Correlated 2D Electron Aspects of the Quantum Hall Effect
Outline:

I. Introduction: materials, transport, Hall effects

II. Composite particles – FQHE, statistical transformations

III. Quasiparticle charge and statistics

IV. Higher Landau levels

V. Other parts of spectrum: non-equilibrium effects, electron solid?
   A. Overview
   B. High magnetic fields – the electron solid
   C. Low magnetic fields – response to radiation

VI. Multicomponent systems: Bilayers
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

A. overview: now examine physics at the extreme ends of the
magnetic field spectrum

2) A new finding near zero B-field, radiation induced
oscillations

1) An old problem at high B-field, the
proposed electron solid: what
happens out here

FIG. 1. Diagonal resistance $R_{xx}$ of a sample of $n = 1.0 \times 10^{11}$ cm$^{-2}$ and $\mu = 10 \times 10^6$ cm$^2$/V sec. Arrows mark several
key fractional Landau level filling factors.

Pan PRL '02
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

A. overview: some numbers

Contrast high and low field values using standard density $1 \times 10^{11} \text{cm}^{-2}$

<table>
<thead>
<tr>
<th></th>
<th>0.1 Tesla</th>
<th>15 Tesla</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length scales</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>magnetic length</td>
<td>813 Å</td>
<td>66 Å</td>
</tr>
<tr>
<td>cyclotron radius</td>
<td>5220 Å</td>
<td>35 Å</td>
</tr>
<tr>
<td>inter-particle spacing</td>
<td>350Å</td>
<td></td>
</tr>
<tr>
<td><strong>Energy scales</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coulomb energy</td>
<td>16 K</td>
<td>193 K</td>
</tr>
<tr>
<td>spin gap</td>
<td>0.03 K</td>
<td>4.3 K</td>
</tr>
<tr>
<td>cyclotron energy</td>
<td>2 K</td>
<td>301 K</td>
</tr>
</tbody>
</table>

Different processes to consider:
- high field = magnetic localization => loss of collective effects
- low field = no kinetic energy quenching => single particle physics
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. High magnetic fields – the electron solid?

Competition between collective effects (solid or liquid) and magnetic freeze-out

Ground state is expected to be Wigner solid for sufficiently large $r_s$ at given small filling factor:

The FQHE states may be in competition with this, or

The ordering may be interrupted by defects, which if sufficiently strong, could localize the electrons.

Solid = both positional and orientational order

uncorrelated

solid

Pinned solid

Magnetically localized, or severely pinned = glass
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

B. High magnetic fields – the electron solid?

The ordering may be interrupted by
defects, which if sufficiently strong, could
localize the electrons.

Magnetically localized, or
severly pinned = glass
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

Given $r_s$ and filling factor $\nu$, what is expected for the phases and in transport?

Predictions for the electron solid: (magnetically induced electron solid)

phase diagram line

- increase in mass pushes onset to larger $r_s$
- increase $r_s$ by decreasing density $n$
V. Other parts of the spectrum:
from the electron solid to non-equilibrium effects

B. the electron solid?

The pinned electron state should have two characteristics
In transport:
1) Insulating in the d.c. limit.
2) Oscillation modes of the electron solid about the pinning sites.
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

For pinned solid or glassy solid, expect transport to show insulated properties

These are indeed apparent

FIG. 1. Diagonal resistance $R_{xx}$ vs magnetic field at $T = 90$ mK. Data are taken on a square sample so that $\rho_{xx} = aR_{xx}$ with $a = 1$. At $v = \frac{1}{2}$, $\rho_{xx} \to 0$ indicating that the $v = \frac{1}{2}$ quantum liquid forms the ground state. The resistivity $\rho_{xx}$ in the sharp spike at $v = 0.21$ and for all $v < \frac{1}{2}$ is rising exponentially on lowering the temperature. All FQHE features at lower magnetic field are well developed but practically invisible on this scale. Inset: Result of a calculation for the total energy per flux quantum of the solid ($E_{\text{tot}}$) and interpolated $1/m$ quantum liquids ($E_{\xi}$) as a function of filling factor (Ref. 4). A classical energy ($E_{\text{class}} = -0.782133v^{-1/2}$) is subtracted for clarity. The dashed lines represent the cusp in the total energy (Ref. 13) of the liquid at $v = \frac{1}{2}$. Its extrapolation intersects the solid at $v = 0.21$ and 0.19 suggesting two phase transitions from quantum liquid to solid around $v = \frac{1}{2}$.

