Sudden parameter change in isolated systems

$|\psi_0\rangle$ initial wavefct

change abruptly system parameter

evolution under Hamiltonian H_f

<u>**questions**</u>:

- what is the short time dynamics of the system
- How do correlations evolve ?
- Does the system reach a steady state and if yes what are the properties of the steady state

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |n\rangle \quad , \quad c_n = \langle n|\psi_0\rangle$$

expectation values

$$\langle O(t)\rangle = \sum_{n,m} c_m^* c_n \langle m|O|n\rangle e^{i(E_m - E_n)t}$$

$$= \underbrace{\sum_n |c_n|^2 \langle n|O|n\rangle}_{\substack{\text{time independent} \\ \text{'diagonal part'}}} + \underbrace{\sum_{n\neq m} c_m^* c_n \langle m|O|n\rangle e^{i(E_m - E_n)t}}_{\text{oscillating}}$$

assume nondegeneracy

Here example of Bosons in optical lattices

<u>Periodic potentials</u> Mermin & Wagner (lattice)

PP4A

(P2)

$$H = \frac{\hbar^2}{2m} \nabla^2 - V_0 \sin^2(kx) \qquad k = \frac{2\pi}{\lambda}, \quad d = \frac{\lambda}{2}$$

eigenfct $\Psi_{q,n}(x)$ must satisfy Bloch theorem

$$\Psi_{q,n}(x) = \underset{\uparrow \text{ plane wave}}{e^{iqx/\hbar}} \; \underset{\uparrow \text{ Blochfunction}}{u_{q,n}(x)} \quad \text{with} \quad \underbrace{u_{q,n}(x+d) = u_{q,n}(x)}_{\text{periodicity of lattice}}$$

q = quasimomentum ranging from $[-\hbar k; \hbar k]$

n = band index labeling several solutions with same q

$\Rightarrow$ energy values & Blochfct can be determined going to frequency space

(Fourier series: $\Psi_{q,n}(x) = e^{iqx/\hbar} \sum_{l \in \mathbb{Z}} c_l^{q,n} e^{i2lkx}$

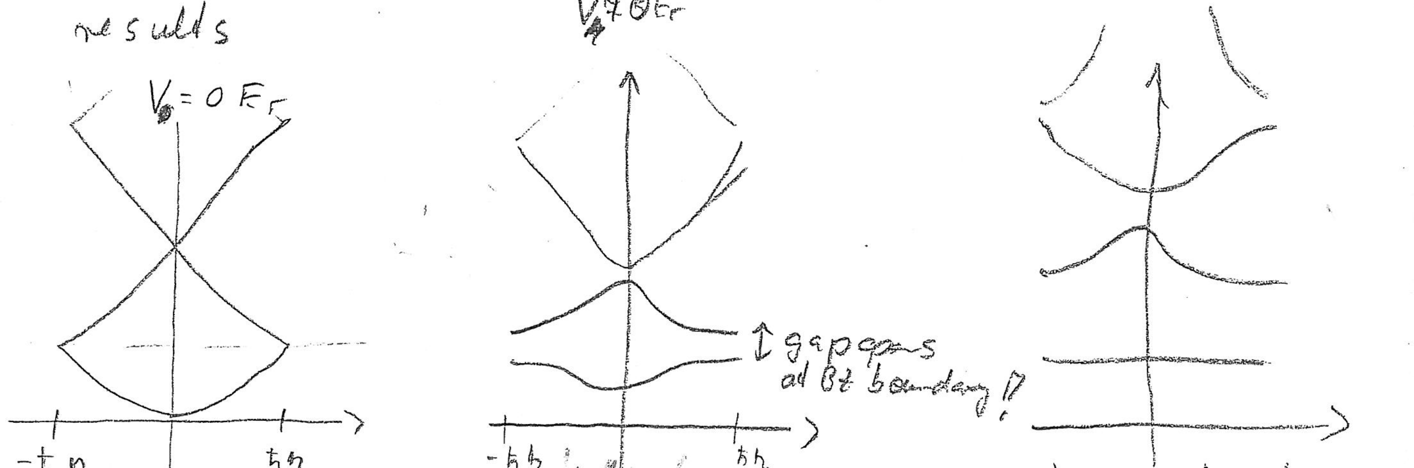
$$-V_0 \sin^2(kx) = \sum_{r \in \mathbb{Z}} V_r e^{i2rkx} \quad \text{with} \quad \begin{aligned} V_0 &= -V/2 \\ V_{-1} = V_1 &= V/4 \\ V_{|r|>1} &= 0 \end{aligned}$$

