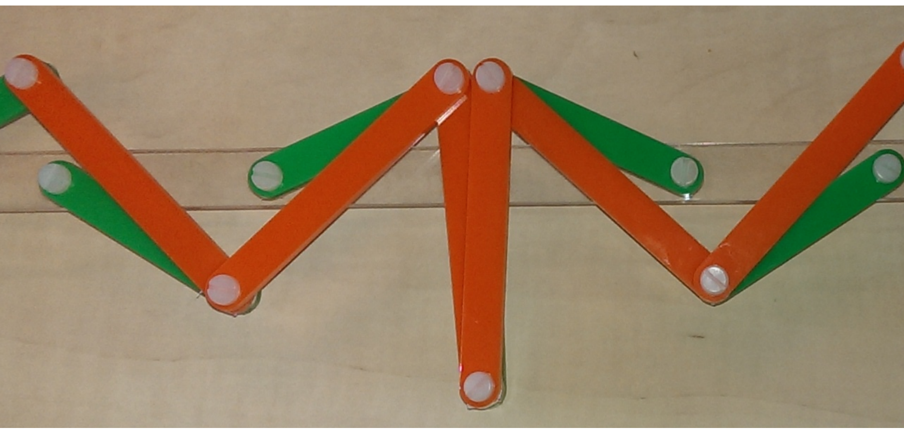
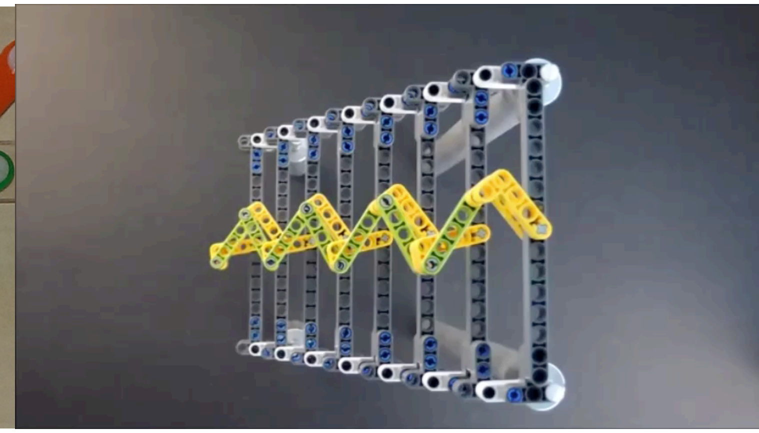
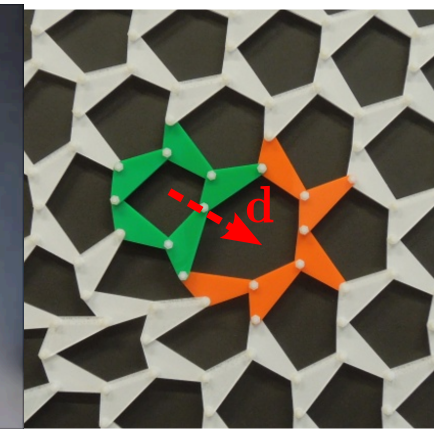
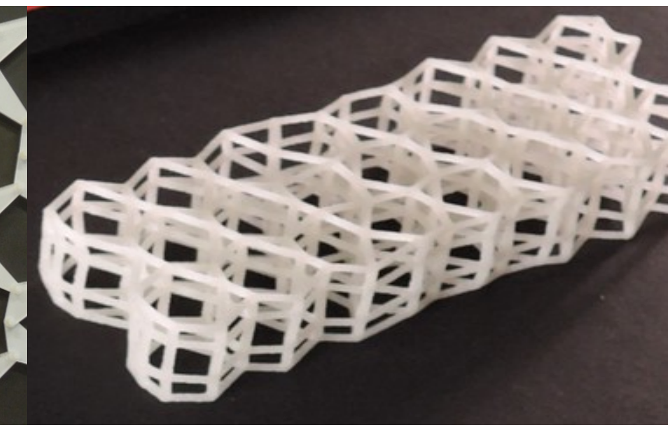
Topological Mechanics



Vincenzo Vitelli

Instituut-Lorentz, Leiden

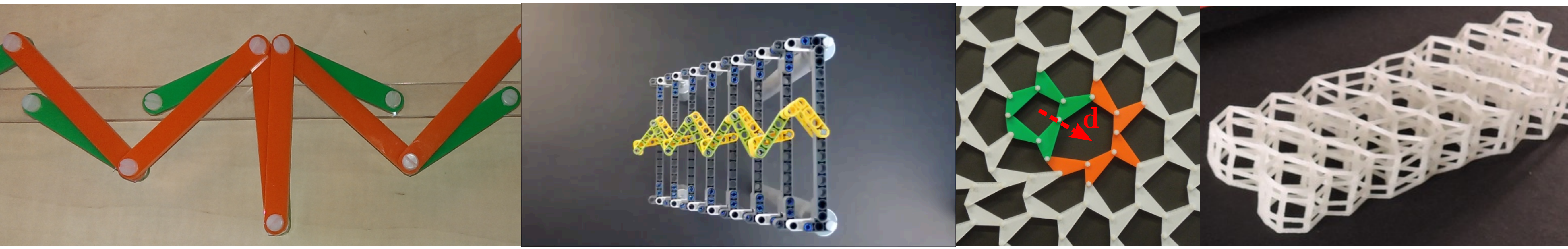
Chen, Upadhyaya, Vitelli, *PNAS* (2014).

Paulose, Chen, Vitelli, *Nat. Phys.* (2015).

Paulose, Meussen, Vitelli, *PNAS* (2015).

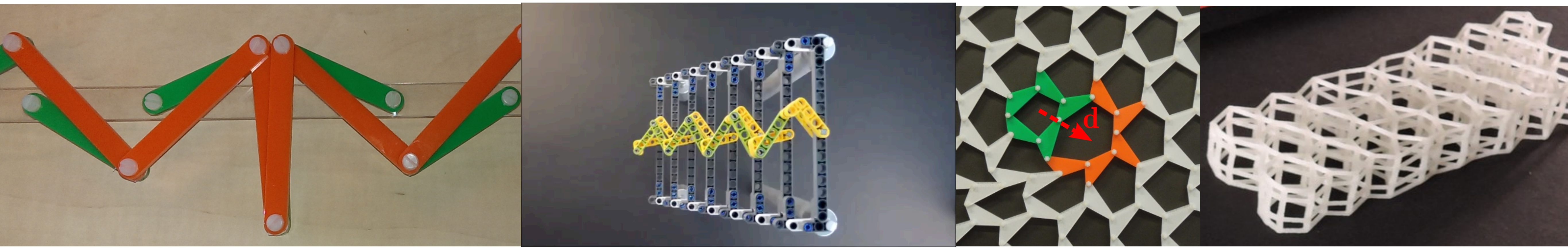
videos: youtube VitelliLab

Topological Mechanical Metamaterials



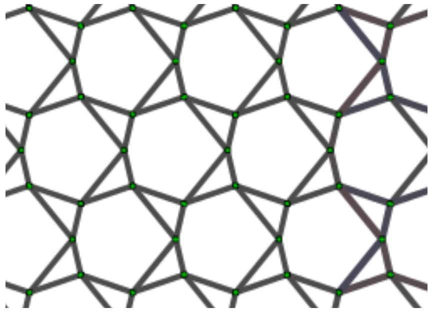
Tunable response
(acoustic and failure)

Topological Mechanical Metamaterials



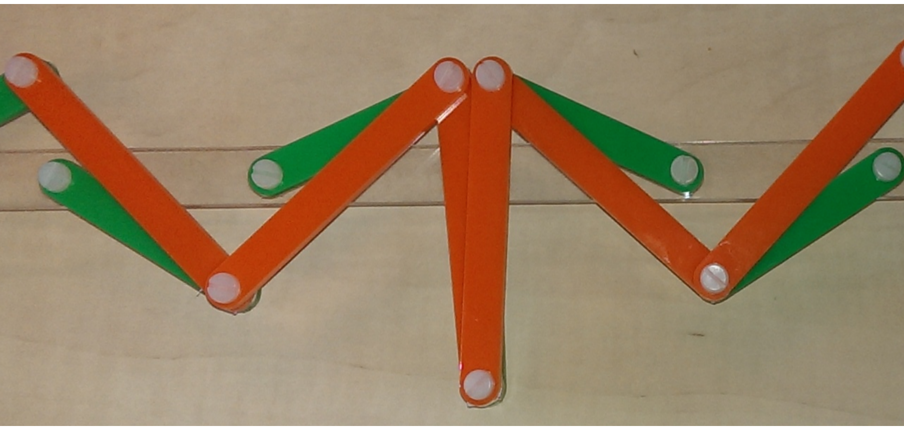
Tunable response

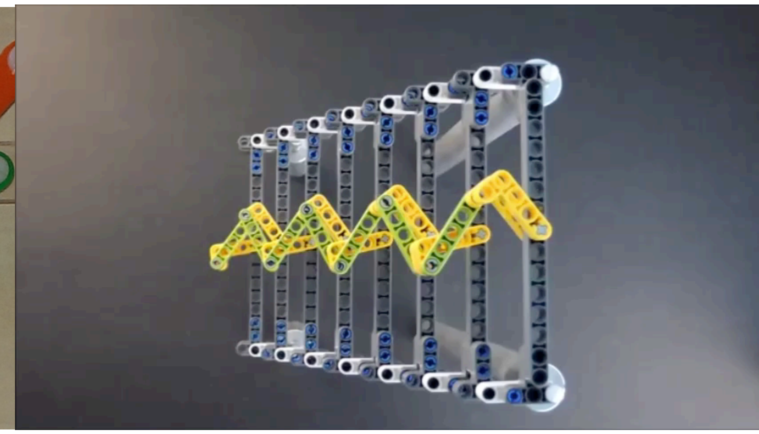
(acoustic and failure)

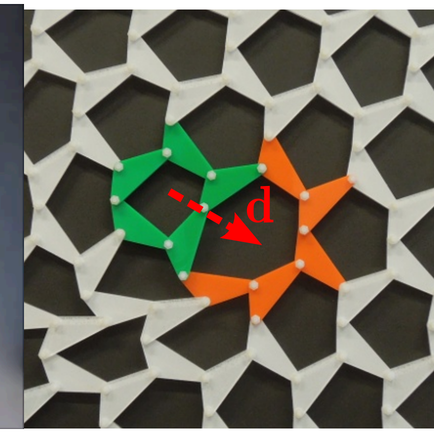


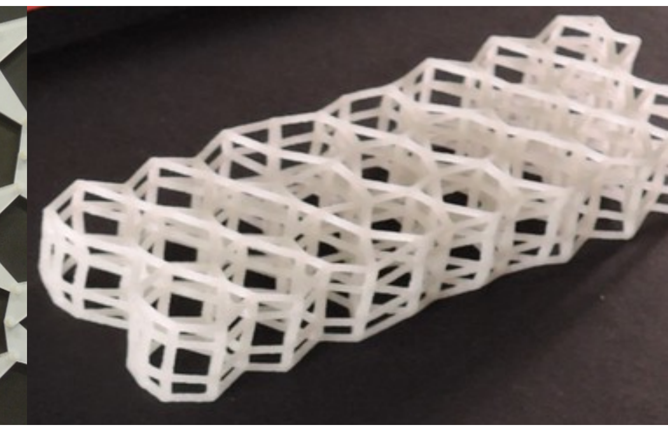
unit-cell geometry

Topological Mechanical Metamaterials



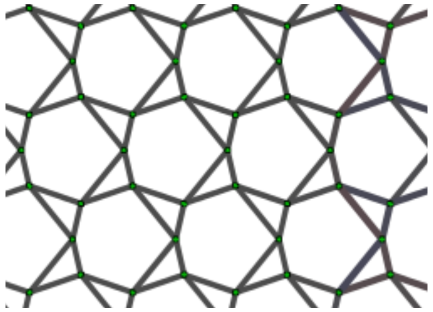






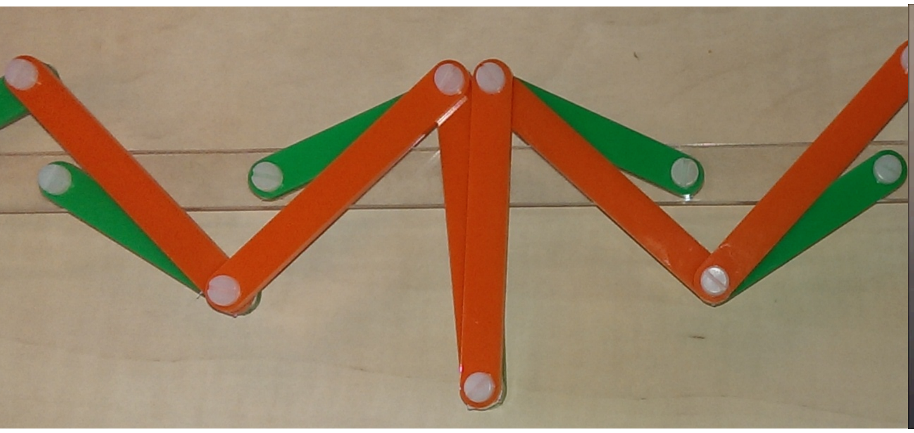
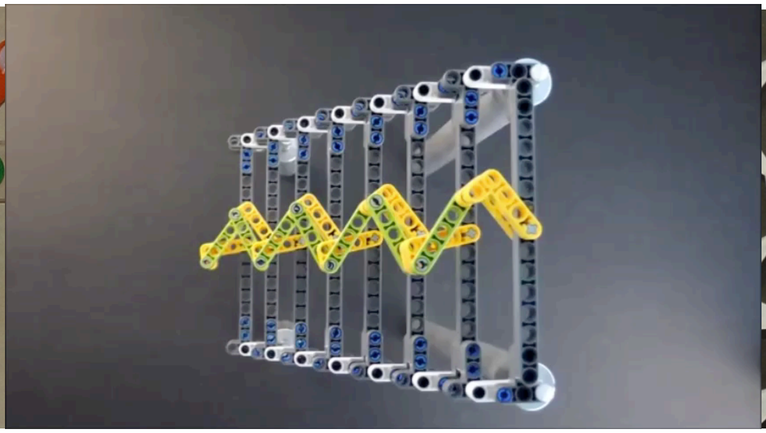
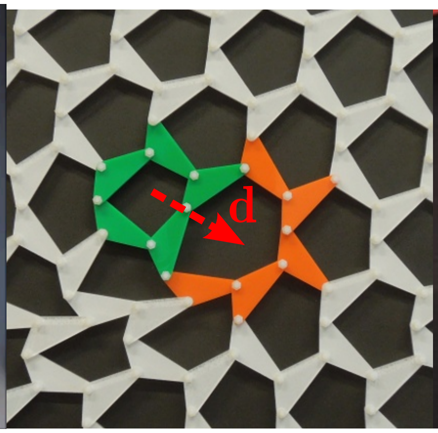
Robust
(insensitive to smooth changes in material parameters)

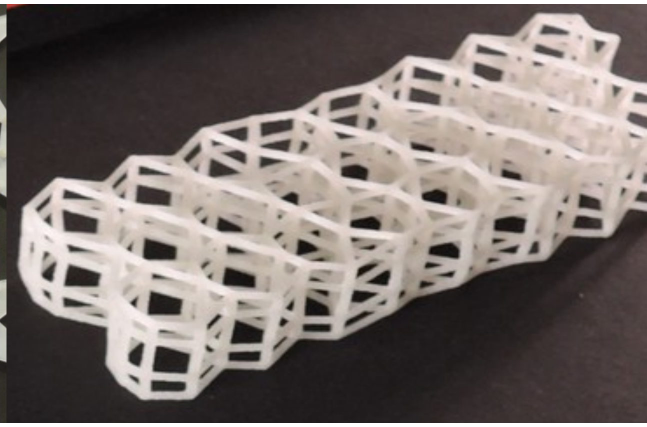
Tunable response
(acoustic and failure)



unit-cell geometry

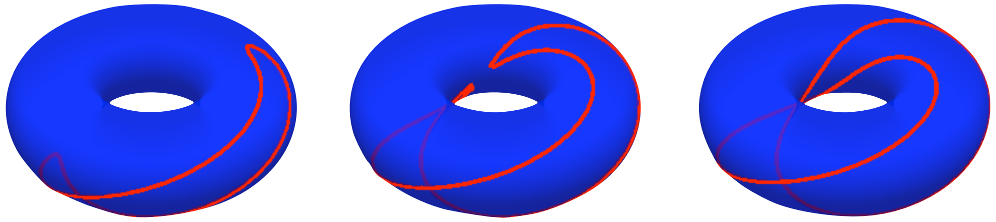
Topological Mechanical Metamaterials

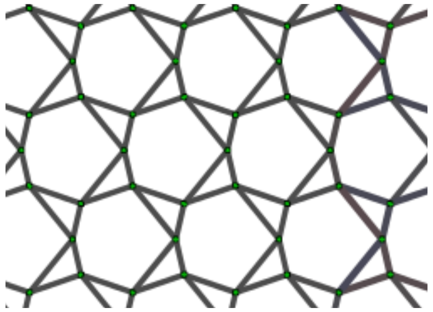






Robust
(insensitive to smooth changes in material parameters)

Tunable response
(acoustic and failure)

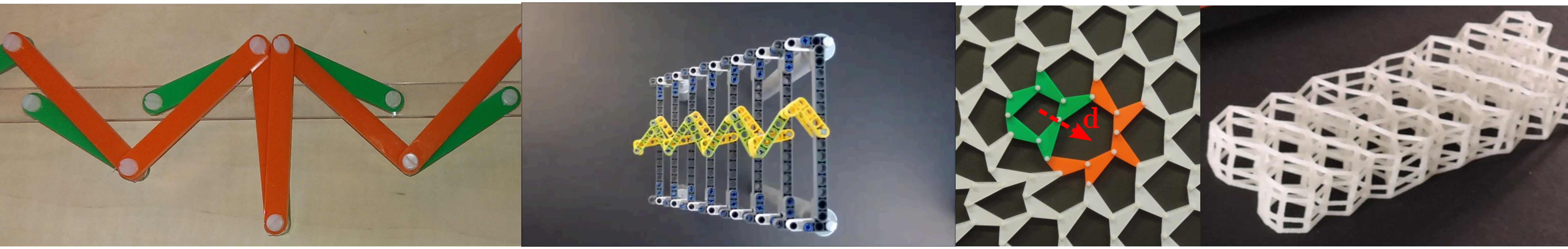




topological invariants

unit-cell geometry

Topological Mechanical Metamaterials



Robust
(insensitive to smooth changes in material parameters)

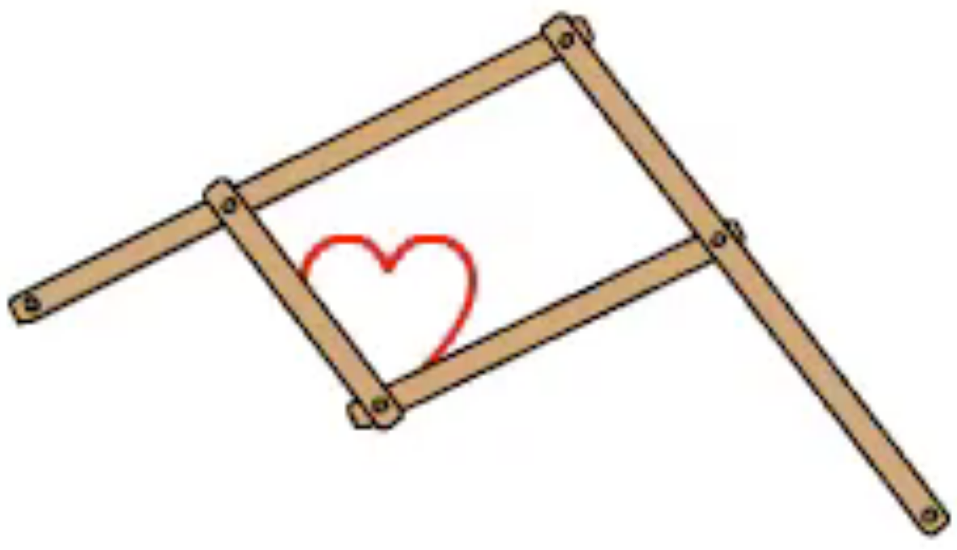
&

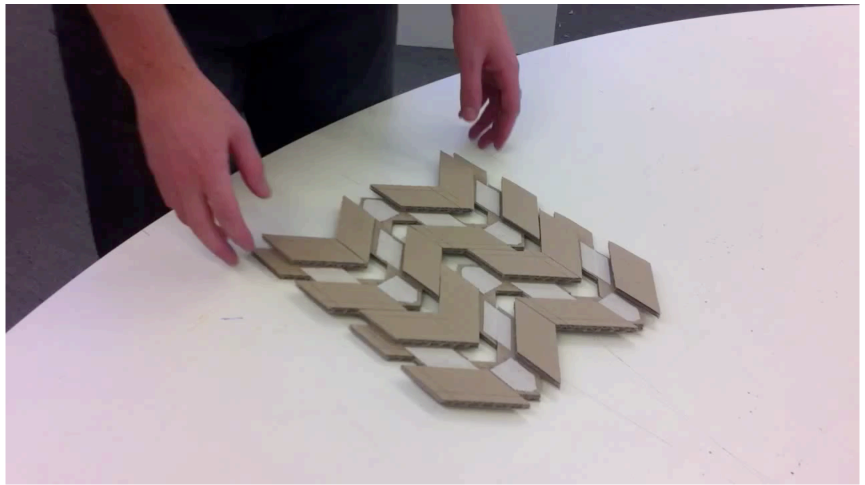
Tunable response
(acoustic and failure)

Can you have both ?

Mechanisms: propensity for motion

zero modes



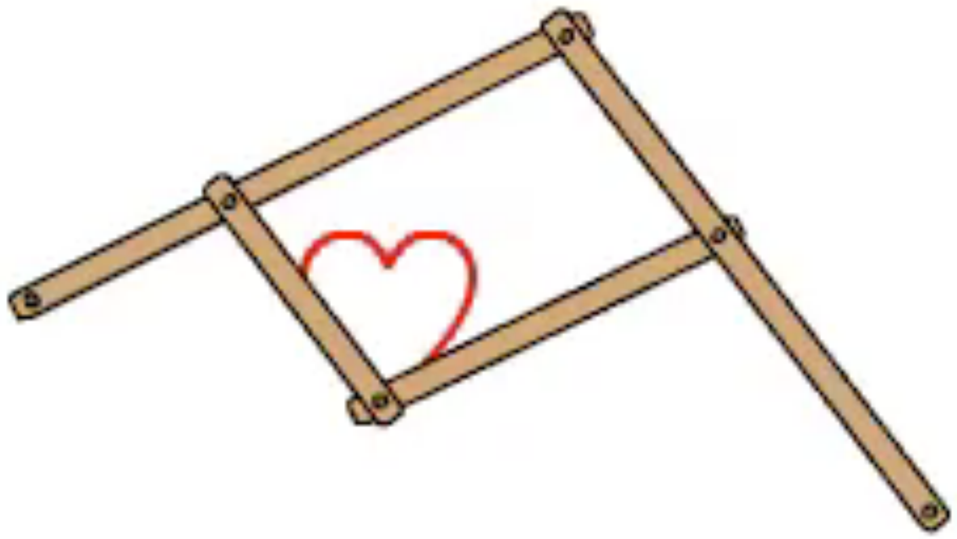


Rigid Miura Origami

Finite motions that obey constraint of zero stretching energy

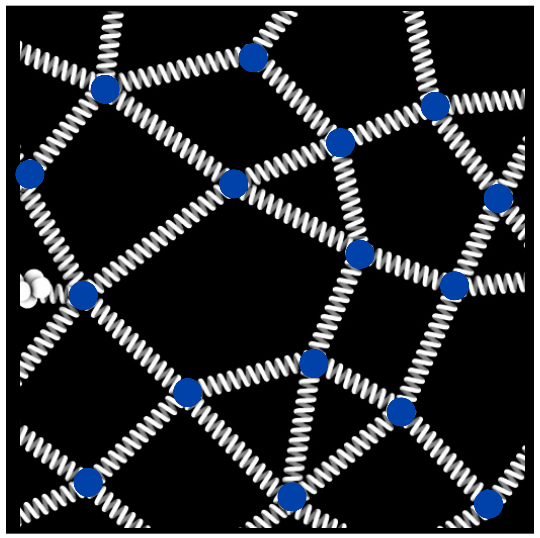
Mechanisms: propensity for motion

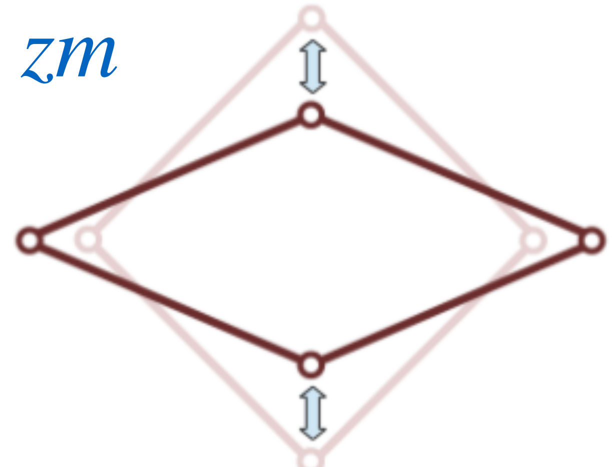
zero modes



Activated mechanisms: building blocks of robots

Maxwell counting





$$e_i = Q_{ij}^T u_j$$

degrees of freedom - #constraints = n_{zm}

Maxwell 1865
Calladine 1978

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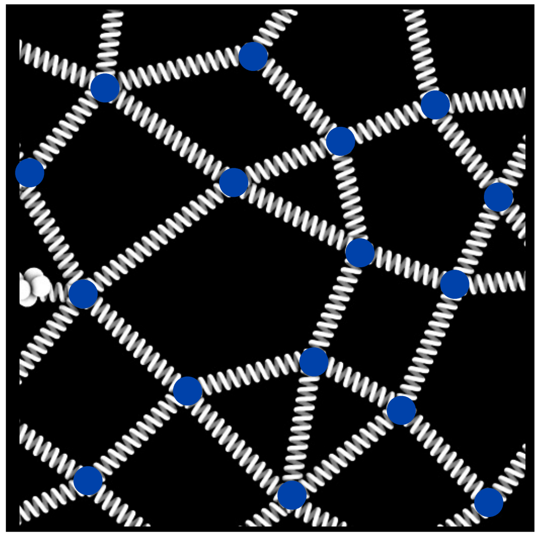
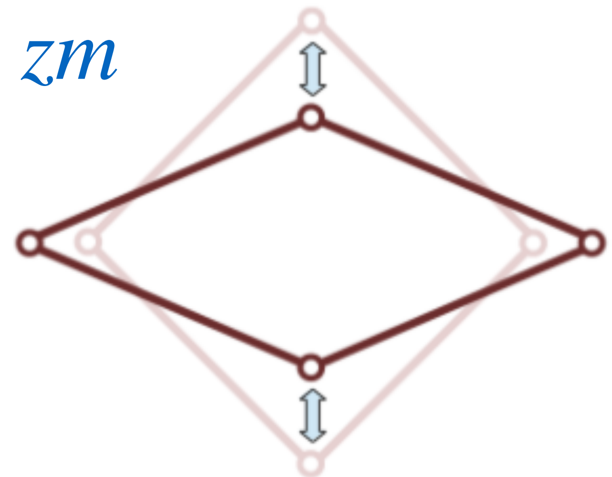
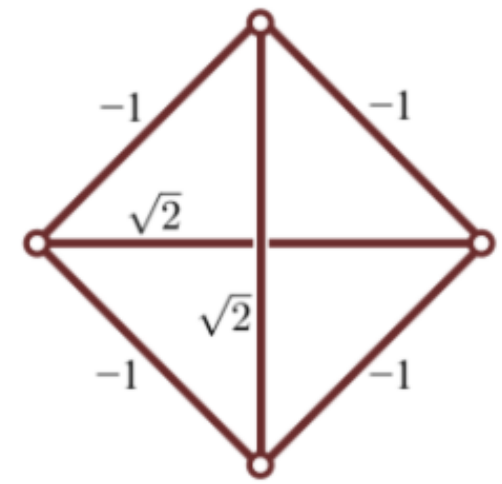
Maxwell counting