Jiang PRL ’89
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

For pinned solid or glassy solid, expect transport to show insulated properties

These are indeed apparent

Temperature dependence of this large resistance part of the spectrum is

$T \sim \exp(\frac{2}{kT})$

What are the current carrying excitations in this system?
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

If this represents a correlated state, then its energy must be close to that of the FQHE states over some B-field range.

Re-entrance of 1/5 state in insulating background observed for electrons.
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

At low densities (large $r_s$) re-entrance observed around 1/3 in holes, due to the larger effective mass.
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

At low densities (large $r_s$) re-entrance observed around $1/3$ and $2/5$: leads to an interesting phase diagram

FIG. 4 (color online). The boundary of the insulating phases (solid symbols) and the $\nu = 1/3$ and $2/5$ FQHL (open symbols) in the $\nu - r_s$ phase space. Lines are guides to the eye. Dotted contours are loci of points obeying $E_c = \lambda \hbar \omega_c$ (see text).

Csathy PRL ‘05
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

Transition from liquids to insulating complex: Higher temperatures can reveal FQHE states

FIG. 1. Diagonal resistance $R_{xx}$ of a sample of $n = 1.0 \times 10^{11}$ cm$^{-2}$ and $\mu = 10 \times 10^6$ cm$^2$/V sec. Arrows mark several key fractional Landau level filling factors.

FIG. 2. $R_{xx}$ above 20 T at various temperatures. The vertical, dashed lines show the positions of the Landau level filling factors $\nu = 1/5, 2/11, 3/17, 3/19, 2/13$, and 1/7. The inset summarizes high-$T$ limits (open squares) and low-$T$ limits (solid dots) for the observation of features in $R_{xx}$ at various $\nu$. The dashed line is only a guide to the eye. The low-$T$ limits may be viewed as the melting line from Wigner crystal to FQHE liquids and the high-$T$ limits are measures of the energy gap of FQHE states (see text for details).

Pan PRL '02
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

B. the electron solid?

For pinned solid or glassy solid, expect transport to show insulated state

What other properties distinguish a solid from a liquid? What other tests can be performed?

Jiang PRL '89
V. Other parts of the spectrum: 
from the electron solid to 
non-equilibrium effects

B. the electron solid?

The pinned state should have two characteristics 
In transport: 
1) Insulating in the d.c. limit. 
2) Oscillation modes of the electron solid about 
the pinning sites.

Are there simple models for pinned electron solid?

Mode may be complicated due to 
the mixing by the magnetic field
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

The pinned state should have two characteristics

In transport:
1) Insulating in the d.c. limit.
2) Oscillation modes of the electron solid about the pinning sites.

Wavevector dependence should be observed if the electron solid domains are sufficiently large
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

The pinned state should have two characteristics

In transport:
1) Insulating in the d.c. limit.
2) Oscillation modes of the electron solid about the pinning sites.