inserting into Seq

$$H \Psi_{q,n} = E_{q,n} \Psi_{q,n}$$

$$\rightarrow \sum_{l'} e^{i(q+2l'\hbar k)x} \underbrace{\left[\left(\frac{(q+2l'\hbar k)^2}{2m} - E_{q,n} \right) c_{l'}^{q,n} + \sum_r V_r \, c_{l'-r}^{q,n} \right]}_{\stackrel{!}{=} 0 \text{ using orthogonality of plane waves}} = 0$$

set of linear equations usually only need few l' to get good results



parabola

moving lowest band

II

becomes close to harmonic oscillator

(123)

<u>Tight binding approximation</u>

aim: lattice hamiltonian

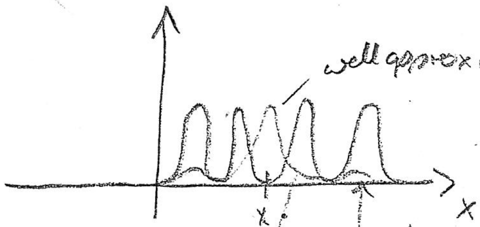
~~lattice~~ [redacted]

Jaksch et al.
PRL 81, 3108 (1998)

change basis from delocalized Bloch fct's
to localized Wannier fcts (complete orthogonal set)

$$w_n(x-x_j) = \underset{\text{Normalisation}}{N^{-1/2}} \sum_q e^{-iqx_j}\, \psi_{q,n}(x)$$

x_j denotes Min around which w_n is localized



well approximated with Gaussian for deep lattices

not captured by Gaussian

rather exponential decay

Expand Hamiltonian in Wannier fct

$$\psi(x) = \sum_j w^0(x-x_j)\, b_j$$

• b_j bosonic operator on site j

• here only lowest band taken into account, needs to be checked!

kinetic energy:

$$\int\!\left(-\frac{\hbar^2}{2m}\right)\psi^\dagger(x)\frac{\partial^2}{\partial x^2}\psi(x) \;\rightarrow\; \sum_{j,j'} \underbrace{-\frac{\hbar^2}{2m}\int d^3x\, w^*(x-x_{j'})\frac{\partial^2}{\partial x^2}w(x-x_{j'})}_{=:\,-J_{j'j}}\; b^\dagger_{j'} b_j$$

$$g\int dx\; \psi^\dagger(x)\psi(x)\psi^\dagger(x)\psi(x) \;\rightarrow\; \sum_{j_1,j_2,j_3,j_4} b^\dagger_{j_1} b_{j_2} b^\dagger_{j_3} b_{j_4} \underbrace{\left(g\int d^3x\; w^*(x-x_{j_1})\, w(x-x_{j_2})\, w^*(x-x_{j_3})\, w(x-x_{j_4})\right)}_{=:\,U_{j_1,j_2,j_3,j_4}}$$

Usually only take

$J := J_{j,j+1}$ can be calculated alternatively by

$$J = \max(\varepsilon_q^{(1)}) - \min(\varepsilon_q^{(1)}) / 4$$

for $V_0/E_r \gg 1$

$$J/E_r = \frac{4}{\sqrt{\pi}}\left(\frac{V}{E_r}\right)^{3/4} e^{-2\sqrt{V/E_r}}$$

