OUTLINE

I. Introduction
   A. Simple Ideas
   B. Granular Phases

II. Key Ideas
   A. Dilatancy
   B. Friction
   C. Janssen Effect
   D. Indeterminacy
   E. Nonlinearity of Contacts
   F. Granular Temperature
   G. Granular Entropy (?)

III. Dense Phases
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   C. Force Propagation
   D. History Dependence
   E. Stick Slip
   F. Fluctuations
   G. Jamming
   H. Compaction
   I. Shearing
Overview
Reviews

de Gennes, Rev. Mod. Phys. 71, S374, 1999

Kadanoff, Rev. Mod. Phys. 71, 435, 1999


Herrmann, Hovi and Luding, Physics of Dry Granular Media, Proceedings, NATO Advance Study Institute, Cargèse, Corsica, France, 1997

Behringer and Jenkins, eds. Powders and Grains '97, Balkema, 1997


Jaeger, Nagel and Behringer, Rev. Mod. Phys. 68, 1259 (1996)

Jaeger, Nagel and Behringer, Physics Today, April, 1996


Bideau and Hansen eds. Disorder and Granular Materials, Random Materials and Processee Series (North-Holland, NY), 1993

Thorton, ed. Powders and Grains '93, 1993
IV. Gas Phase
   A. Collapse
   B. Clustering
   C. Granular Temperature
   D. Hydrodynamics
   E. Melting in 2D
   F. Velocity Distributions

V. "Phase Transitions"
   A. Horizontal Shaking
   B. Shear Banding
   C. Fluidization

VI. Pattern Formation
   A. Shaking
   B. Segregation
   C. Avalanching
   D. Hopper Flow

VII. Conclusions/Summary
Mixing/Segregation


Shearing—2D


S. Luding et al. Granular Matter

S. Bardenhagen et al. to appear, Phys. Rev. E

Shearing—3D


Granular Gases


Self Organized Criticality


Hopper Flows


Janssen Effect

E. Kolb et al. Submitted

Models

A. Tkachenko and T. Witten, submitted (adaptive network)


Sandpiles


Historical

O. Reynolds, Phil Mag. S5. 20, 469 (1885)

Shaken Systems

Motion on a Surface
B. Painter and R. Behringer, to appear
Friction

Shear Bands

Granular Phase Transitions

Multiphase Flows

Techniques
P. Dettu, Gletechnique 18, 50 (1968)

MD Modeling
S. Luding and S. McNamara, Granular Matter
Soil Mechanics


M. Oda, Soils and foundations, 12, 17 (1972)

Friction


Shear Bands


Granular Phase Transitions

E. Aharonov and D. Sparks, Phys. Rev. E 60, 6890 (1999)

Multiphase Flows


Techniques

P. Dantu, Géotechnique 18, 50 (1968)


MD Modeling


S. Luding and S. McNamara, Granular Matter
Hunting Drums

Jannning
C. O'Hern et al. to appear

Force Fluctuations
Geotechnical


Rotating Drums


Jamming


C. O'Hern et al. to appear

Force Fluctuations


1. Consists of a large number of individual solids

2. Grain-grain interactions are purely classical

\[ \lambda = \frac{h}{mv} - 10^{-27} \text{ m} \quad \lambda \text{ - system size, quantum important} \]

3. Grains exert forces only when in contact

4. Collisions between grains are generally inelastic

5. Grains are surrounded by a fluid or a vacuum
For a fluid: \[ \text{mgd} \ll k_bT \quad (\text{mgd} \sim 10^{13} k_b T) \]

Thermal energy allows exploration of many different states.

Fluid finds equilibrium lowest free energy state,
Thermodynamics applicable.

For a granular material: \[ \text{mgd} \gg k_bT \quad (\text{mgd} \sim 10^{35} k_b T) \]

Consequently, the material does not sample other configurations unless energy is put into the system.

This can lead to metastable states that persist indefinitely.
2. Continuum dynamics

What are right models? When do they apply?...

