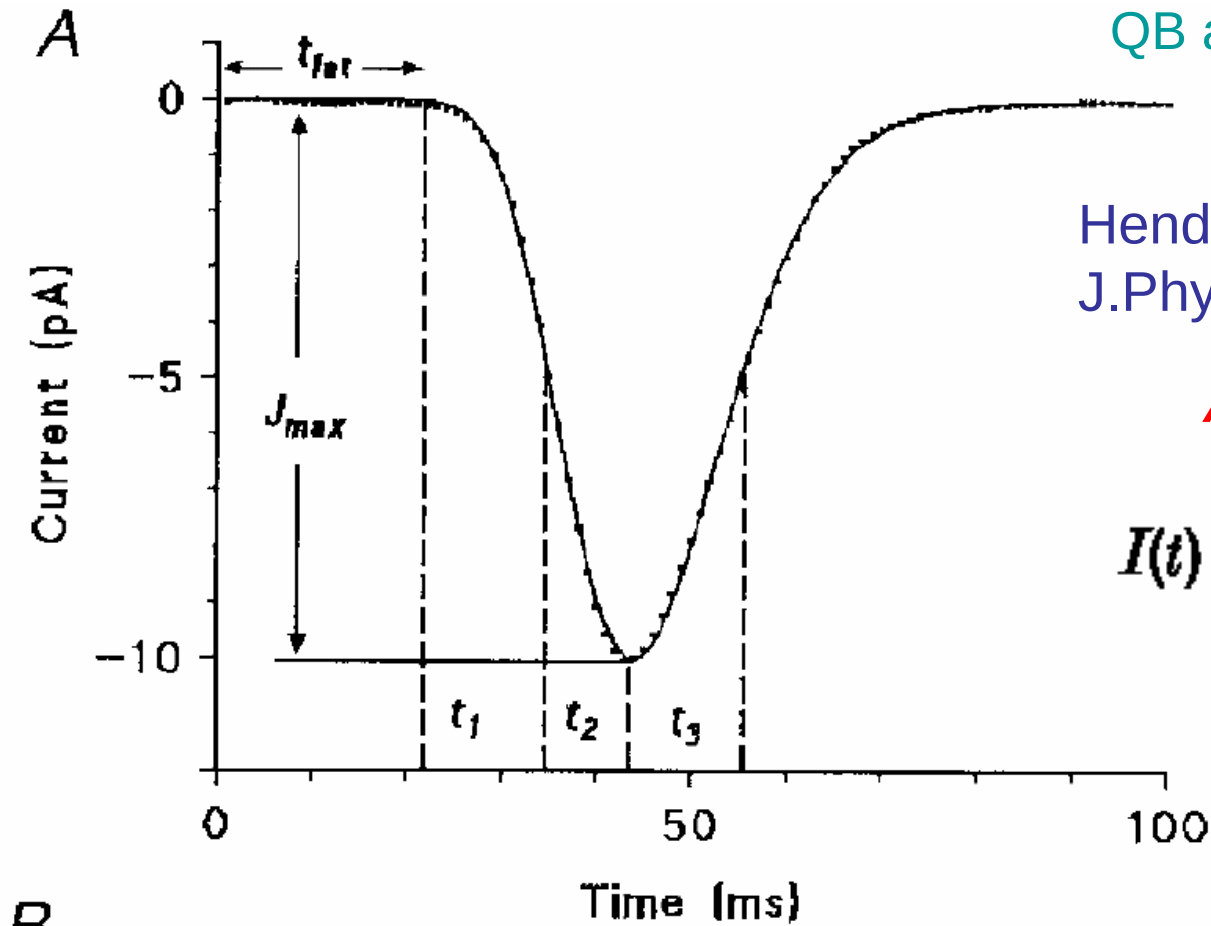
Linearity of macroscopic response



*Linear
summation
over microvillae*

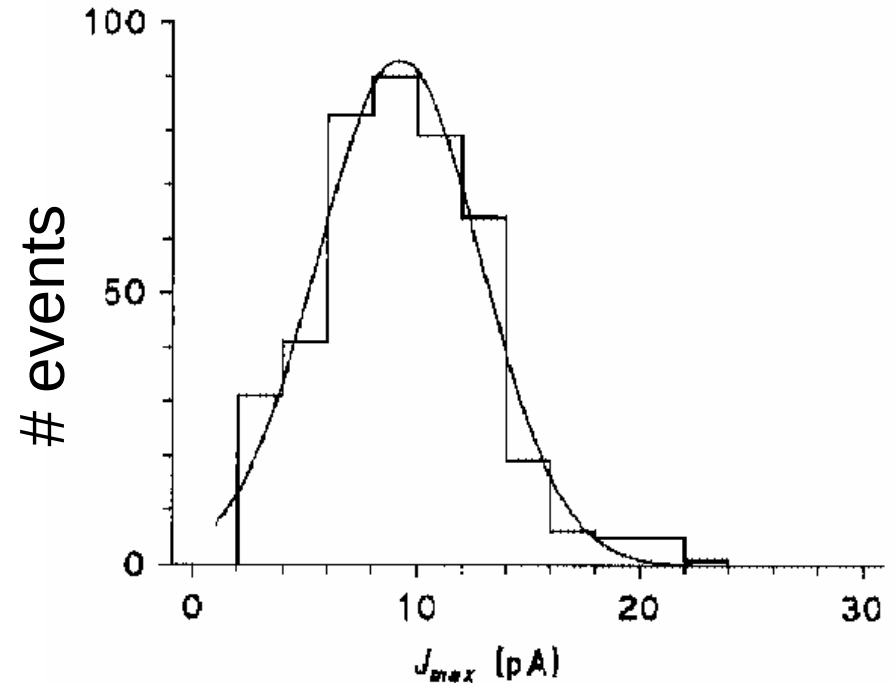
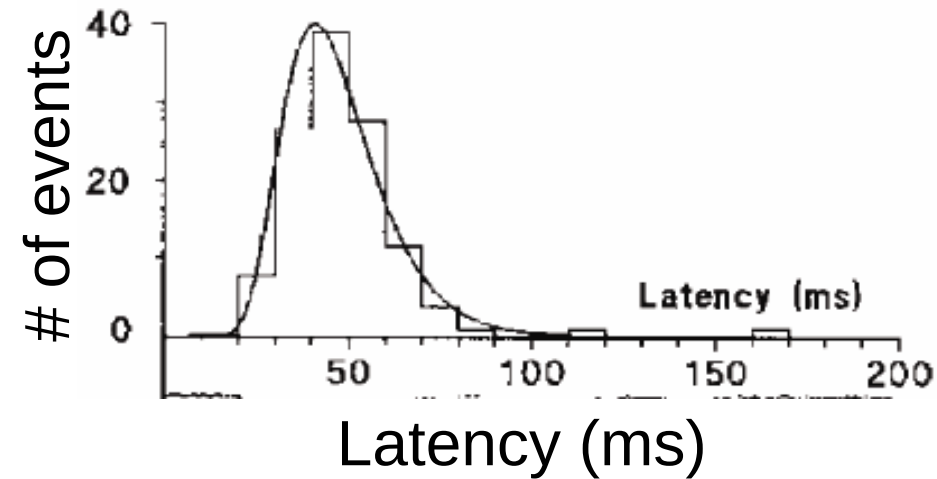


Average QB wave-form



QB variability

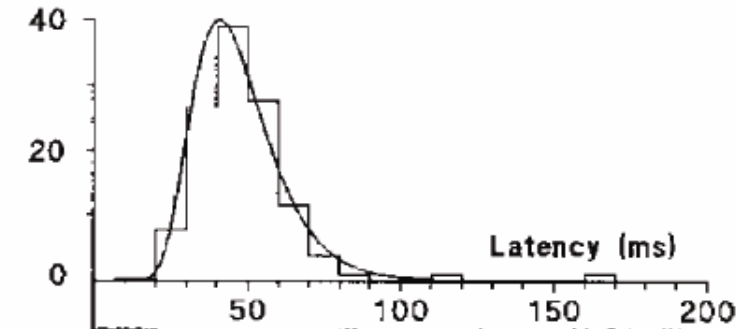
Latency distribution



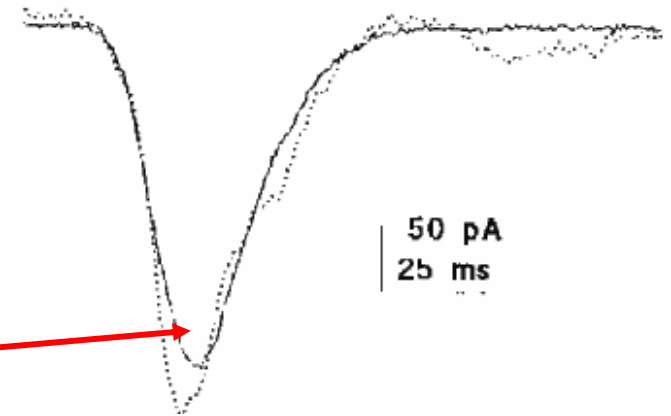
Peak current $J_{\max}$ (pA)

$$\frac{\text{rms}(J_{\max})}{\text{mean}(J_{\max})} \approx 0.4$$

Multi-photon response



Macroscopic response
= average QB



QB
waveform

Convolution
with latency
distribution

*Latency distribution
determines the average
macroscopic response*

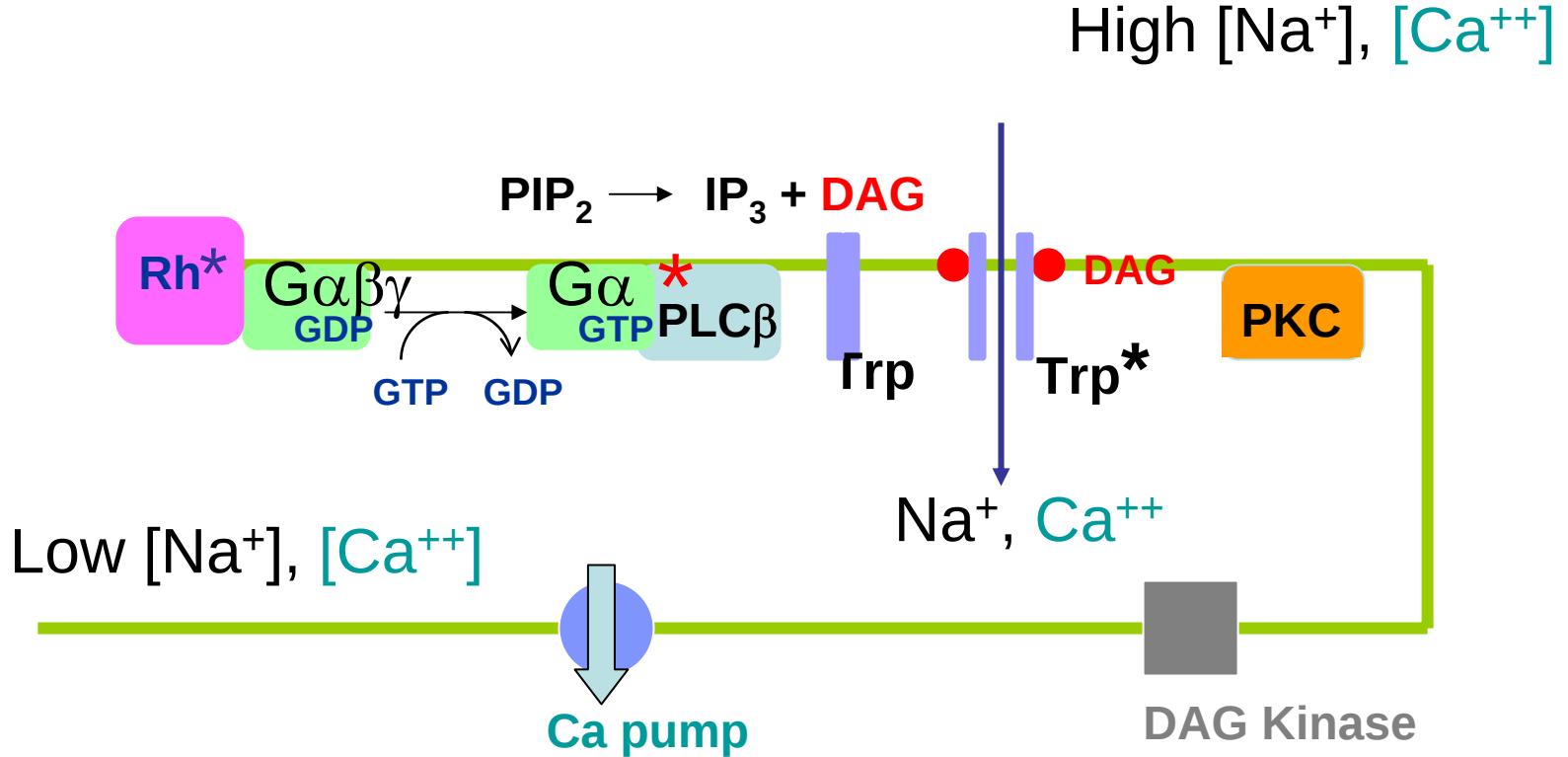
!!! Fluctuations control the mean !!!