<u>onsite U</u>:

$$\underset{\text{Gauss}}{U/E_r} = \sqrt{8/\pi}\; k a_s \left(\frac{V_x V_y V_z}{E_r^3}\right)^{1/4}$$

4 transparency

Jaksch PhD

p4 * lattice p3

→ Bose-Hubbard model

$$H = -J \sum b_j^{\dagger} b_{j+1} + U/2 \sum n_j (n_j - 1) + \sum_j \varepsilon_j n_j - \mu \sum_j n_j$$

kinetic energy interaction trapping

famous models in condensed matter physics

Literature: Fisher et al PRB 40, 546 ('89)
Zwerger J Opt B: Quant Semiclass. Opt 5 (2003) S9-S16

Quantum phases:

consider two limits:

$\underline{J = 0}$

$$H = U/2 \sum n_j (n_j - 1) + \sum_j \varepsilon_j n_j$$

→ eigenstates = Fock states
without trap ($\varepsilon_j = 0$)

commensurate filling $n = n_0 \in \mathbb{N}$ eg $n_0 = 1$ $|1 1 1 \dots 1\rangle$

(atomic Mott state)

Mott state
- localized particles ($\langle b_i^{\dagger} b_{i+1} \rangle = 0$ only valid for $J \neq 0$ (and for $J \ll U$))
- interaction energy ($\langle n_i (n_i - 1) \rangle = n_0 (n_0 - 1)$)
- energy gap to particle-hole excitations with energy $\sim U$

E $\} U$

incommensurate filling $n = n_0 \notin \mathbb{N}$

superfluid state:
- delocalized states $|110 \dots\rangle + |1010 \dots\rangle + |10010 \dots\rangle$
- no gap above the ground state

$\underline{U = 0}$ $H = -J \sum b_i^{\dagger} b_{i+1} = -2J \sum \cos(k)\, b_k^{\dagger} b_k$

→
- delocalized states, s
- no energy gap above ground state

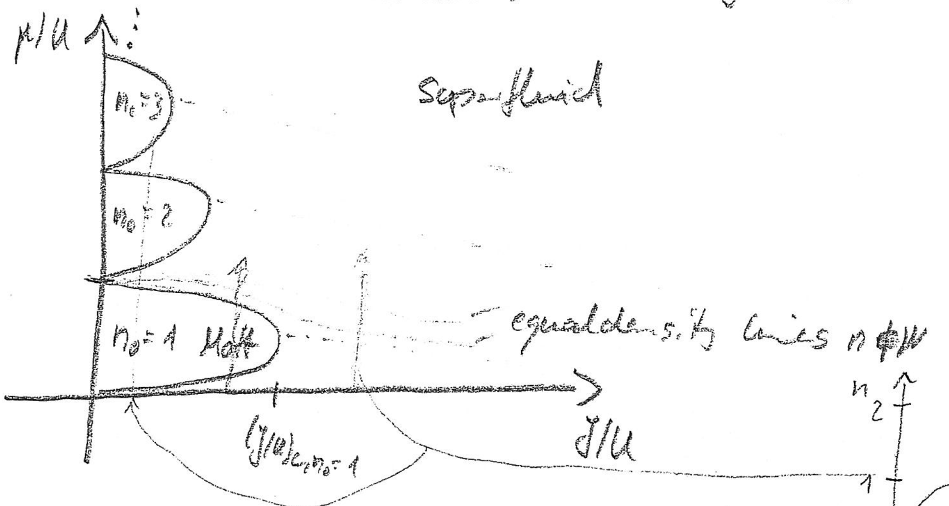
real states for $J \neq 0, U \neq 0$ more interesting.
fluctuations present