3. Member of class of similar systems:

Foams, Glasses, Colloids, Quantum flux lattices

4. Important applications

Coal and grain transport...
Pharmaceuticals
Xerography
Soils
Avalanches
Mixing
Earthquakes and mudslides...

5. $$$!! Lots of them Claim: ~1 STrillion/year in US for bulk solids handling

Industrial facilities operate well below design efficiency
2. Continuum dynamics

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   Industrial facilities operate well below design efficiency
FROM THE GEOGRAPHIC ARCHIVES

Changing the Channel

Construction crews in 1913 had to clear frequent landslides (top) Culebra Cut (later called Gaillard Cut) across the Continental Div. area Canal navigable in William Joseph Showalter’s February 191
Slow landslide forces freeway section closed

Southbound repairs to take 2 months

by ROBBY MOORE

Cas of Durham's newest section of freeway was abruptly closed Monday afternoon when road workers discovered a moving landslide posed a threat to motorists.

An embankment supporting the southbound lanes of the Durham Freeway began sliding about four weeks ago, encroaching the freeway's right lane to curb in sight. Highway crews had closed the lane while workers planned a way to halt the slide.

On Monday, with rain expected and a severe Monday morning storm apparently made the embankment more unstable. When the earth began moving Monday, engineers decided it wasn't safe to wait even one more day.

By Wednesday afternoon, the depression in the right lane was about 20 feet long and 3 feet deep - and growing.

"It represents an immediate hazard to the traveling public," said Gary S. Martin, chief engineer for the state Division of Highways.

Martin had been watching for the embankment since it began sliding about 10 days ago. The slide is 400 feet long and 10 feet deep.

"The landslide will probably stay on the right side of the road, but the embankment on the left side is likely to continue sliding," said Martin.

Southbound motorists will have to take U.S. 70-296 bypass around Durham's freeway. Southbound motorists will use the exit ramp at Edgewood Avenue to get back on the freeway.

The embankment problem - technically called "a slope failure" - came from a 5 1/2-foot layer of clay about 10 to 20 feet below the surface of the embankment.滑坡问题 - 技术上称为 "斜坡滑动" - 来自于大约20到50英尺的粘土层。
I. Different regimes or states:

A. 'Solid' regime
   Persistent contacts, slow flows
   Compact structure
   Strong force fluctuations
   Spatially variable
   Temporally intermittent

   Industrial applications—hoppers...

B. 'Fluid' or 'Gas' regime
   Energetic flows
   Binary dissipative collisions
   Granular 'temperature' $T_g < v^2$

   E.g. avalanches, shaken systems...

C. Transition regimes
   'Gas' $T_g \rightarrow 0$: clustering/collapse
   'Solid' $\rightarrow$ Plastic under shear (Jamming)
   'Solid' + Energy $\rightarrow$ fluid
   'Solid' & 'Fluid' often close spatially
II. Statistical properties

A. Solid

1. Disorder and fluctuations

   - Strong force fluctuations—only weak packing fluctuations
   - Small grain displacements \(\Rightarrow\) large force changes
   - Length and time scales of fluctuations an open question

   Continuum Limit?

2. Friction and contacts

   - Coulomb friction is indeterminate \(\Rightarrow\) History Dependence
   - Persistent contacts \(\Rightarrow\) limited sampling of phase space

B. 'Fluid'

   - Instabilities: clustering, hydrodynamics
   - Existence of \(T_c\)?

   Segregation and mixing phenomena
1. New states of matter

- Ordinary Matter
  - SOLIDS
  - LIQUIDS
  - GASES

- Granular Matter
  - "SOL-QUIDS"
  - "COALESING GASES"
  - Boltzmann distribution
  - (Reynolds dilatancy)

- Resists shear only up to a point

- Dissipative collisions
  - Clustering
Solid-like up to yield
Then dilates & flows
O. Reynolds (Plastic?)

"GASES THAT CLUSTER"

Clustering in 2-D system, from m-D distribution?