Advantages of *Drosophila* photo-transduction as a model signaling system:

- Input: Photons
- Output: Changes in membrane potential
- Single receptor cell preps
- *Drosophila* genetics

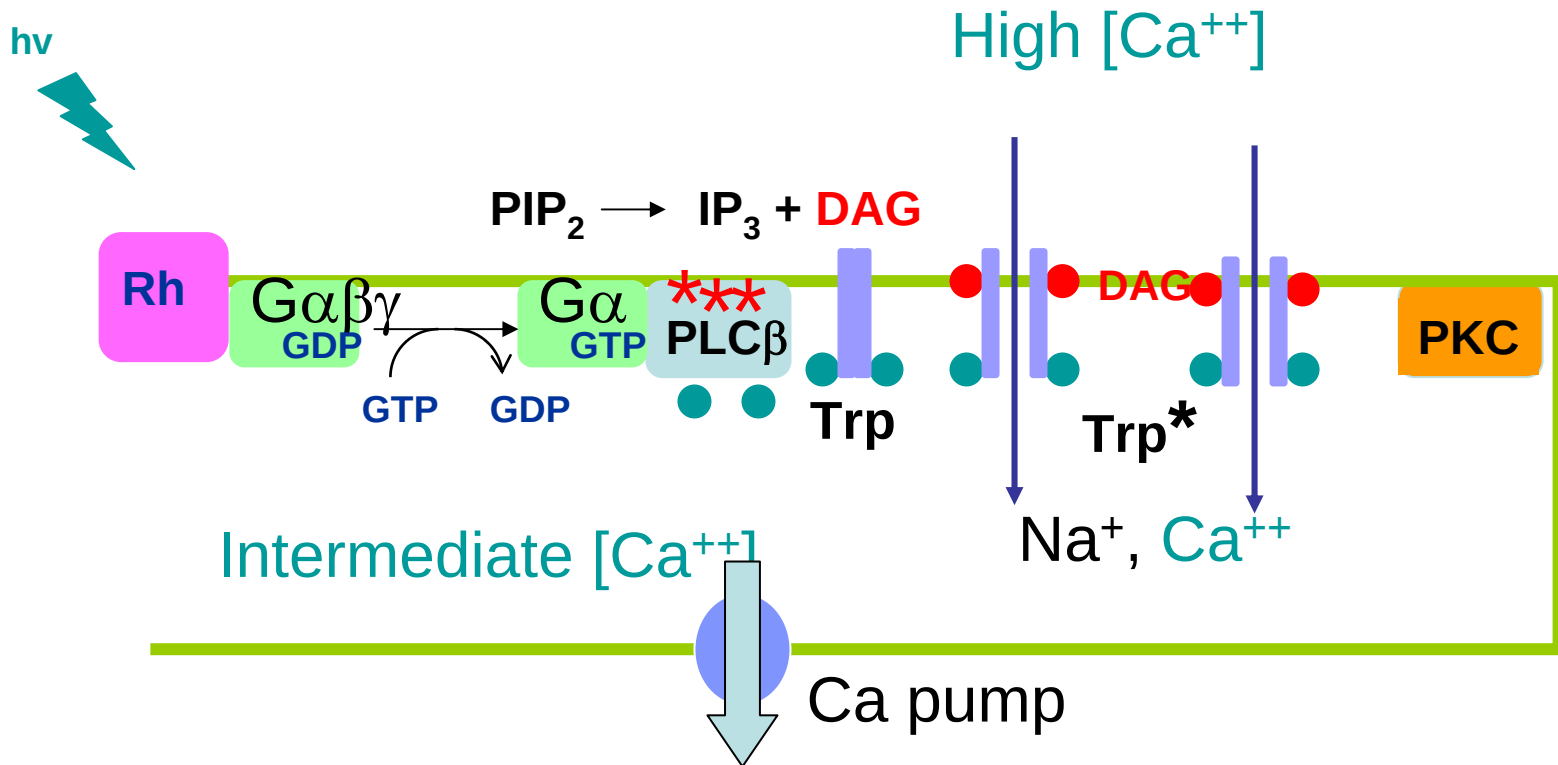
*Molecular mechanism of
fly phototransduction*

Response initiation



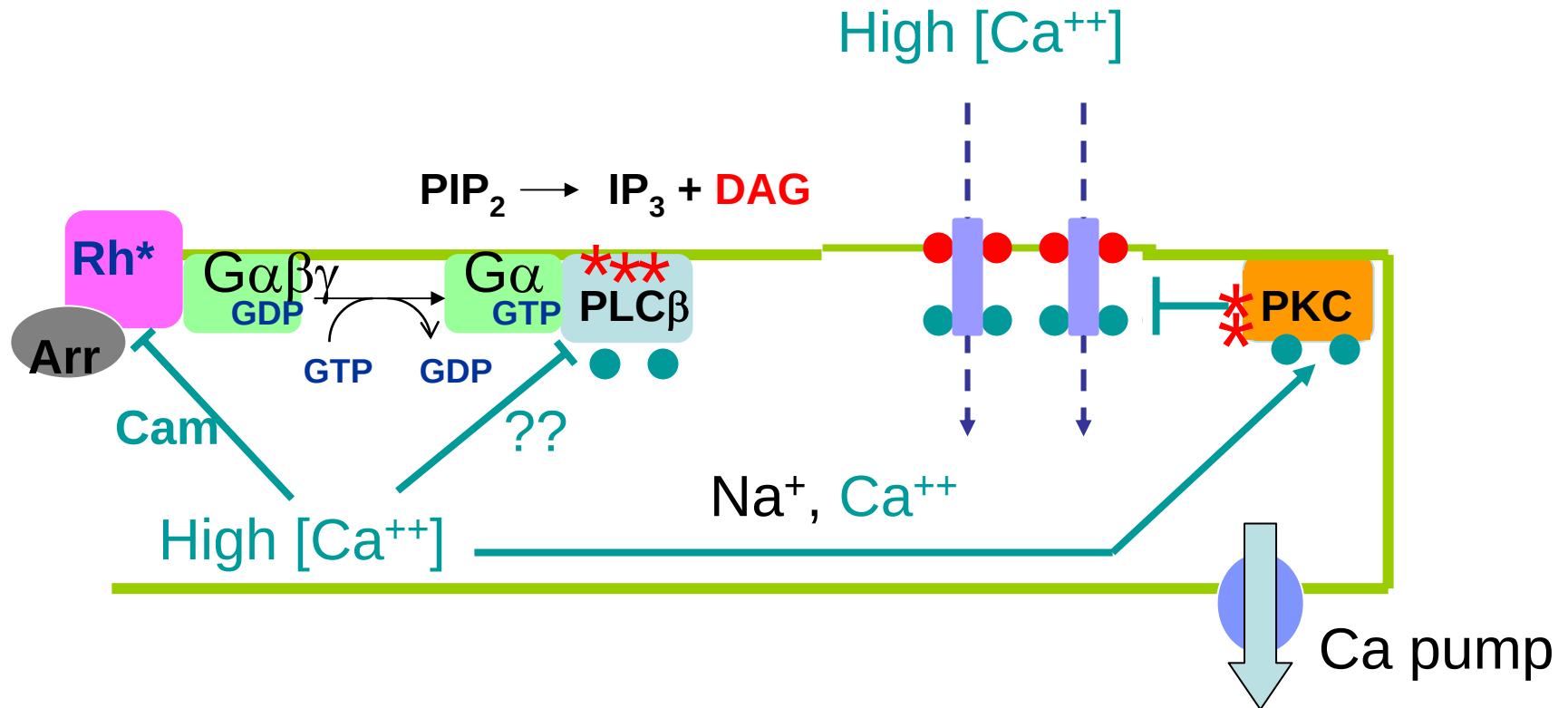
Cast: Rh = Rhodopsin; $G_{\alpha\beta\gamma}$ = G-protein
PIP₂ = phosphatidyl inositol-bi-phosphate
DAG = diacyl glycerol
PLCβ = Phospholipase C -beta ;
TRP = Transient Receptor Potential Channel

Positive Feedback



Intermediate $[Ca]$ facilitates opening of Trp channels and accelerates Ca influx.

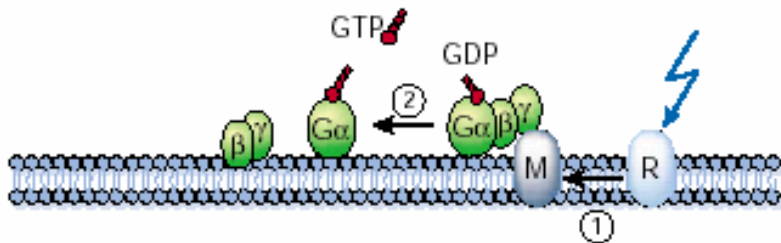
Negative feedback and inactivation



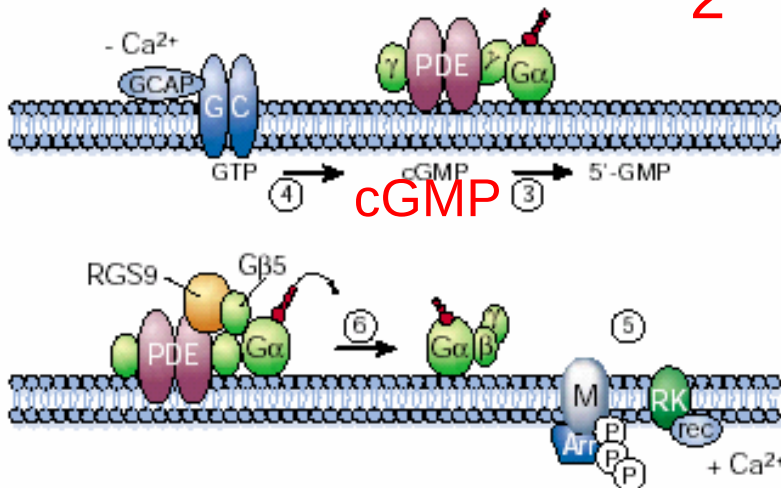
Cast: Ca^{++} acting directly and indirectly
e.g. via **PKC** = Protein Kinase C
and **Cam** = Calmodulin
Arr = Arrestin (inactivates **Rh***)

Comparison of early steps

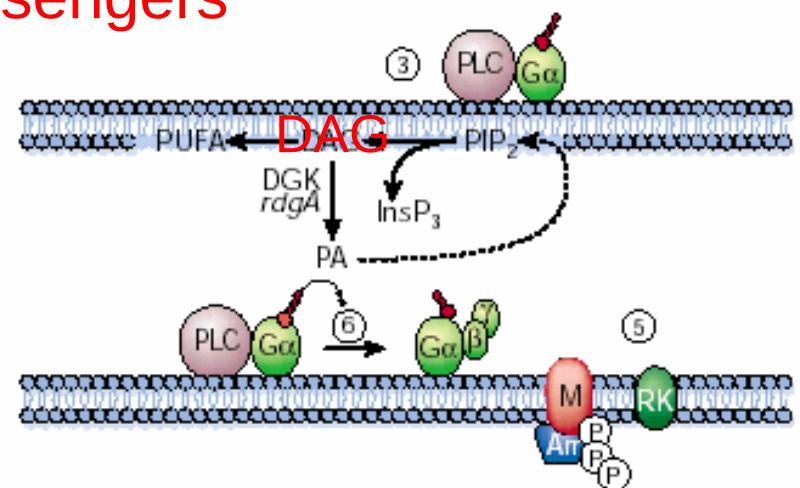
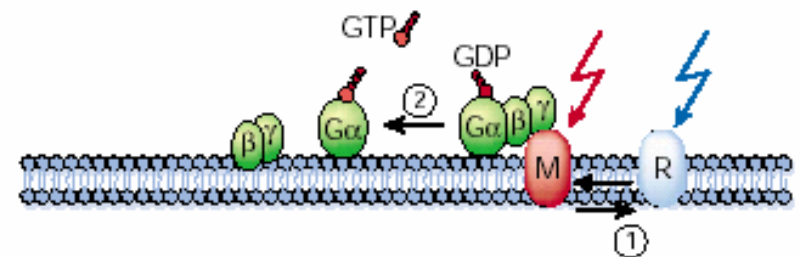
Vertebrate