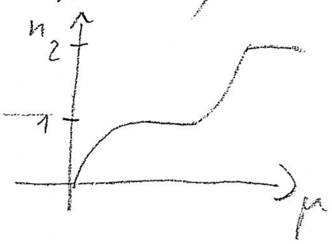
25

~~* lattice p4~~ p5 p22a

phase diagram homogeneous system

~~Fisher et al PRB 40, 546 (1989)~~
~~Zwerger J. Opt B: Quantum Semiclass. Opt. 5, (2003) S9-S16~~





mean field approximation:

$(U/J)_c = 5.8\,z$ for $\bar{n} = 1$, $(U/J)_c = 4\bar{n}\,z$ for $\bar{n} \gg 1$

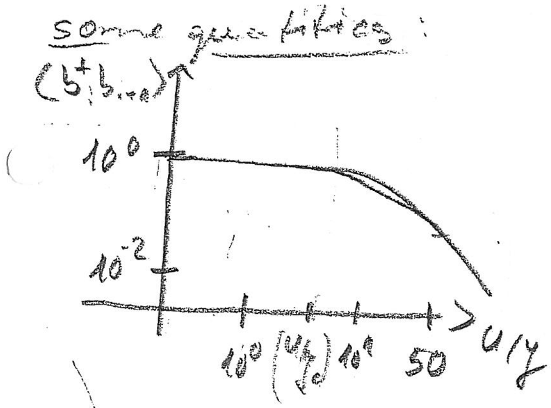
z coordination no

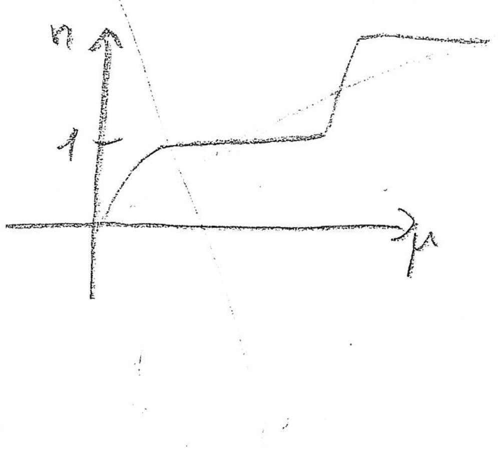
1D: $(U/J)_c = 3.84$ for $\bar{n} = 1$ Kühner & Monien PRB 58 R14741 (1998)

Kosterlitz-Thouless transition

now exist QMC calculations for many quantities.

some quantities: no one-dimensional





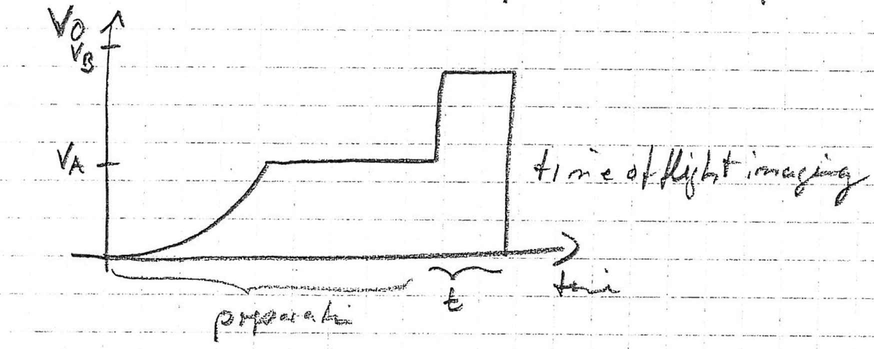
Collapse & Revival

Example to a 'not intuitive' non-equilibrium phenomenon

M. Greiner et al Nature 419, 51 (2002)

3D optical lattice of height V_0, ^{87}Rb (bosons)

sudden increase in the potential depth



V_A corresponds to superfluid state

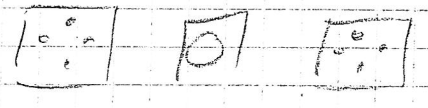
V_B " Mott insulating ground state

Q: What do you expect?