zm





ss

$$e_i = Q^T_{ij} u_j \qquad F_i = Q_{ij} t_j$$

\# degrees of freedom - #constraints = n_{zm} - n_{ss} global

right hand side does not change unless you cut links

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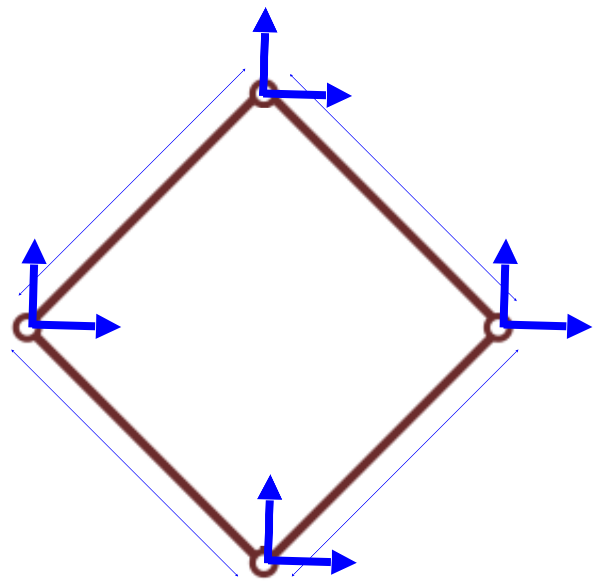
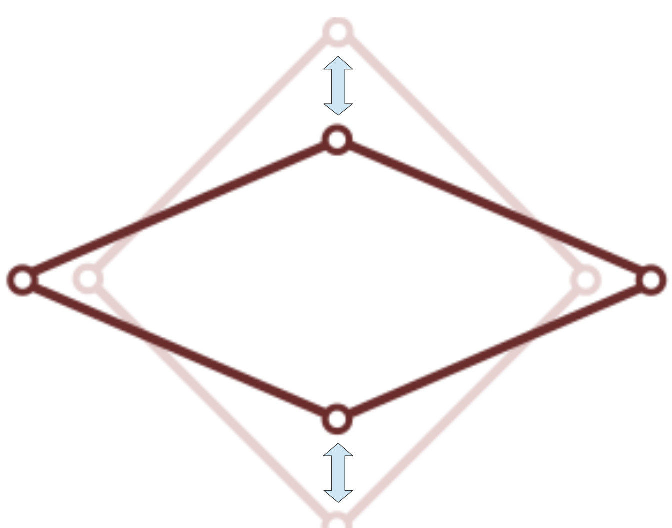
What determines motion in a structure?



8 degrees of freedom
4 constraints

#d.o.f. – #constraints = **4** = #zero modes



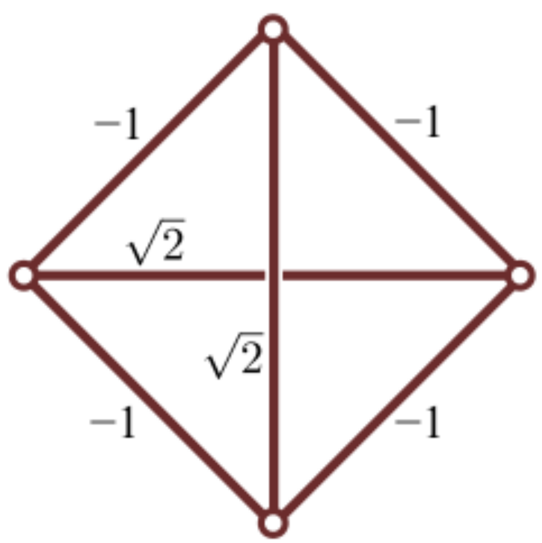
3 trivial (translations + rotation)
1 nontrivial

Maxwell 1865

What determines motion in a structure?

8 degrees of freedom
6 constraints

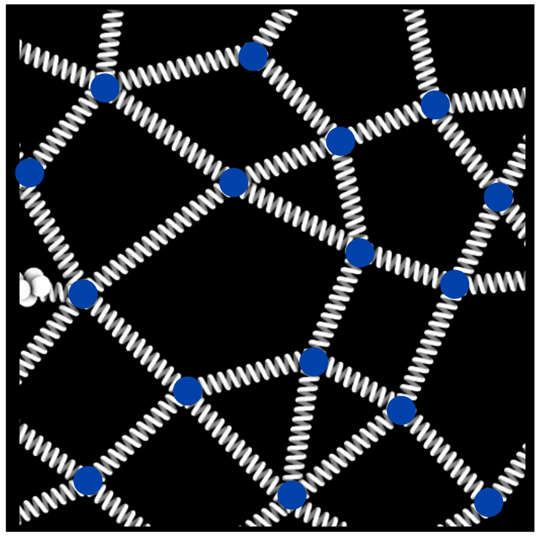
#d.o.f – #constraints **=** **2** **=** #zero modes – #states of self-stress

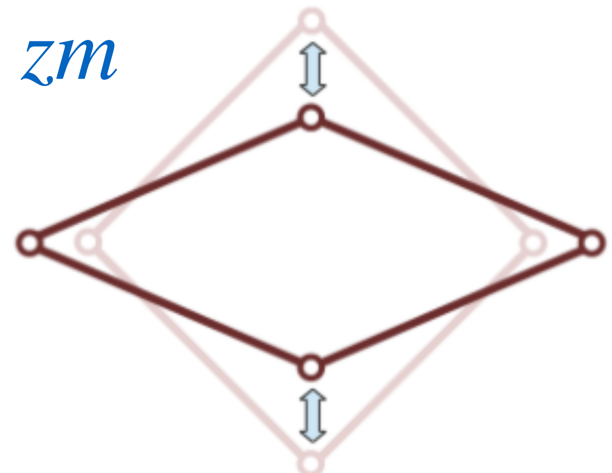


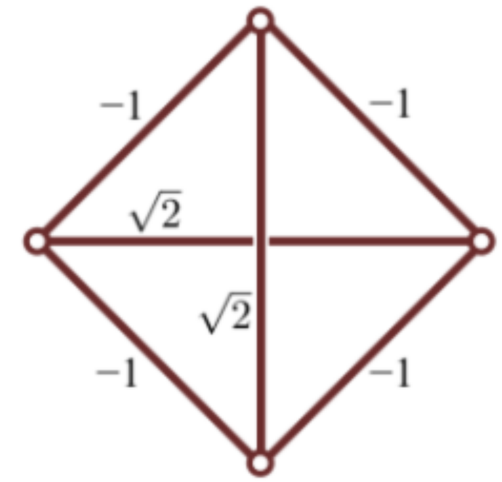
3 trivial zero modes (translations + rotation)
1 state of self-stress (redundant constraint)

Maxwell 1865
Calladine 1978

Maxwell counting







$$e_i = Q^T_{ij} u_j \qquad\qquad F_i = Q_{ij} t_j$$

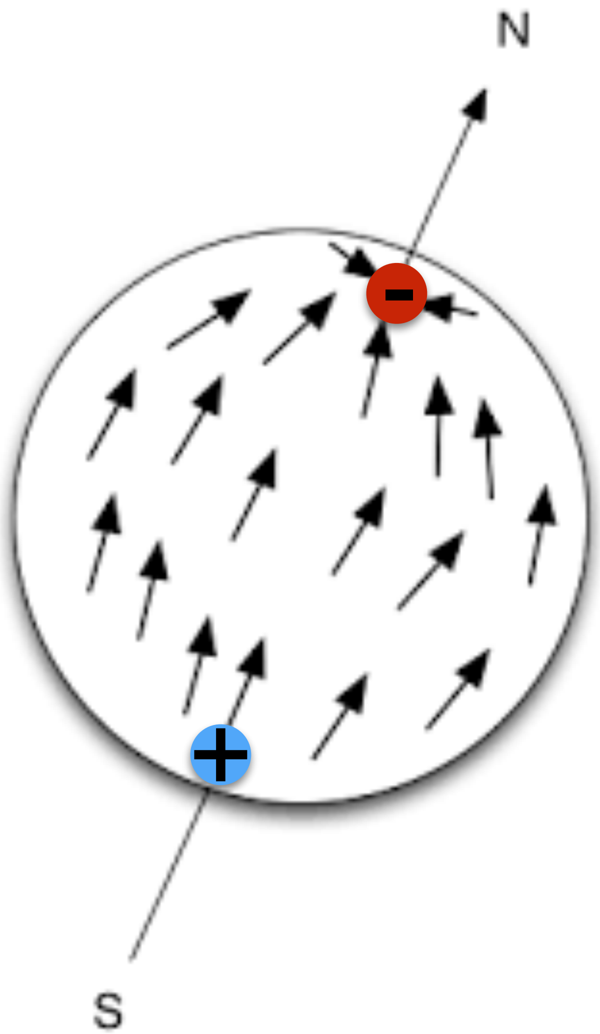
0

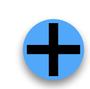
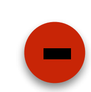
\# degrees of freedom - #constraints = n_{zm} - n_{ss} global

Index theorem

charge neutrality ?

Electrostatic analogy



0  

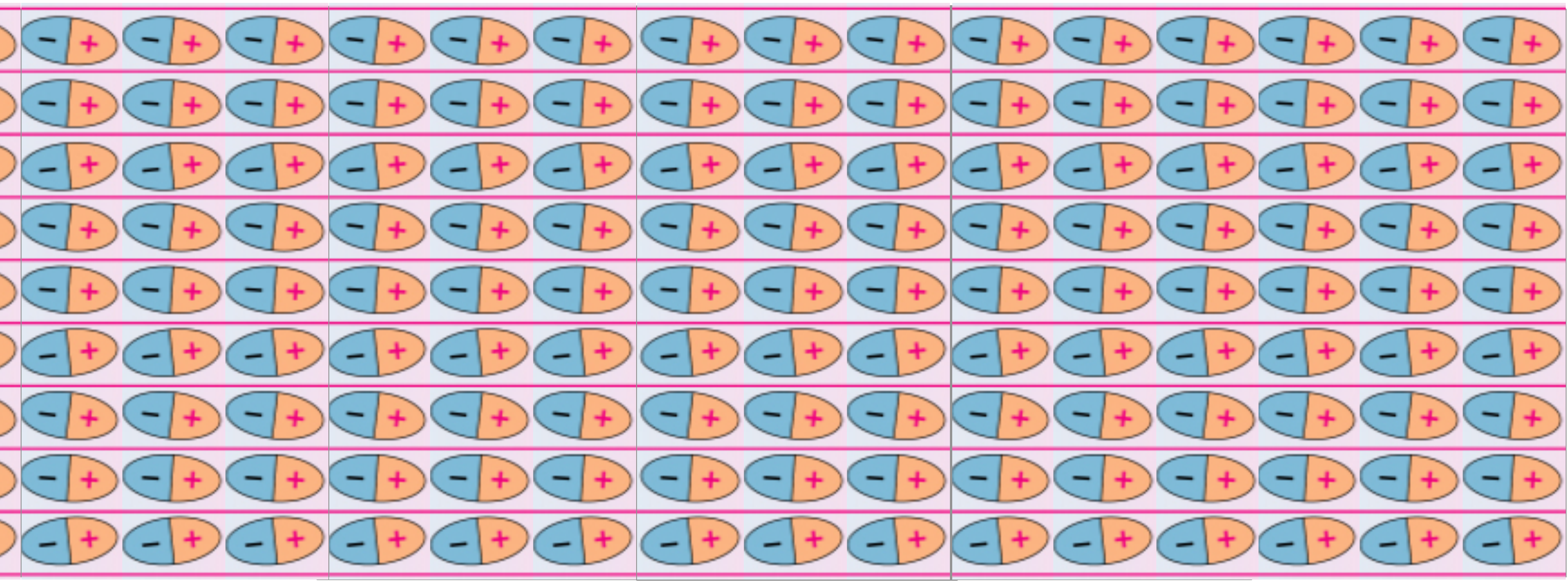
degrees of freedom - #constraints = n_{zm} - n_{ss} global

Index theorem

charge neutrality ?

Polarized medium

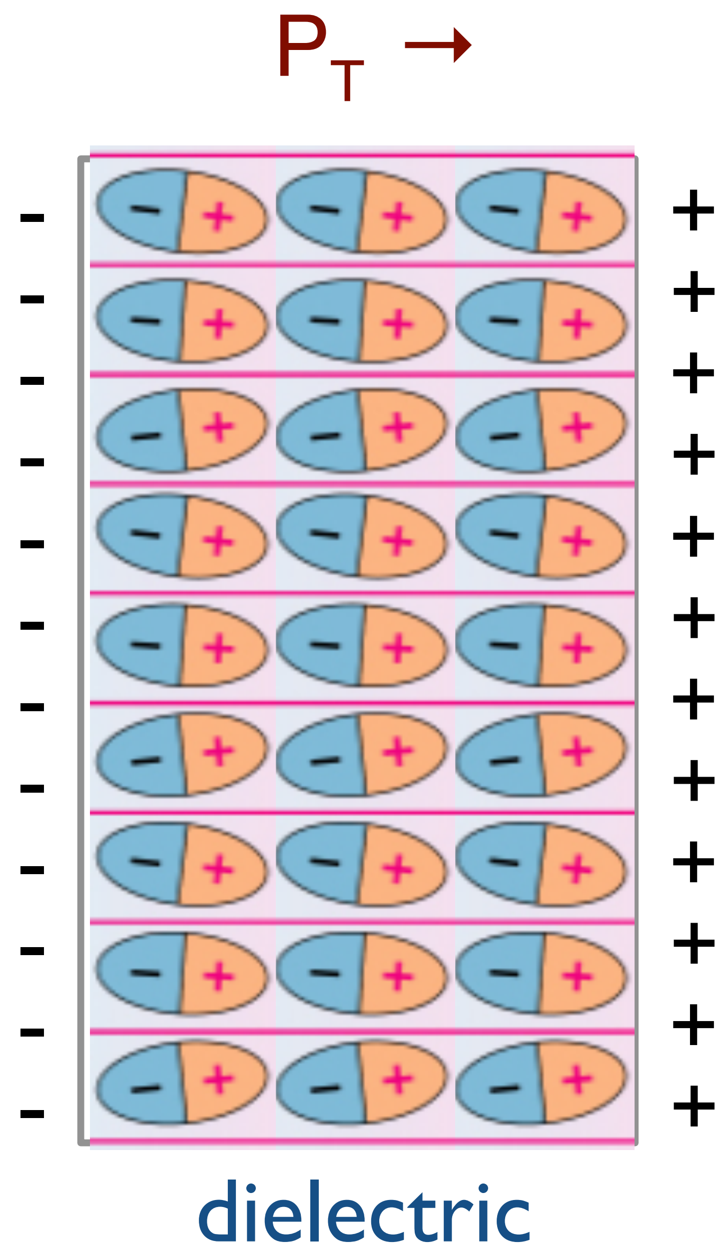
$P_T \rightarrow$



dielectric

charge neutrality

Consider a finite patch: introduce edges



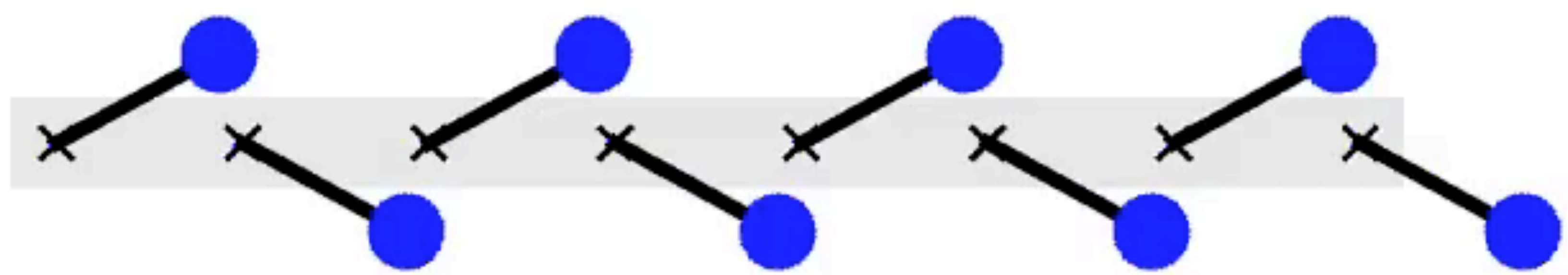
0

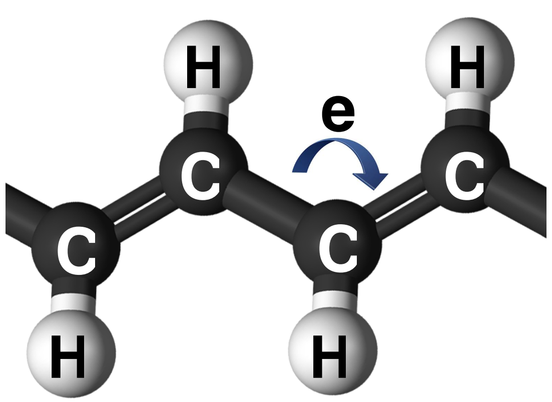
degrees of freedom - #constraints = n_{zm} - n_{ss}