2nd messengers

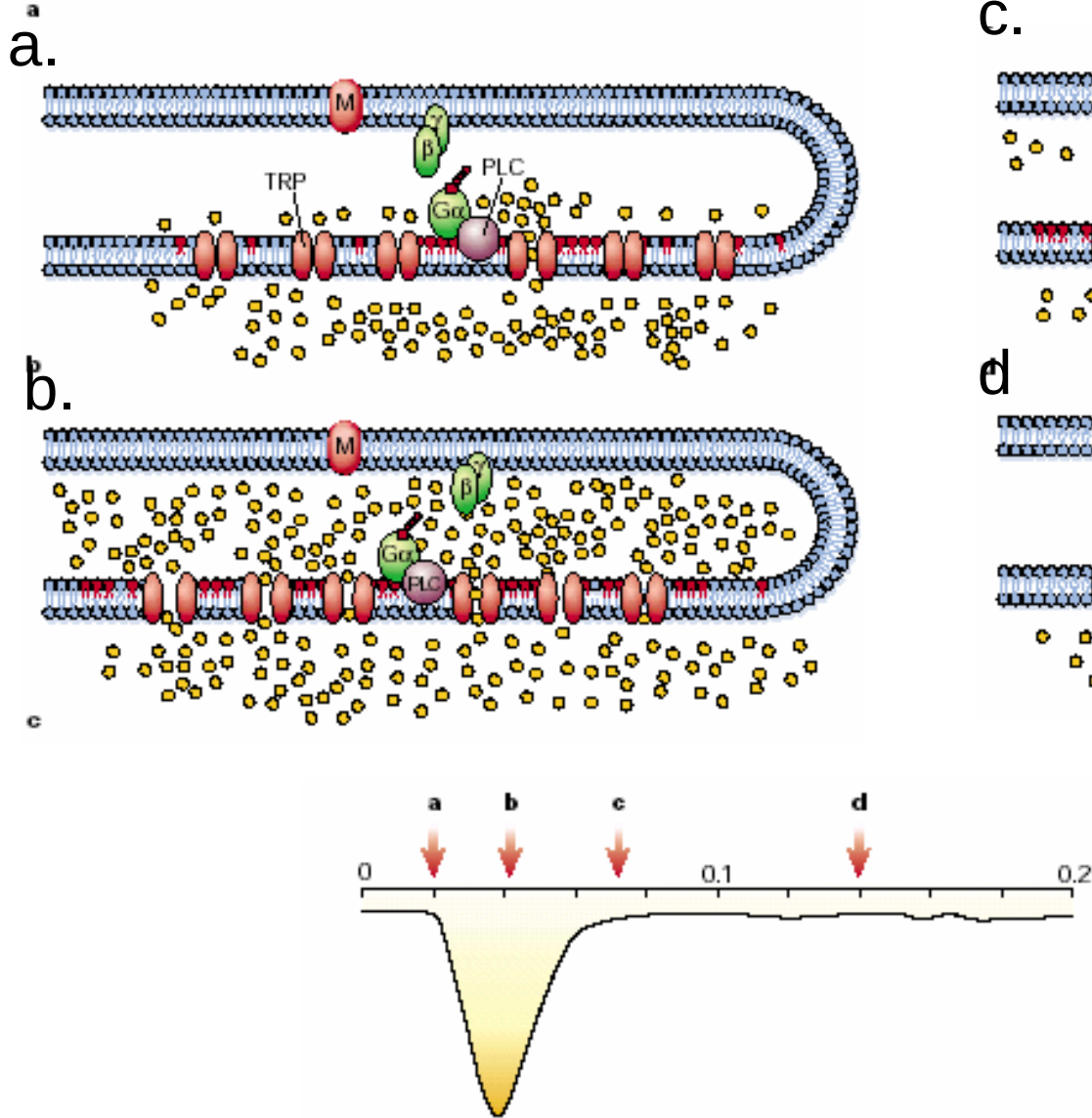


Drosophila



From Hardie and Raghu, Nature 413, (2001)

...and another cartoon

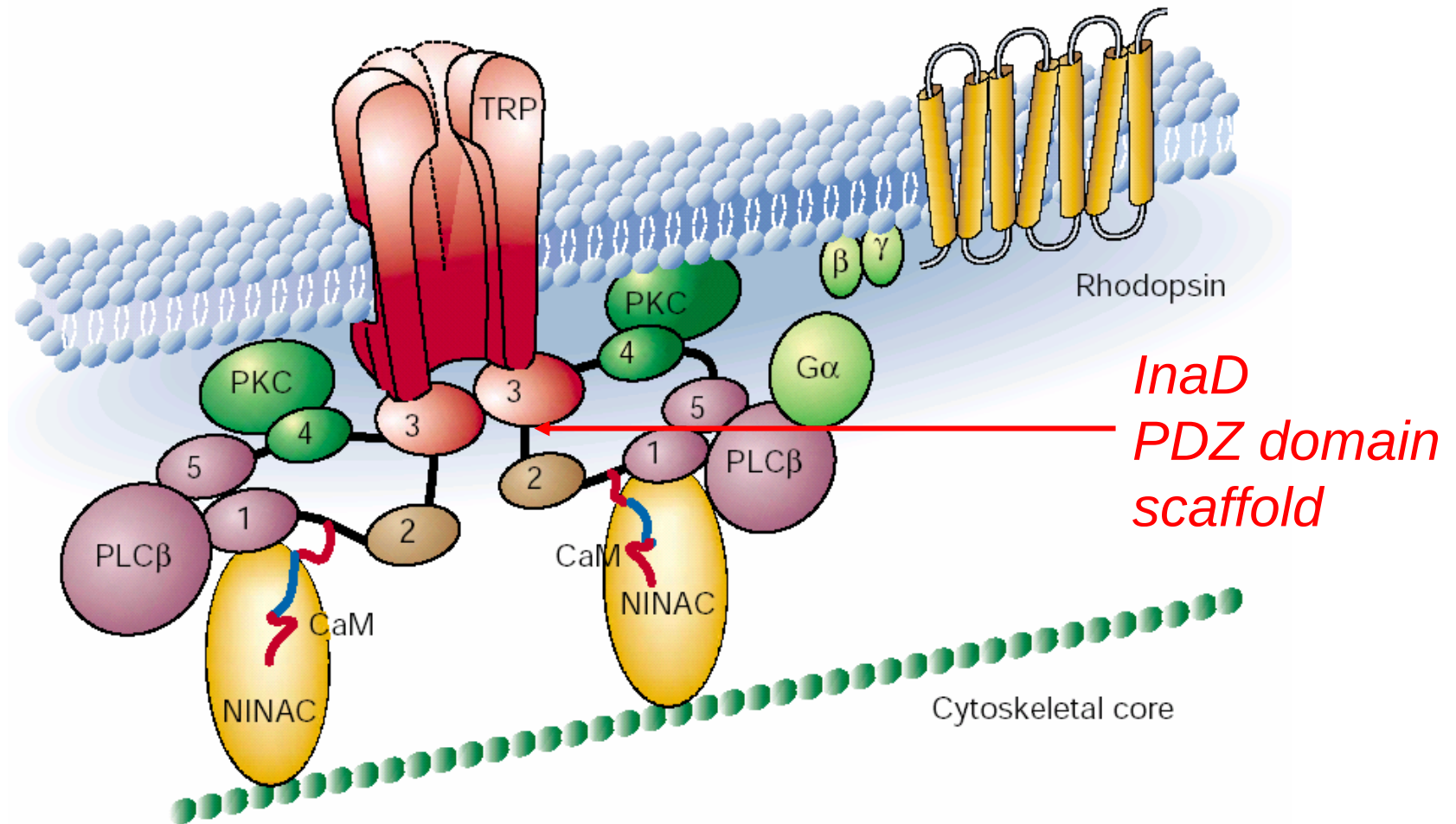


c.

d

From Hardie and Raghu, Nature 413, (2001)

InaD signaling complex



From Hardie and Raghu, Nature 413, (2001)

Speed and space: the issue of localization and confinement.

Order of magnitude estimate of activation rates:

$$\dot{G}^* \sim \dot{PLC}^* \sim k [G] \sim 10 \mu\text{m}^2/\text{s} \quad 100 / .3 \mu\text{m}^2 > 1 \text{ ms}^{-1}$$

Diffusion limit on reaction rate Protein (areal) density ! Fast Enough !

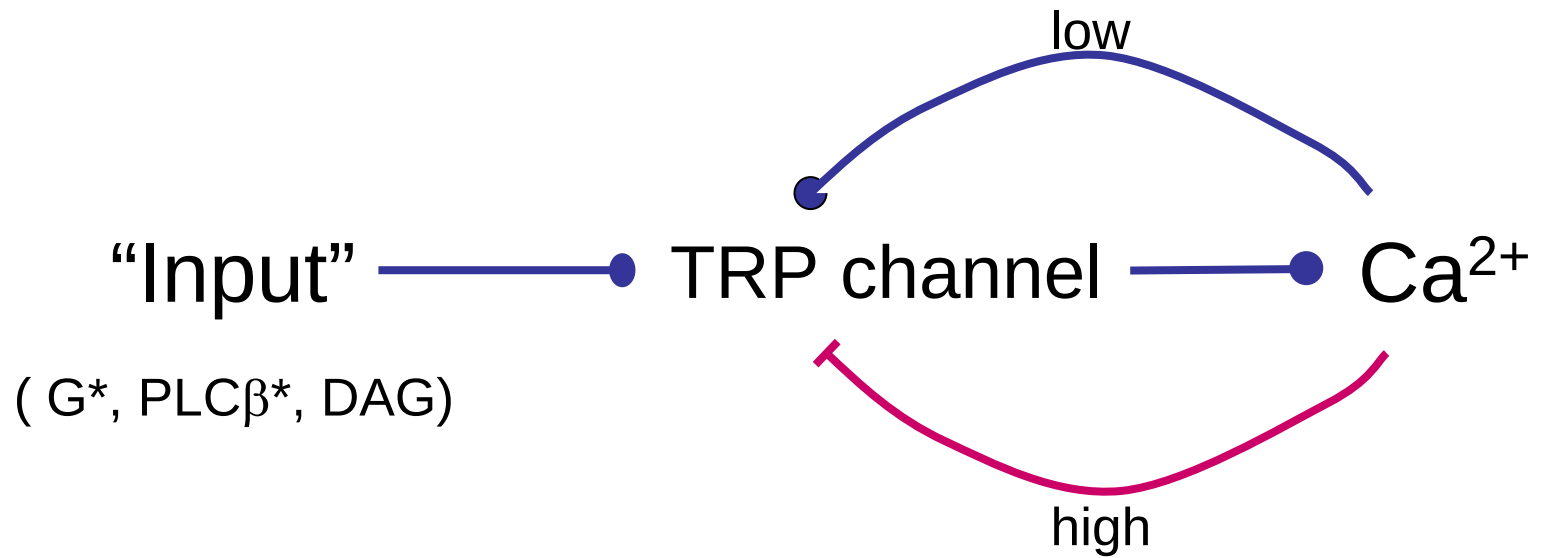
However if: $\sim 1 \mu\text{m}^2/\text{s} \quad 10 / .3 \mu\text{m}^2 = .03 \text{ ms}^{-1} \ll 1 \text{ ms}^{-1}$

! Too Slow !

Possible role for InaD scaffold !