Goldhirsch, Tan, Zanetti

Physical Systems: Gravity compacts
\[ m = C \cdot D^{2.96} \]

\( m \) independent of fill height!

\( m \) measured here
Why is height independent?

Very

Arches carry stresses to sidewalls

Grains fall out near exit

\[ V \approx (gD)^{\frac{1}{2}} \]

\[ M \approx D^2 \quad V \approx D^{5/2} \]

(+ Boundary layer effect)
\[ F_N = mg \]

\[ F_f = \begin{cases} 
F_h & F_h < \mu_k F_N \\
\mu_k F_N & \text{Otherwise} 
\end{cases} \]
ESSAI
Sur une application des règles de Maximis & Minimis
à quelques Problèmes de Statique, relatifs à
l'Architectura.
Par M. CORLORS, Ingénieur du Roi.

INTRODUCTION.

On a procédé à des recherches, montant que le
nombre des Calculs & de la Physique peuvent le par-
전학, l'effet est du force et de la résistance, dans
des quelques problèmes de Statique. Voici une légère analyse
des divers objets qu'il contient.

Après quelques observations préliminaires sur la
séance, & quelques expériences sur le même objet, l'on détermine
la force d'un pilier de maçonnerie; le poids qu'il peut porter,
preférant la longueur de la pièce, dans lequel il doit se
ranger. Comme ce problème réagit que des considérations
elles seules, qui seront à faire expérimenter dans les autres
parties de cet Essai, disons de développer les principes de
la force.

Si l'on suppose un pilier de maçonnerie compris par un
plus indiférent à l'horizon, en sorte que les deux parties de ce
pilier soient unis dans ce pilier, par une colonne donnée,
toujours que tout le tube de la toile s'entrelace lalachte, ou
lié par une adhérence infinie, qui trouve un charge ce pilier d'un
poids; ce poids tend à faire courir la partie supérieure du
pilier fort le plus incliné, par lequel à mettre la partie infé-
riore. Ainsi, dans le cas d'équilibre, la position de la colonne,
qui agit parallèlement à la séance, est exactement égale à
la colonne. Si l'on remarque s'acheminer, dans le cas de
l'homogénèse, que l'adhérence de pilier est sensiblement égale.
Janssen Effect

\[ P = P_0 \times \exp \left( -\frac{z}{\lambda} \right) \]

\[ \lambda = \frac{R}{2 \mu} \]

\[ P_0 = \rho g \lambda \]

Effective Wall Friction Coefficient \( \mu \)

Circumference = \( 2 \pi R \)

Area = \( A = \pi R^2 \)
It is near
by dependence

\[ F_g \approx \mu F_N \]

For each disk:
\[ \sum F = 0 \]

3 Unknowns \( \xi_j \)
4 Constraints \( \xi_j \)
6 Unknowns

9 Constraints
12 Unknowns
\[ F = 0 \quad \text{if } x < 0 \]
\[ F = x^{3/2} \quad \text{if } x > 0 \]
Class of systems that are constrained or jammed

Granular Materials
Foams
Colloids
Glasses


Note also

Ngadii + Raschenbach
PRL 81, 1541 (1998)

Chicago group
PRE 53, 4673 (1996)

Duran et al.
Science, 275, 1920 (1997)
F. Radicai, M. Jean,
S.-I. Moreau, S. Roux
PRL 77, 264 (1996)
Sand Types

1. Smooth, spherical d = 0.01 cm (Chow)
2. Rough d = 0.06 cm
3. Dune-rough, d = 0.02 cm

Baxter et al.

Europhysics Lett. 21, 569 (1993)
1. Chains deform, break, reform

2. Result: large spatio-temporal force fluctuations
Some Models for Stress Propagation
In Granular Materials

(Static)

1. Elasto-Plastic (Classical theories of soils—Coulomb)

   Plus variations:  Rigid Plastic
                   Incipient Failure Everywhere (IFE)
                   Mixed Elastic and plastic
                   ...