Flux of polarization gives net charge

INSTITUUT LORENTZ

The simplest topological metamaterial

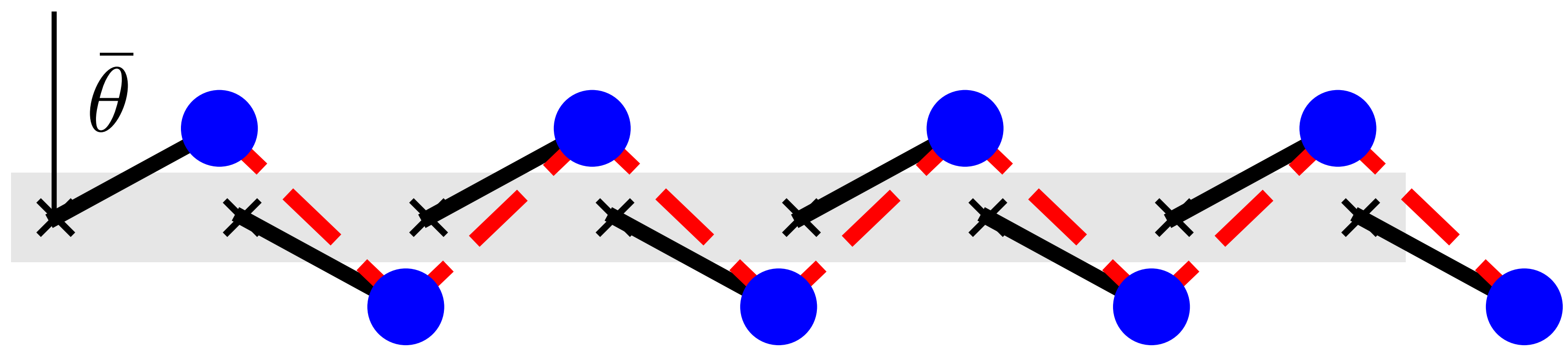




Kane and Lubensky, Nature Physics 2014

The simplest topological metamaterial

$P_T \rightarrow$

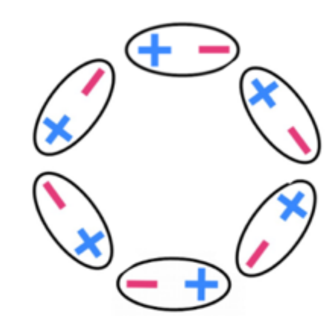


vibrations gapped

$\bar{\theta} \neq 0$

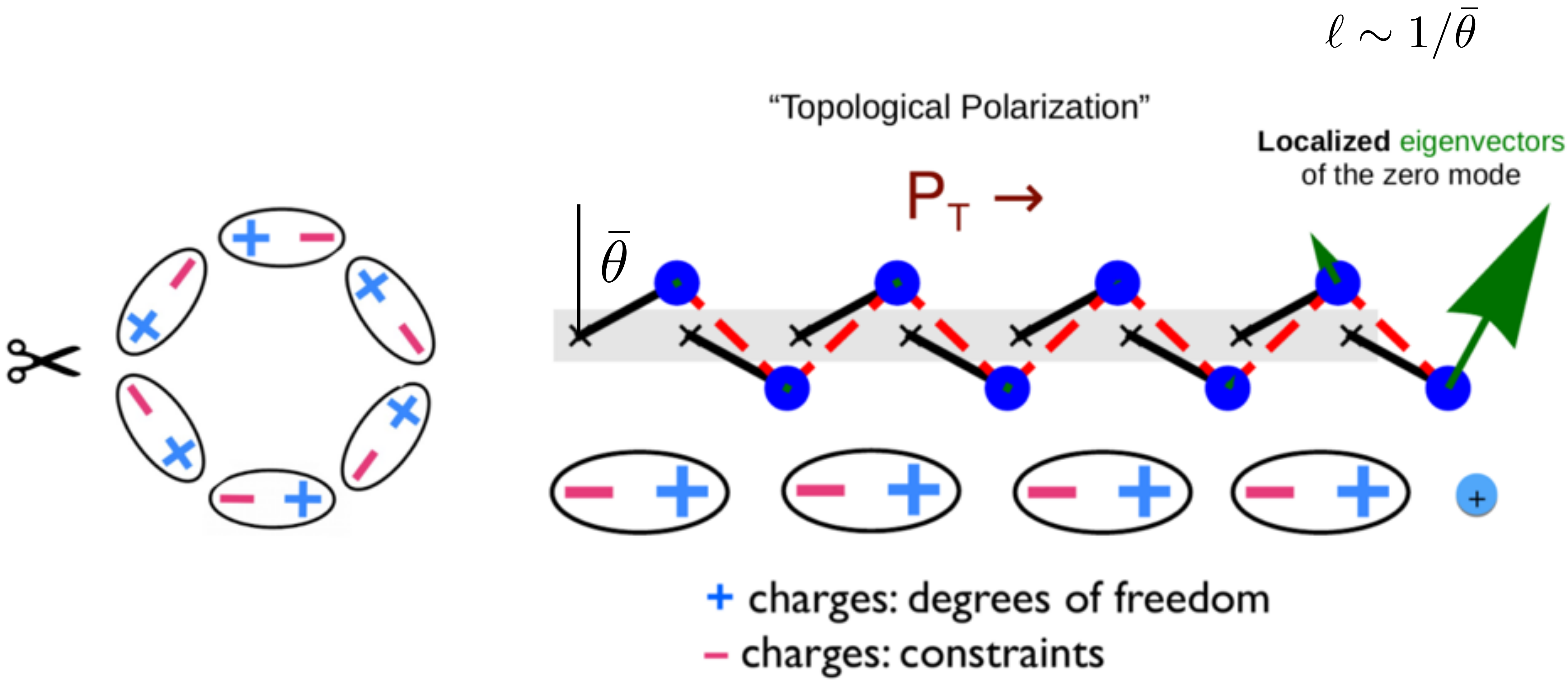
$$\omega^2 = c^2 k^2 + \omega_0^2$$

\# degrees of freedom = # constraints, in the bulk



Kane and Lubensky, Nature Physics 2014

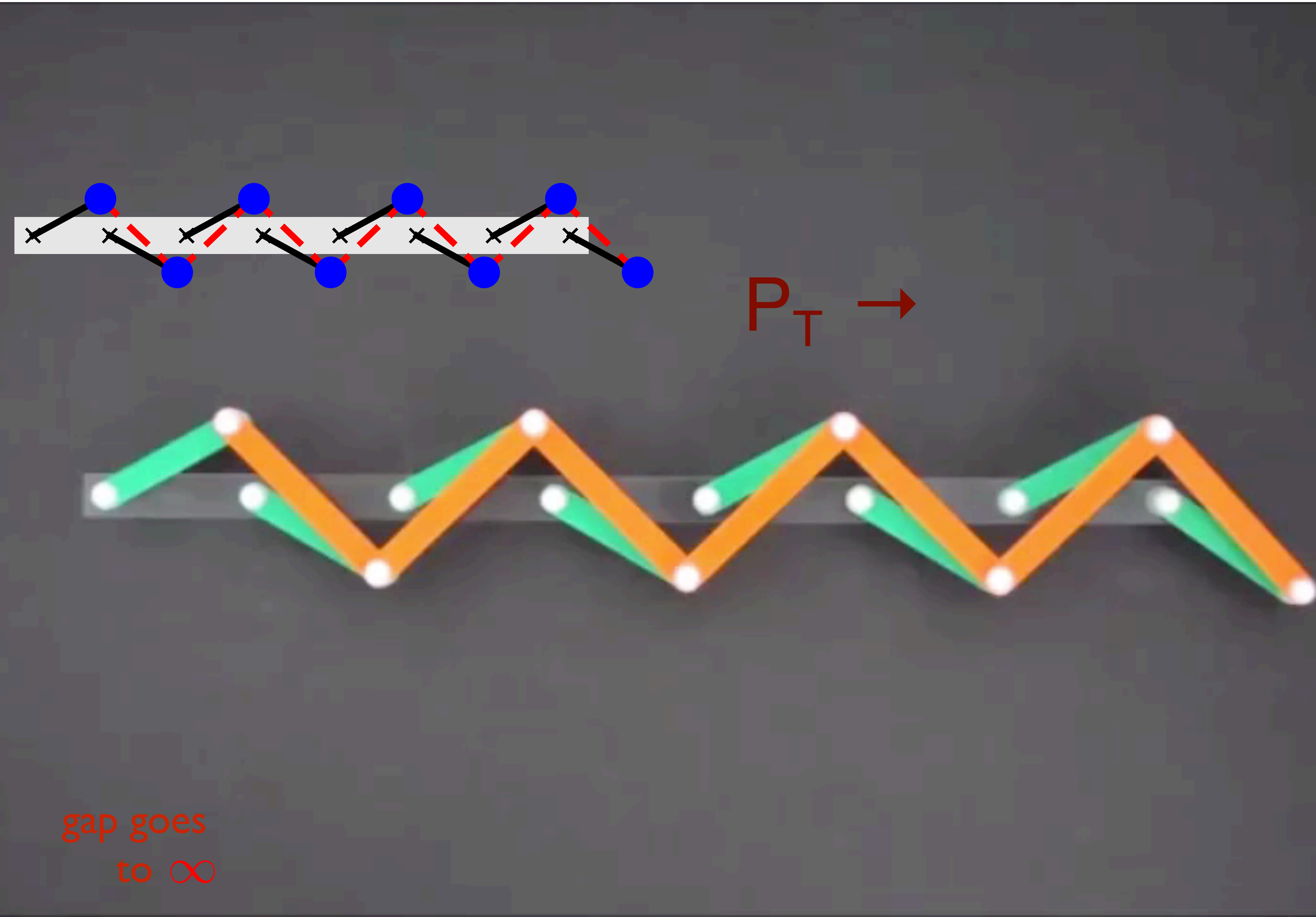
The simplest topological metamaterial



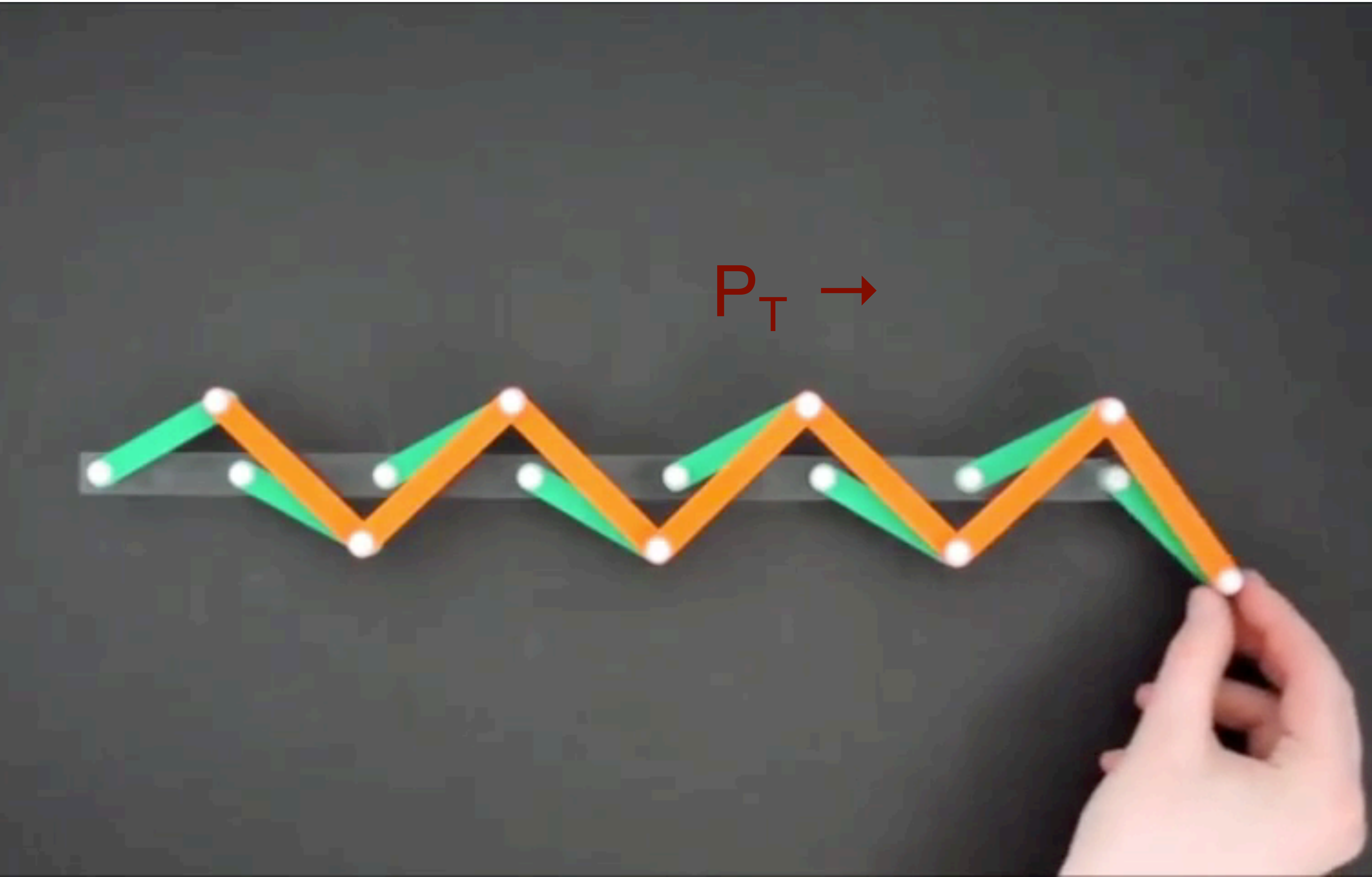
Zero energy vibrational mode localized at only one edge

Kane and Lubensky, Nature Physics 2014

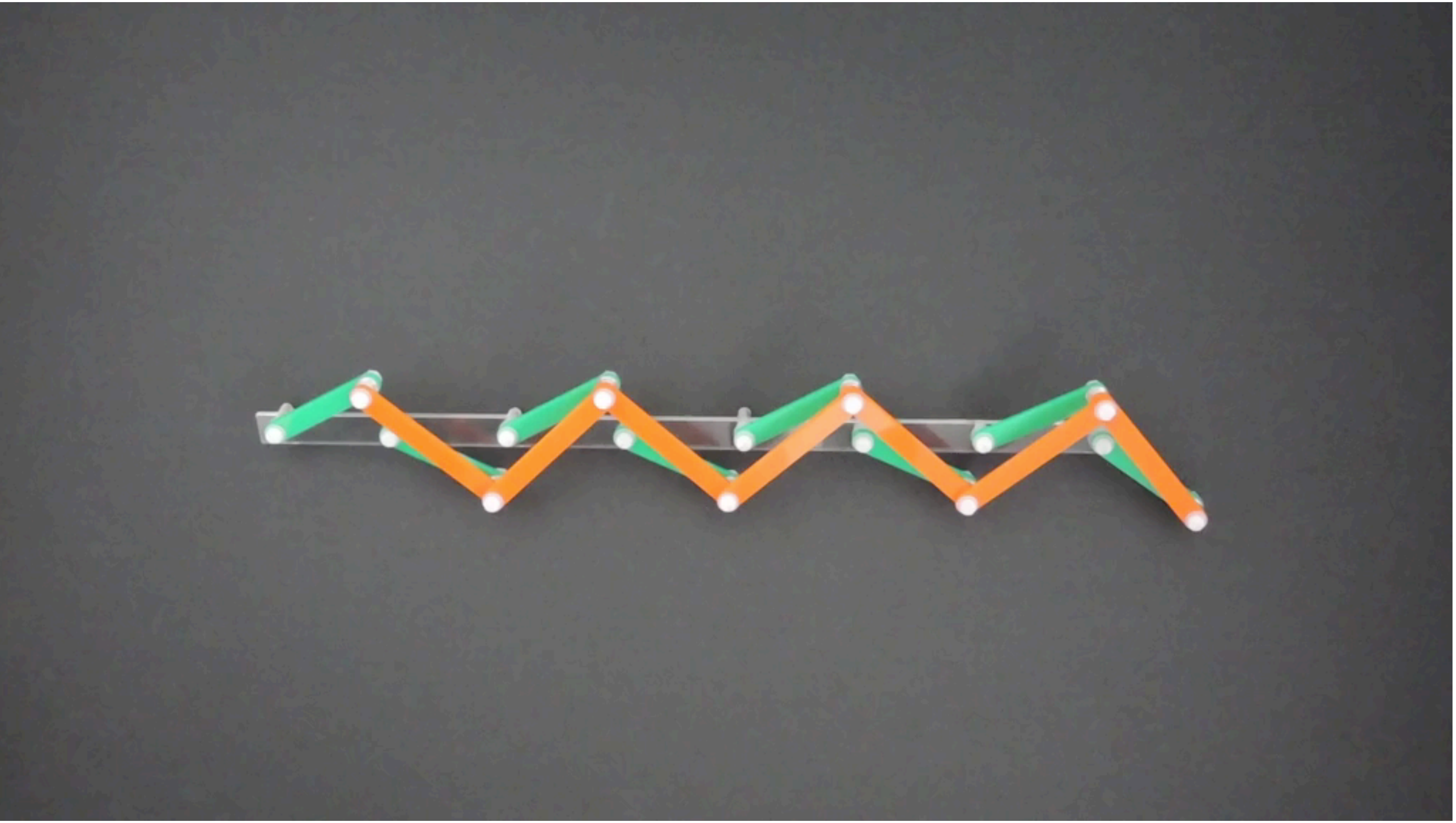
Linkages: 1D origami



Soft motion localized at right edge chosen by P_T



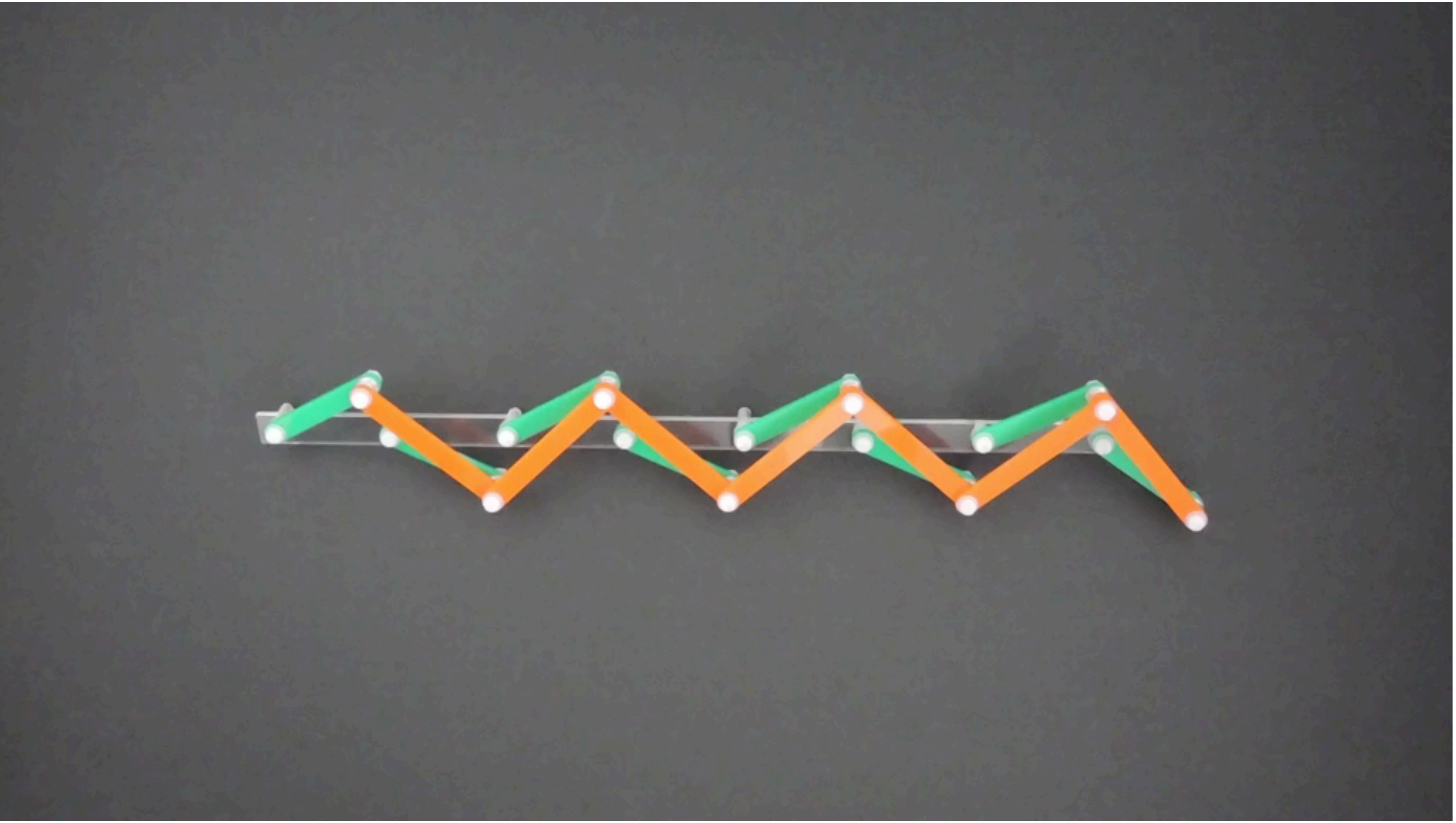
Mechanical insulator within harmonic theory



What happens when we excite the zero energy mode ?

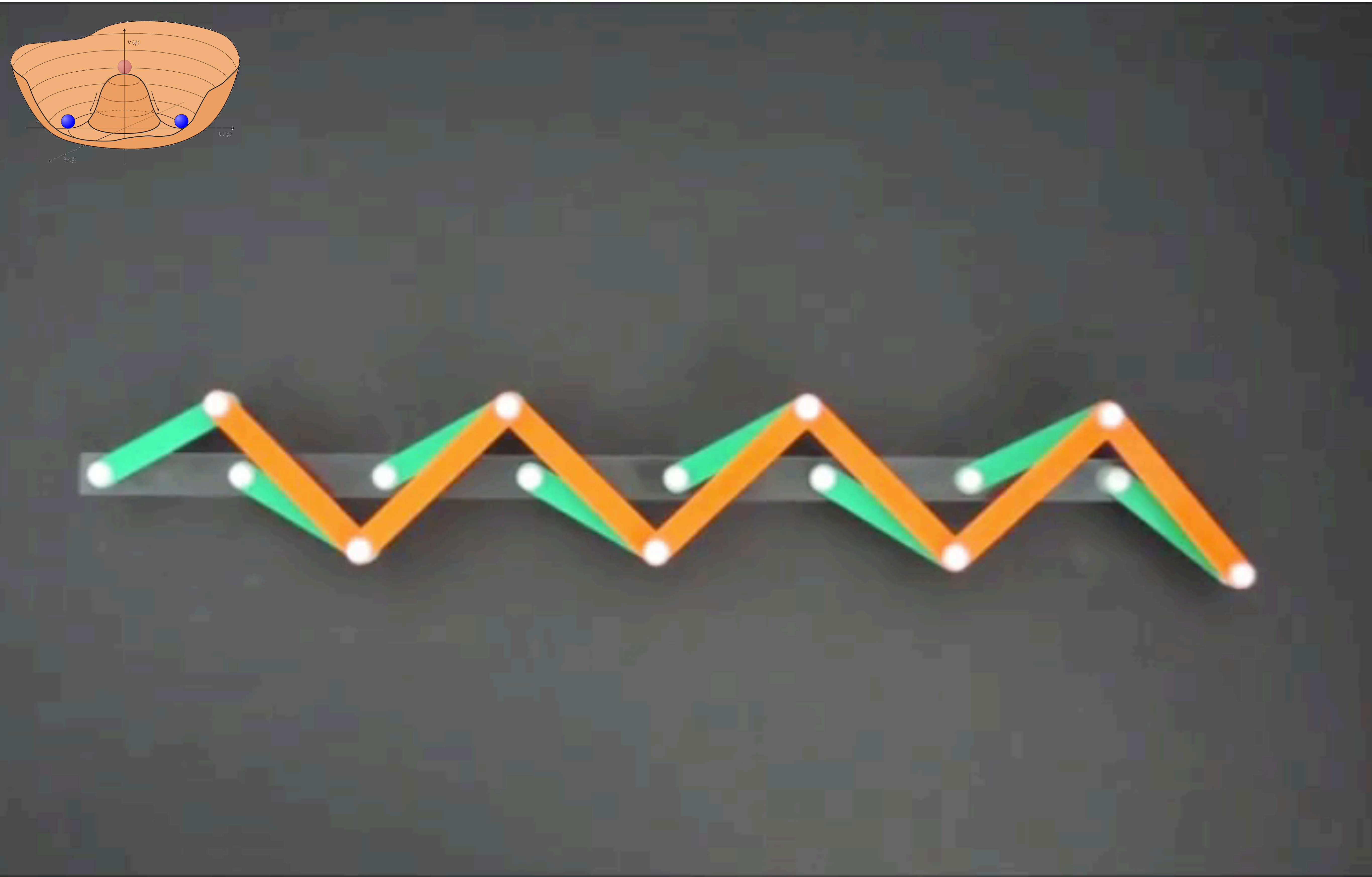
go beyond linear analysis

The chain *conducts* mechanical energy !

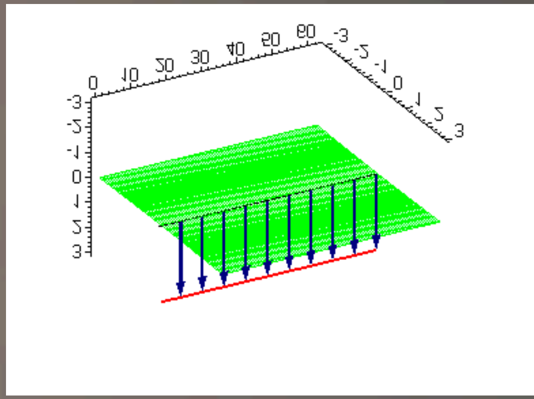


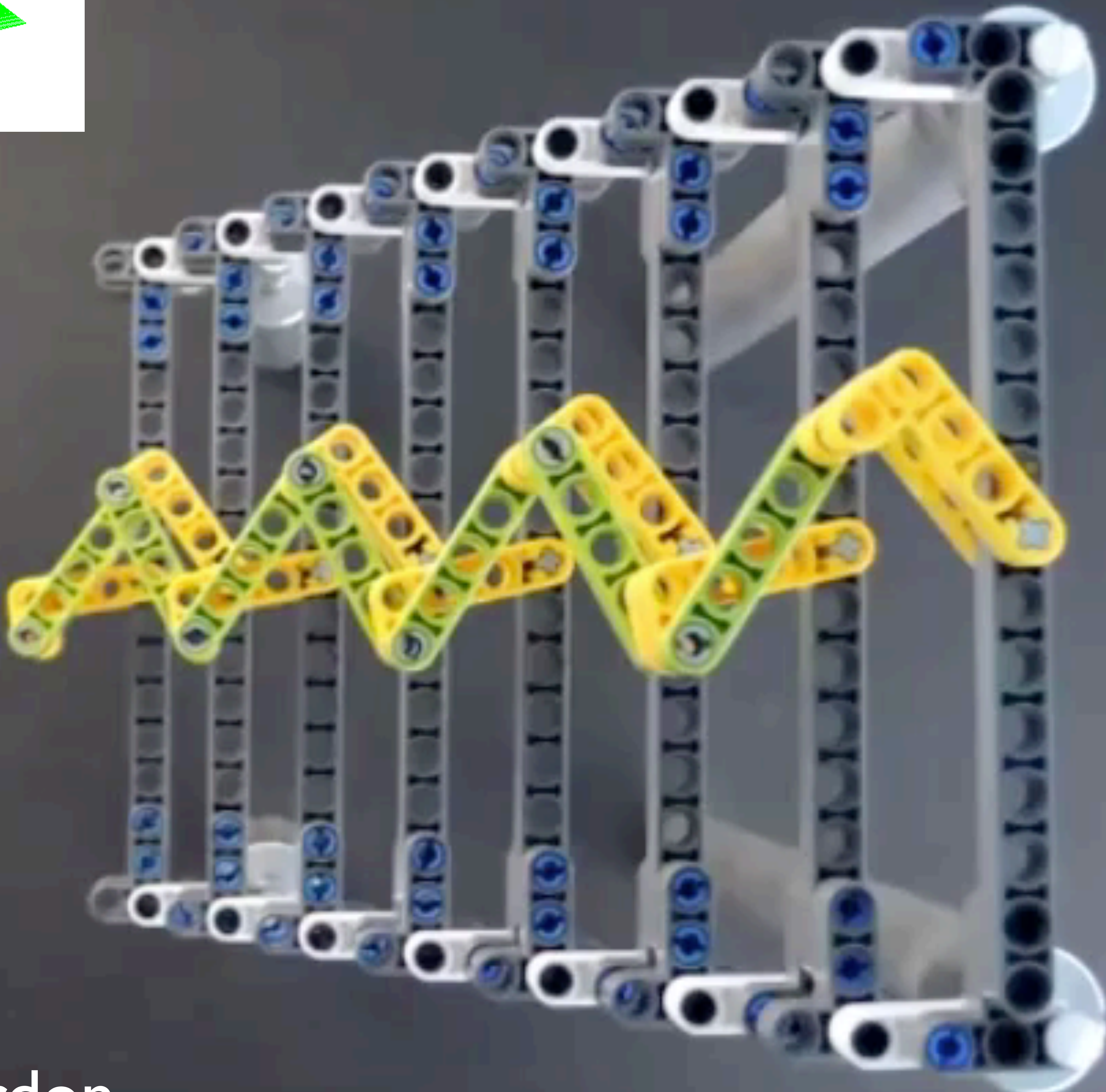
An insulator at harmonic level becomes a conductor
in non-linear theory

How does the edge mode move?