*How "complex" should the model
of a complex network be?*

A naïve model



Kinetic equations:

Activation stage (G-protein; PLC β ; DAG):

$$\frac{d}{dt}A = -\tau_A^{-1}A + I \quad \text{Input (Rh activity)}$$

QB “generator” stage (Trp, Ca⁺⁺):

open channels

Positive and negative feedback

$$\frac{d}{dt}Trp^* = A^n F_+(Ca)Trp - \tau_{Trp^*}^{-1} F_-(Ca)Trp^*$$

$$\frac{d}{dt}[Ca] = \sigma Trp^* ([Ca_{ex}] - [Ca]) - \tau_{Ca}^{-1} ([Ca] - [Ca_0])$$

Ca⁺⁺ influx via Trp*

Ca⁺⁺ outflow/pump

Feedback Parameterization

$$F_{\alpha}(Ca) = 1 + g_{\alpha} \frac{([Ca] / K_{D\alpha})^{m_{\alpha}}}{1 + ([Ca] / K_{D\alpha})^{m_{\alpha}}}$$

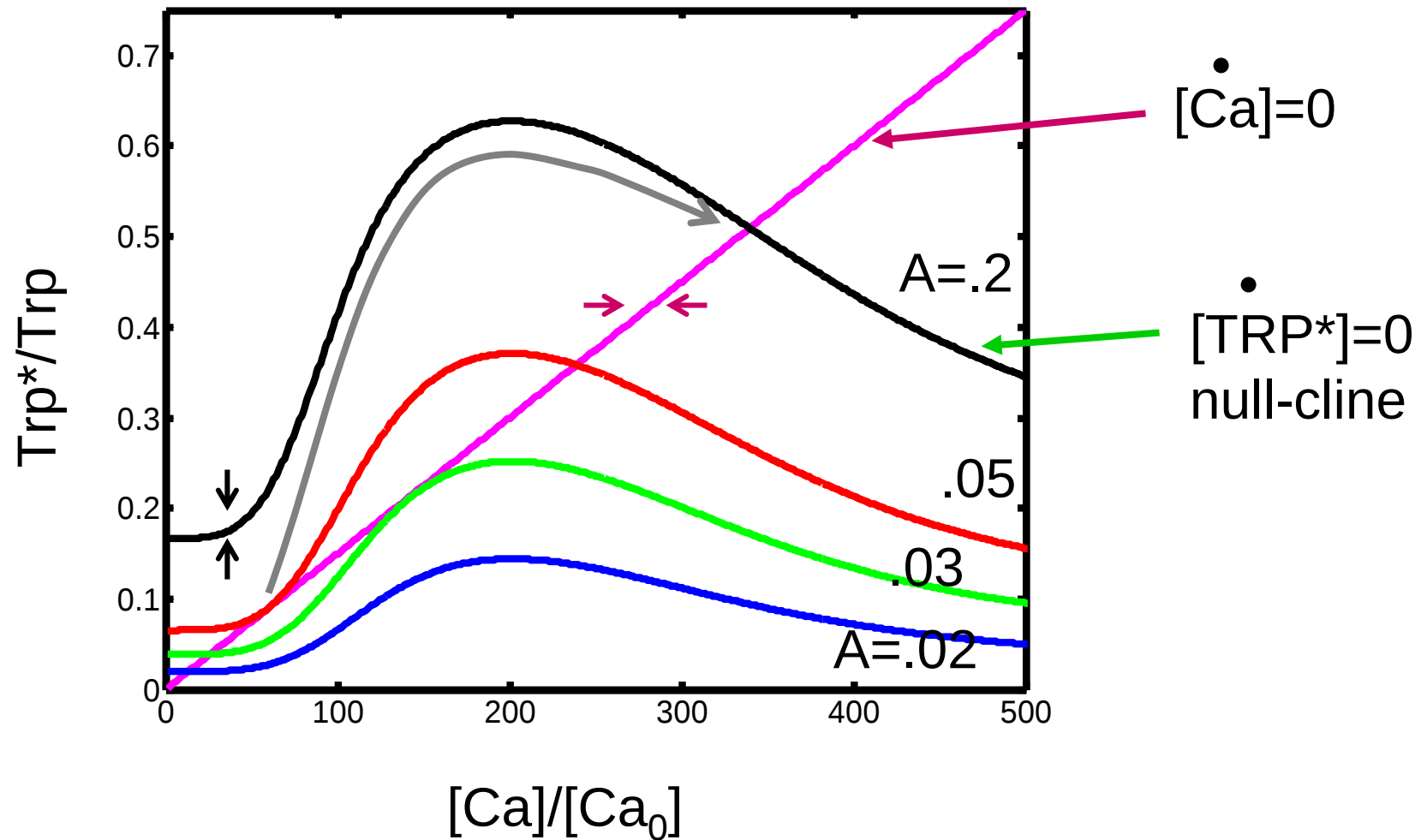
Parameterized by the “strength” g_{α} ($\sim$ ratio at high/low [Ca])

Characteristic concentration $K_{D\alpha}$

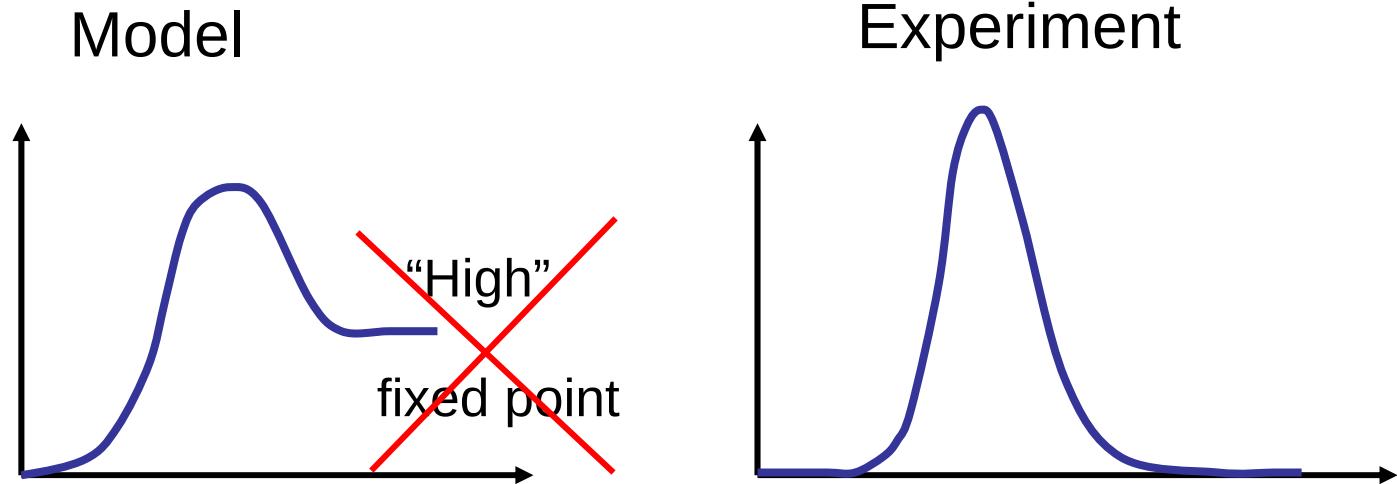
and Hill constant m_{α}

Note: this has assumed that feedback is instantaneous...

Null-clines and fixed points



Problems with the simple model



- In response to a *step* of Rh^* activity (e.g. in Arr mutant) QB current relaxes to zero
- Ca dynamics is fast rather than slow ➡ no “overshoot”
- Long latency is observed

Order of magnitude estimate of Ca fluxes

$[Ca]_{dark} \sim .2\mu\text{M}$ $\longleftrightarrow$ 1 Ca ion / microvillus

$[Ca]_{peak} \sim 200\mu\text{M}$ $\longleftrightarrow$ 1000 Ca ion / microvillus

Note: μ -villus volume $\sim 5 \cdot 10^{-12} \mu\text{l}$

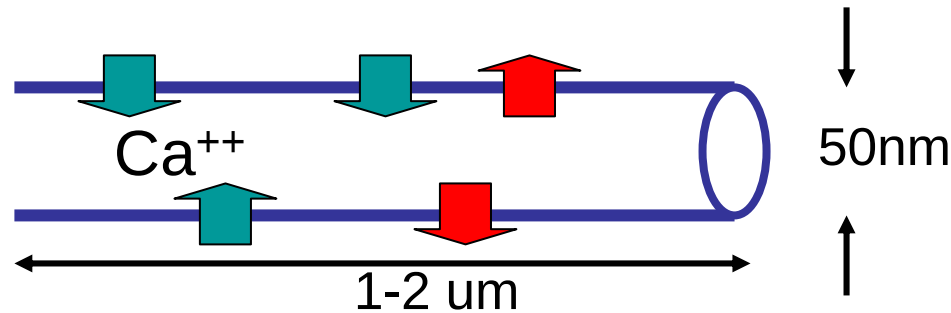
30% of 10pA $\longleftrightarrow$ Influx $10^4 \text{ Ca}^{2+} / \text{ms}$

Hence, Ca is being pumped out very fast $\sim 10 \text{ ms}^{-1}$



$[Ca]$ is in a quasi-equilibrium

Microvillus as a Ca compartment



Compare 10 ms^{-1} with diffusion rate across the microvillus:

$$\tau^{-1} \sim D_{\text{Ca}} / d^2 \sim 1 \mu\text{m}^2/\text{ms} / .0025 \mu\text{m}^2 = 400 \text{ ms}^{-1}$$

But diffusion along the microvillus:

$$\tau^{-1} \sim 1 \mu\text{m}^2/\text{ms} / 1 \mu\text{m}^2 = 1 \text{ ms}^{-1}$$

is too slow
compared to 10 ms^{-1}

Hence it is decoupled from the cell.

Note: microvillae could not be $> .3 \mu\text{m}$ in diameter,
i.e it is possible the diameter is set by diffusion limit

Slow negative feedback

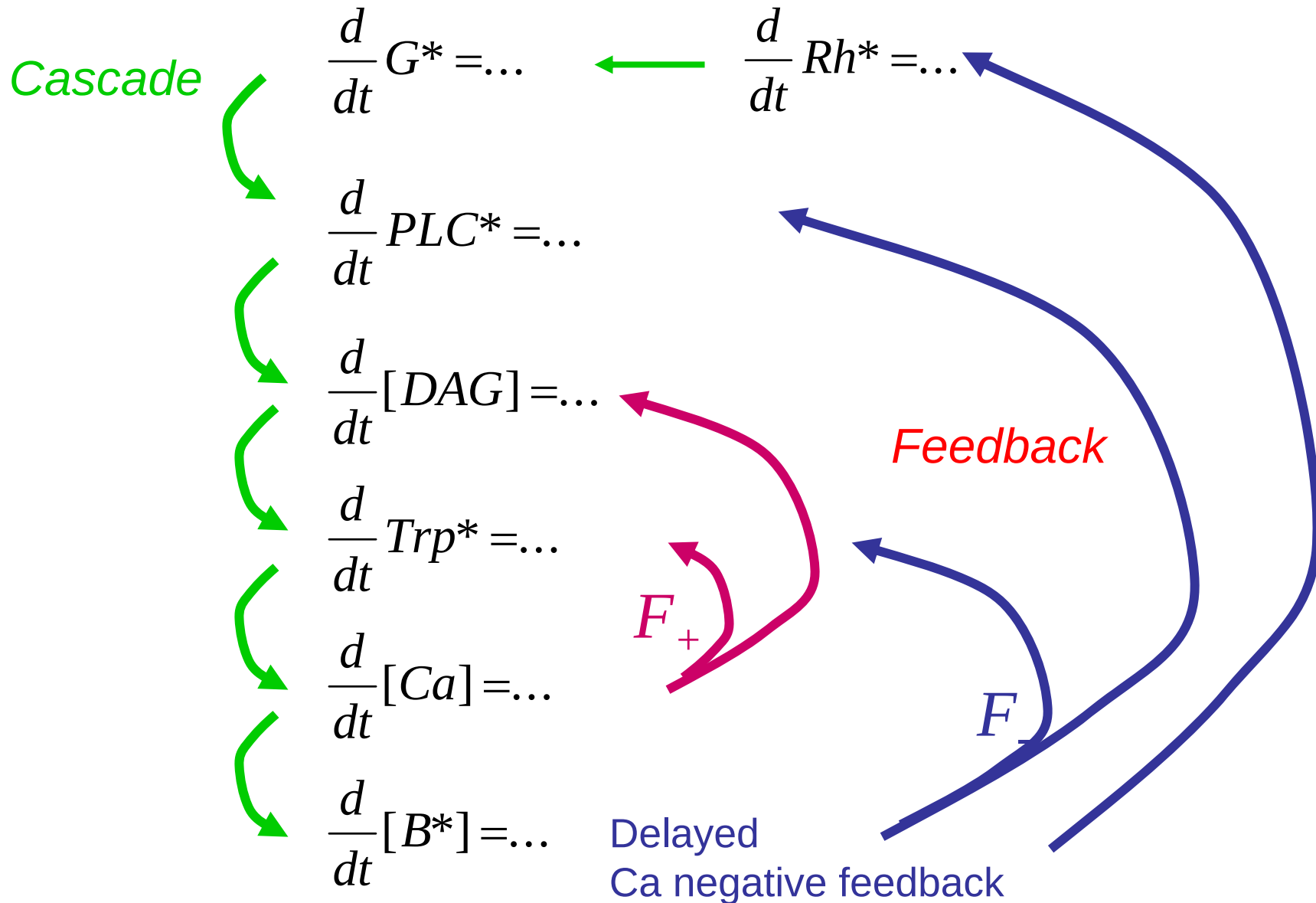
Assume negative feedback is mediated by a Ca-binding protein (e.g. Calmodulin??)

$$\frac{d}{dt} B^* = k_x [Ca] B - \tau_x^{-1} B^*$$

Slow relaxation

$$F_-(B^*) = 1 + g_- \frac{([B^*] / K_{D-})^{m_-}}{1 + ([B^*] / K_{D-})^{m_-}}$$

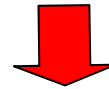
A more 'biochemically correct' model:



Stochastic effects

Numbers of active molecules are small !

e.g. 1 Rh*, 1-10 G* & PLC*, 10-20 Trp*

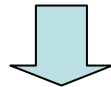


Chemical kinetics

Reaction “shot” noise.