Naive expectation: destruction of coherence

results:

transparency I



revival of coherence after a time T

PRG P7

Theoretical understanding of revival

initial state $|\psi_A\rangle$ superfluid state

Hamiltonian after increase of interaction

$$H_B \approx U_B/2 \sum \hat{n}_j (\hat{n}_j - 1)$$

neglect hopping term => only 'dephasing' in the wells

$$|\psi(t)\rangle = e^{-i\, t/\hbar\, U_B \underbrace{\sum 1/2\, n_j(n_j-1)}_{m \in \mathbb{N}}} |\psi_A\rangle$$

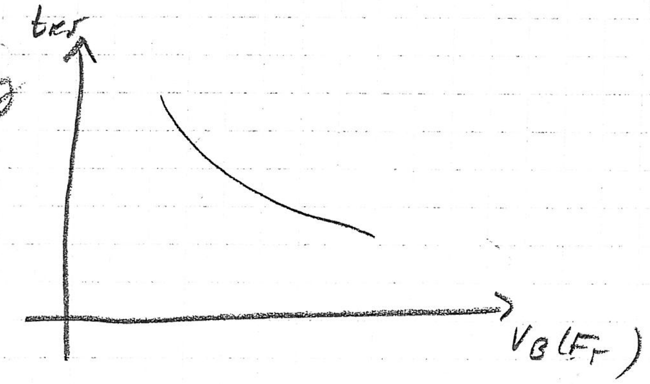
Revival if $2\pi = T/\hbar\, U_B \cdot m$

$$T_m = \frac{h}{U_B} \frac{1}{m}$$

if $T = \frac{h}{U_B}$ for all integers fullfilled

=> full revival of the wavefunction after $T = h/U_B$

=> Can be used to 'measure' interaction strength



MI -> SF

quasi-particle approach inside MI — PRA Barmettler et al

assume MI with filling $\bar{n}$

in MI low fluctuations ⇒ truncate local basis to $\bar{n}, \bar{n} \pm 1$ particles $|\bar{n}\rangle, |\bar{n}+1\rangle, |\bar{n}-1\rangle$

introduce auxiliary boson operators

$a^\dagger_{j,+} |\bar{n}\rangle_j = |\bar{n}+1\rangle_j$ creates excess particle

$a^\dagger_{j,-} |\bar{n}\rangle_j = |\bar{n}-1\rangle_j$ creates hole

the original operator can be reexpressed by

$$b^\dagger_j = \sqrt{\bar{n}+1}\, a^\dagger_{j+} + \sqrt{\bar{n}}\, a_{j-}$$

$a_\pm$ obey bosonic commutations

additional constrain

$$(a^\dagger_{j,\sigma})^2 = (a_{j,\sigma})^2 = 0 \quad (*1) \qquad \& \qquad a^\dagger_{j+} a_{j+} + a^\dagger_{j-} a_{j-} = 0 \quad (*2)$$

well known update very useful — context of coldatoms E Altman & Auerbach, S. Huber, G. Blatter, ...

in equilibrium typically very good to obtain results

during quench problem that (*1) can grow a lot and dominate the results ⇒ need something better

Jordan Wigner trafo in order to introduce fermionic operators which fulfill (*1) automatically

$$z_{j\sigma} c_{j,\sigma} = a_{j,\sigma}, \quad z_{j+} = e^{i\pi \sum_{\sigma, j'<j} n_{j',\sigma}}, \quad z_{j-} = z_{j+} e^{i\pi n_{j+}}$$

why are these auxiliary operators helpful ?

$$H = \sum_j \mathcal{P} \Big\{ -J(\bar{n}+1)\, c^\dagger_{j,+} c_{j+1,+} - J \bar{n}\, c^\dagger_{j+1,-} c_{j,-}$$
$$- J \sqrt{\bar{n}(\bar{n}+1)} \left(c^\dagger_{j,+} c^\dagger_{j+1,-} - c_{j,-} c_{j+1,+} \right) + h.c.$$
$$+ \frac{U}{2} (n_{j+} + n_{j-}) \Big\} \mathcal{P}$$