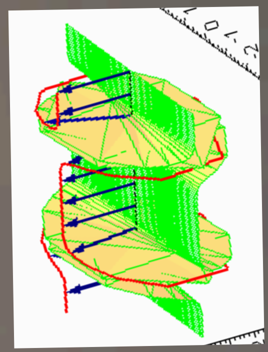


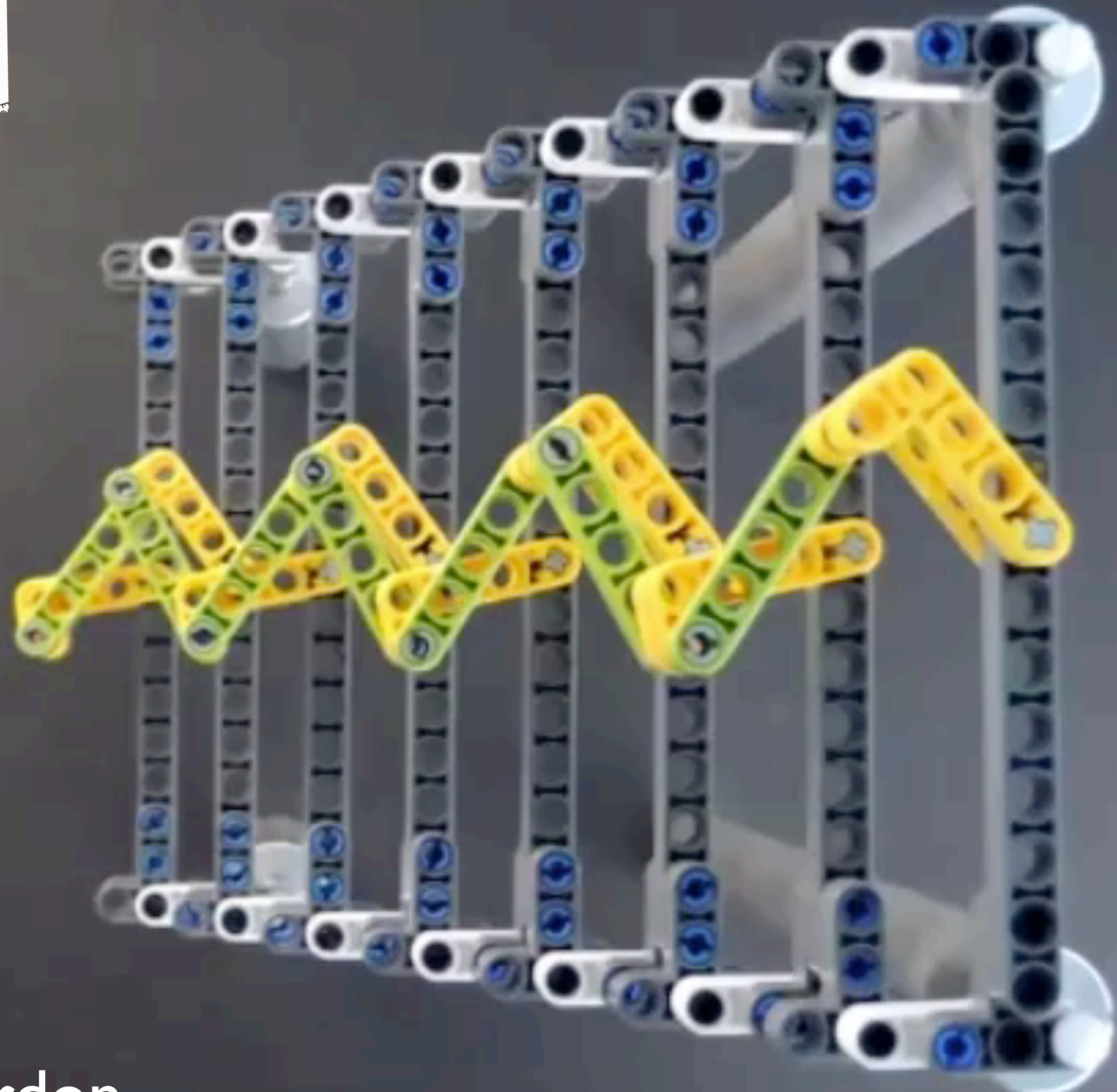
Spinner





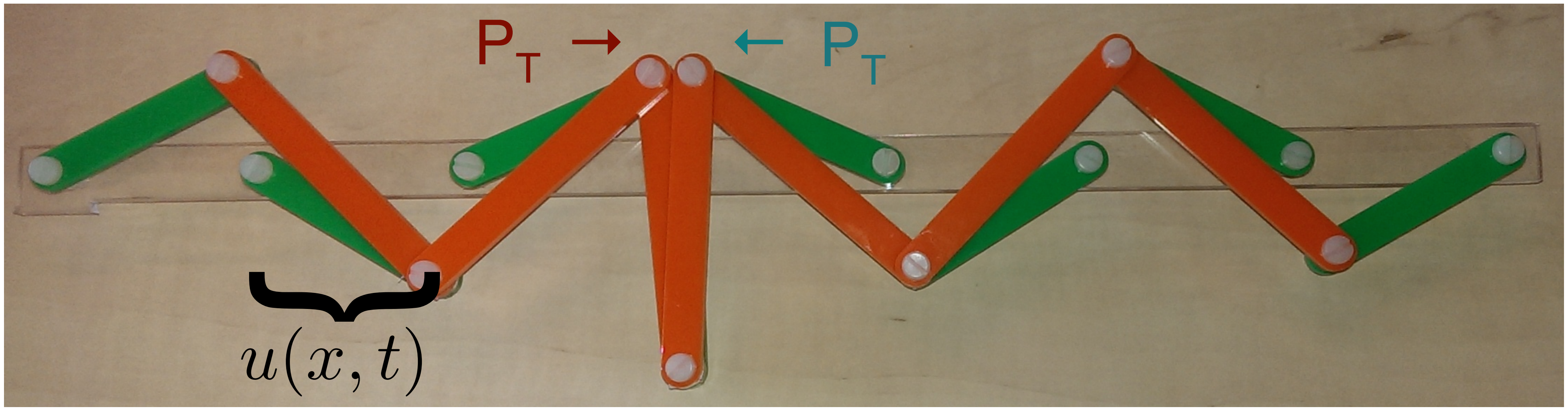
Sine-Gordon

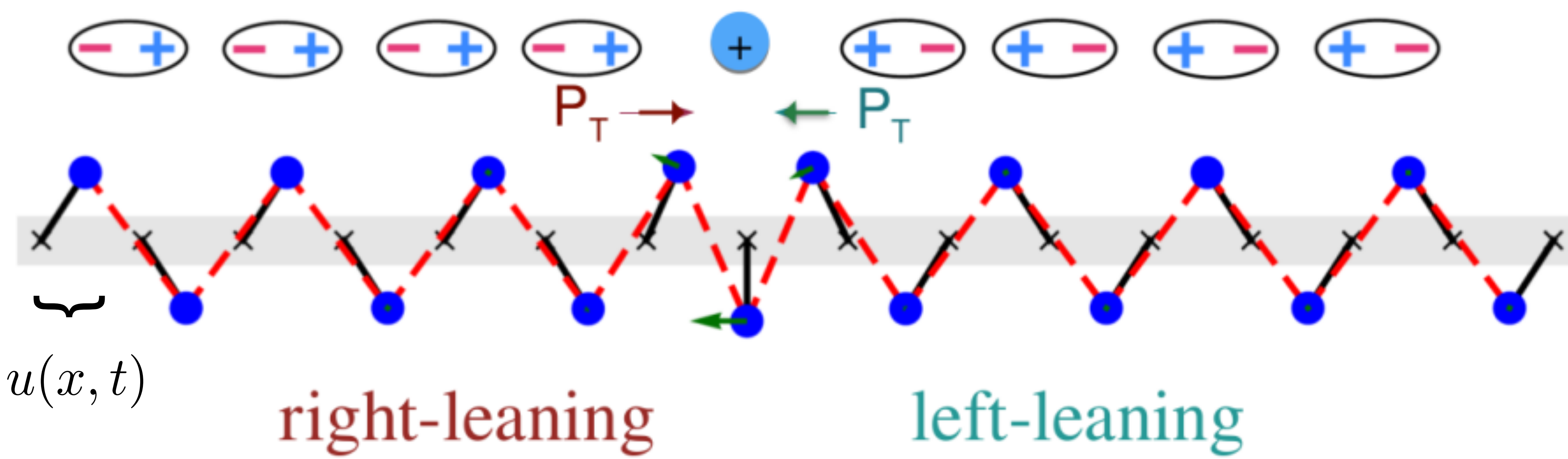




Sine-Gordon

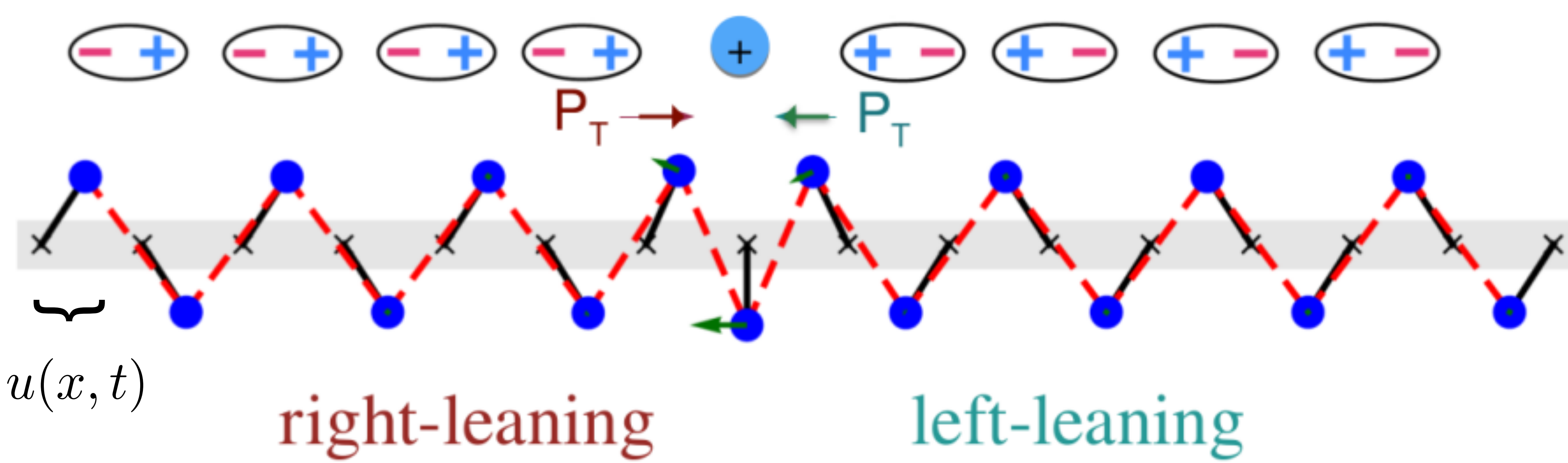
Zero energy kink that harbors a soft motion



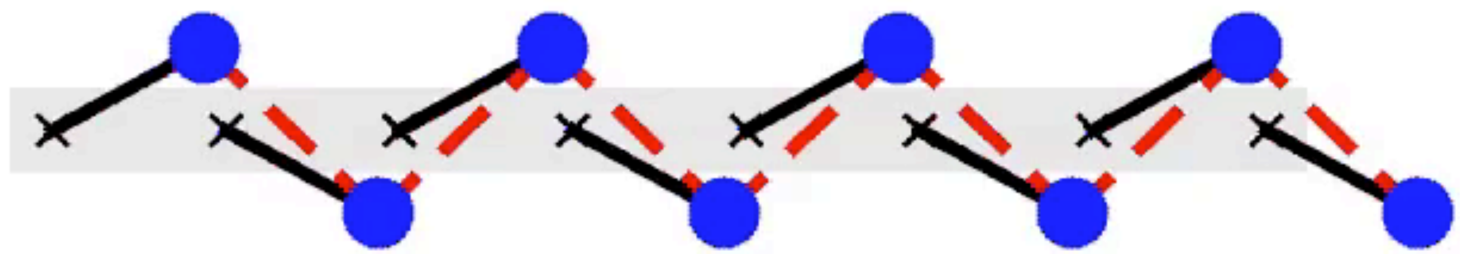


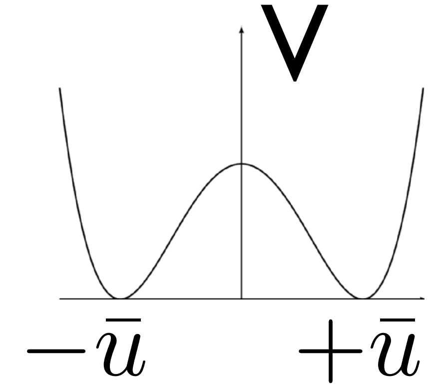
Zero energy kink that harbors a soft motion

degrees of freedom - #constraints = n_{zm} - n_{ss}

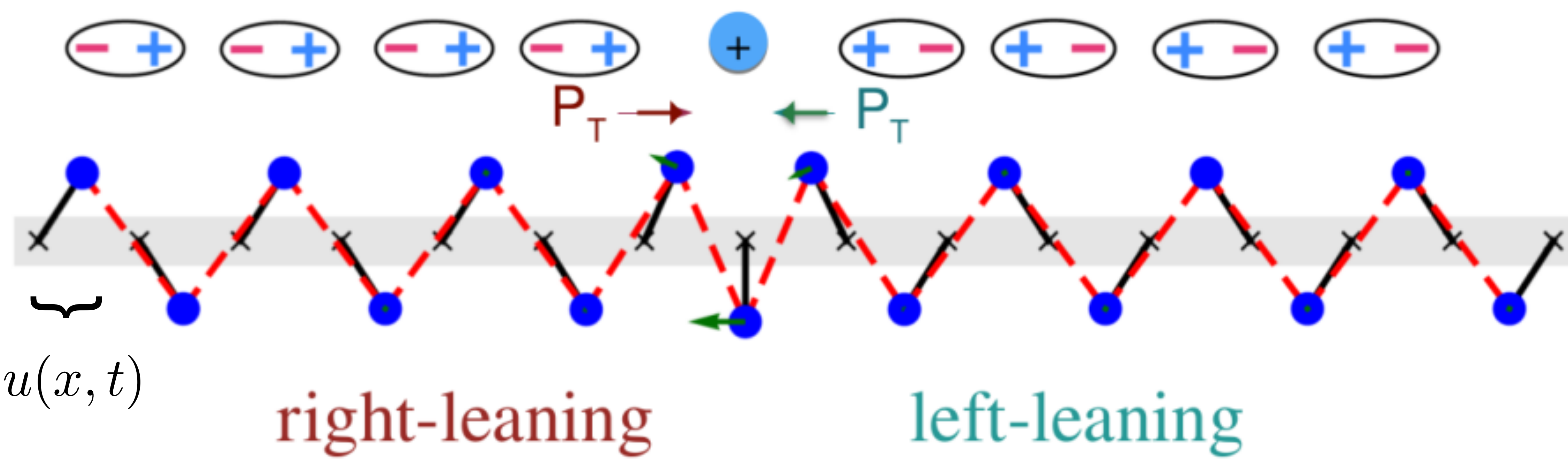


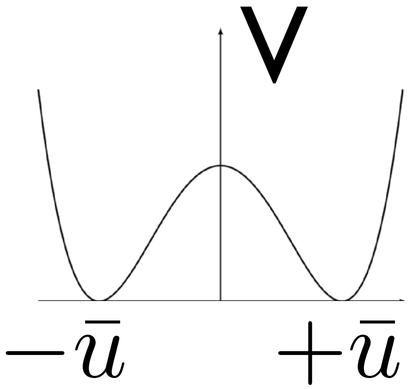
Restore springs



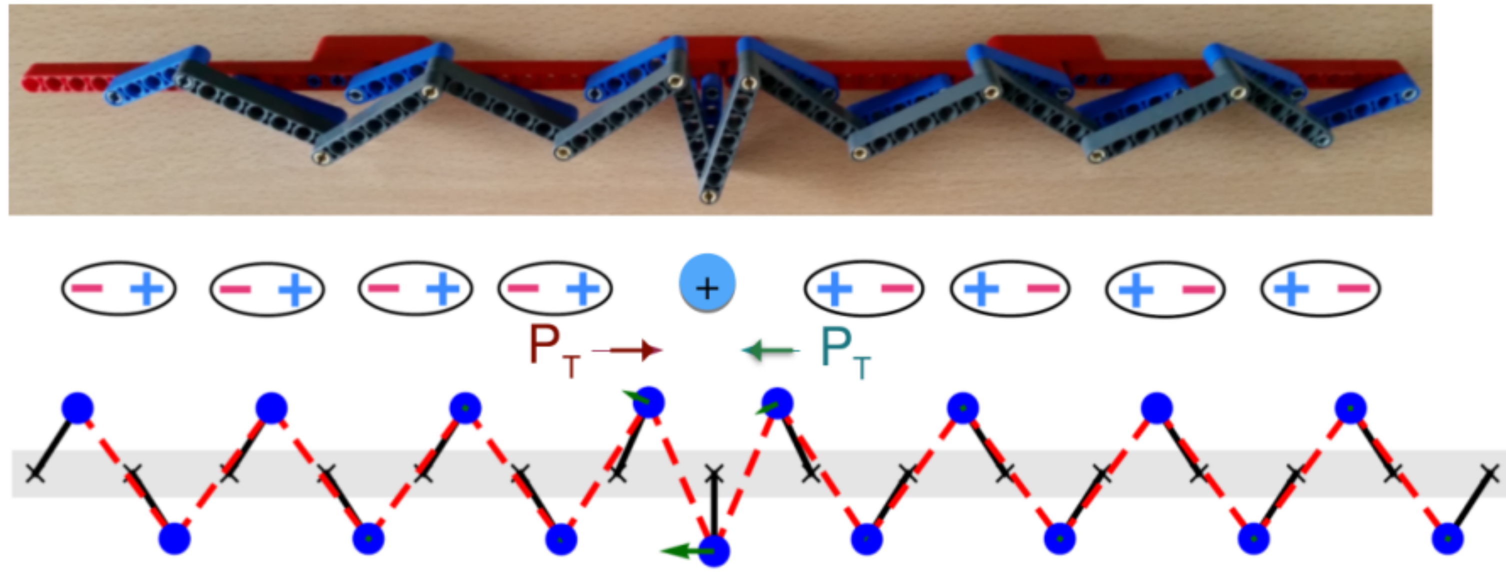


$$V(u) = \mathrm{k}\,(u^2 - \bar{u}^2)^2$$





Continuum theory



right-leaning left-leaning

E cannot be zero unless $u = \pm\bar{u}$

spring constant

potential energy

$$E = k \int dx \left[\left(\frac{\partial u}{\partial x} \right)^2 + (u^2 - \bar{u}^2)^2 \right]$$

$$A^2 \quad + \quad B^2$$

Chen, Upadhyaya, Vitelli, *PNAS* (2014).

initially ignore kinetic term

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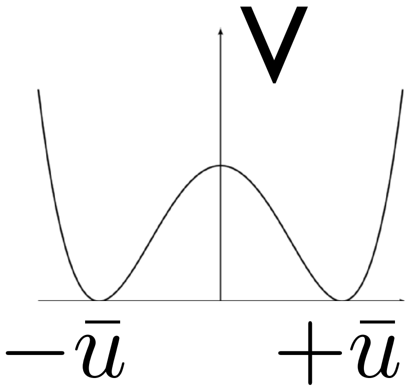
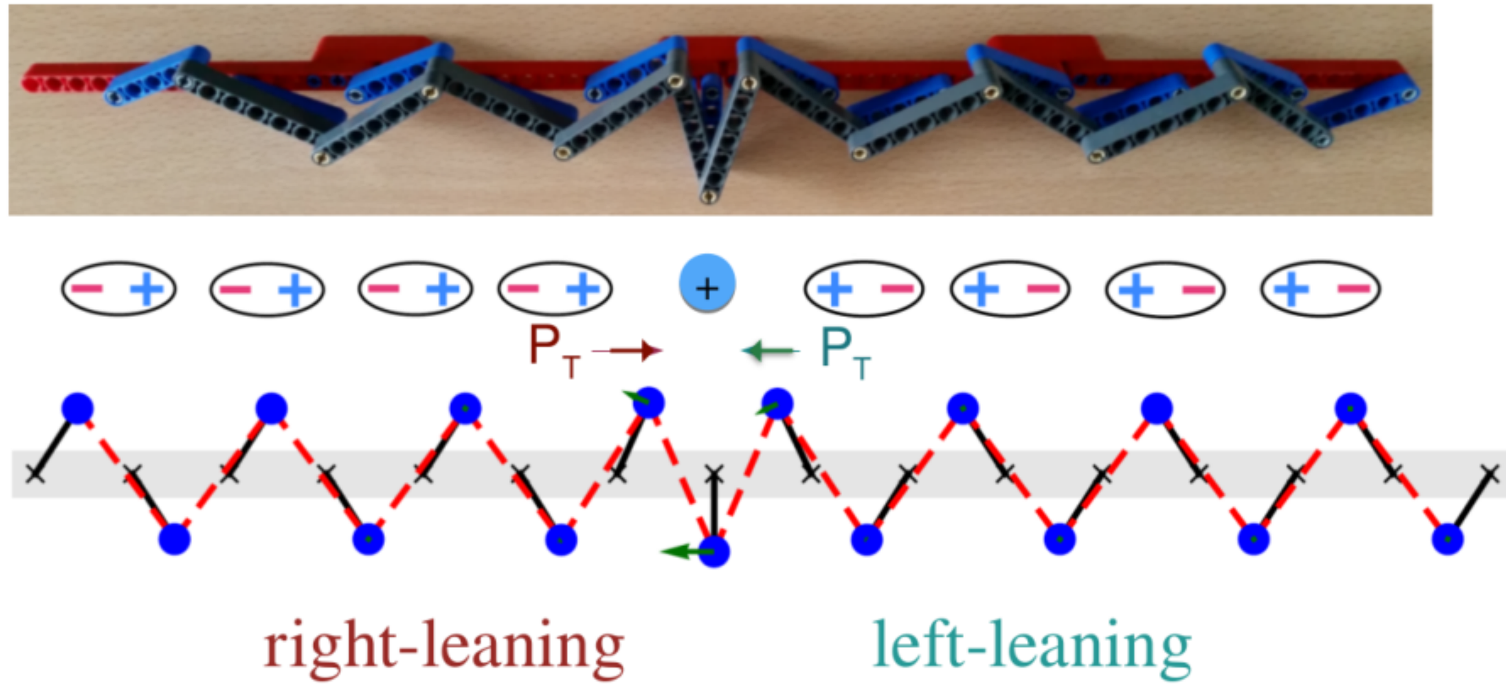
Topological boundary term





BPS state

construct a perfect square

$$E = k \int dx \left[\left(\frac{\partial u}{\partial x} \right)^2 + (u^2 - \bar{u}^2)^2 \qquad \right]$$

$$A^2 \quad + \quad B^2 \quad \pm \quad 2AB$$

Chen, Upadhyaya, Vitelli, *PNAS* (2014).

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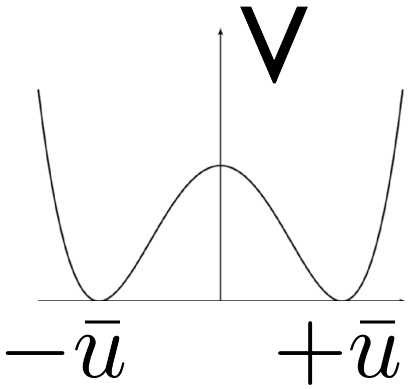
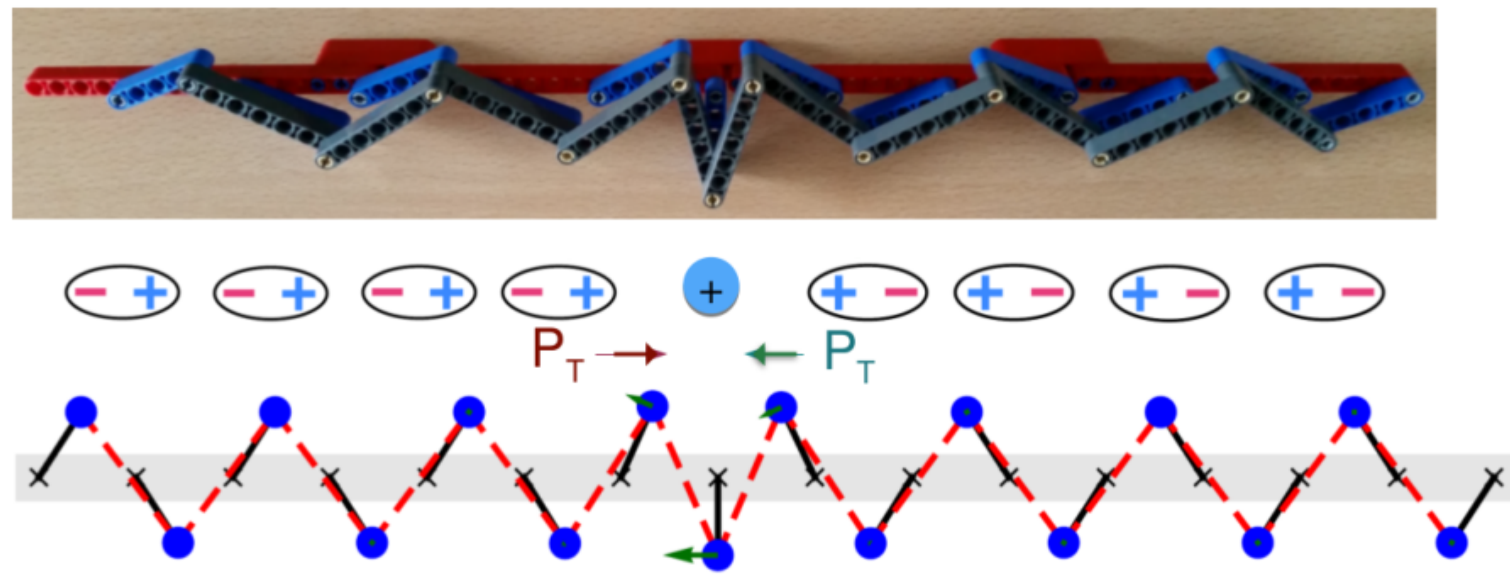


Topological boundary term



right-leaning left-leaning

BPS state

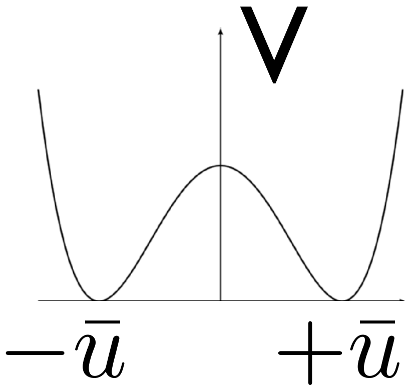
sign of flux of polarization

$$E = \mathrm{k} \int dx \left[\left(\frac{\partial u}{\partial x} \right)^2 + (u^2 - \bar{u}^2)^2 - 2(u^2 - \bar{u}^2) \frac{\partial u}{\partial x} \right]$$

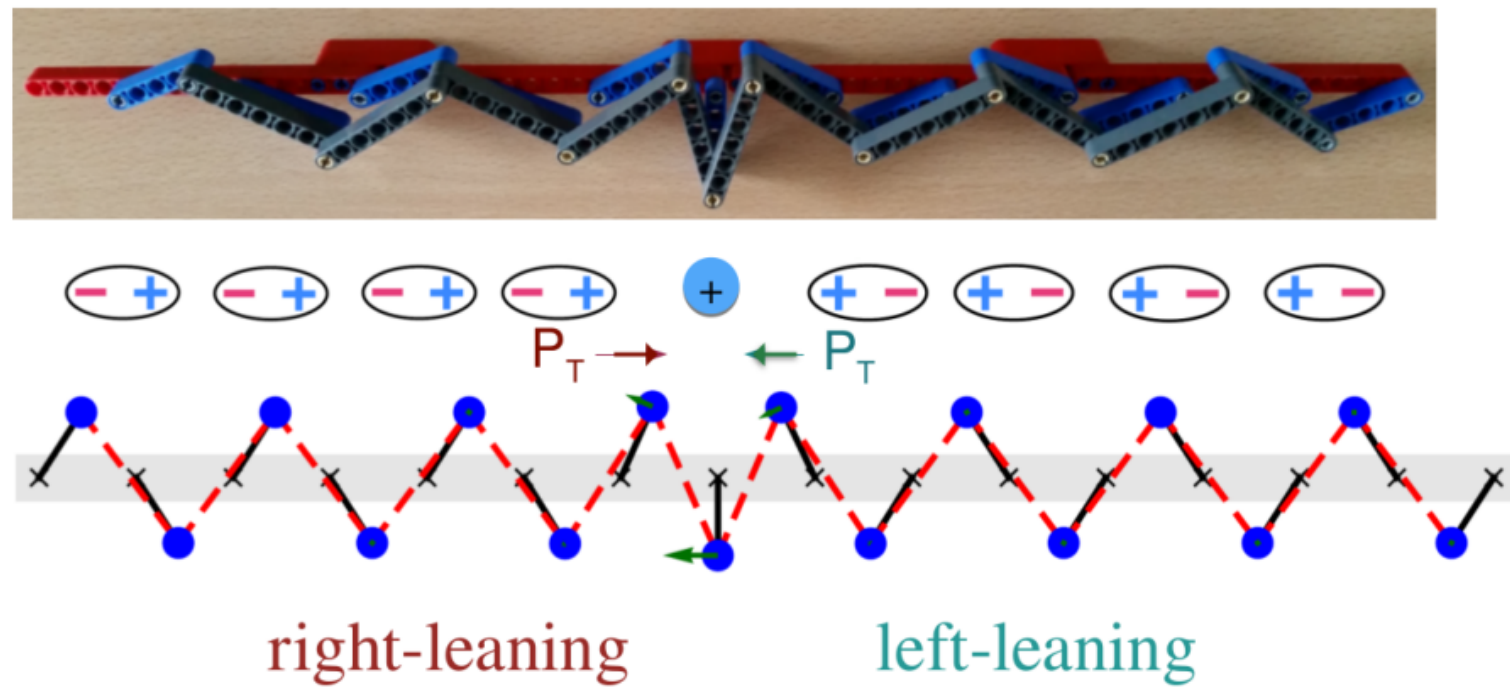
$$A^2 \quad + \quad B^2 \quad - \quad 2AB$$

topological
boundary term

Chen, Upadhyaya, Vitelli, *PNAS* (2014).