Master equation



Numerical simulation

Gillespie, 1976, J. Comp. Phys. 22, 403-434

see also

Bort, Kalos and Lebowitz, 1975, J. Comp. Phys. 17, 10-18

Stochastic simulation

Event driven Monte-Carlo simulation
a.k.a. Gillespie algorithm

Gillespie, 1976, J. Comp. Phys. 22, 403-434

see also

Bort, Kalos and Lebowitz, 1975, J. Comp. Phys. 17, 10-18

Numbers of molecules (of each flavor) $\#X_a(t)$ are updated

$$\#X_a(t) \longrightarrow \#X_a(t) \pm 1 \text{ at times } t_{a,i}$$

distributed according to independent Poisson processes
with transition rates $\Gamma_{a,\pm}$. Simulation picks the
next “event” among all possible reactions.

Note: simulation becomes very slow if some of the
Reactions are much faster than others. Use a “hybrid” method.

The model is phenomenological...

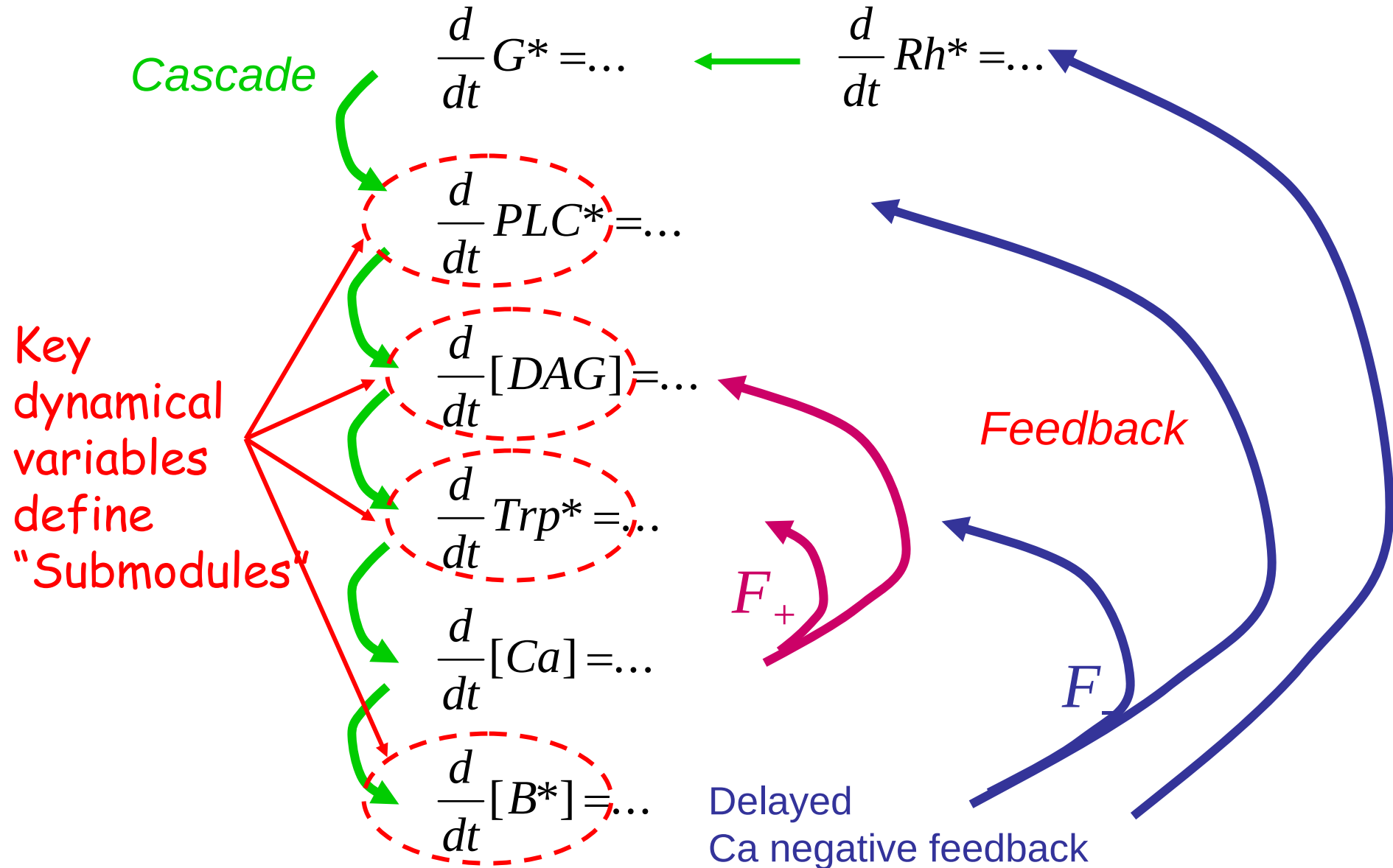
Many (most?) details are unknown:

e.g. Trp activation may not be directly by DAG,
but via its breakdown products;
Molecular details of Ca-dependent feedback(s)
are not known;
etc, etc

BUT

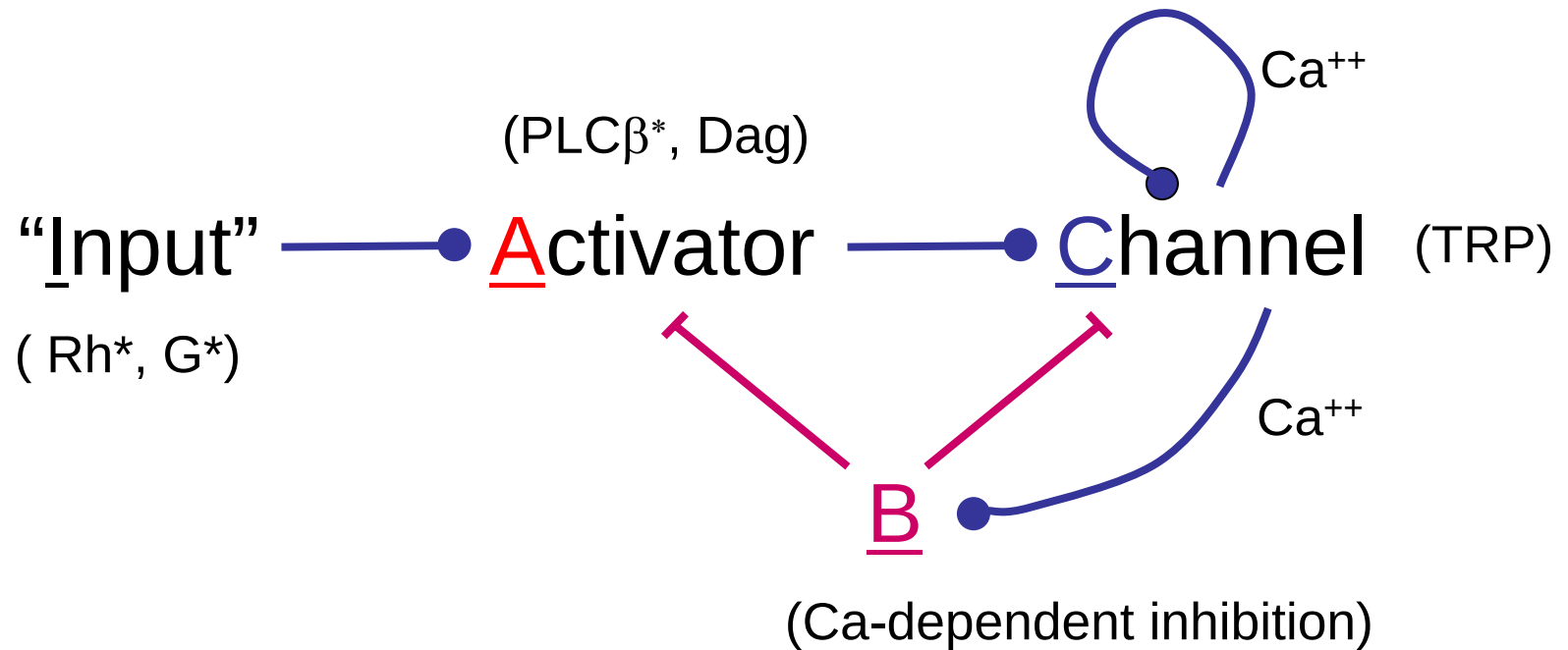
there's much to be explained on a qualitative and
quantitative level...

Identifying “submodules”

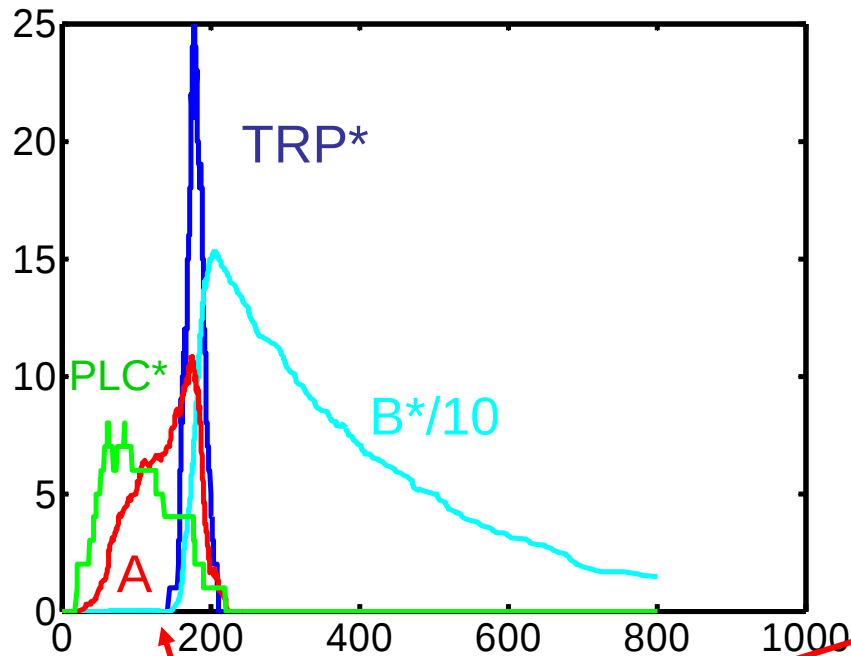


Rephrased in a “Modular” form:
the “ABC model”

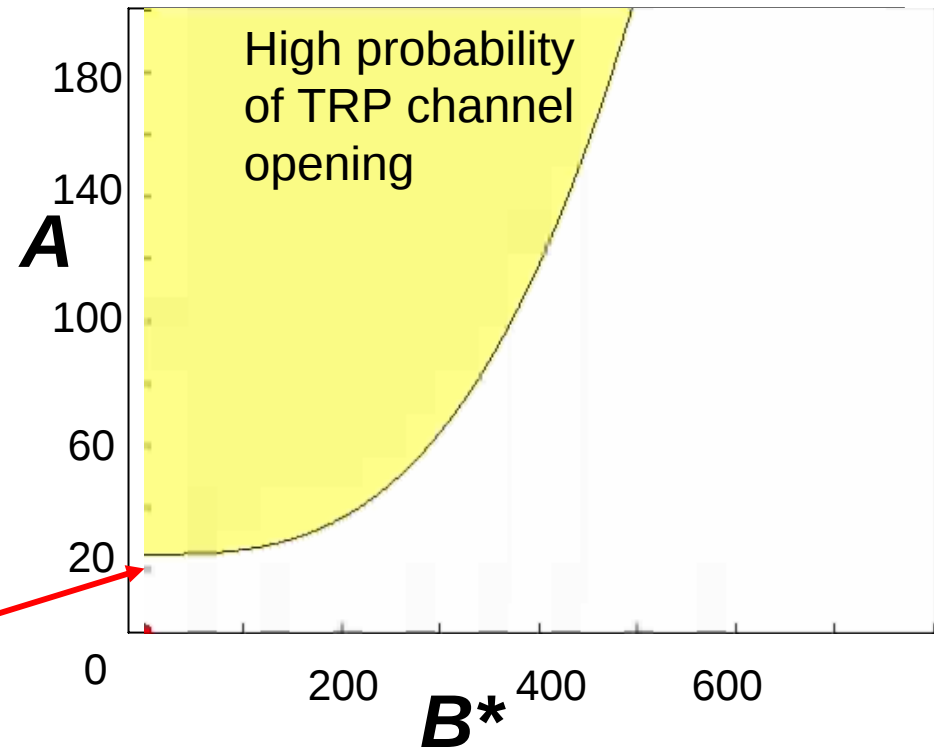
Activator – Buffer – Ca-channel



Quantum Bump generation



(A, B) - "phase" plane



Threshold for QB
generation

*"INTEGRATE & FIRE"
process*

What about null-cline analysis?

