Betterton

Molecular Motors

Boulder School 2007

Order of magnitude estimates

Free Energy of ATP hydrolysis $ATP \to ADP + P_i$

$$\Delta G = \Delta G_0 + kT \ln \underbrace{\frac{[ATP]}{[ADP][P_i]}}_{\text{Concentrations in M}}$$

- 2nd term = 0 if all concentrations = 1 M

$\Delta G_0 = 13.5\ kT$ (at room temperature)

Typically ATP is in excess

$\Rightarrow \Delta G \approx 20\ kT$

Drag coefficient of a protein

6nm

$\to$ sphere, radius 3 nm

$\gamma = 6\pi\eta r$ - Stokes' law

$$\sim (18)\underbrace{\left(10^{-2}\ \frac{g}{cm\ s}\right)}_{\text{Water}}(3 \times 10^{-7}\ cm)$$

$$\sim 50 \times 10^{-9}\ \frac{g}{s}$$

$$\boxed{\gamma \sim 5 \times 10^{-8}\ \frac{g}{s}}$$

Diffusion coefficient of a protein (in water)

Stokes-Einstein relation

$$D = \frac{kT}{\gamma}$$

At room temperature, write kT in various units:
↖ 25° C

$$kT = 4.1 \times 10^{-21}\ J$$
$$= 4.1 \times 10^{-14}\ erg\ \left(\frac{g\ cm^2}{s^2}\right)$$
$$= 4.1\ pN\ nm$$

Then $D = \frac{kT}{\gamma} = \frac{4.1 \times 10^{-14}\ g\ cm^2/s^2}{5 \times 10^{-8}\ g/s}$

$$\boxed{D \sim 8 \times 10^{-7}\ \frac{cm^2}{s}}$$

Time to diffuse 1 m
↖ protein moves from spine to fingertip inside a neuron

$$t \sim \frac{L^2}{D} \sim \frac{10^4\ cm^2}{8 \times 10^{-7} cm^2/s} \sim 10^{10} s \sim \boxed{10^4\ years}$$

Note: crowding in cells makes this time even longer (lower D)

Kinesin numbers

$v \sim 10^2 - 10^3$ nm/s

$\delta \sim 10$ nm step size

$F_s \sim 6$ pN stall force

$K_m \sim 10^2$ μM for ATP

Energy and maximum stall force

We have $\Delta G \sim 20\ kT$ for single ATP hydrolysis

~~Work~~ ~ 100 pN·nm

Work done per step $= F\delta$

For a perfectly efficient motor, $F\delta = \Delta G$

$$\Rightarrow \text{Maximum force } F_{max} = \frac{\Delta G}{\delta}$$

$$\sim \frac{100 \text{ pN}\cdot\text{nm}}{10 \text{ nm}}$$

$$\boxed{F_{max} \sim 10 \text{ pN}}$$

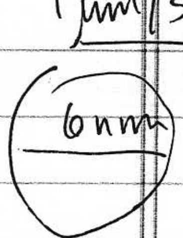
$\Rightarrow$ Kinesin is close to theoretical maximum

Efficiency: $\frac{\text{work done against external load}}{\text{available free energy}}$

$$\epsilon = \frac{F\delta}{\Delta G} \sim \frac{(6 \text{ pN})(8 \text{ nm})}{100 \text{ pN nm}}$$

$$\boxed{\epsilon \sim 0.5}$$ high!

Reynolds number: ratio of inertial to viscous force



$$Re = \frac{\rho L v}{\eta} \sim \frac{\left(1 \frac{g}{cm^3}\right)\left(3 \times 10^{-7}\, cm\right)\left(10^{-4} \frac{cm}{s}\right)}{\left(10^{-2} \frac{g}{cm\, s}\right)}$$

$$\boxed{Re \sim 3 \times 10^{-9}} \ll 1$$

$\Rightarrow$ Highly dissipative
Can neglect inertia

Coasting distance if motor stops

$$F = -\gamma v$$

$$m \frac{dv}{dt} = -\gamma v$$

$$\dot{v} = -\frac{\gamma}{m} v \Rightarrow v(t) = v_0 e^{-\frac{\gamma}{m} t}$$

Then $x(t) = \int v(t)\, dt = x_0 + \frac{m}{\gamma} v_0 e^{-\frac{\gamma}{m} t}$

Stopping time $t_s \sim \frac{m}{\gamma} \sim \frac{2 \times 10^{-19}\, g}{5 \times 10^{-8}\, g/s}$

$\sim 0.5 \times 10^{-11}$

$$\boxed{t_s \sim 50 \times 10^{-10}\, s}$$

Stopping distance $x_s \sim v_0 t_s$

$$\sim \left(1 \frac{\mu m}{s}\right)\left(5 \times 10^{-10} s\right)$$

$$\sim 5 \times 10^{-10} \mu m$$

$$\boxed{x_s \sim 5 \times 10^{-7} \, nm}$$

→ Tiny!

Effect of thermal fluctuations on motion

- Time to diffuse 1 radius = 3 nm

$$t \sim \frac{L^2}{D} \sim \frac{\left(3 \times 10^{-7} cm\right)^2}{8 \times 10^{-7} cm^2/s}$$

$$\boxed{t \sim 10^{-7} s \sim 0.1 \, \mu sec} \rightarrow \underline{\text{fast}}$$

Enzymatic time scales are μsec – msec

⇒ Motor is greatly aided by a <u>track</u>