* interaction is quadratic

$\mathcal{P}$ stands for the projection to physical subspace
ie to fulfill ($*_2$)

neglecting $\mathcal{P} \equiv 1$ H can be solved by Bogolyubov transformation $\gamma^\dagger_+, \gamma^\dagger_-$

-> time-evolution can be calculated 'exactly' within the made approx

$$|\Psi(t)\rangle = \prod_k \left(\bar{u}(k) - \bar{v}(k) e^{-i 2\omega(k) t}\, \gamma^\dagger_{k,+} \gamma^\dagger_{-k,-} \right) |\Psi_0\rangle$$

dispersion of Bogolyubov particles

take large $U/J \gg 1$ limit, $|\Psi_0\rangle$ atomic MI

$$|\Psi(t)\rangle = |\bar{n}\rangle + i \frac{2\sqrt{\bar{n}}J}{U} \sum_k \sin(k)\, c^\dagger_{k+} c^\dagger_{-k,-} |\bar{n}\rangle$$
$$- i\, 2\sqrt{\bar{n}}J/U \sum_k \sin(k)\, e^{i 6 J \cos(k) t/\hbar}\, c^\dagger_{k,+} c^\dagger_{-k,-} |\bar{n}\rangle$$

note occupation of unphysical states remains small ($*_2$)

however at large time ~~[illegible]~~ error can grow large by multiparticle within

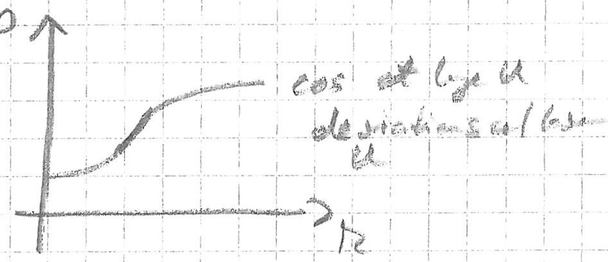
slides

Deviations from light cone

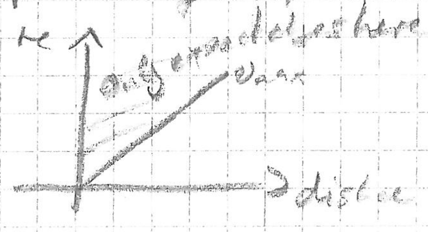
we found wave packet of quasi-particle pairs which propagate

velocity given by dispersion

$v(k) = 2 \frac{\partial}{\partial k} \omega(k)$



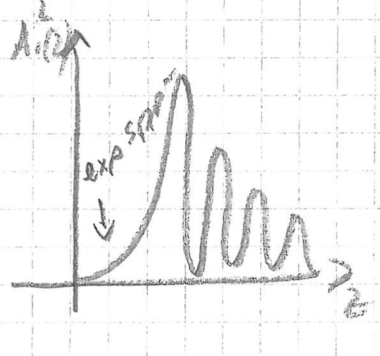
↳ it exists maximal velocity of these quasi-particles
sets a cone - in spin time



approx 2 of correlations.

$$C_{d \gg 1} = -\left(\frac{2 d^{2/3}\, 2^{1/3} J h}{3 U t}\right)^2 Ai^2\left(-\left(\frac{2}{d}\right)^{1/3} \underbrace{\left(\frac{6 J t}{\hbar} - d\right)}_{\text{light cone}}\right)$$

↳ deviations



Findings much more general

max velocity for spreading in spin lattice systems

Lieb & Robinson Commun Math Phys 28 (1972)

many extensions and applications: Bravyi, Hastings, Nachtergaele, Sims, Eisert ...

(as eg. exponential clustering theorem: presence of gap ⇒ exp decay of spatial correlations)