Modeling signal transduction

Phototransduction from frogs to flies

Eye as an evolutionary challenge

"To suppose that the eye, with all its inimitable contrivances ..., could have been formed by natural selection, seems, I freely confess, absurd in the highest possible degree. Yet reason tells me, ... "

C. Darwin, The Origin of Species:

Darwin goes on to describe how eye may have evolved through accumulation of gradual improvements

A number of different designs exist: e.g. vertebrate, molluskan or jellyfish "camera" eyes or insect "compound eye"

The eye may have evolved independently 20 times!! (read "Cells, Embryos and Evolution" by Gerhart&Kirschner)

Eyes are present in most metazoan phyla: how did they evolve?



See Gerhart and Kirschener, "Cells, Embryos and Evolution"

The grand challenge:

To understand the evolutionary processes that underlie the appearance of a complex organ (an eye).

To what extent is the similarity between different eyes (e.g. vertebrate and mollusk) is due to common ancestry or is the result of evolutionary convergence due to physical constraints?

What can we learn from the similarities and differences in the architecture/anatomy, development and <u>molecular machinery</u> of eyes (or light sensitive cells) in different species?

BUT before we can address the question of evolution we need first to learn how it works...

More broadly:

Molecular pathway(s) of phototransduction are similar to many other signaling pathways: olfaction, taste, etc

"Comparative Systems Biology"

Modeling molecular mechanisms of photo-transduction

Case studies: 1) vertebrate and 2) insect





Phototransduction as a model signaling system

Vertebrate Rods and Cones



terminal

Vertebrate photoreceptor cell



Intracellular Voltage in Rods and Cones





Measuring currents





Patch clamp





Fig. 2. Response of outer segment to a series of forty consecutive dim flashes. Local illumination; 20 msec flash delivering 0.029 photons μm^{-2} at 500 nm, flash timing monitored below; saturating response 12 pA. Same cell as Fig. 1. Low pass filtered at 30 Hz.



In the dark a standing current circulates through the rod.

Electrophysiology of Rods



Na,Ca channels of outer segment need cGMP to stay open

Light detector protein: Rhodopsin



Rhodopsin undergoes light-induced conformational change

chromophore, 11-cis retinal

opsin, a membrane protein with 7 transmembrane segments

cis/trans -retinal transition



Light-induced conformational transition





Rhodopsin is a G-Protein coupled receptor





Rhodopsin is a G-Protein coupled receptor



G-proteins



<u>α subunit</u>: 35-45 kD <u>β subunit</u>: 35-40 kD <u>γ subunit</u>: 6-12 kD

At least 15 different genes for α subunit and several different genes for β and γ subunits



GDP -- GTP exchange at nucleotide binding site activates the G protein



2.



Where did cGMP come from?





Ion channels: e.g. K, Ca and Na channels

Shut-off



Rhodopsin inactivation



Transducin is inactivated by the α intrinsic GTPase activity which hydrolyzes GTP to GDP



The intrinsic GTPase activity of the α subunit is regulated by a GAP (GTPase accelerating protein) RGS-9



Recovery to the resting state requires resynthesis of the cGMP that had been lost to hydrolysis.

This is accelerated by low Ca (feedback signal!) that stimulates Guanylate Cyclase

GTP

cGMP + PPi

Ca- the second "2nd messenger"



Negative feedback





Signaling cascade

Converts a microscopic stimulus activation of a single molecule – -into a macroscopic response

Alberts et al, "Mol Bio of the Cell"









Enzymatic amplifier modules



1st stage: G-protein (Transducin) activation and deactivation

2nd stage: cGMP hydrolysis and synthesis



Note: [GTP] / [GDP] & [GMP] acts a metabolic "power supply"

More examples of amp modules



General "push-pull" amplifier circuit



"Power supply": **a+b → c** maintains high [c]/[a]

$$\frac{d}{dt}[X^*] = k[X][E_{act}] - k'[X^*][E_{de-act}]$$

Steady state:
$$[\overline{X}^*] = \frac{k[E_{act}]}{k[E_{act}] + k'[E_{de-act}]} [X_{tot}]$$