For disordered systems, the pinning mode will be wavevector independent:

Resonance mode should be present
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

B. the electron solid?

An electron solid can demonstrate shear mode propagation or the wavevector independent pinned mode:

This may be detected using surface acoustic waves:
Contribution to \( ?'(?) \), \( ?''(?) \) when SAW dispersion and electron solid modes cross
Also true for other frequency, wavevector dependent techniques
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

Instead of distinct mode crossings, a broad resonance structure was found near 1GHz

![Graph showing magnetic-field dependence of dc resistance ρ_xx, normalized SAW attenuation, and velocity shifts in sample 1 at 80 mK and 235 MHz. The theory lines are calculated from Eq. (1) using ρ_xx(dc).]
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

As temperature is lowered, the onset to this mode can be traced over filling factor range:

Temperature versus $f$ phase diagram

---

**FIG. 2.** Temperature dependence of normalized velocity shift and attenuation in sample 2 at 160 kG ($\nu=0.167$) and 91 MHz. The theory lines are based on Eq. (1) and on the simultaneously measured $\rho_{xx}(\text{dc})$ values.

**FIG. 4.** High-frequency conductivity $\sigma_{xx}(k,\omega)$ of a 2DEG in the Wigner crystallization regime as a function of inverse temperature. The inset shows $T_c$ as a function of filling factor $\nu$. 

Temperature versus $f$ phase diagram
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid? coplanar waveguide can also be used to look for $\chi_{xx}(?)$:

Power absorption $P = \exp\{ (2/Z_0/w) \Re \left[ \chi_{xx}(?) \right] \}$, $w =$ width, $l =$ length of waveguide

$\Re(\chi_{xx}(?)) \sim \ln(\text{absorbed power}, P)$

$q \sim \chi/w$, $w = 30-80\text{?m}$
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

With cleaner samples and in hole samples this mode structure is observed

\[
\text{coplanar waveguide: } \text{Re}(\sigma_{xx}(\omega)) \sim \ln(\text{absorbed power})
\]

Chen PRL '97
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

Higher mobilities show sharper, multiple modes at lower frequencies: (may reflect fewer pinning sites)

One mode at B-field lower than 1/5, disappears at high fields

Also, possible wavevector dependence

Coplanar waveguide technique

Chen PRL ’04
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

The pinned state should have two characteristics:

In transport:
1) Insulating in the d.c. limit.
2) Oscillation modes of the electron solid about the pinning sites.
3) Depinning, or non-linear I-V

\[
\left( \frac{\alpha}{\sqrt{\pi}} \right)^2 e E_T = \alpha \eta q
\]

with \((\frac{\alpha}{\sqrt{\pi}})^2\) electrons/domain, and \(n\) the integer crystal shear modulus:

\[
\eta = 0.12 e^2 n^2 q / 4 \pi \omega^2
\]

with \(\omega\) a numerical product of 0.1 \(\text{[Hamer, } \text{92,}} \text{ Bassel, } \text{77} \text{, Esfand} \text{91]}\).

With above and \(E_T \sim 1 \text{MV/cm}\)

\(\Rightarrow \beta / \alpha > 10\)

In CO3 (Bassel, '88), relate pinning frequency \(\omega\) to threshold field \(E_T\):

\[
e E_T \approx \eta / \omega^2 \approx 0.5 \text{MV/cm}
\]

But a) + b) above produce \(\beta / \alpha \sim 1\), and \(\omega\) from measurements \(\Rightarrow E_T\) too large ?!!

Depinning thresholds and microwave resonances not consistent
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

The pinned state should have two characteristics:

1) Insulating in the d.c. limit.
2) Oscillation modes of the electron solid about the pinning sites.
3) Depinning, or non-linear I-V

Non-linear I-V observed

Shayegan '93

Depinning thresholds and microwave resonances not consistent in models
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

B. the electron solid?

Summary:
1) Insulating state observed at small filling factors
2) Competition between insulating state and fractional quantum Hall states
3) Radio frequency mode observed – broad and high frequency for low mobility, narrow for cleaner samples
4) Non-linear I-V

NO DIRECT OBSERVATION OF ORDERING: we still don’t know if we are observing a true ordered electron system or a glass
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Pertinent numbers for small magnetic fields: 0.1T, $1 \times 10^{11}$ cm$^{-2}$

1) Fermi energy 41 K
2) Spin gap 0.03 K
3) Cylotron energy 2.0 K
4) Coulomb energy 16 K
5) Cyclotron radius 5200 Å

With little magnetic field to quench kinetic energy, single particle effects more prominent
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Apply high frequency radiation (50-120GHz) through waveguide onto 2D electron system and look at low B-field end
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Upon radiation on a high mobility sample with GHz radiation, oscillations are observed near zero magnetic field.