2. Q-model (Coppersmith et al.)

   Microscopic model—introduces randomness
   Improvements by Socolar, Bouchaud et al.

   In continuum limit, Q-model is diffusional

3. Oriented Stress Linearity—OSL
   Bouchaud, Cates Wittmer, Claudin

   Builds on intuitive idea of stress chains

   Continuum model—Hyperbolic Partial Differential Eqs.
EQUATIONS OF MOTION FOR A GRANULAR MATERIAL

° conservation of mass:

\[ \frac{\partial \rho}{\partial t} + \partial_i (\rho v_i) = 0. \]

° conservation of energy:

material properties not affected by energy dissipation.

Dissipation by sliding friction
Typical Elasto-plastic Model

EQUATIONS OF MOTION FOR A GRANULAR MATERIAL - SLOW FLOW

(Compare to Newtonian Fluid)

1. Conservation of mass:
\[ \frac{\partial \rho}{\partial t} + \partial_j (\rho v_j) = 0. \]

2. Conservation of energy:
- Material properties not affected by energy dissipation.

Dissipation by sliding friction

\[ \text{Shear stress} \ 
\begin{array}{c}
\Rightarrow \\
\text{Normal Stress}
\end{array} \]

\[ \text{no flow until } \tau \text{ reaches } Y.S. \]
conservation of momentum:

\[ \rho \frac{dv_i}{dt} = -\partial_j T_{ij} \]

\( T_{ij} = \) stress tensor

For a "simple" model:

\[ T_{ij} = P \delta_{ij} + kPV_{ij}/|V|. \]

\( V_{ij} = -(\partial_j v_i + \partial_i v_j)/2 \) (The strain rate tensor)

\[ |V|^2 = \sum V_{ij}^2. \] (\( |V| = \text{norm}(V) \))

compare to Newtonian fluid:

\[ T_{ij} = P \delta_{ij} + 2\eta [V_{ij} - (1/3)\text{Tr}(V)] + (2\zeta/3)\text{Tr}(V). \]

\( \zeta \) 1st order homogeneous
Fluctuations and lattice-based models

Suppose the following distribution of forces $P(f)$:

$$P(f) = A f^2 \exp\left(-\frac{f}{f_{\text{avg}}}\right)$$  (Coppersmith et al.)

where $f_{\text{avg}} = \text{mean force}$, $\sigma = \text{rms about } f_{\text{avg}}$

$$P( f > f_{\text{avg}} + \sigma ) = 0.149$$

$$P( f > 2 f_{\text{avg}} ) = 0.062$$

$$P( f > 3 f_{\text{avg}} ) = 0.0062$$

Continuum equations used in engineering design only model the mean behavior.
Q-Model

Model for static stress propagation and fluctuations

\[ P(F) = C F^2 \exp(-F/F_0) \]


*** Experiments support exponential distribution at large F

Improvements by Socolar (torque and vector force bal.)

Similar results by ‘Contact Dynamics’ Radjai et al. PRL 77, 264 (1996).
Force distribution in a granular medium

Daniel M. Mueth, Heinrich M. Jaeger, and Sidney R. Nagel
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(Received 18 August 1997)

We report on systematic measurements of the distribution of normal forces exerted by granular material under uniaxial compression onto the interior surfaces of a confining vessel. Our experiments on three-dimensional, random packings of monodisperse glass beads show that this distribution is nearly uniform for forces below the mean force and decays exponentially for forces greater than the mean. The shape of the distribution and the value of the exponential decay constant are unaffected by changes in the system preparation history or the boundary conditions. An empirical functional form for the distribution is proposed that provides an excellent fit over the whole force range measured and is also consistent with recent computer simulation data.

PACS number(s): 81.05.Rm, 46.10.+k, 80.40.+b

FIG. 1. Sketch of the apparatus used for experiments with “floating walls.” The lower piston is fixed and the cylinder is supported by friction with the bead pack. A load is applied to the upper piston and the beads press the carbon paper into white paper, leaving marks which are used to determine the contact forces. A detail of the obtained raw data is shown in the photograph (field of view: 76 mm across).