Note: $[X^*]/[X] \neq e^{-\beta \Delta E}$ because the system is held out of equilibrium by fixed [c]/[a] maintained by metabolism

Amplifier gain and time constant

Small change in [X*] induced by a small change in $[E_{act}]$:

$$\frac{d}{dt}\delta[X^*] + \tau^{-1}\delta[X^*] = k[X_{tot}]\delta[E_{act}]$$

Linear response (in the Fourier domain):

Note: $g \sim \tau$ the gain-bandwidth "theorem" !!

Linear analysis of vertebrate phototransduction

Rh* deactivation + 2 amplifier "modules" with negative feedback via $[Ca^{++}]$

Good approximation to weak flash (single photon) response

Linear analysis of the cascade

$$\frac{d}{dt}\delta G^* = -\tau_G^{-1}\delta G^* + \tau_G^{-1}g_G\delta Rh^*$$
$$\frac{d}{dt}\delta[cGMP] = -\tau_{cGMP}^{-1}\delta[cGMP] - \tau_{cGMP}^{-1}g_{cGMP}\delta G^*$$

Assuming stoichiometric processes of PDE* and Ch* activation are "fast" :

 $\delta PDE^* \sim \delta G^* \qquad \delta Ch^* \sim \delta cGMP^*$

$$\delta Ch^* \sim \delta [cGMP](\omega) = \frac{g_G g_{cGMP} \delta Rh^*(\omega)}{(1 + i\omega\tau_G)(1 + i\omega\tau_{cGMP})}$$

"Hard" and "Soft" parameters

Kinetic constants/affinities -- "hard" parameters change on evolutionary time scale

Enzyme concentrations -- "soft" parameters that can be regulated by the cell.

e.g.
$$g = \tau k[\overline{X}]$$
 $\tau = k[E_{act}] + k'[E_{de-act}]$

Golden rule of biological networks: "anything worth regulating is regulated"

General behavior

$$\frac{\delta X^*(\omega)}{\delta Y^*(\omega)} = \frac{g_1 \dots g_n}{(1 + i\omega\tau_1) \dots (1 + i\omega\tau_n)}$$

Impulse response in the time domain:

$$\frac{\delta X^{*}(t)}{\delta Y^{*}(0)} = \begin{cases} g_{1} \dots g_{n} \frac{t^{n}}{\tau_{1} \dots \tau_{n}} & \text{at short times} \\ g_{1} \dots g_{n} e^{-t/\tau_{Max}} & \text{at times times} \end{cases}$$
For:

$$\tau_{1} = \dots = \tau_{n} = \tau \qquad \frac{\delta X^{*}(t)}{\delta Y^{*}(0)} \sim g \frac{t^{n}}{\tau^{n}} e^{-t/\tau} \qquad \text{With n=3} \\ -\text{ good fit single-phot} \end{cases}$$

Case of feedback

$$\tau_{G} \frac{d}{dt} \delta G^{*} = -\delta G^{*} + g_{G} \delta Rh^{*}$$

$$\tau_{cGMP} \frac{d}{dt} \delta [cGMP] = -\delta [cGMP] - g_{cGMP} \delta G^{*} + g_{F} \delta [Ca]$$

$$\tau_{Ca} \frac{d}{dt} \delta [Ca] = -\delta [Ca] + g_{Ca} \delta [cGMP]$$
feedback
via δCh^{*}

 $\frac{\delta c GMP(\omega)}{\delta Rh^*(\omega)} = -\frac{g_G g_{cGMP}(1+i\omega\tau_{Ca})}{(1+i\omega\tau_G)[(1+i\omega\tau_{cGMP})(1+i\omega\tau_{Ca}) - g_{Ca}g_F]}$

Consequences of negative feedback



Signal and Noise: How much gain is enough?