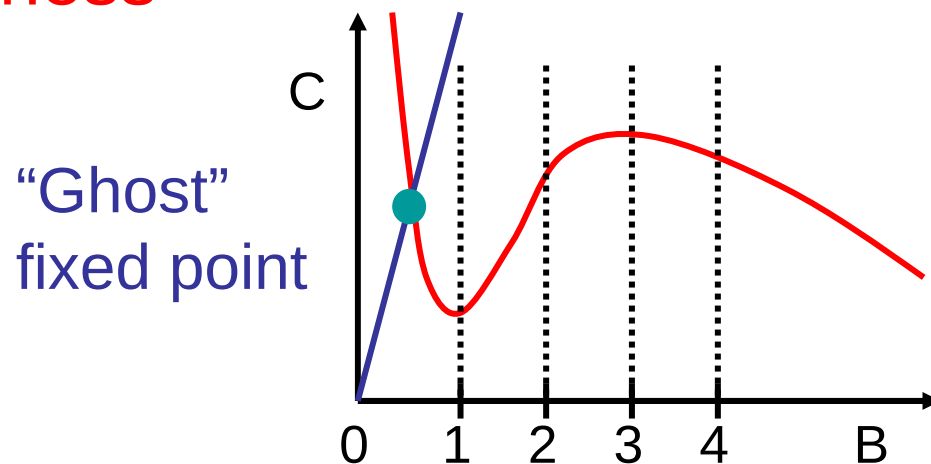
Problems:

- 3 variables A,B,C
- Stochasticity

Generalized "Stochastic Null-cline"

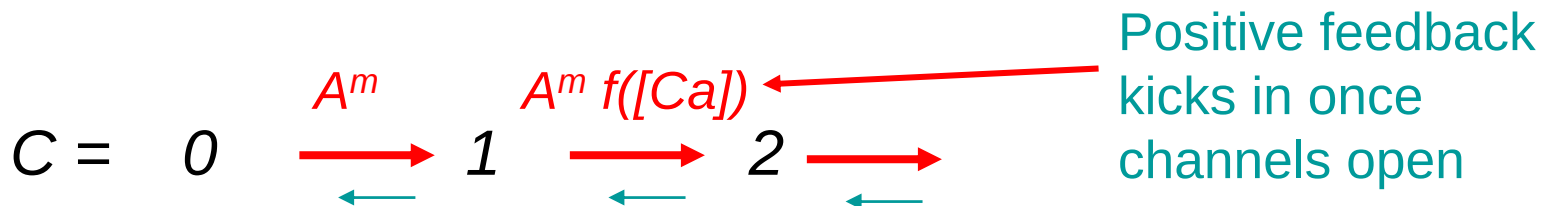
$$\frac{dX}{dt} = 0 \rightarrow \text{Prob}(x \rightarrow x+1) = \text{Prob}(x \rightarrow x-1)$$

- Discreteness



Can one calculate anything?

E.g. estimate the threshold for QB generation:



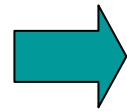
Threshold $A = A_T$ such that

$$\text{Prob}(A_T \rightarrow A_T + 1) = \text{Prob}(C=0 \rightarrow C=1)$$

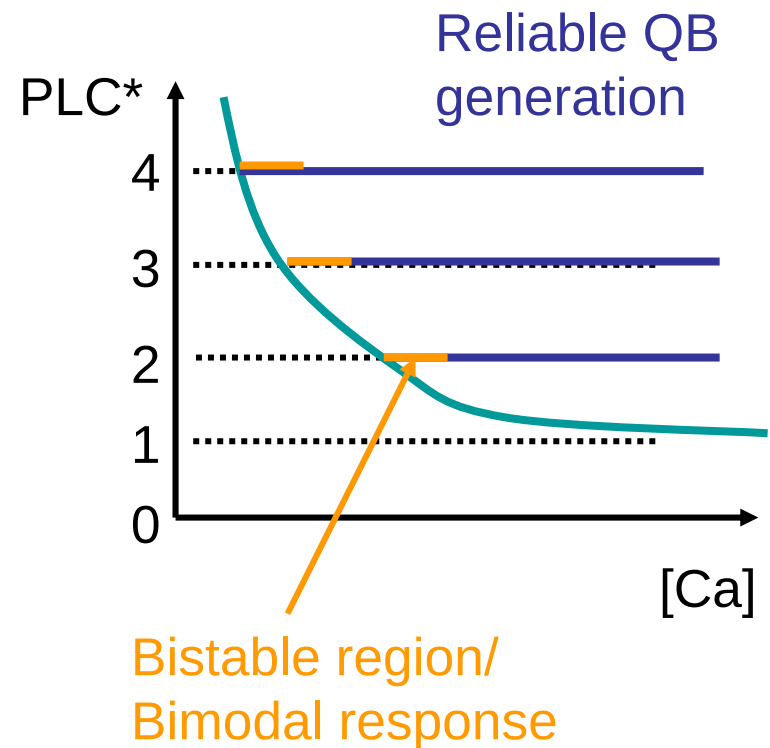
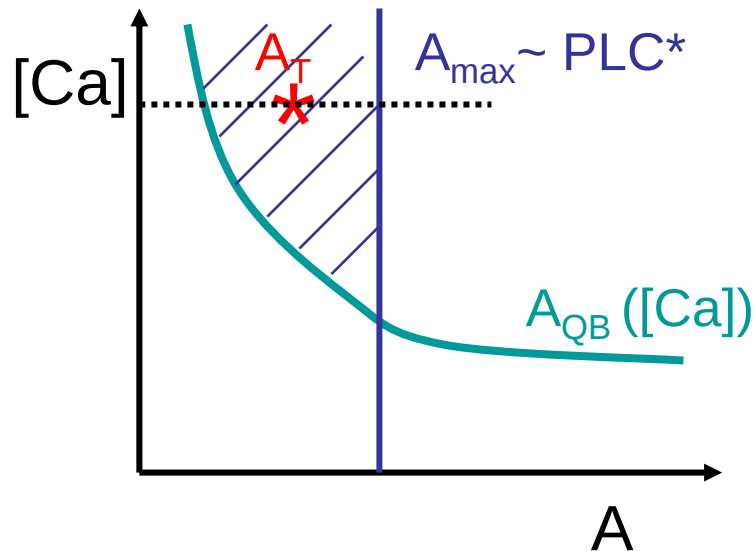
NOTE: Better still to formulate as a "first passage" problem

Condition for QB generation

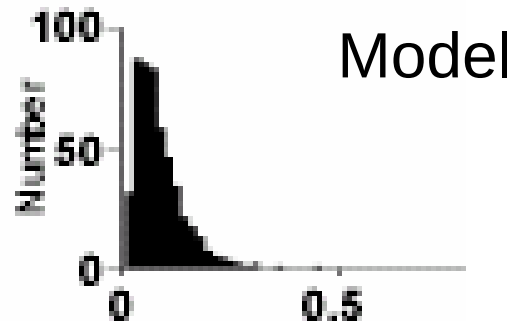
$$\text{Prob} (C=1 \rightarrow C=2) > \text{Prob} (C=1 \rightarrow C=0)$$



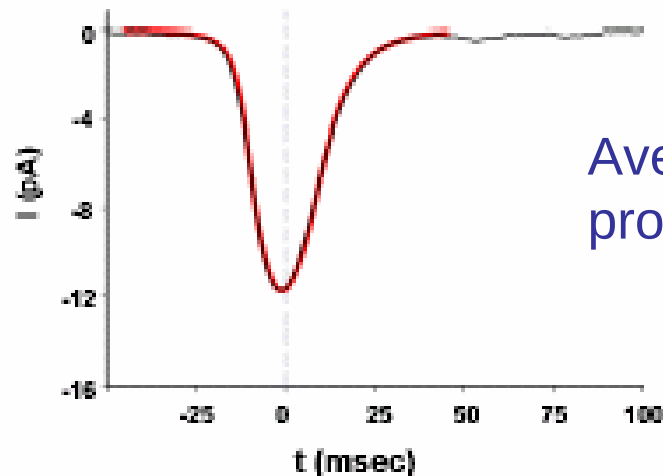
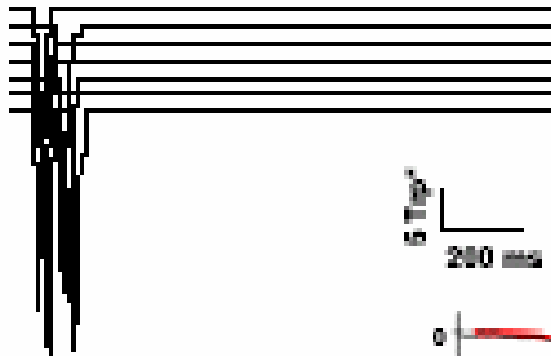
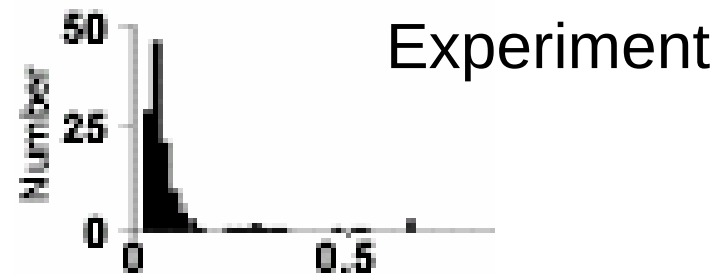
$$A_T > A_{QB} ([Ca])$$