FIG. 3. The distribution $P(f)$ of normalized forces $f$ against the top piston (open circles), the bottom piston (diamonds), and the walls (solid circles). The upper panel shows $P(f)$ for the pistons, averaged over fourteen identical experiments. The curve drawn is a fitting function as explained in the text [Eq. (1)]. The lower panel shows the same data, but with data from the walls included as well.
Q-Model

Model for static stress propagation and fluctuations

\[ P(F) = C F^2 \exp(-F/F_0) \]


*** Experiments support exponential distribution at large F

Improvements by Socolar (torque and vector force bal.)

Similar results by ‘Contact Dynamics’ Radjai et al. PRL 77, 264 (1996).
New Continuum Models

1. Q-model → in continuum limit:

   Diffusion of Forces/stresses

   \[
   \frac{\partial F}{\partial x} = \sigma
   \]

2. Oriented Stress Linearity (OSL): based on idea that chains can slip wrt each other.


   Combines static stress balance plus

   \[ T_{xx} = \eta T_{zz} + \mu T_{xx} \]

   \( \eta \) and \( \mu \) history-dependent

   \[
   \left( \partial_x + c_x \partial_x \right) \left( \partial_x + c_x \partial_x \right) T_{ij} = 0
   \]

   (hyperbolic—propagating stresses)

3. Kenkre, Scott, Pease and Hurd—suggest mixed diffusion/propagation

Note: Soil mechanics models can have elliptic behavior
Models of stress fluctuations in granular media

P. Claudin and J.-P. Bouchaud
Service de Physique de l'Etat Condensé, CEA, Orme des Merisiers, 91191 GIF-sur-Yvette, Cedex, France

M. E. Cates and J. P. Wittmer
Department of Physics and Astronomy, JCMB King's Buildings, University of Edinburgh, Mayfield Road, Edinburgh EH9 3JZ, United Kingdom
(Received 19 November 1997)

We investigate in detail two models describing how stresses propagate and fluctuate in granular media. The first one is a scalar model where only the vertical component of the stress tensor is considered. In the continuum limit, this model is equivalent to a diffusion equation (where the role of time is played by the vertical coordinate) plus a randomly varying convection term. We calculate the response and correlation function of this model and discuss several properties, in particular related to the stress distribution function. We then turn to the tensorial model, where the basic starting point is a wave equation that, in the absence of disorder, leads to a raylike propagation of stress. In the presence of disorder, the rays acquire a diffusive width and the angle of propagation is shifted. A striking feature is that the response function becomes negative, which suggests that the contact network is mechanically unstable to very weak perturbations. The stress correlation function reveals characteristic features related to the raylike propagation, which are absent in the scalar description. Our analytical calculations are confirmed and extended by a numerical analysis of the stochastic wave equation. [S1053-651X(98)07004-4]

PACS number(s): 81.05.Rm, 46.10.+z, 05.40.+j, 83.70.Fn

\[
    w(i,j) = w_s + q_+(i-1,j-1)w(i-1,j-1) \\
    + q_-(i+1,j-1)w(i+1,j-1),
\]

(1)
Proposed Model:

Two-branch **Convection-Diffusion Equation**.
Assuming the stress satisfies the following PDE:

\[ O^+ O^- \sigma = 0 , \]

where \( O^\pm = \partial h - D \partial xx \pm c \partial x \), with \( c \) and \( D \geq 0 \).