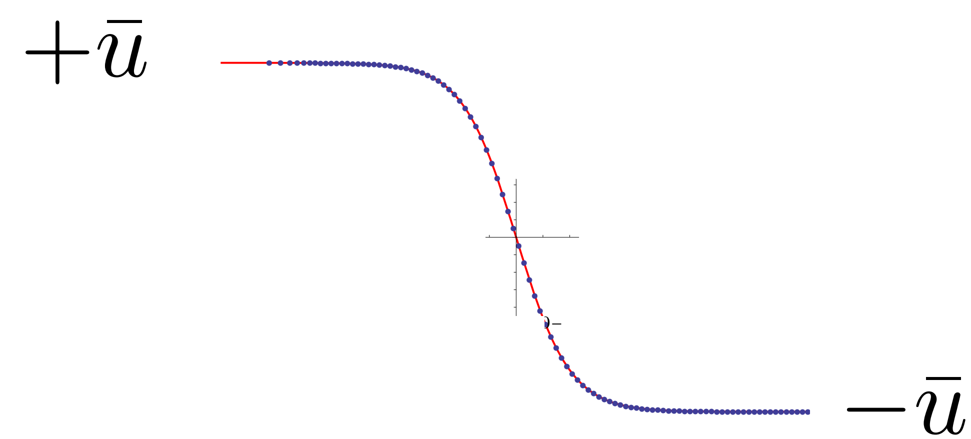
Kink costs zero energy



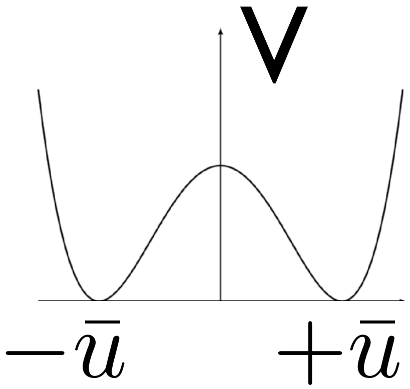
set to 0 gives constraint

$$E = \mathrm{k}\int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2)\right]^2$$

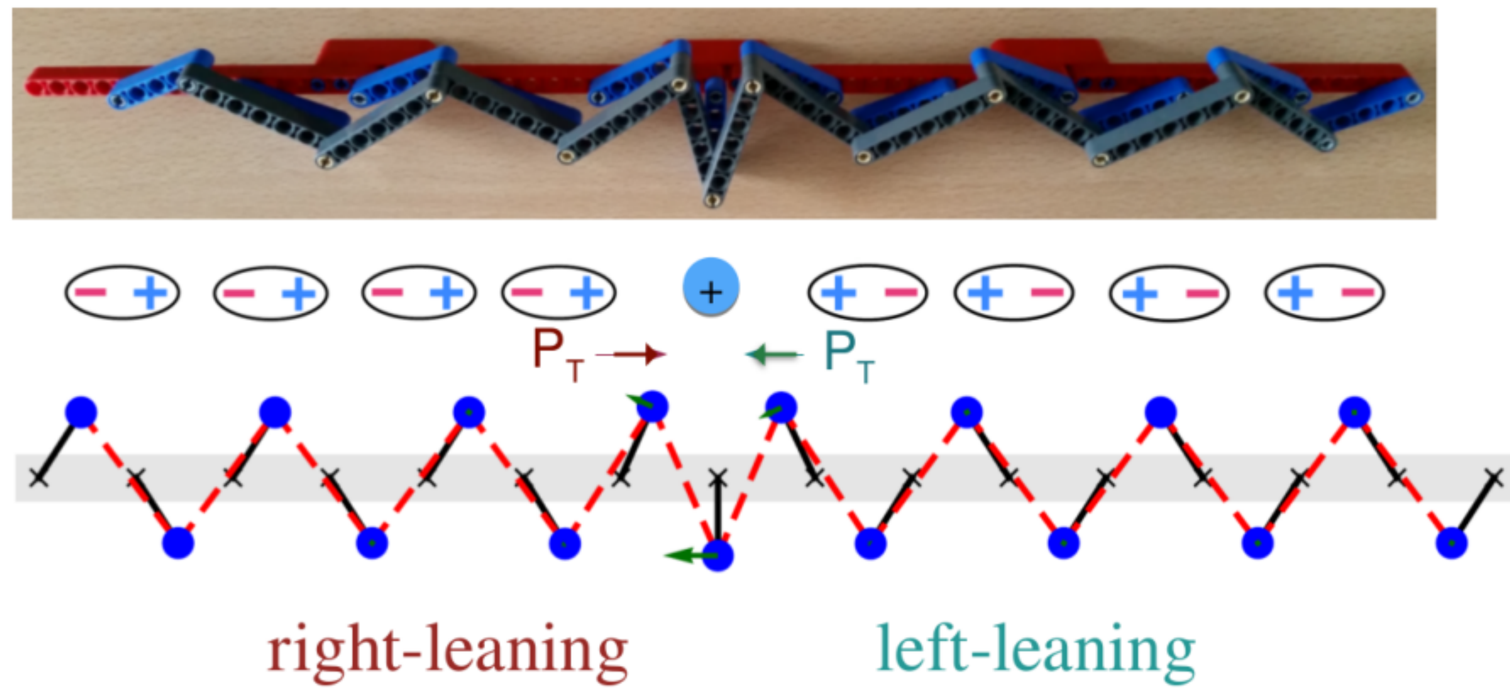
minus sign
chooses kink



Chen, Upadhyaya, Vitelli, *PNAS* (2014).



Kink costs zero energy



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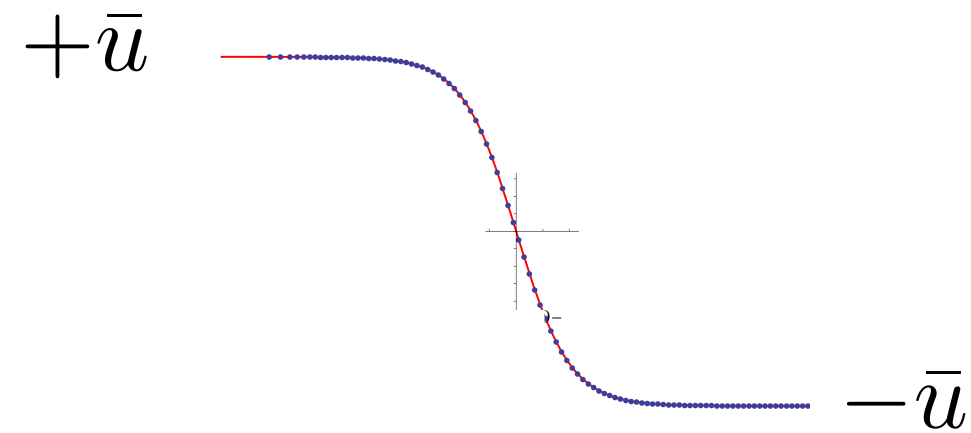
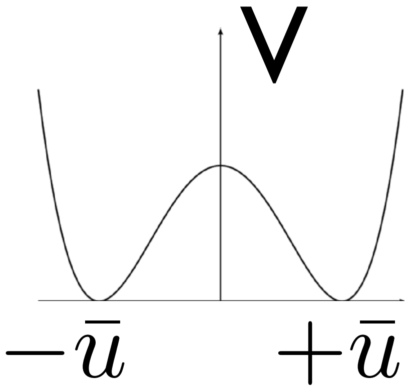
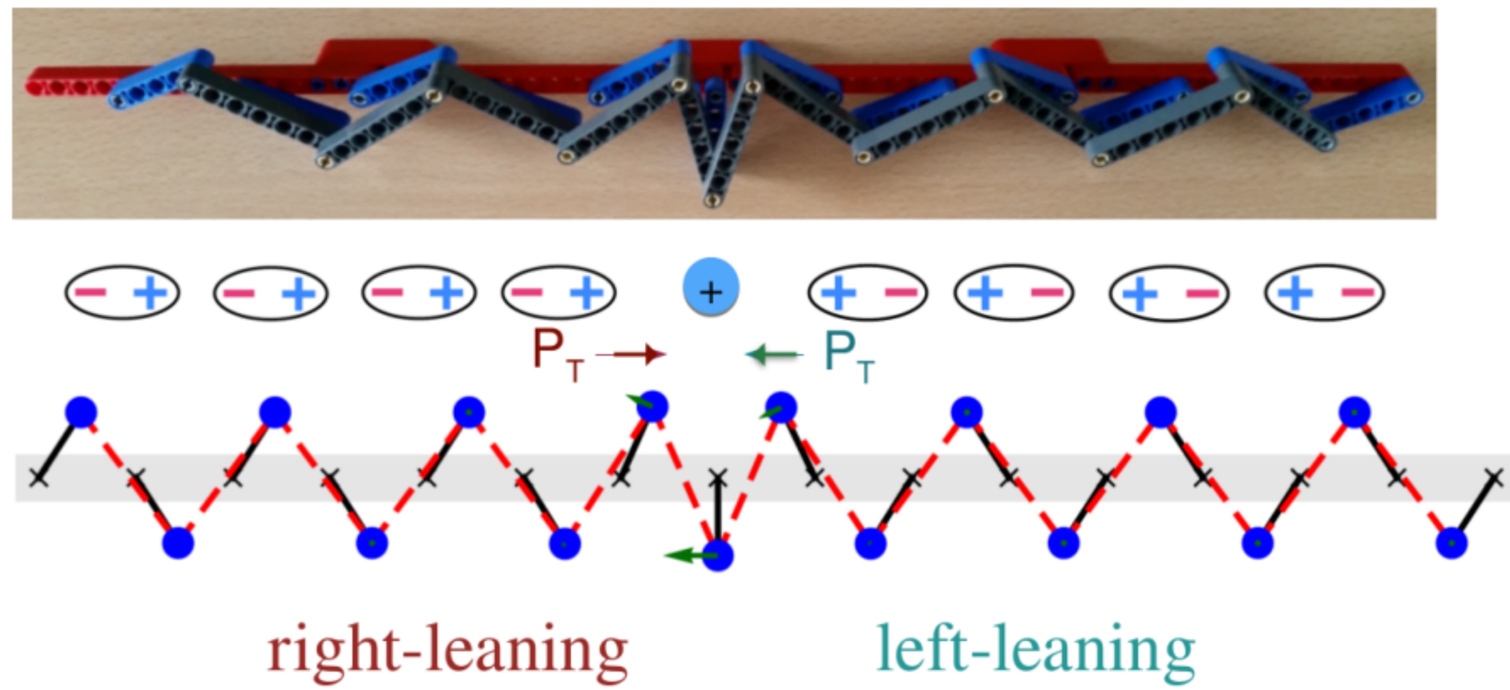


Chen, Upadhyaya, Vitelli, *PNAS* (2014).

Kink costs zero energy

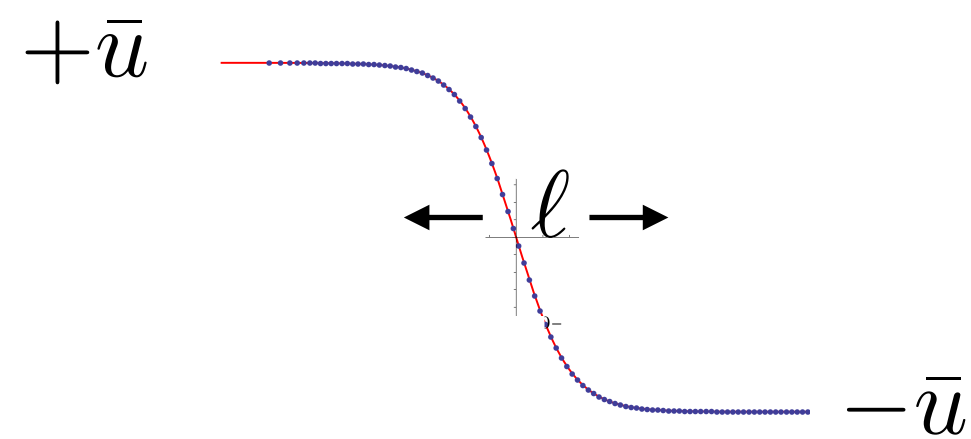




set to 0 gives constraint

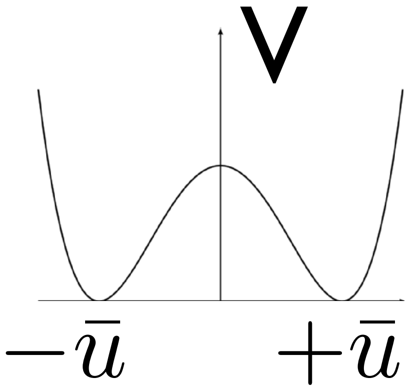
$$E = k \int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2) \right]^2$$

minus sign chooses kink

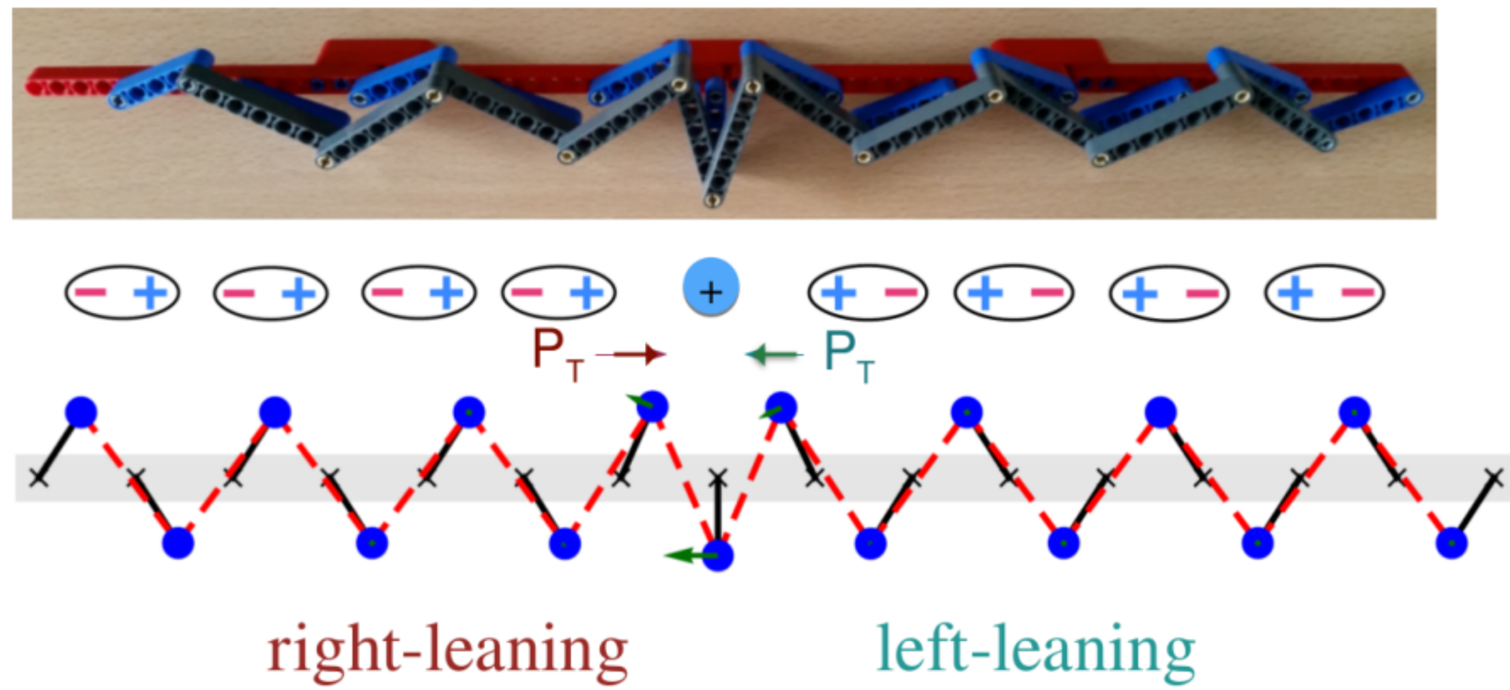


$$\ell \sim 1/\bar{u}$$

Kink width diverges as gap closes



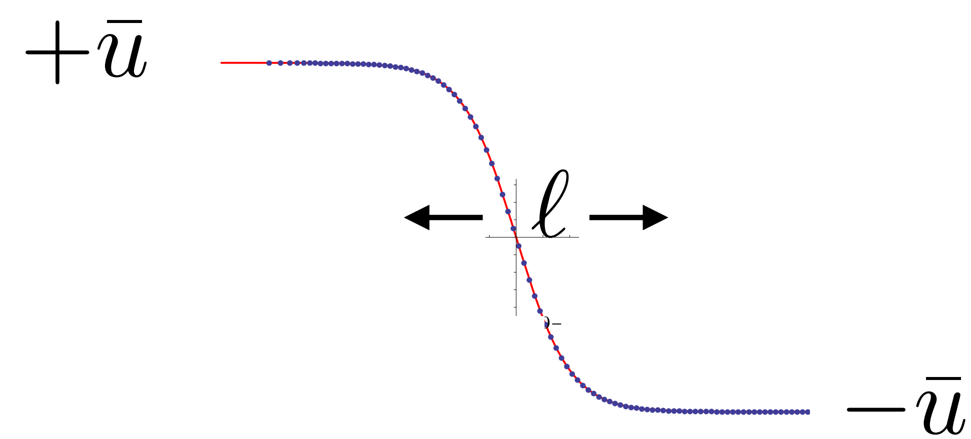
Kink costs zero energy



set to 0 gives constraint

$$E = \mathrm{k} \int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2) \right]^2$$

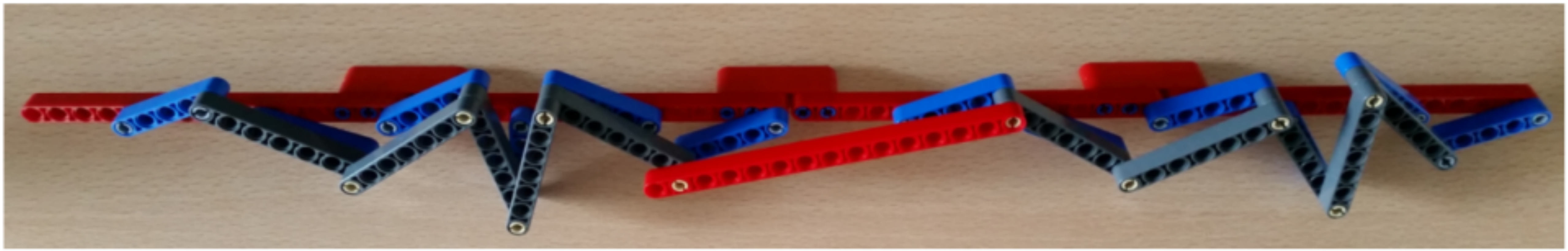
plus sign
chooses antikink

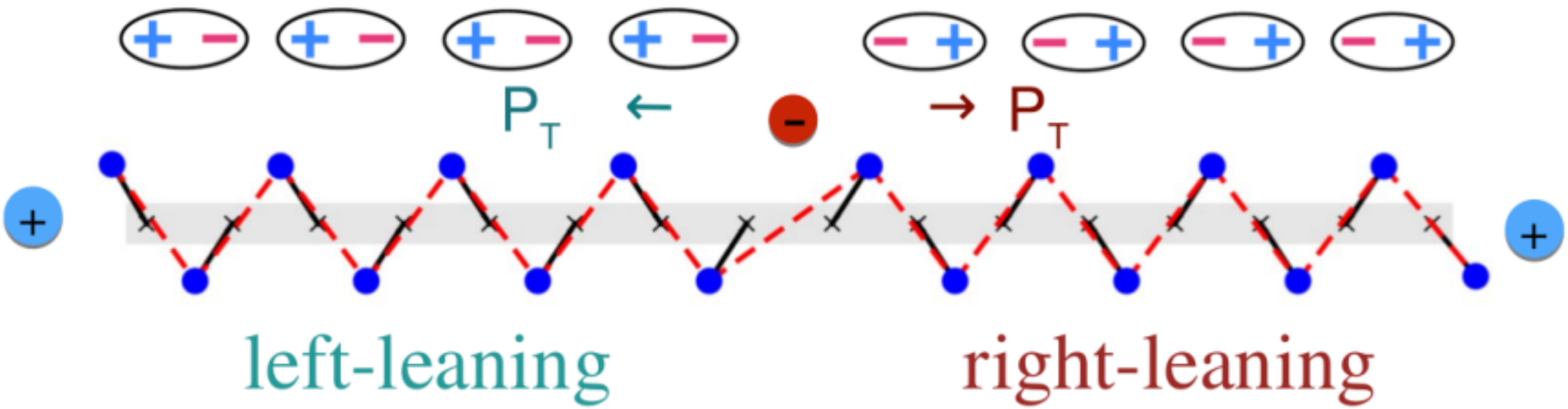


kink costs zero stretching energy ☑

anti kink suppressed ☑

Anti-kinks harbour states of self-stress





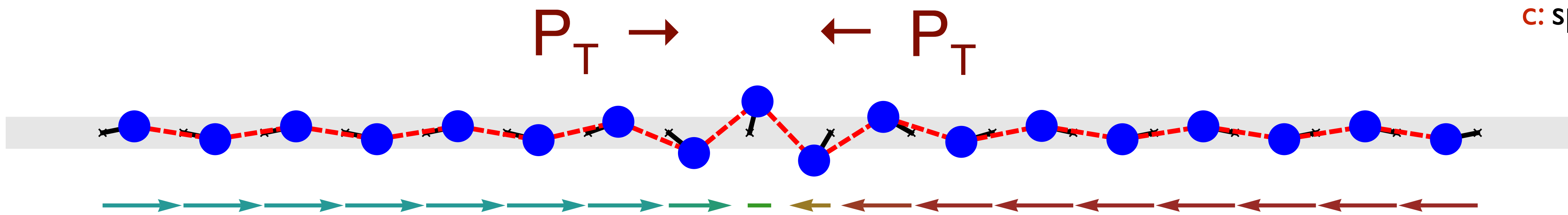
1

2

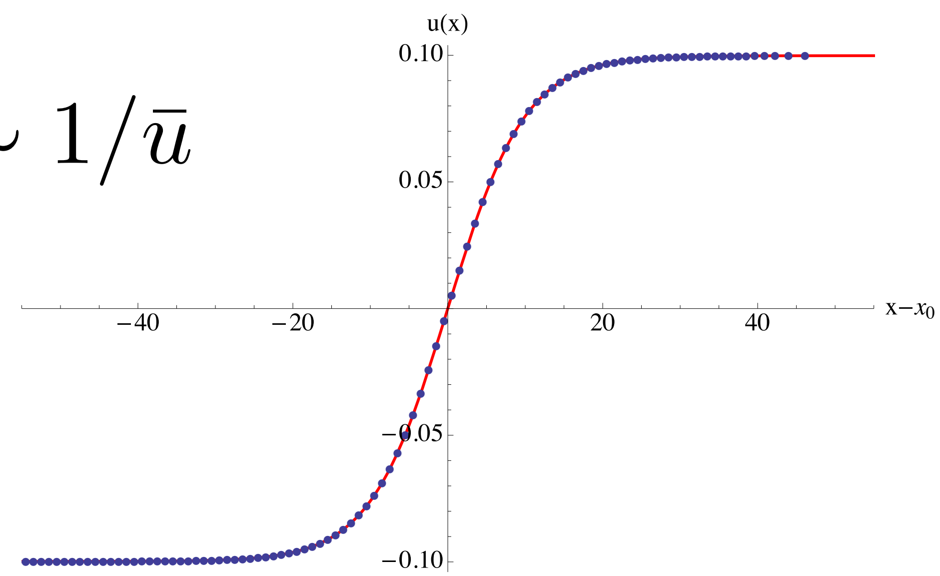
degrees of freedom - #constraints = n_{zm} - n_{ss}

Restore kinetic term

a: lattice spacing
v: kink speed
c: speed of sound



$$\ell \sim 1/\bar{u}$$



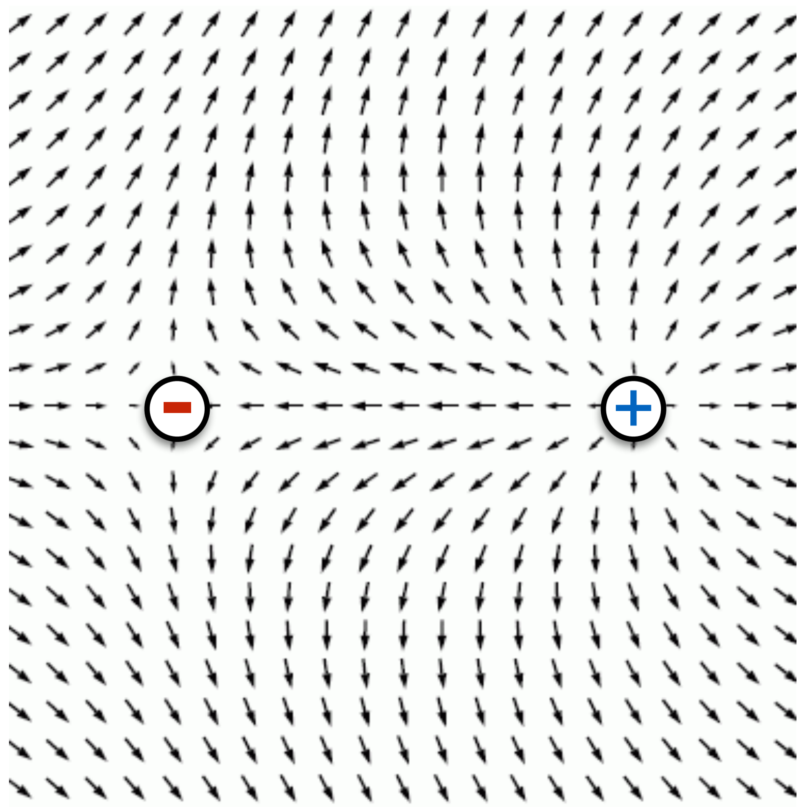
$$u = \bar{u}\tanh\left[\frac{x - x_0 - vt}{\frac{a^2}{2\bar{u}}\sqrt{1-\frac{v^2}{c^2}}}\right]$$