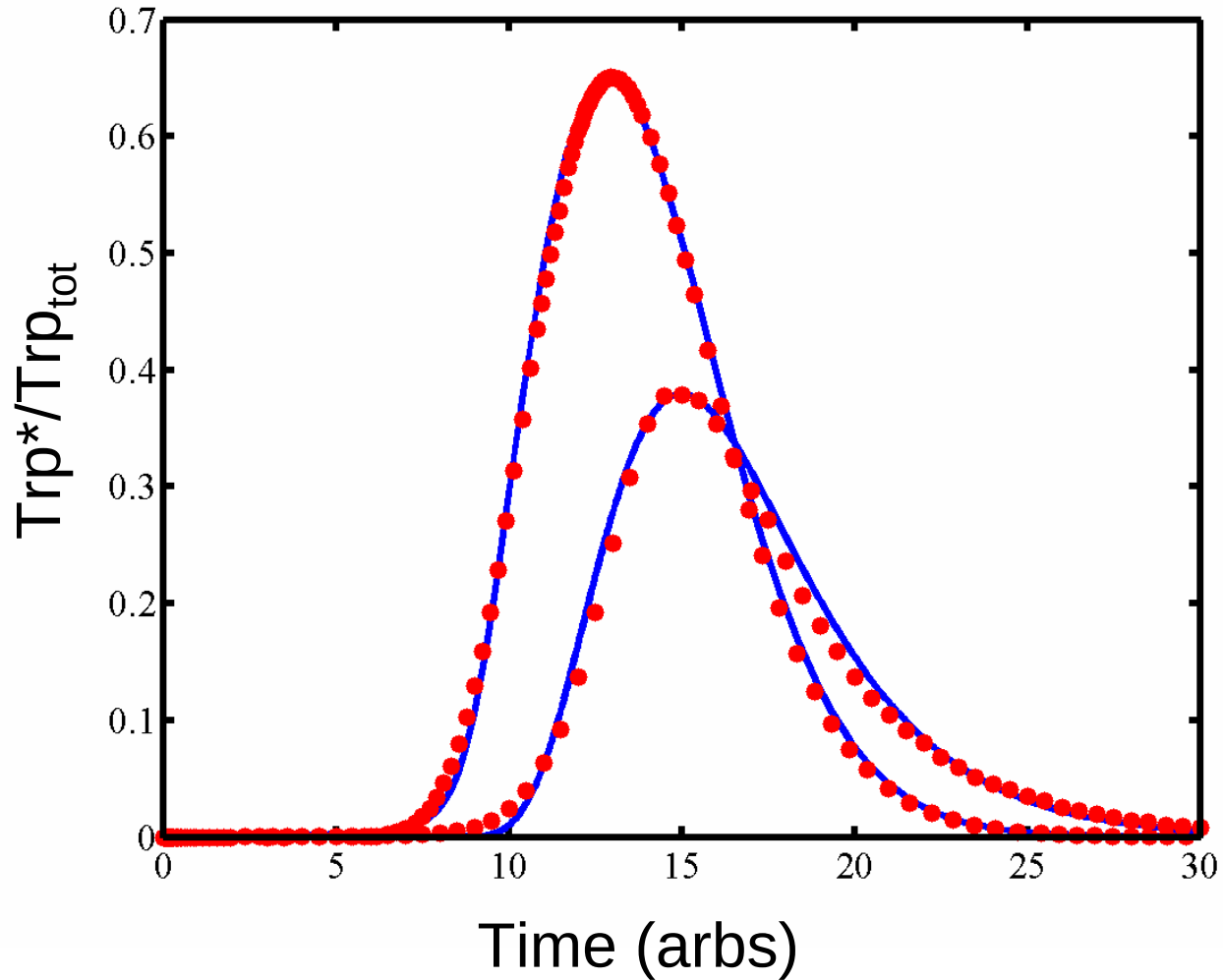
Quantum Bump theory versus reality



Latency
histogram



Fitting the data: QB wave-form

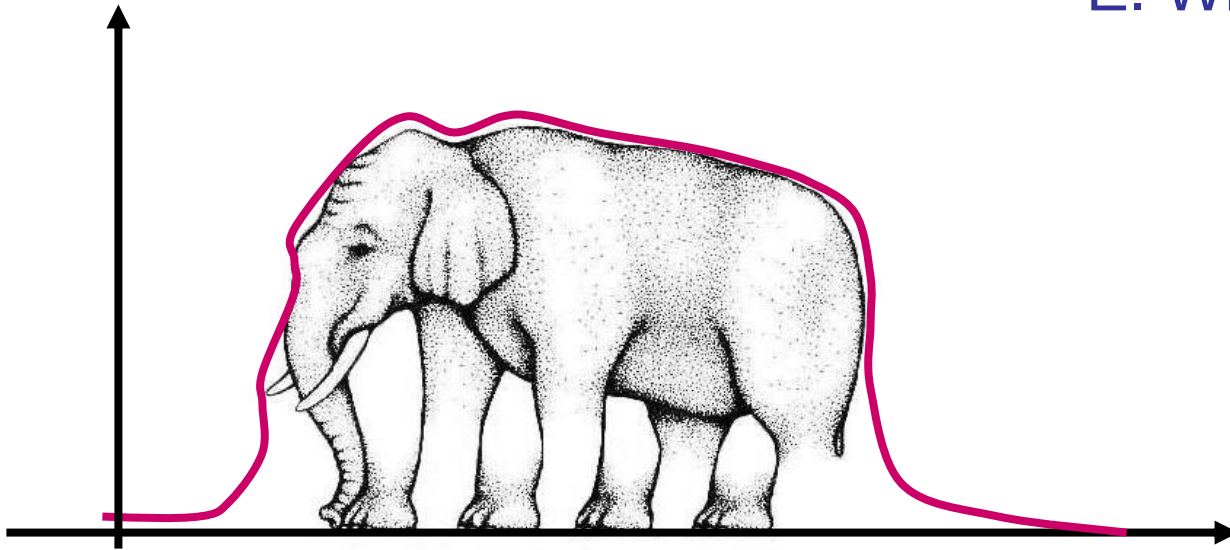


There is a manifold
of parameter values
providing good fit
for $\langle \text{QB} \rangle$ shape !!

So what ???

“With 4 parameters I can
fit an elephant and with
5 it will wiggle its trunk.”

E. Wigner



Non-trivial “architectural” constraints

Despite multiplicity of fits, certain constraints emerge:

- Trp activation must be cooperative
 - Activator intermediate must be relatively stable:
“integrate and fire” regime.
 - Negative feedback must be delayed
 - Multiple feedback loops are needed
- Etc, ...

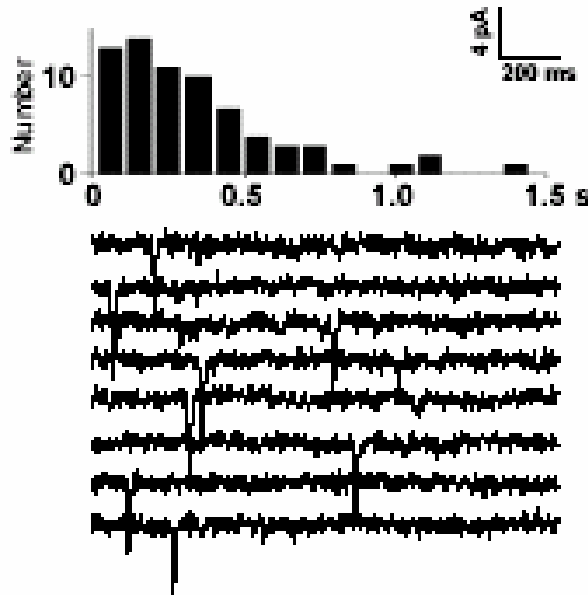
Furthermore:

Fitting  certain relation between parameters:
“phenotypic manifold”
- the manifold in parameter space
corresponding to the same quantitative
phenotype.

*Many more features to explain
quantitatively!*

Constraining the parameter regime...

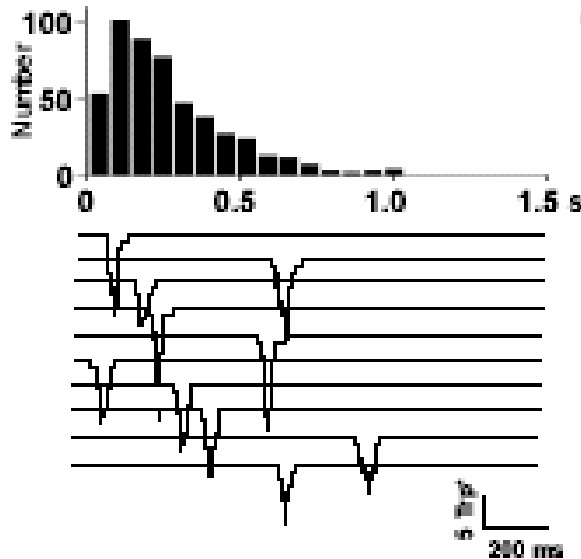
Help from the data on *G*-protein hypomorph flies:



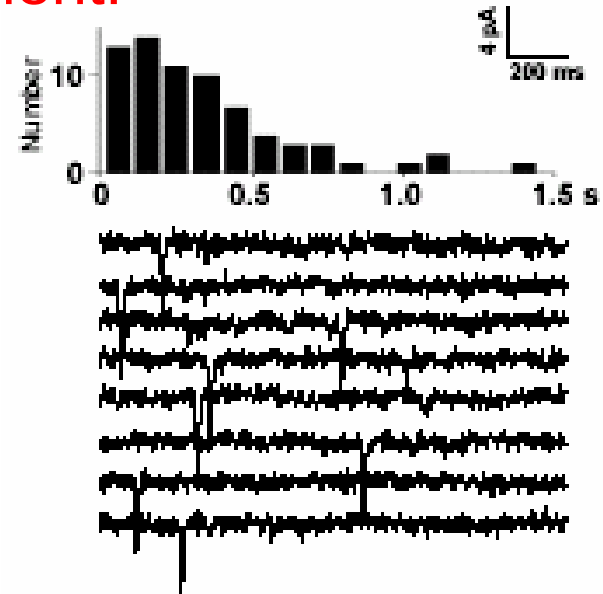
- # of G-proteins reduced by ~ 100
- QB “yield” down by factor of 10^3
- Increased latency (5-fold)
- Fully non-linear QB with amplitude reduced about two-fold

G-protein hypomorph

Model:



Experiment:



- Single G^* and PLC^* can evoke a QB !!
- Reduced yield explained by PLC^* deactivating before A reaches the QB threshold
- Relation between yield reduction and increased latency. # PLC^* ~ 5 for WT