For a \( \delta(x, 0) \) initial condition, the solution to this equation is:

\[ \sigma_{zz} = \frac{F}{2} \left( \frac{1}{2\sqrt{\pi Dz}} e^{-\frac{(x-cz)^2}{4Dz}} + \frac{1}{2\sqrt{\pi Dz}} e^{-\frac{(x+cz)^2}{4Dz}} \right) \]

- Biased Random-Walk Model and DoubleY Model

\[ D \rightarrow c \Rightarrow OSL \text{ model} \]

\[ O^+ O^- : (e^{2c \Delta x})(e^{-c \Delta x}) \]
Contrast types of P.D.E.'s

1. $\Phi$-model (continuum limit):
   Parabolic

2. OSL hyperbolic:

3. Elasto-plastic:
   elliptic up to yield.
Other Features

1. Hindrance Effects

2. Transition regimes

\[ T = 0 \quad \text{Intermediate} \quad T >> 0 \]

Many flows contain all ranges of granular "temperature" simultaneously
Other Features

1. Hindrance Effects

- Dense ordered
- Low density disordered

2. Transition regimes

\[ T = 0 \quad \text{Intermediate} \quad T \gg 0 \]

- Plastic
- Gas

Many flows contain all ranges of granular "temperature" simultaneously
Fig. 1. A 16-mm camera operating at about 1440 frames per second or 60 times the normal projection rate documents a typical disperse flow in the experimental chute. Plastic spheres 6 mm in diameter contained in a hopper above and behind the upper end of the apparatus feed onto a gently inclined tray, organizing into a single layer that spills into the head of the 6.7-mm-wide chute, where they form an essentially two-dimensional grain flow. The chute is 3.7 m long and 0.5 m deep and can be tilted to any inclination. The bed for the flows is the light-colored strip inside the chute, which can be independently adjusted through a small range of angles to fine tune the inclination. The flows approach a nominally steady, uniform condition rapidly, typically in 10-15 flow depths. The center of the camera field is about 70 cm upstream of the chute outlet.

Drake: Structural Features in Granular Flows

Fig. 3. Flows can be divided into a frictional region in which contacts between neighboring particles are enduring and frictional and an overlying collisional region in which binary collisional contacts predominate. The regions are further subdivided into zones based on characteristics of the collective and individual motions of particles: the frictional region consists of quasi-static and block-girling zones, and the collisional region consists of grain-layer-girling, chaotic, and saltational zones. The boundaries between regions and zones are gradational, and some flows do not exhibit all the zones. Dashed lines on vertical axis indicate both temporal variation and uncertainty in boundary positions. Groups of shaded particles are blocks (see text). Drawing traced from a single film frame of 6-mm-diameter particles flowing from right to left over the 6-mm bed inclined 38° to the horizontal. The flux is about 2070 particles s⁻¹, determined by counting the number of particles crossing a line perpendicular to the bed in 0.42 s.
Conclusions

Granular systems:

Are complex, rich arena for physics and other disciplines

Have phases analogous to solids and fluids, but...

Require new statistical approaches (no ‘canonical ensemble’ for example)

Have unresolved questions of very simple issues such as how forces ‘propagate’

Show large force fluctuations

Offer great intellectual challenges, but also the potential for real economic payoff
Note also Numerical Studies

1. Contact Dynamics - e.g.
   F. Radjai, M. Jean, J.-J. Moreau,
   S. Roux

2. Molecular Dynamics / Discrete Element Methods
   S. Ludwig, H. Herrmann,
   T. Poeschel
   S. McNamara
“Definition”

- Consists of a large number of interacting macroscopic grains.
- Grain-grain interactions are purely classical.
- **Grains interact only when in contact.**
- Collisions are generally inelastic / dissipative.
- Grains are surrounded by a fluid or vacuum.

*Fig. E. van Doorn*
Why Granular Flows?

1. Interesting statistical questions
   Fluctuations, hydrodynamic limit, e.g. of jamming system, self-organized criticality...

2. Continuum dynamics
   What are right models? When do they apply?...

3. Member of class of similar systems:
   Foams, Glasses, Colloids, Quantum flux lattices

4. Important applications
   Coal and grain transport...
   Pharmaceuticals
   Xerography
   Soils
   Avalanches
   Mixing
   Earthquakes and mudslides...

5. $$$!! Lots of them Claim: ~1 $Trillion/year in US for bulk solids handling
   Industrial facilities operate well below design efficiency