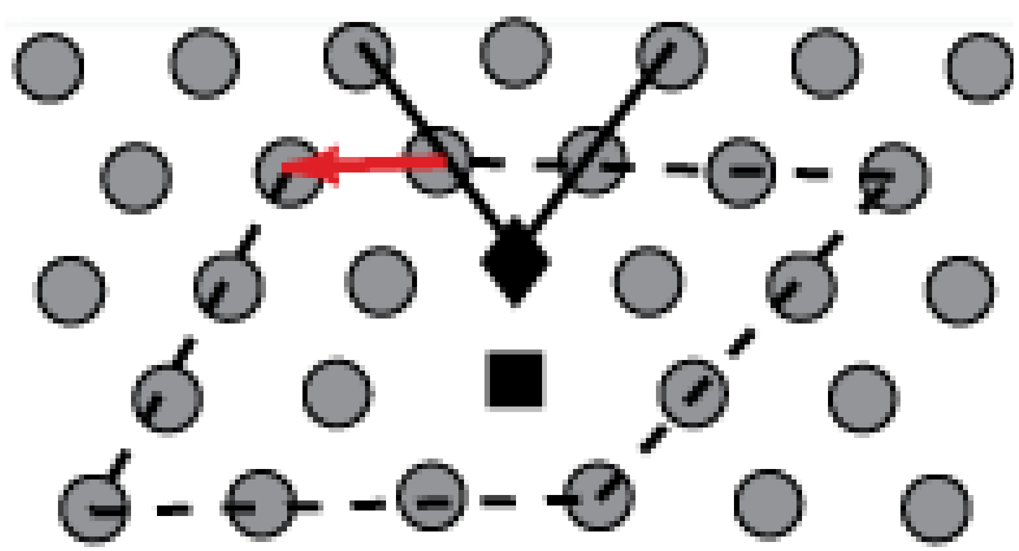
Lorentz contraction of the width

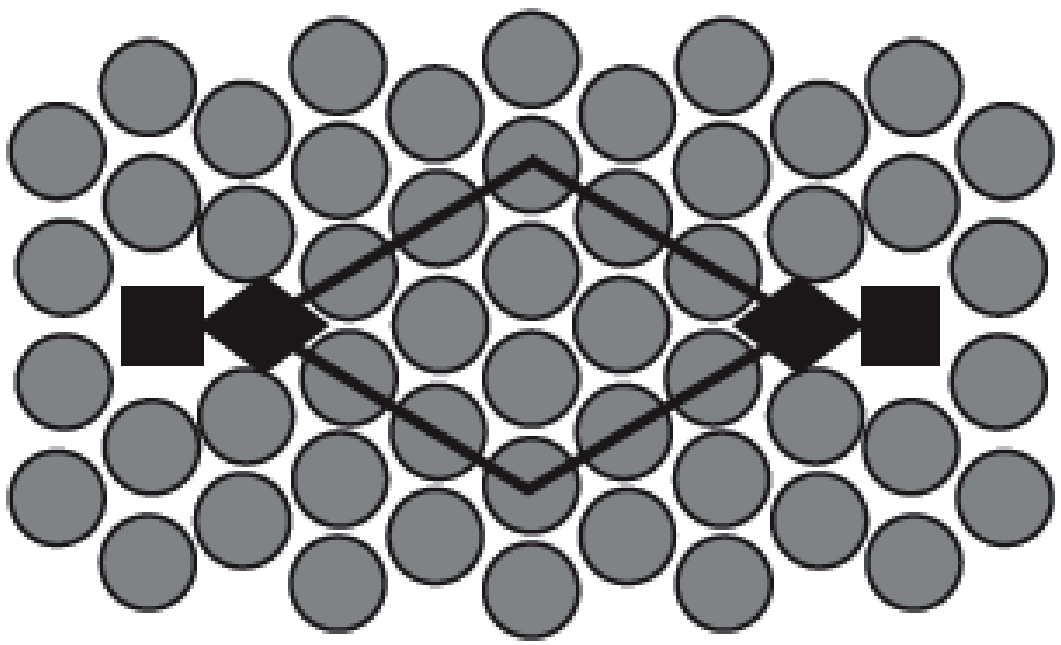
BPS perfect square: enforces constraint

$$\mathcal{L} = \int \frac{1}{2}\frac{Mr^2}{r^2-u^2}\left(\frac{\partial u}{\partial t}\right)^2 - \boxed{\frac{1}{2}K\frac{a^4}{4}\left(\frac{\partial u}{\partial x}\right)^2 - \frac{1}{2}K(\bar{u}^2-u^2)^2 - \quad K\frac{a^2}{2}(\bar{u}^2-u^2)\frac{\partial u}{\partial x}.}\; dx$$

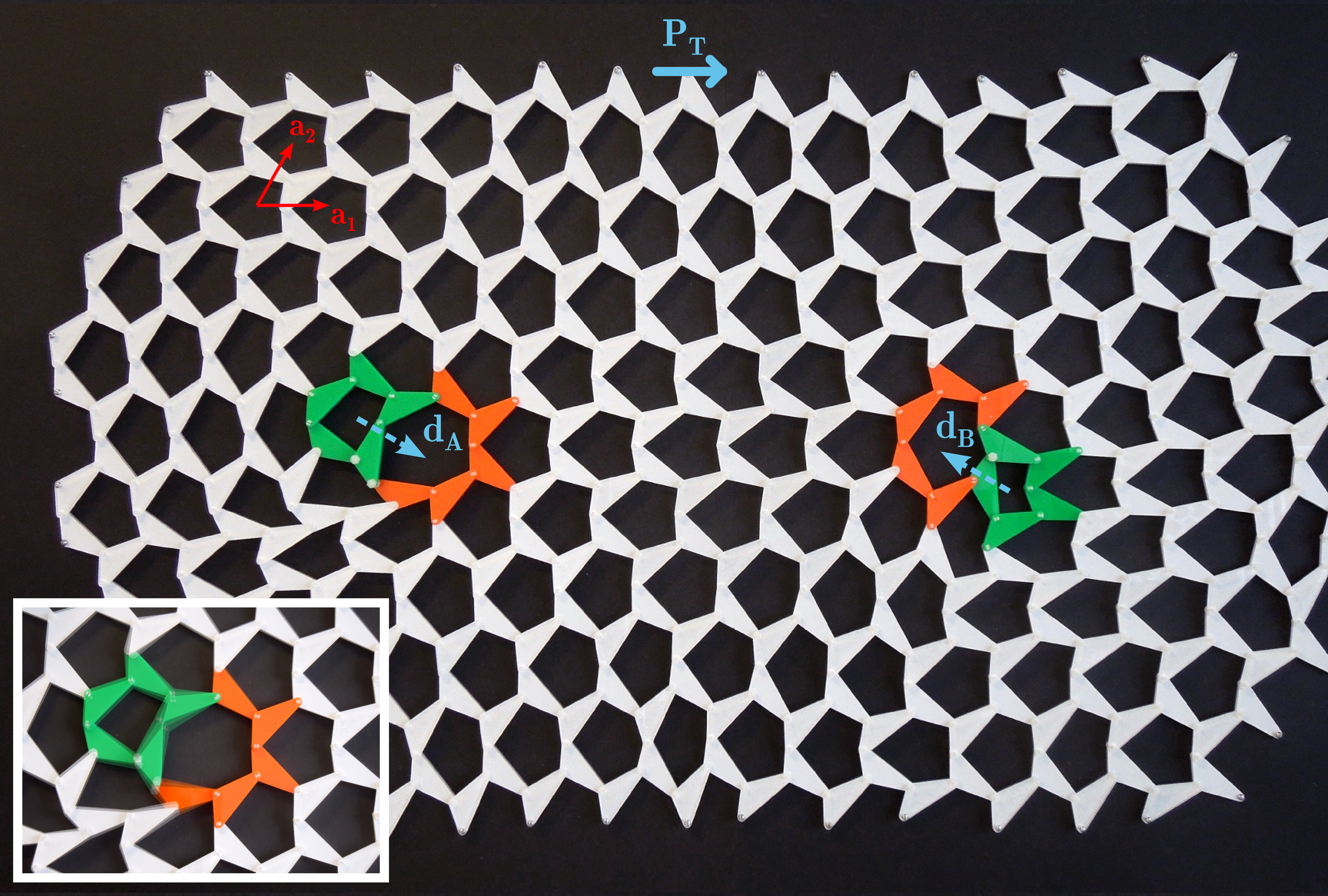
Topological defects

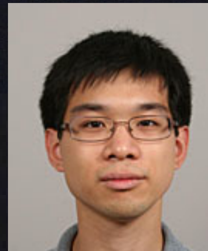






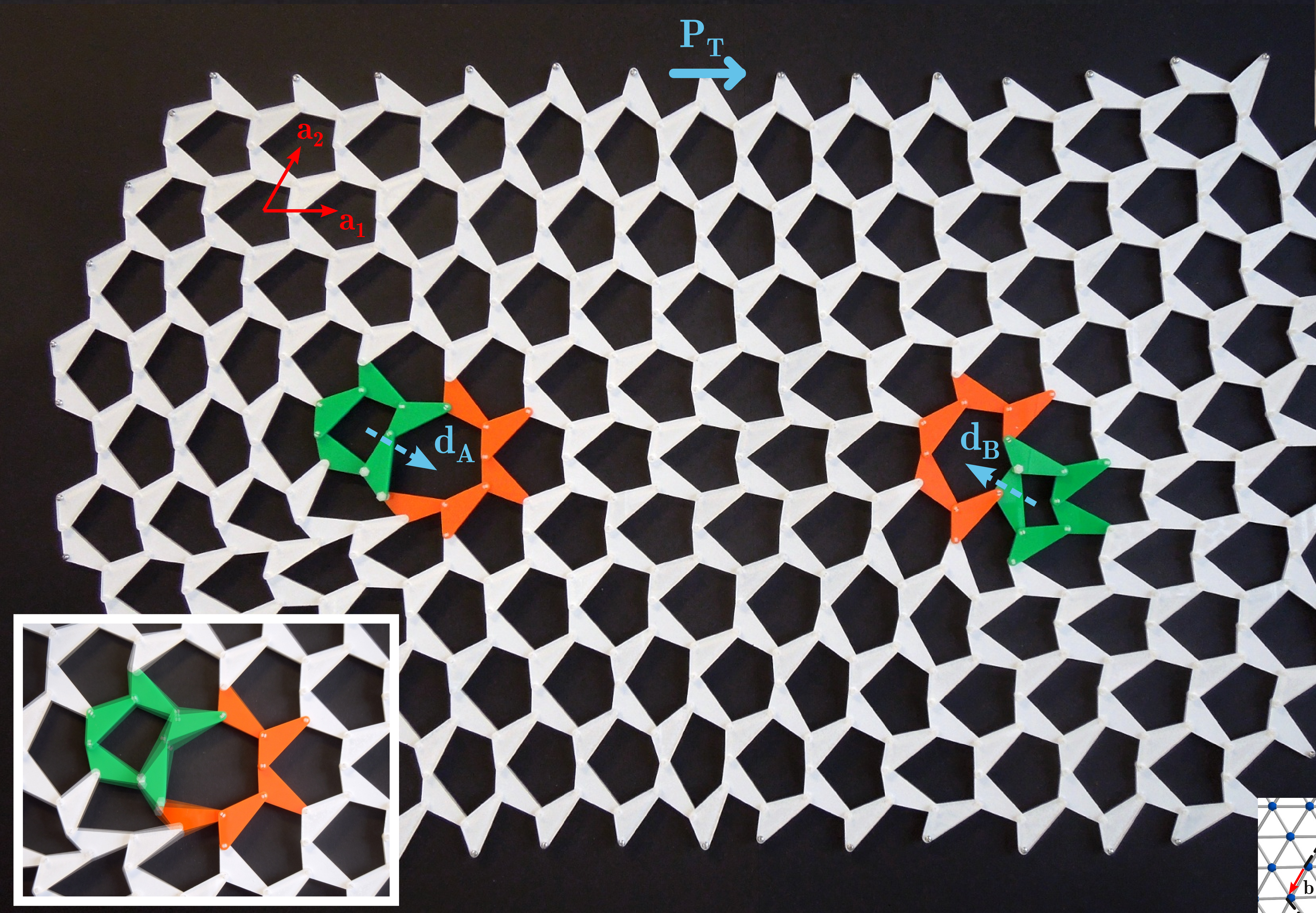
Topological soft modes at topological defects

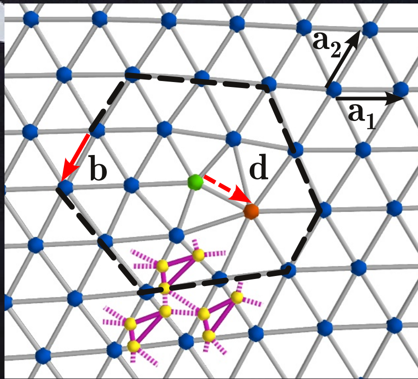




Paulose, Chen, Vitelli, *Nat. Phys.* (2015).

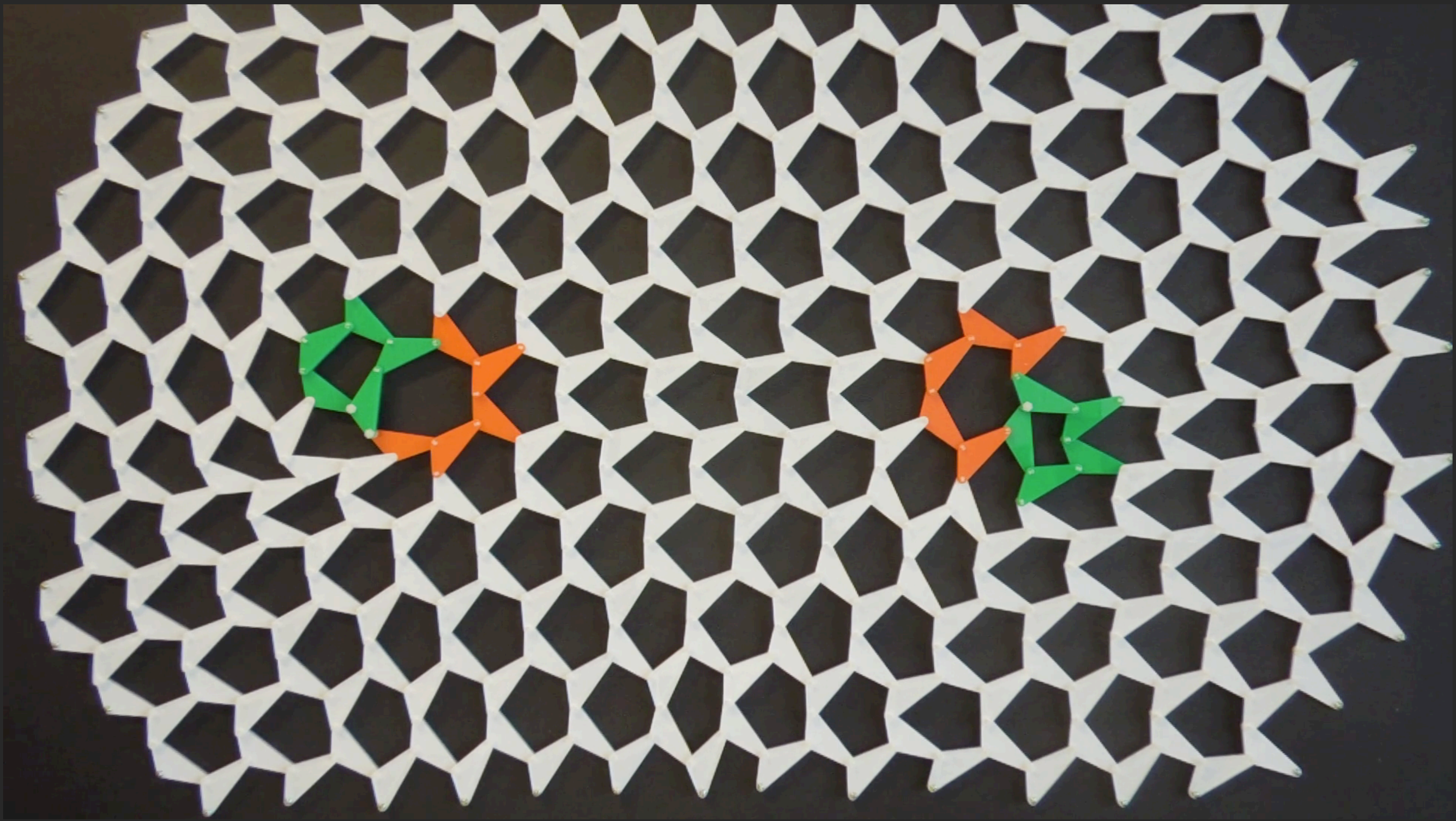
Where are the soft spots ?



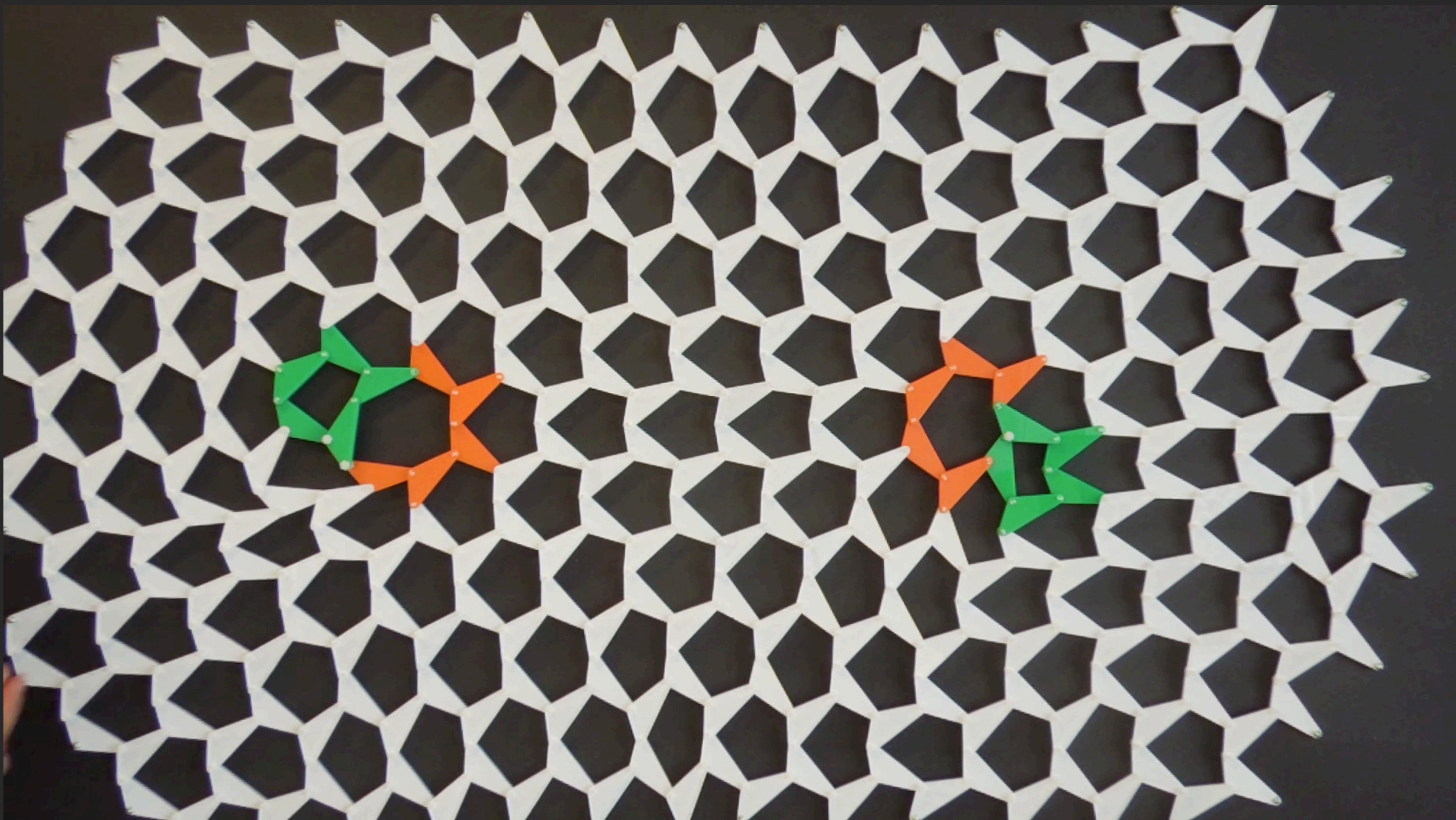


Paulose, Chen, Vitelli, *Nat. Phys.* (2015).

Rigid in the bulk

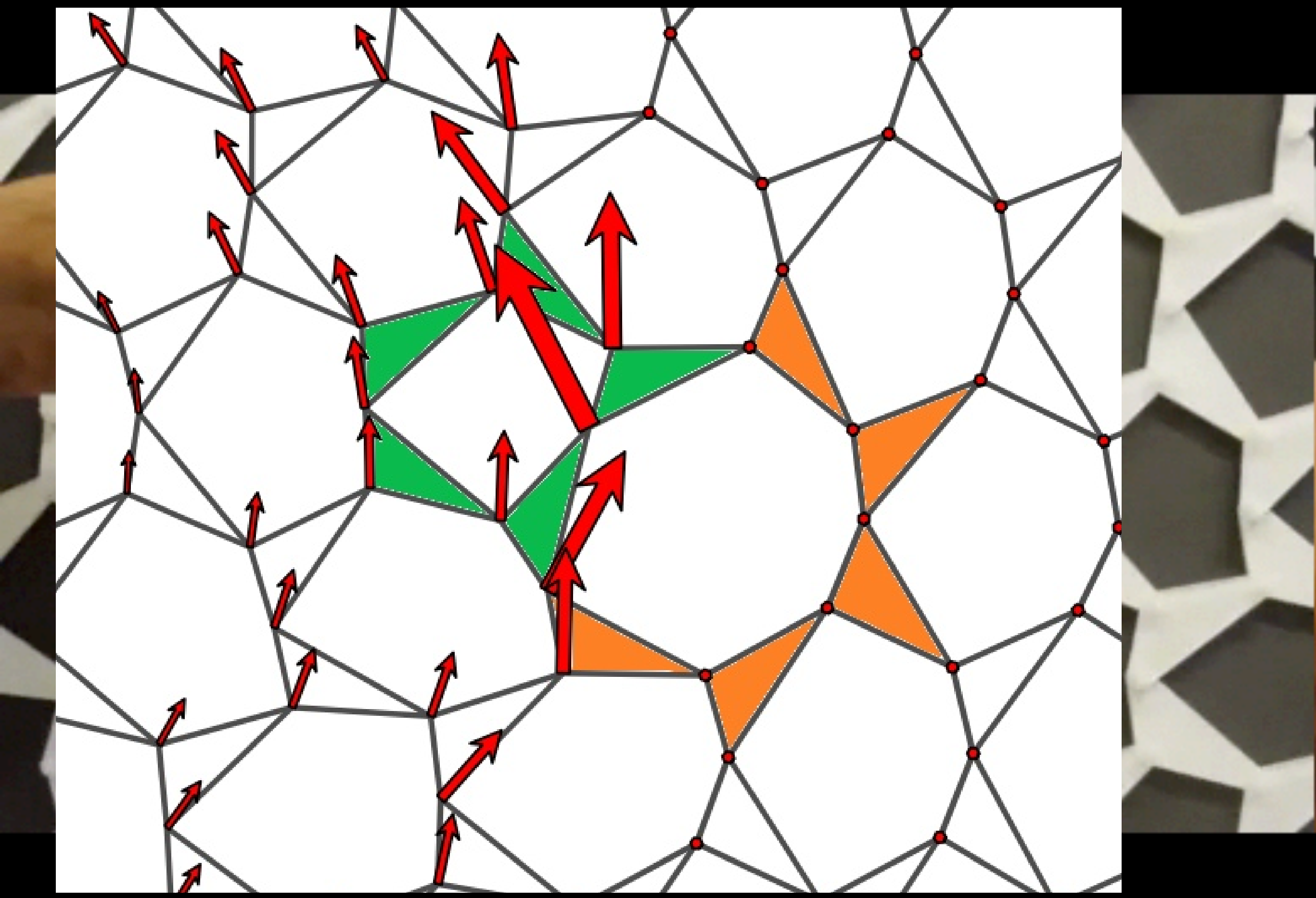


But there is one floppy spot!