*What happens in response to
continuous activation ?*

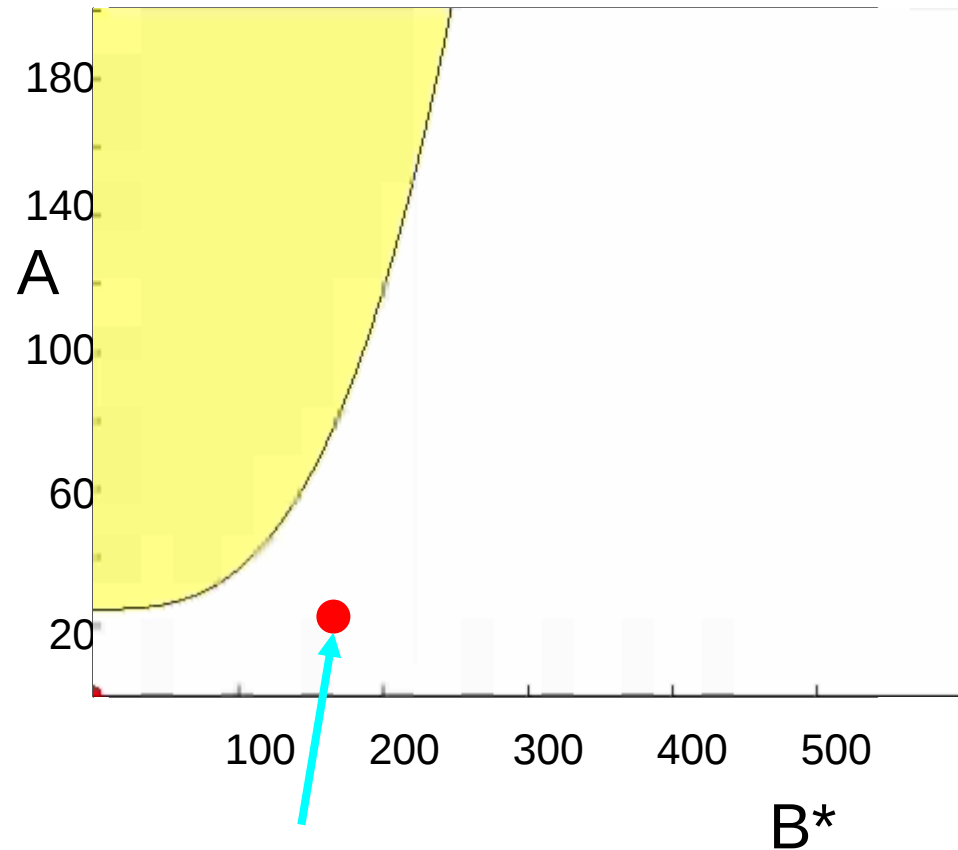
e.g. if Rh^* fails to deactivate

Persistent Rh^ activity*

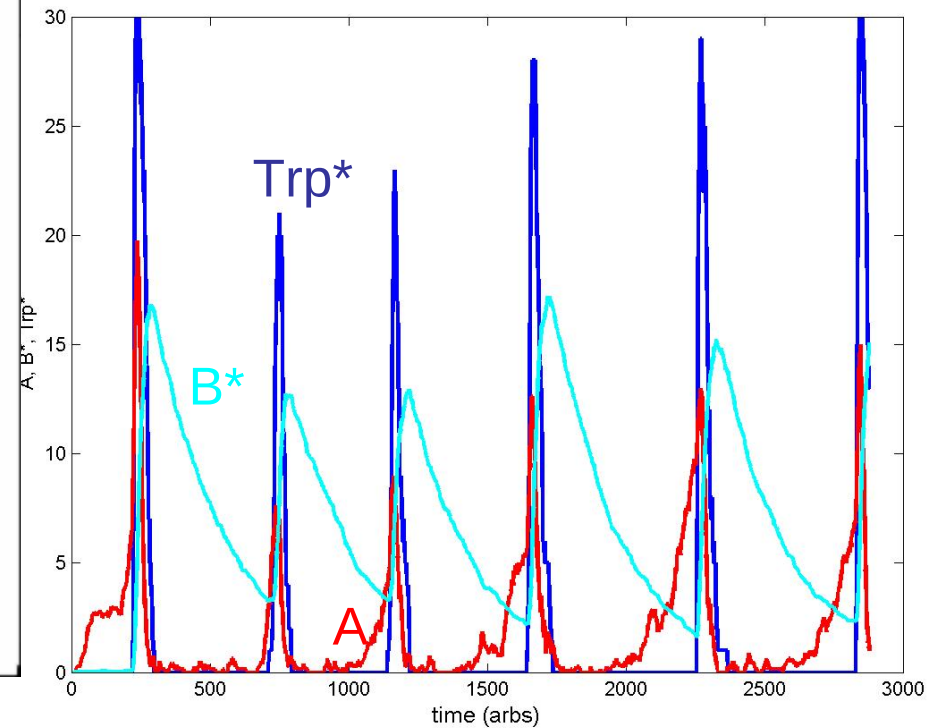


Relaxation Oscillator

(A,B*) phase plane

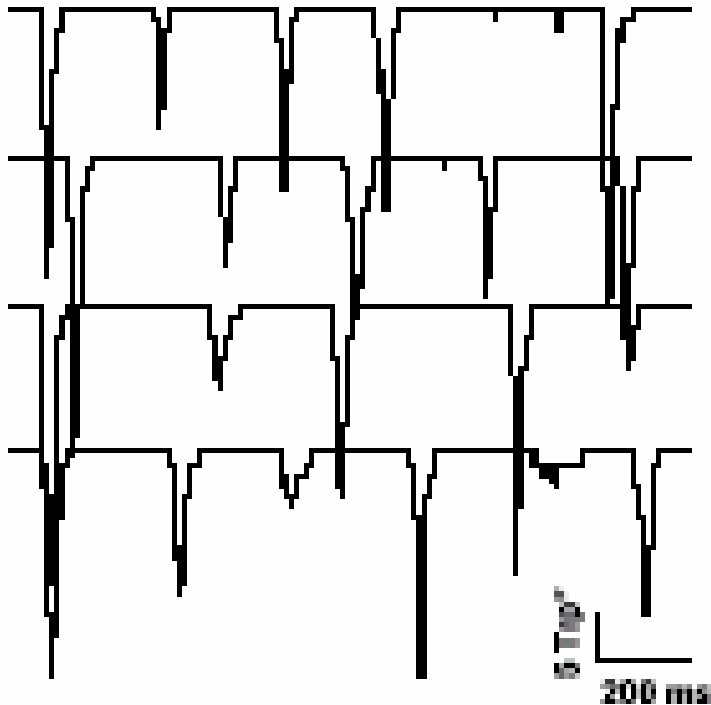


Unstable Fixed Point

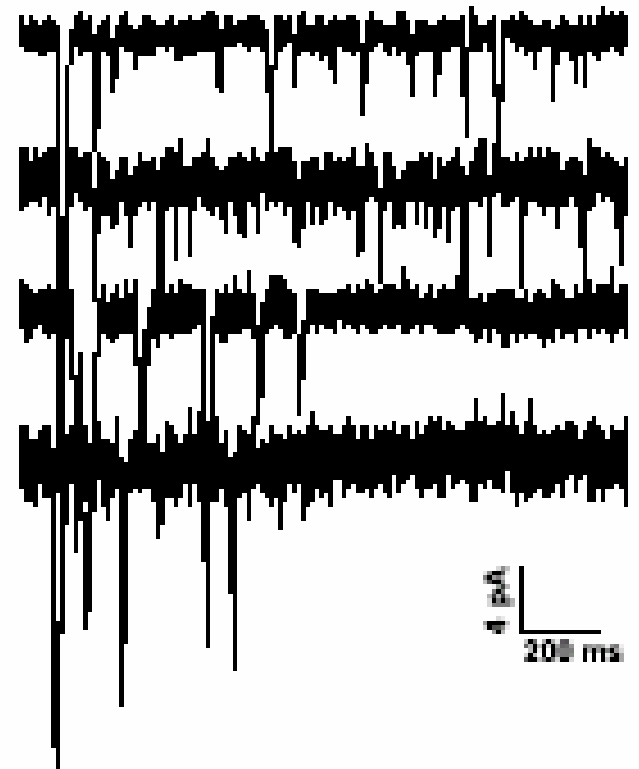


QB trains: theory versus experiment

Model:

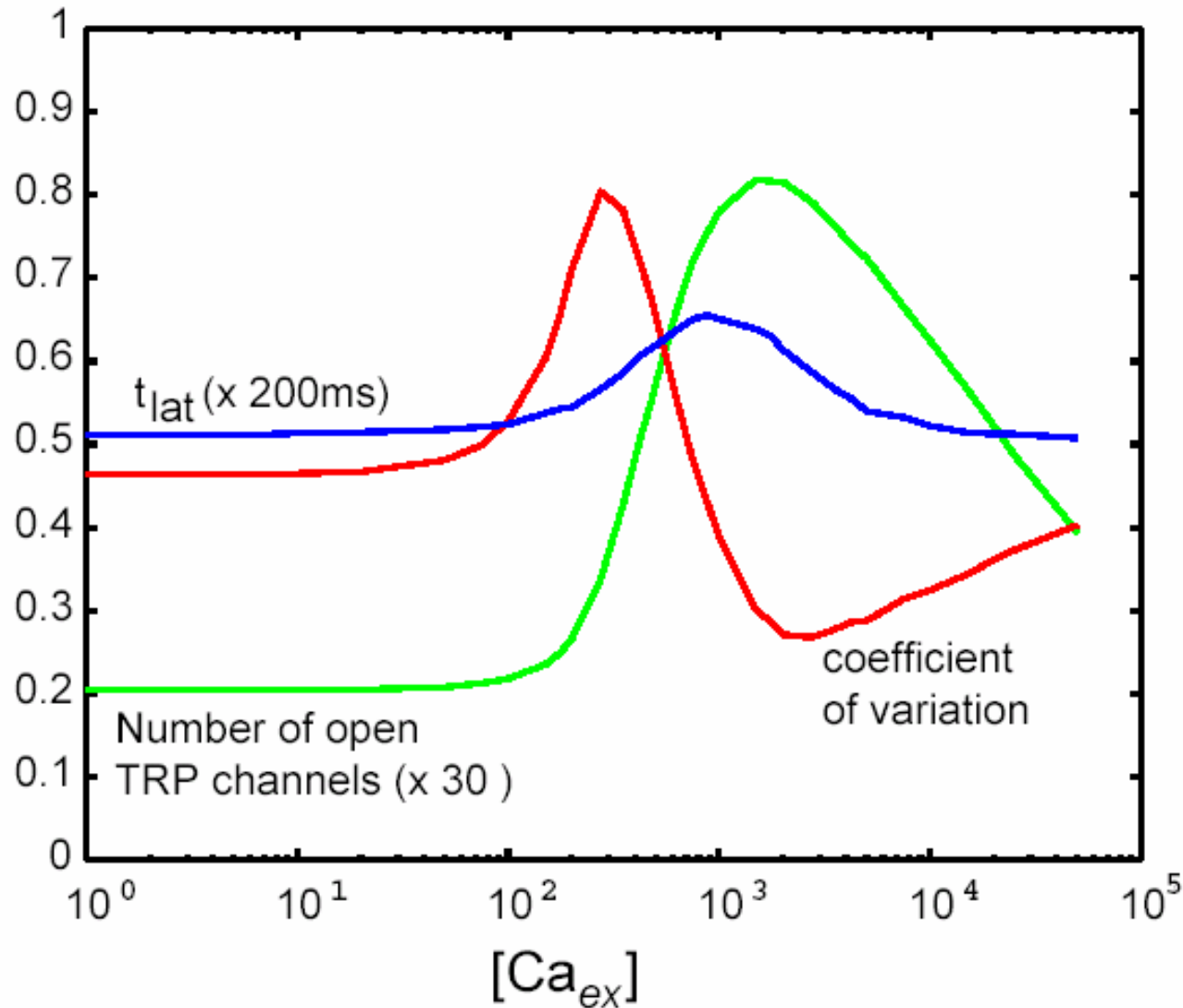


Arrestin mutant
(deficient in Rh* inactivation):

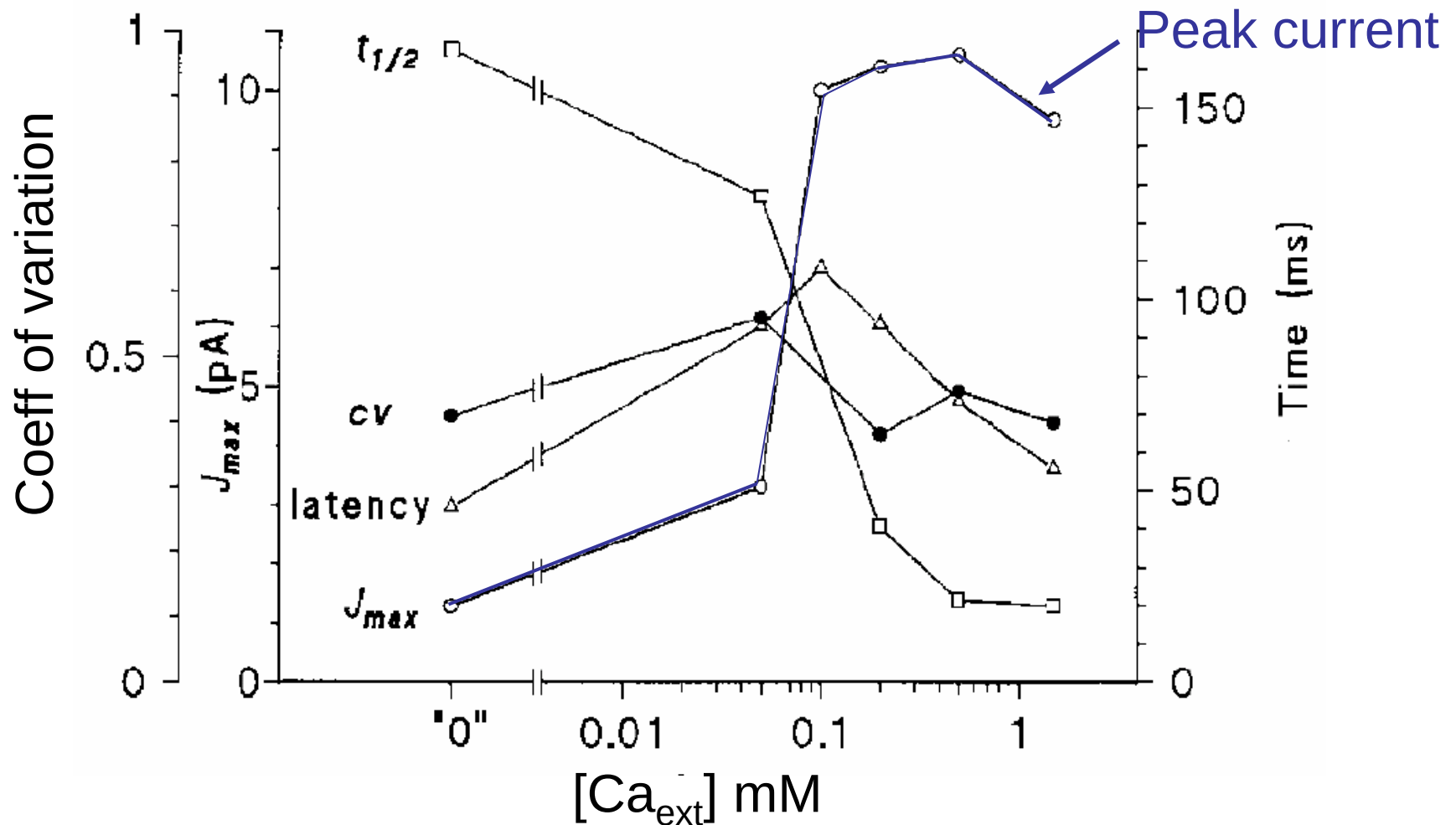


Qualitative but not quantitative agreement so far...

Predicted $[Ca_{ex}]$ dependence



Observed external $[Ca^{2+}]$ dependence



What does one learn from the model?

e.g. Mechanisms/parameters controlling:

Threshold for QB generation.

QB amplitude fluctuations.

Latency.

Yield (or response failure rate)

Latency distribution.

Functional dependences:

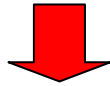
e.g. dependence of everything on $[Ca^{++}]_{ext}$

Modeling methodology questions

- Need an intelligent method of searching the parameter space and of characterizing the parameter manifold ??? How does Evolution search the parameter space?
- Characterizing the “space of models”??
- “Convergence proof”??
Given a model that fits N measurements can we expect that it will fit $N+1$ (even with additional parameters)?
- How accurate should a prediction be for us to believe that the model is correct ?? Unique??

Summary and Conclusion

A phenomenological model can explain observations and make numerous falsifiable predictions (especially for the functional dependence on parameters).



Insight into **HOW** the system works from understanding the most relevant parameters and processes.

?????

Can one get any insight into **WHY** the system is constructed the way it is (e.g. vertebrate versus insect)

?????

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