Single photon "signal":

$$\begin{cases} \delta I \sim 2 \text{ pA} \\ I_{Dark} \sim 60 \text{ pA} \end{cases}$$

$$\delta I / I_{Dark} \sim 3\%$$

What's the dominant source of background noise?

Thermal fluctuations?

$$\frac{CV^2}{2} \to CV_{Dark} \delta V = k_B T \to \frac{\delta V}{V_{Dark}} = \frac{k_B T}{CV_{Dark}^2} \sim 10^{-6}$$

ROS capacitance C ~ 20pF

Negligible !

Reaction shot noise

$$\frac{d}{dt}X^* = \Gamma_+(X,..) - \Gamma_-(X^*,..) + \eta(t)$$
f

of molecules
Langevin noise
(i.e. Gaussian, White)

$$< \eta(t)\eta(0) >= (\Gamma_{+} + \Gamma_{-})\delta(t)$$

#

E.g. if $\Gamma_{-} = 0$ consider ΔX^{*} produced over time Δt $<\Delta X^{*} >= \overbrace{\Delta t} \Gamma_{+}$ $<(\Delta X^{*})^{2} > - <\Delta X^{*} >^{2} = <[\overbrace{\Delta t} \eta(t)]^{2} >= <\Delta X^{*} >$ Poisson process

Channel opening noise

$$\frac{d}{dt}Ch^* = \gamma([cGMP])Ch - \tau_{Ch}^{-1}Ch^* + \eta(t)$$

$$Ch_D^* = \langle Ch^* \rangle = \tau_{Ch} \gamma([cGMP]_D)Ch_{Tot}$$

Channel flicker noise:

$$<(\delta Ch^{*})^{2}>=< Ch^{*}>= Ch_{Dark}^{*}$$
Note: this Poisson law holds generally for mass action
$$<(\delta Ch^{*})^{2}>^{1/2} = \frac{1}{\sqrt{Ch_{D}^{*}}} \sim \frac{1}{\sqrt{10^{4}}} = 1\%$$

$$< 3\% \text{ signal}$$

Plus fluctuations of $\delta[cGMP]$:

$$< (\delta Ch^*)^2 >= Ch_D^* + f(\tau_{cGMP} / \tau_{Ch}) < (\delta [cGMP])^2 >$$

cGMP fluctuations

Note: voltage response is coherent over the whole rod outer segment, hence must consider the whole volume $V_{ros} \sim 10^4 \mu m^3$

$$#cGMP = [cGMP] V_{ros} \sim 10 \mu M \ 10^4 \mu m^3 \sim 10^7$$

Negligible fluctuations ...

Locality of single photon response



Single photon response variability



Baylor-Riecke:

coefficient of variation

 $\frac{<\left[\Delta I_p - <\Delta I_p >\right]^2>^{1/2}}{<\Delta I_p>}$ 0.25

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What is the largest source of response variability?

Spontaneous activation G -> G* ?? < 10⁻⁷ s⁻¹ Negligible even with 10⁸ G molecules !

Variability of Rh^{*} "on"- time Δt ?? $\Delta G^* \sim k \Delta t$ $<\Delta t > = \tau_{Rh^*}$

1st order Poisson process:

$$\frac{\langle (\Delta t)^2 \rangle - \langle \Delta t \rangle^2}{\langle \Delta t \rangle^2} = 1$$

Baylor& Reicke measured: $\frac{\langle (\Delta I_p)^2 \rangle - \langle \Delta I_p \rangle^2}{\langle \Delta I_p \rangle^2} \sim .05$

Poisson process of order 20 !!!

Multistep deactivation of Rh*



Summary:

What we have learned:

- GPCR / cyclic nucleotide cascade
- Enzymatic amplifier module and linear response
- Noise analysis

Tomorrow...

from frog to fly and from linear to non-linear

Acknowledgements

Anirvan Sengupta, (Rutgers)

Peter Detwiler (U. Washington)

Sharad Ramanathan (CGR/Lucent)