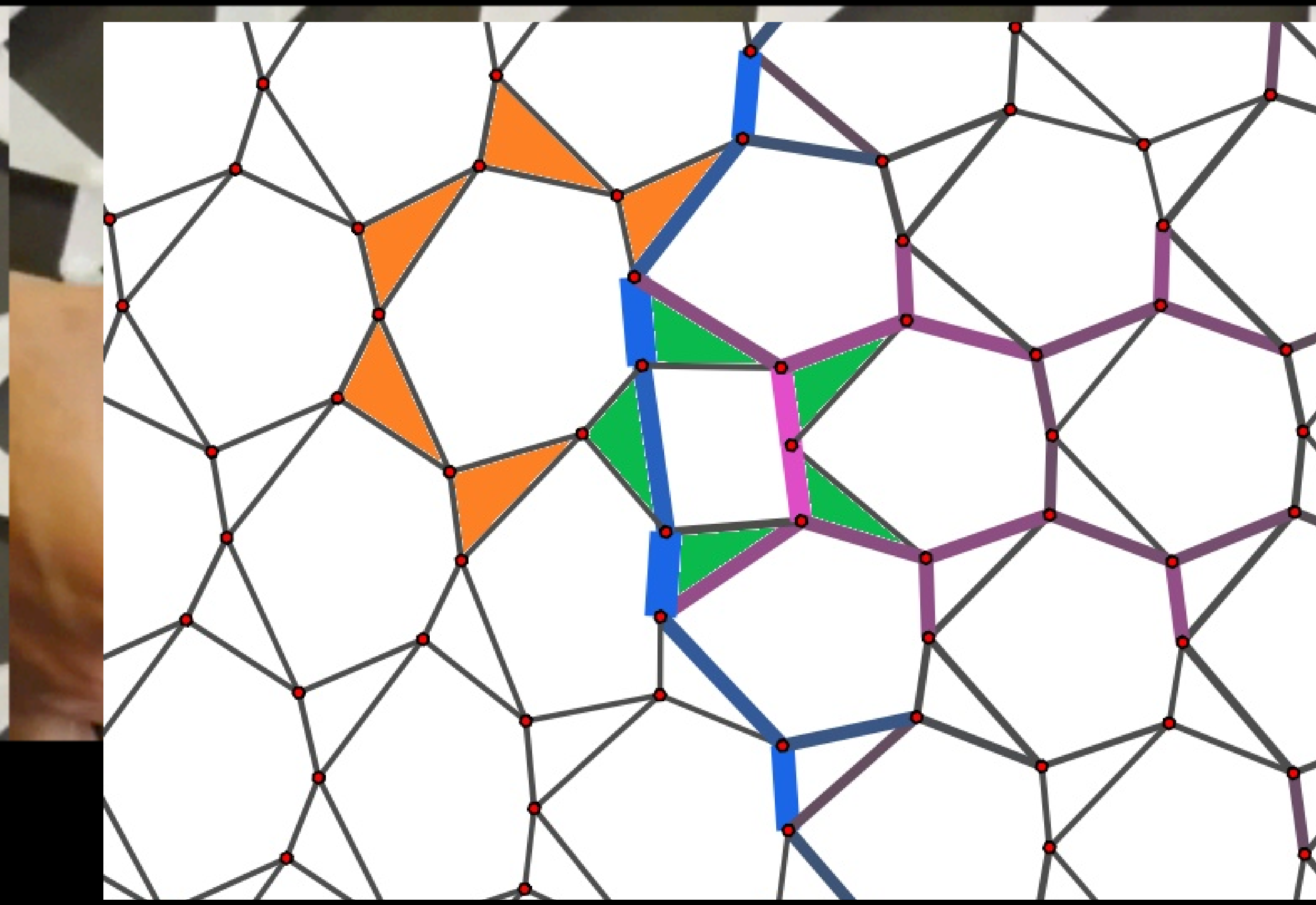
Soft motion at dislocation self stress at anti-dislocation

P_T



floppy mode

P_T



state of self-stress

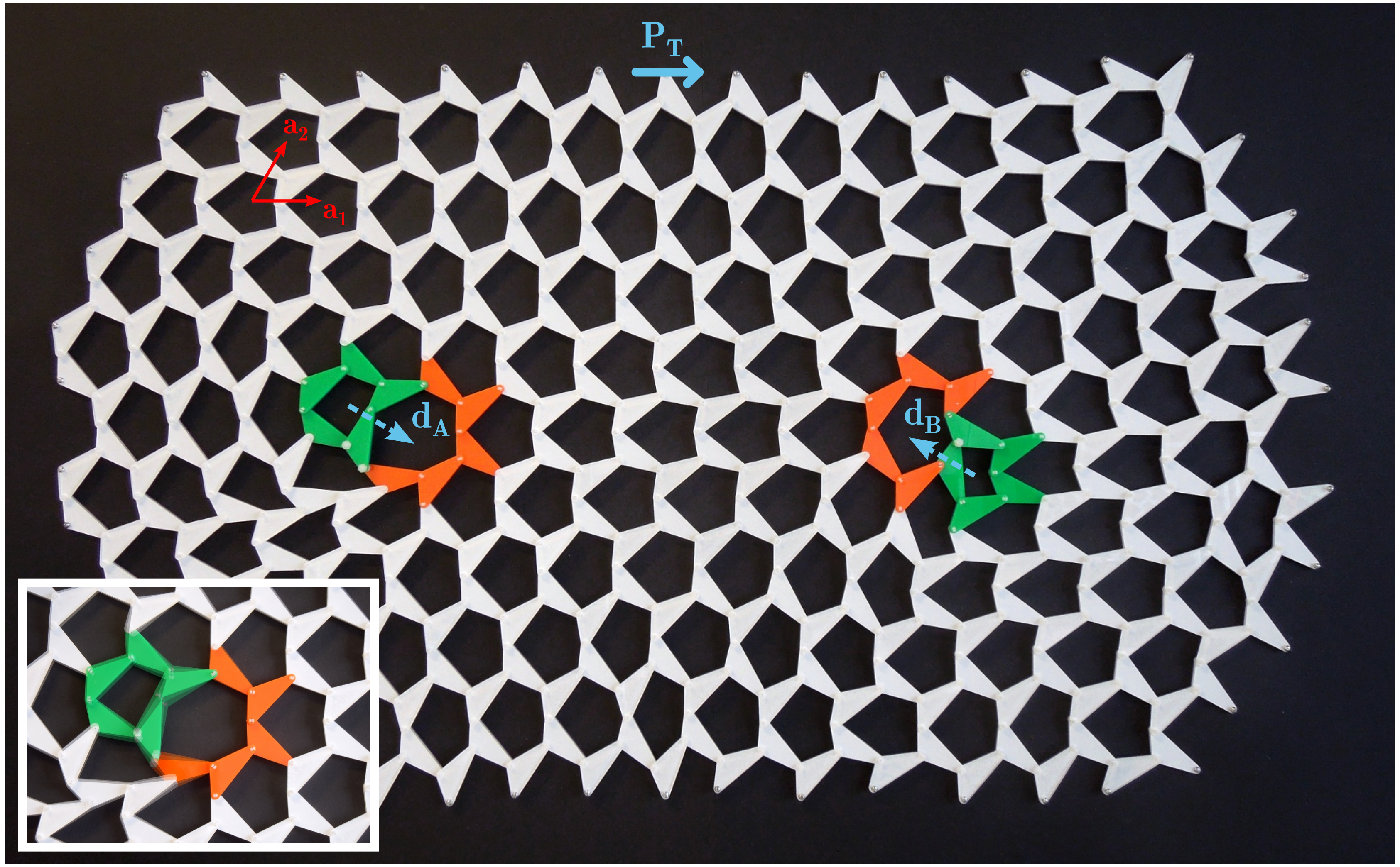
Mode count at dislocation:

$$\boxed{\nu_T^S = \mathbf{P}_T \cdot \frac{\mathbf{d}}{V_{cell}}}$$

+1 zm

-1 ss

a



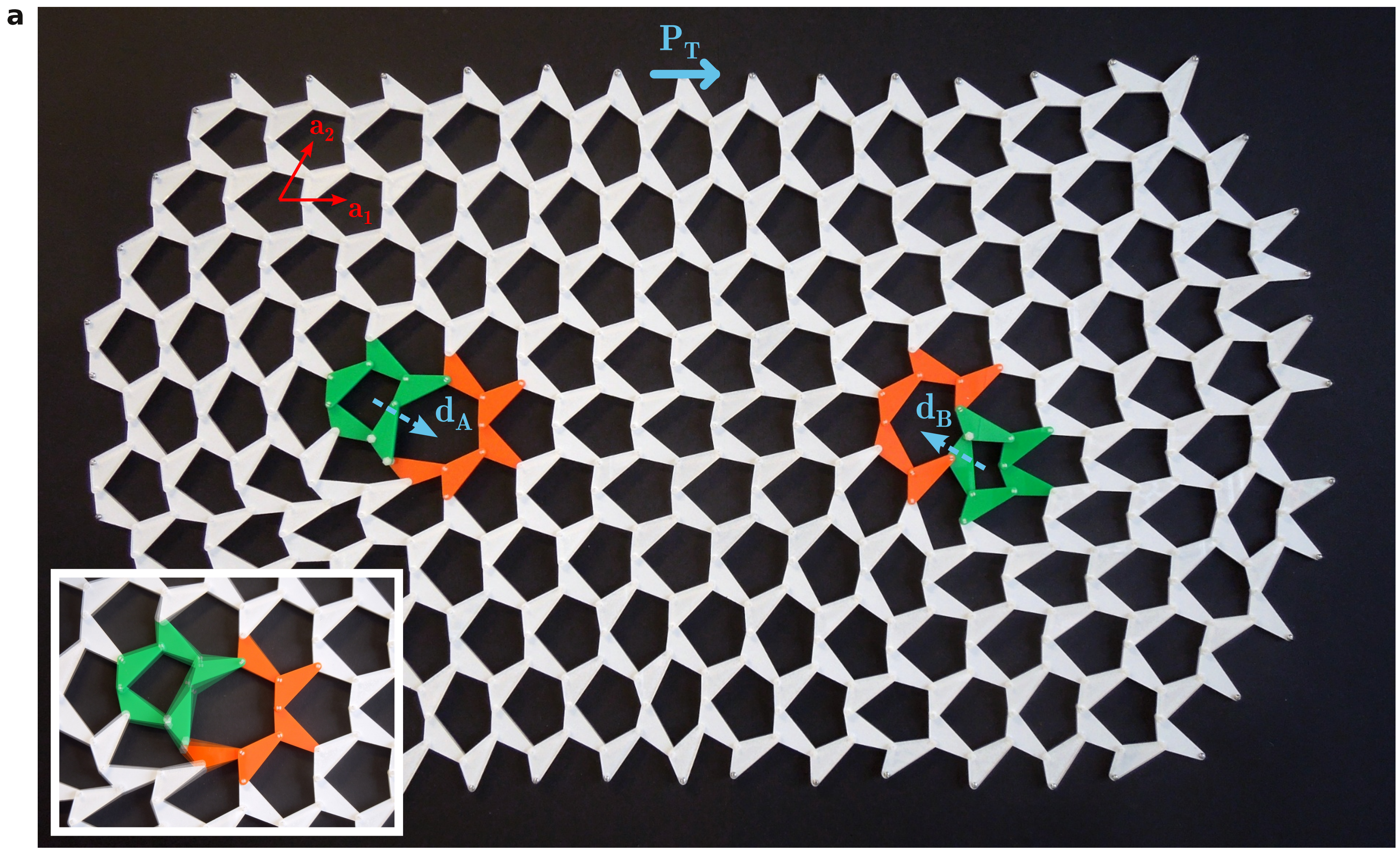
d: dislocation dipole (perpendicular to Burgers vector)

$\mathbf{P}_T$: topological polarization

Mode count at dislocation: $\boxed{\nu_T^S = \mathbf{P}_T \cdot \frac{\mathbf{d}}{V_{cell}}}$

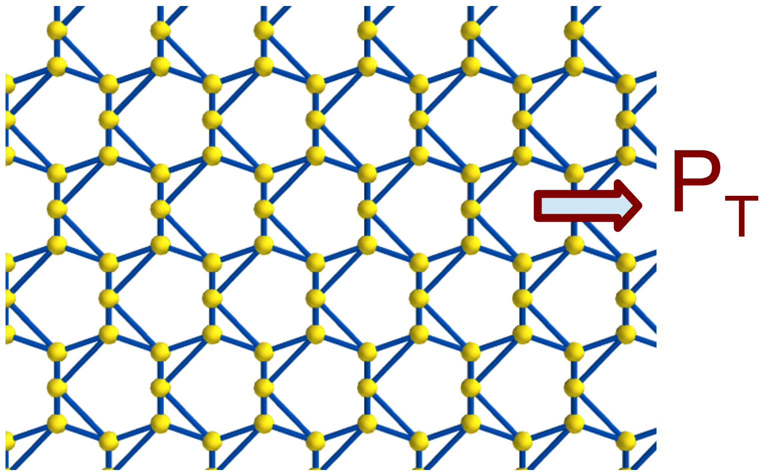
+1 zm

-1 ss



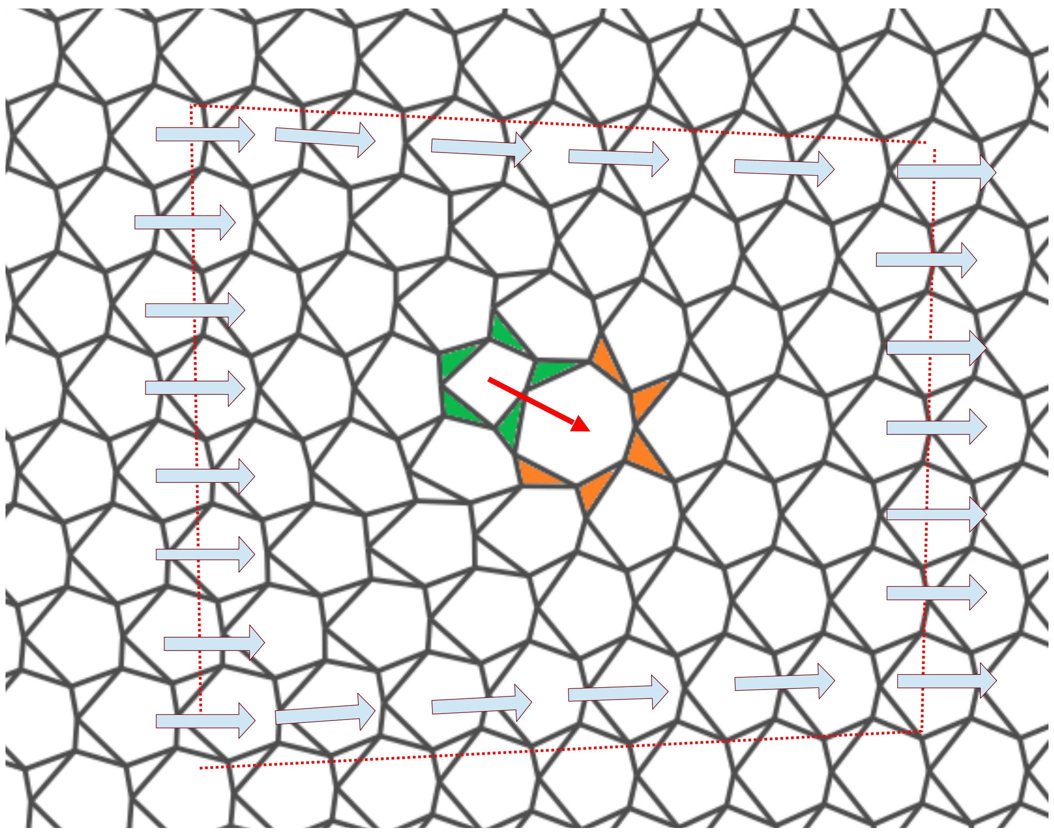
You can insert topologically protected states of motion where you want

Lattice polarization and zero-energy mode count



Zero mode count ↔ Net "charge" in a region

Net charge = Polarization flux into region



Zero mode count:

$$\boxed{\mathbf{P}_{\mathrm{T}} \cdot \frac{\mathbf{d}}{V_{\mathrm{cell}}}}$$

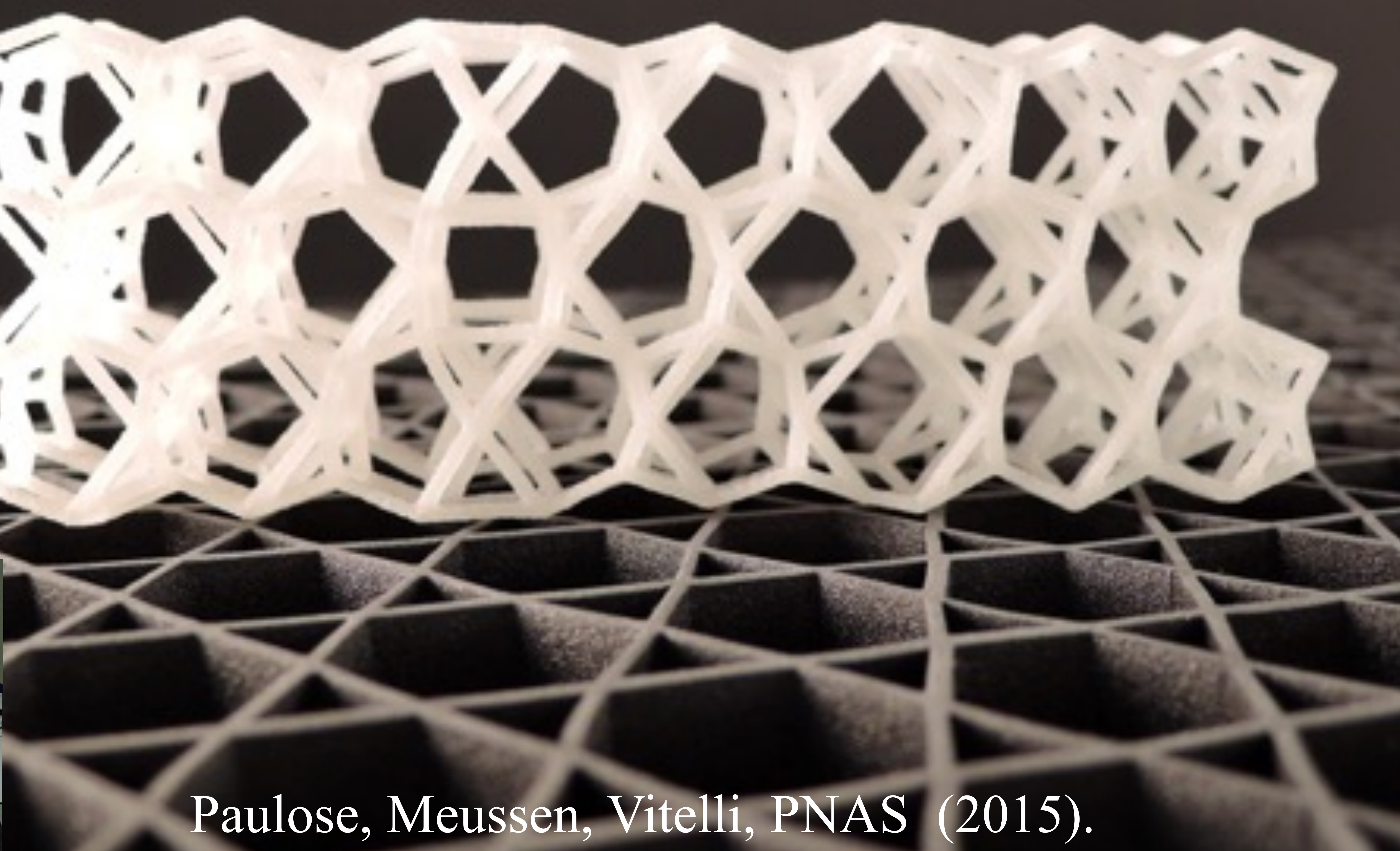
Electronic states in topological insulators:

Ran, Zhang, Vishwanath, *Nat Phys* 2009

Teo and Kane, *PRB* 2010

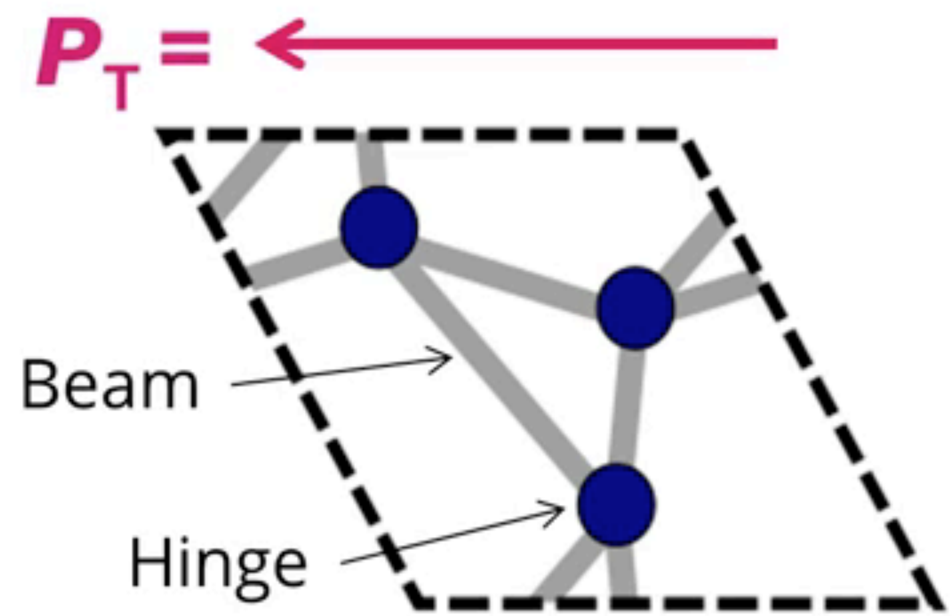
Paulose, Chen, Vitelli, *Nat Phys* (in press) 2014

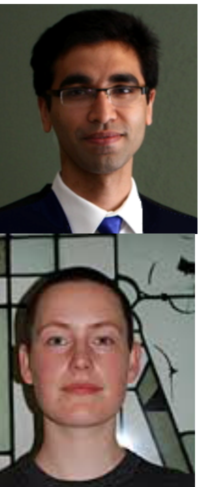
Topological control of material failure



Paulose, Meussen, Vitelli, PNAS (2015).

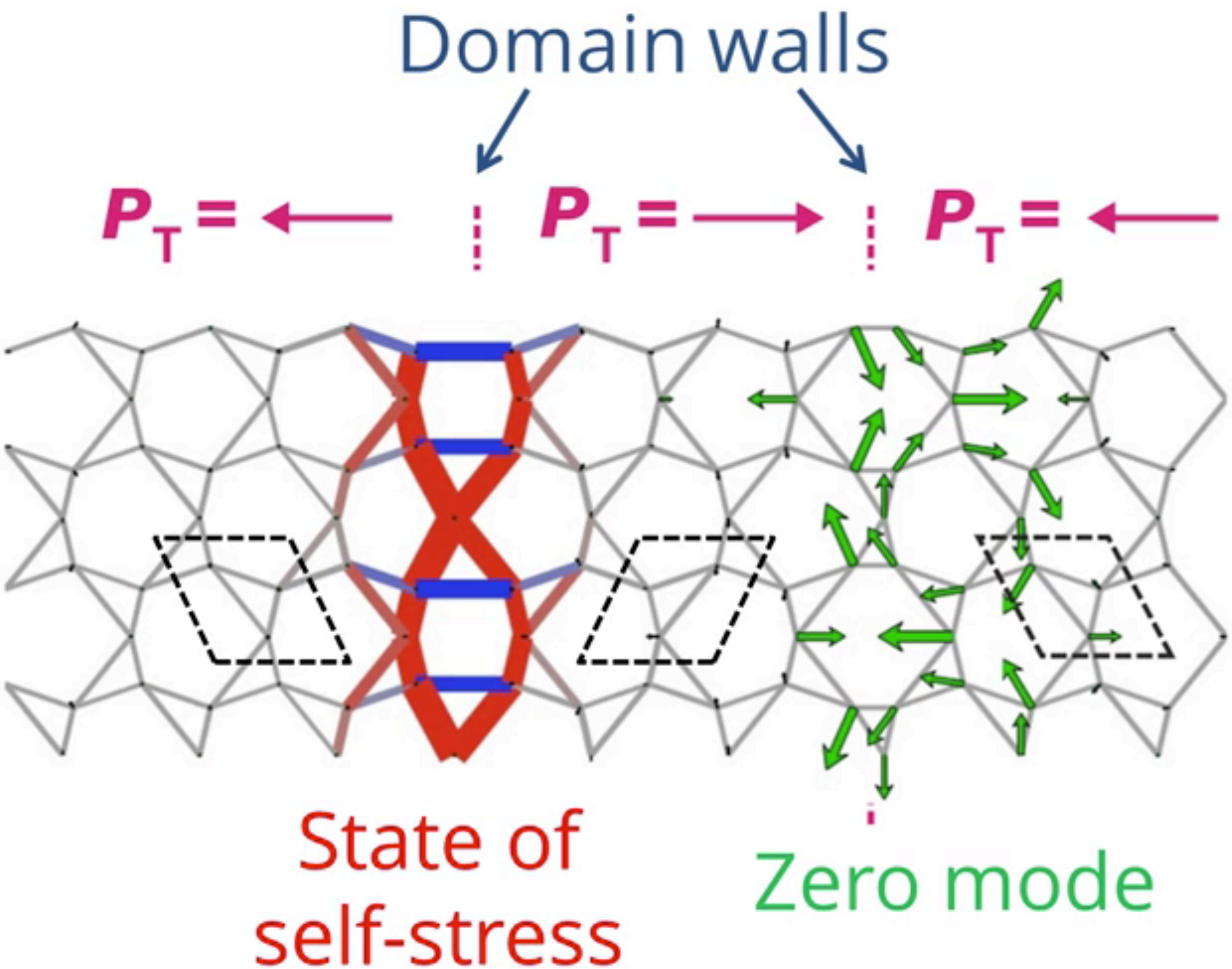
Unit cell with topological polarization of phonon degrees of freedom



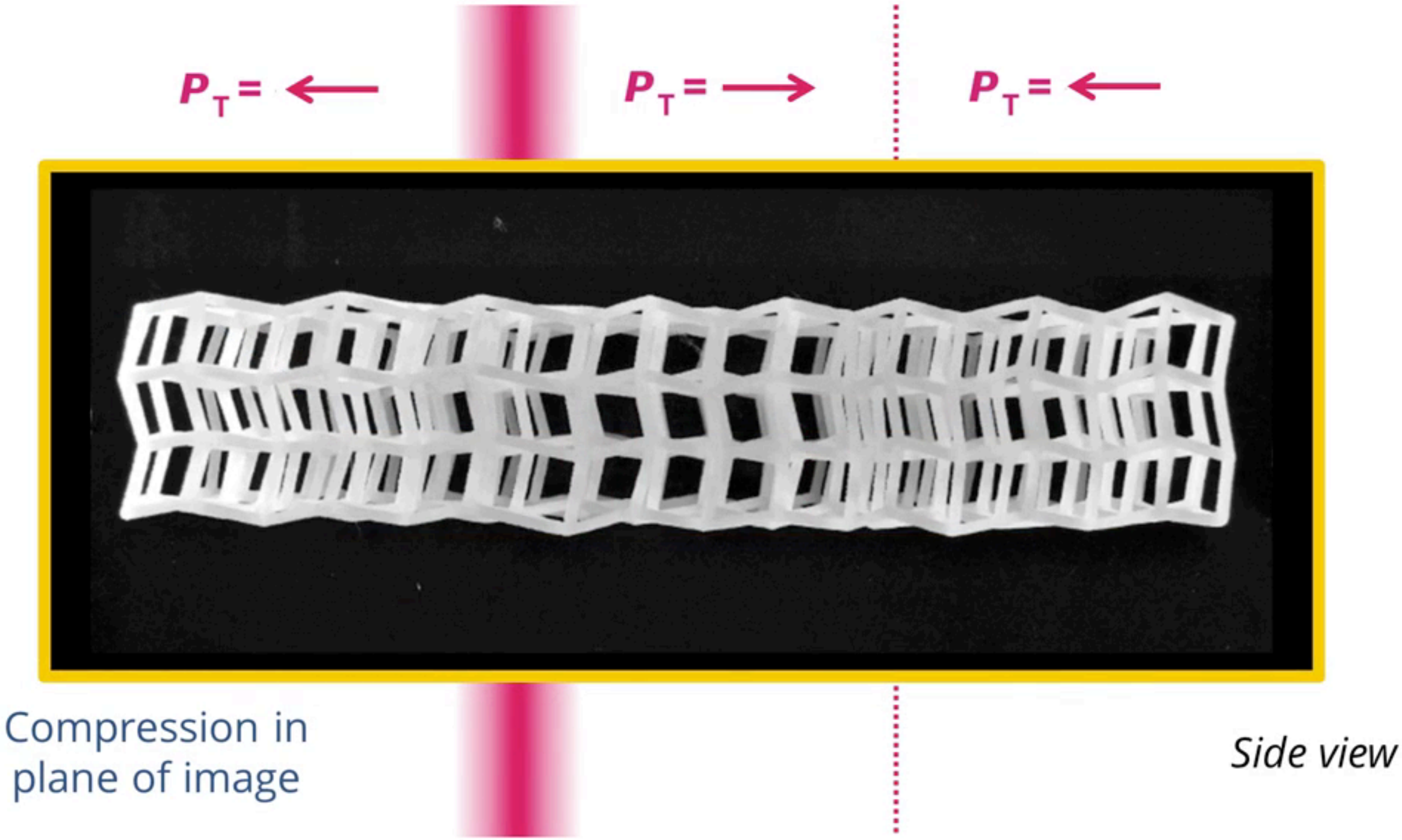


Take one of the lattices Tom described

Paulose, Meussen, Vitelli, PNAS (2015).

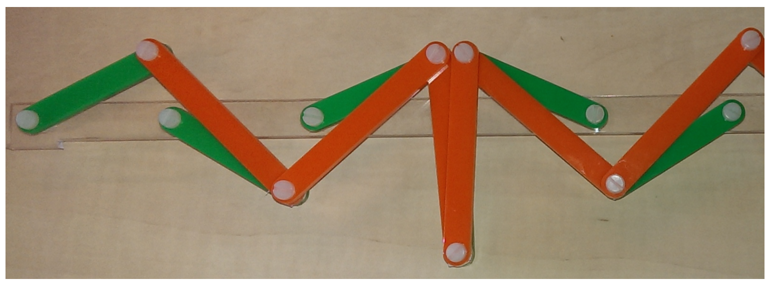


Paulose, Meussen, Vitelli, PNAS (2015).

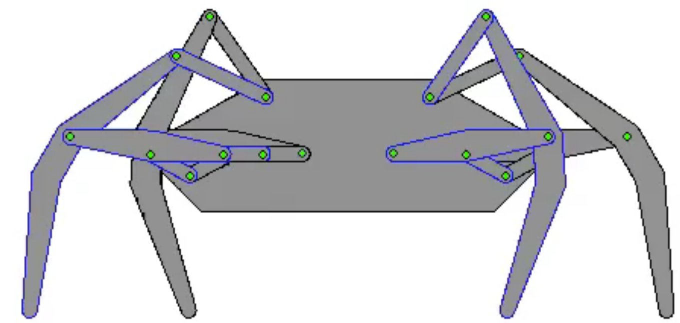


Paulose, Meussen, Vitelli, PNAS (2015).

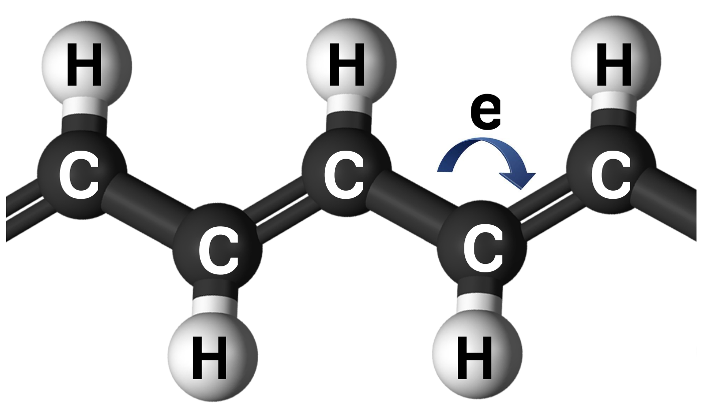
The road ahead



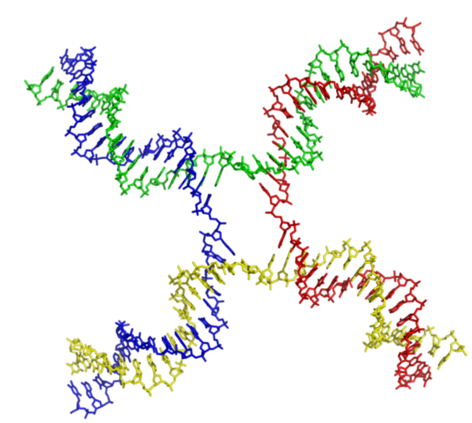
topological mechanisms



activated mechanisms
robots & smart materials



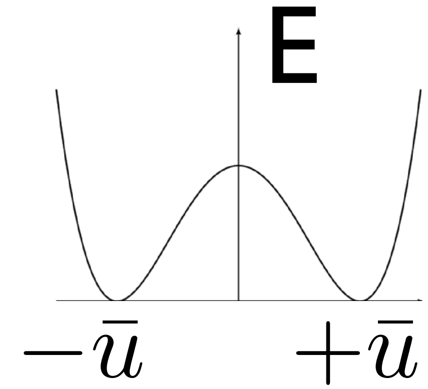
molecular electronics

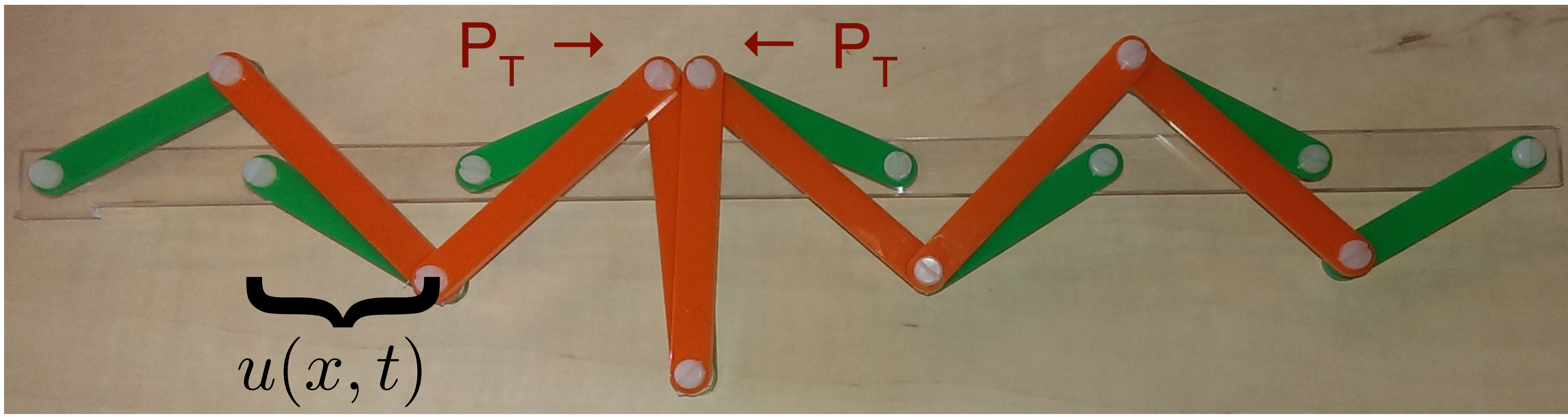


molecular robotics

videos: search youtube for VitelliLab

Kink costs zero energy

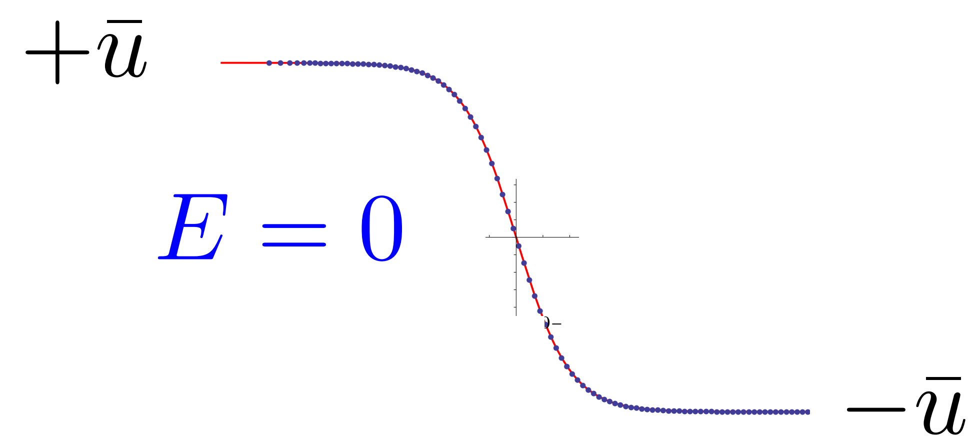




sign of flux of polarization

$$E = \int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2) \right]^2$$

minus sign
chooses kink



kink costs zero stretching energy ☑

anti kink suppressed ☑

Chen, Upadhyaya, Vitelli, *PNAS* (2014).

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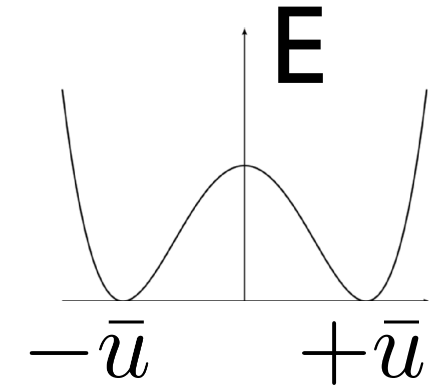
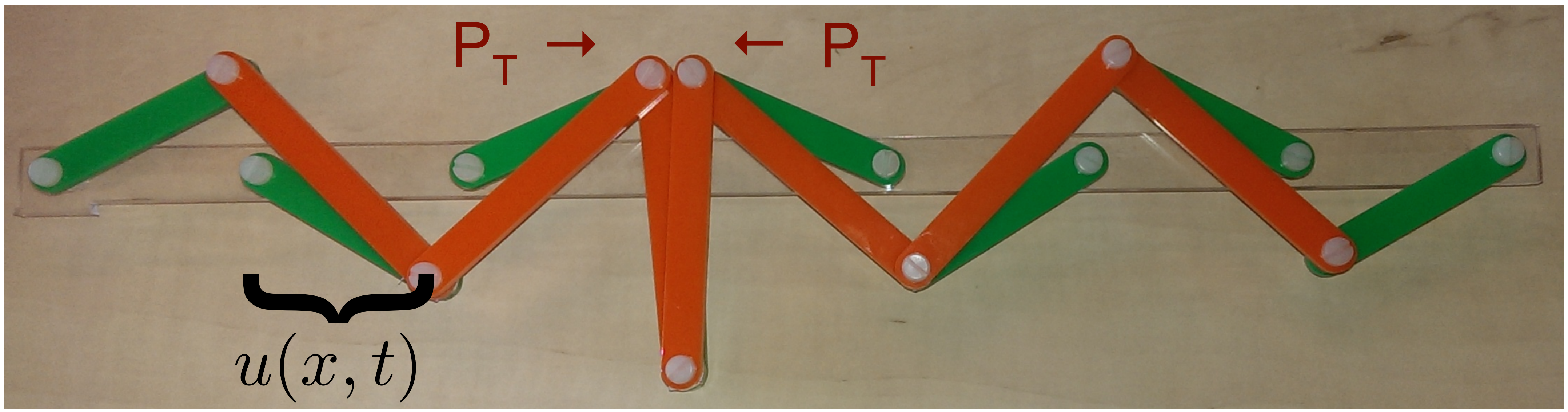
Soft modes on right edge





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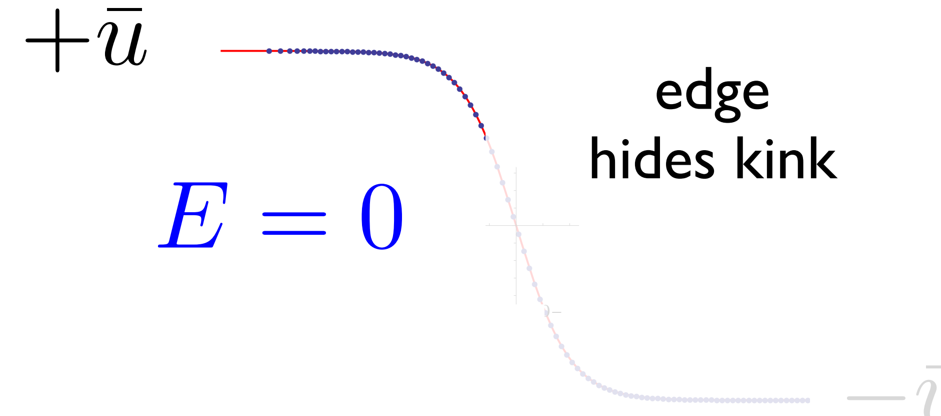

kink costs zero stretching energy ☑

Chen, Upadhyaya, Vitelli, *PNAS* (2014).

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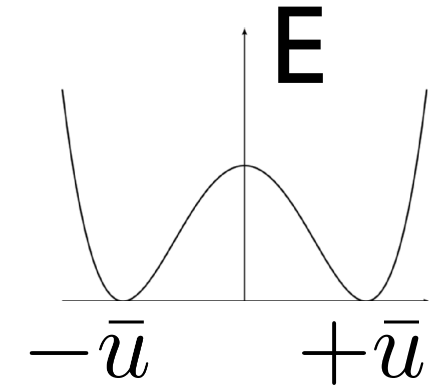
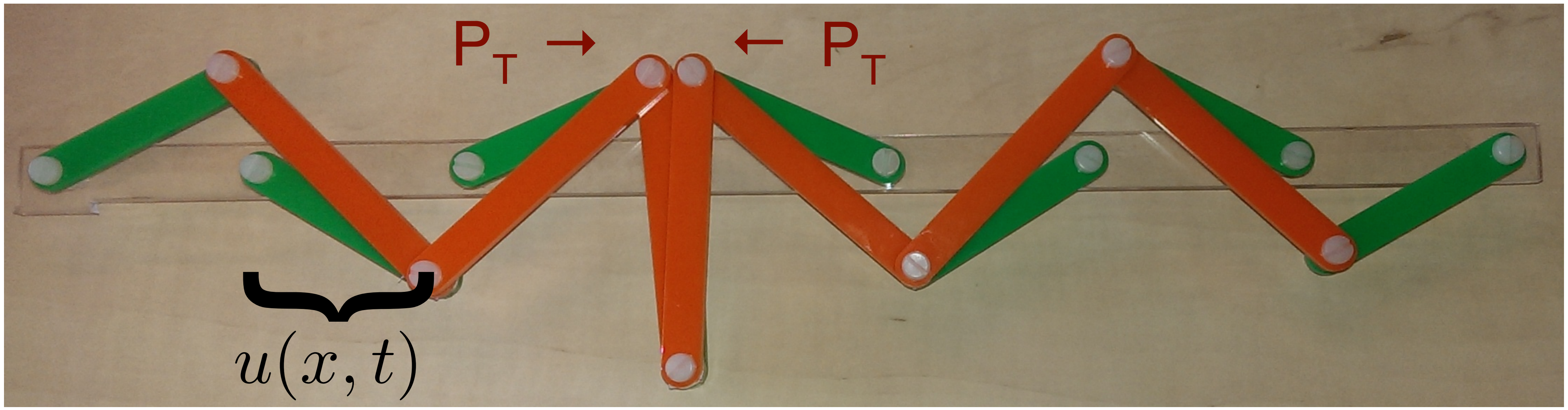
Phase transition when $\bar{u} = 0$





sign of flux of polarization

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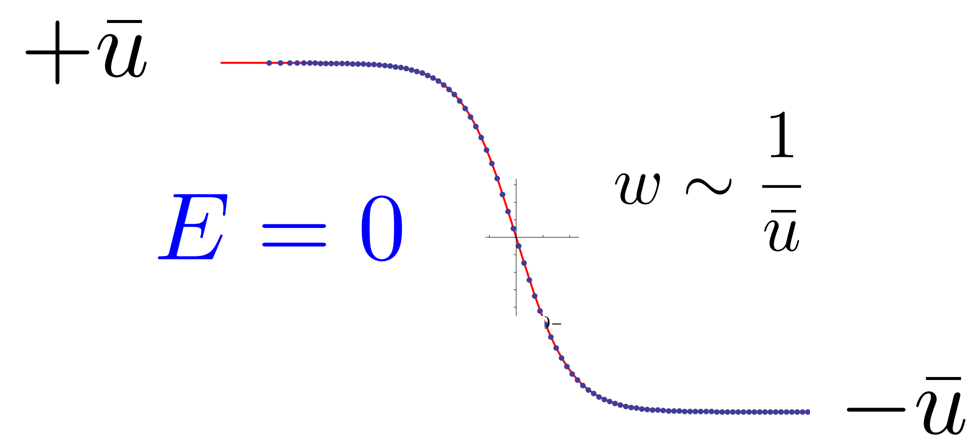
minus sign
chooses kink



kink costs zero stretching energy ☑

Kink width diverges as gap closes

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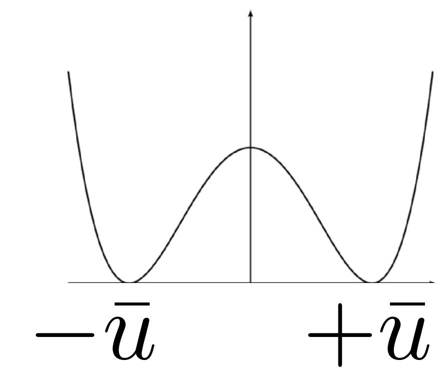
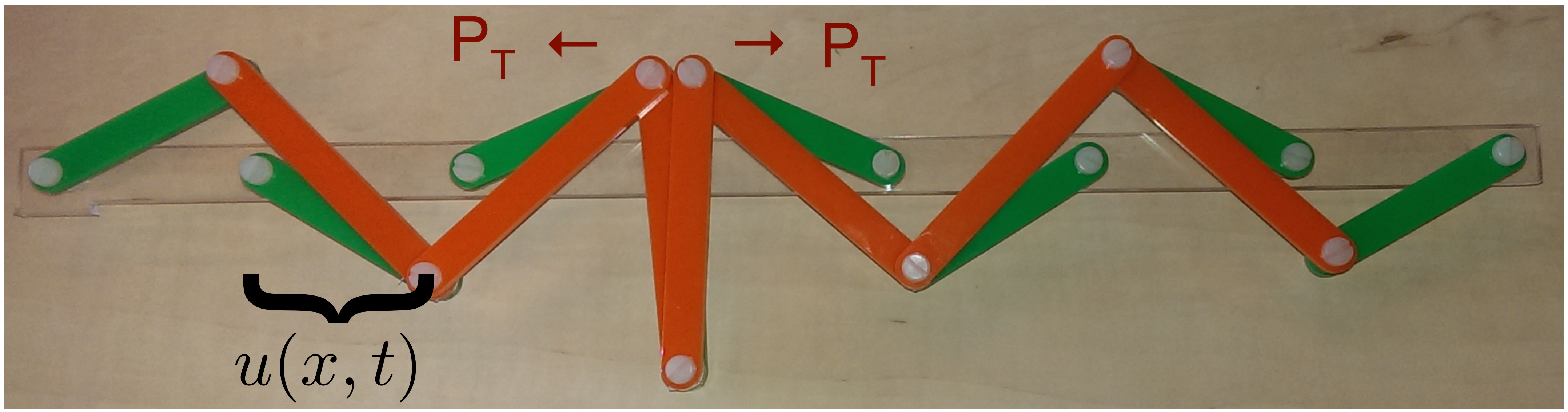
Anti-kink is suppressed

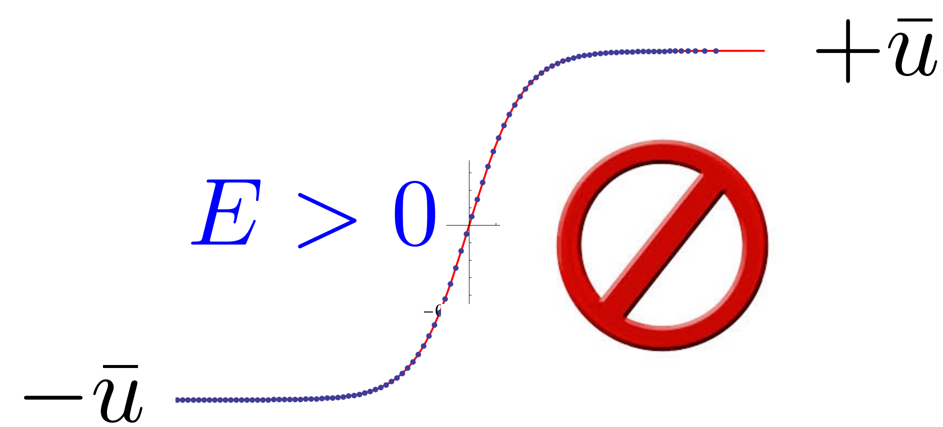




sign of flux of polarization

$$E = \int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2) \right]^2$$

anti-kink needs +

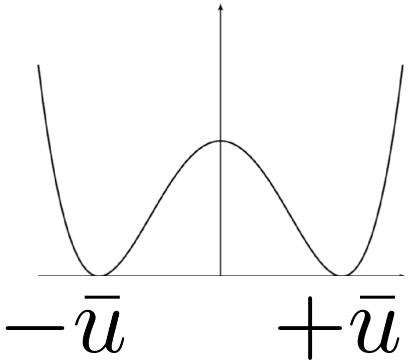


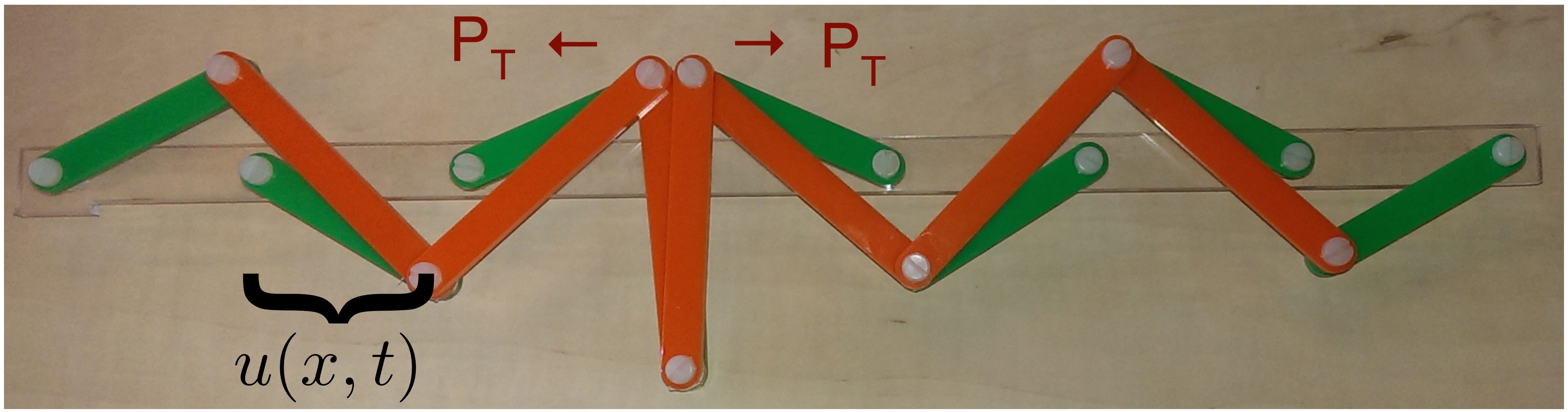
kink costs zero stretching energy ☑

anti kink suppressed ☑

Chen, Upadhyaya, Vitelli, *PNAS* (2014).

No motion from the left

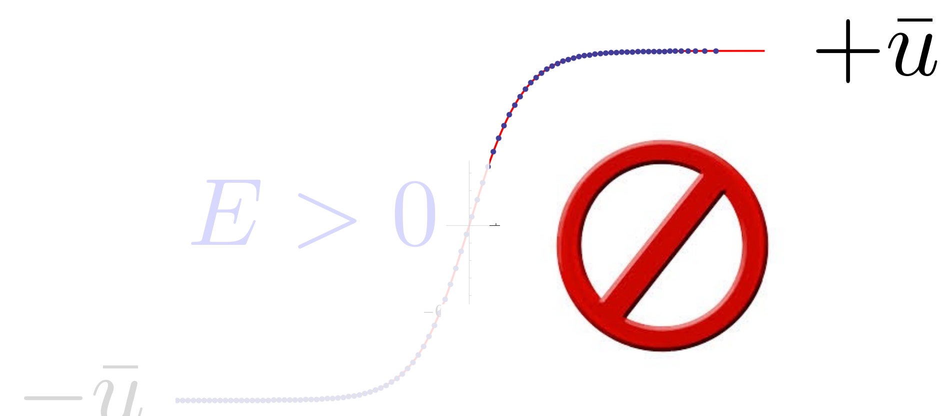




sign of flux of polarization

$$E = \int dx \left[\frac{\partial u}{\partial x} - (u^2 - \bar{u}^2) \right]^2$$

anti-kink
needs +



kink costs zero stretching energy ☑

anti kink suppressed ☑

Chen, Upadhyaya, Vitelli, *PNAS* (2014).