Minima at \( \frac{C}{2} = j + \frac{1}{2}, \ j = 1, 2, 3, \ldots \)
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

oscillations are observed near zero magnetic field

minima at \( C = j + 1/2, \) 
j=1,2,3,...
( \( C = e/Bm^*, \) m* bare GaAs mass)

no dependence upon filling factor (density)

What’s causing this?
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

For higher mobility samples, the minima look like quantum Hall effect zeroes
V. Other parts of the spectrum:
from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Minima show activated temperature dependence:
consistent with an energy gap

Large activation energies:
activation energy $E$
photon energy $E_\gamma$
$E \gg E_\gamma$

Gap energy $E \sim 18\text{K}$, $E_\gamma \sim 3\text{K}$
for $\omega = 57\text{GHz}$, $j=1$

Zudov PRL '03, see also Mani Science '03
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

C. Low magnetic fields: response to radiation

Zudov PRL '03, see also Mani Science '03
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Further work showed that oscillations can be observed at lower frequencies, revealing same physics
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Using lower frequencies, consider possible anomalous current paths due to radiation
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Using lower frequencies, consider possible anomalous current paths due to radiation

By looking at different contact configurations around samples, found

Zeroes not always zeroes

RLW PRL '04
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Using lower frequencies, consider possible anomalous current paths due to radiation

By looking at different contact configurations around samples, found

Zeroes not always zeroes

Common structure in non-zeroes
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

Consider possible anomalous current paths due to radiation

In addition to non-zeroes, found systematic differences in zeroes around sample perimeter
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

In addition to non-zeroes, found systematic differences in zeroes around sample perimeter

And anomalous response to reversing B-field
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

In addition to non-zeroes, found systematic differences in zeroes around sample perimeter

And anomalous response to reversing B-field

Clearly odd current paths at work

Reverse B-field direction

B-field

B-field
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. response to radiation: Theory of oscillations

Among many, two theories developed to explain radiation induced magneto-oscillations:

1) Radiation induced, disorder assisted current

Durst, et. al. PRL ’03

2) Radiation induces change in electron distribution function

Dmitriev, et al. PRL ’03

FIG. 1. Simple picture of radiation-induced disorder-assisted current. Landau levels are tilted by the applied dc bias. Electrons absorb photons and are excited by energy \( \omega \). Photoexcited electrons are scattered by disorder and kicked to the right or to the left by a distance \( \pm d \). If the density of states to the left exceeds that to the right, current is enhanced. If vice versa, current is diminished.

FIG. 2. Magneto-oscillations of the dynamical conductivity for a system with smooth disorder. \( \tau_{e,0}/\tau_{c,0} = 10 \). Solid line: separated LLs, \( \omega_c \tau_{e,0}/\pi = 3.25 \); dashed line: overlapping LLs, \( \omega_c \tau_{e,0}/\pi = 1 \). Inset: \( \sigma_{xx} \) for fixed \( \omega_c \tau_{e,0}/2\pi = 1 \) as a function of \( \omega_c \).

Cyclotron-Resonance Harmonics in the ac Response of a 2D Electron Gas with Smooth Disorder

I. A. Dmitriev,1,4 A. D. Mirlin,1,2,3 and D. G. Polyakov1,4

1Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany
2Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany
(Received 23 April 2003; published 24 November 2003)

The frequency-dependent conductivity \( \sigma_{xx}(\omega) \) of 2D electrons subjected to a transverse magnetic field and smooth disorder is calculated. The interplay of Landau quantization and disorder scattering gives rise to an oscillatory structure that survives in the high-temperature limit. The relation to recent experiments on photoconductivity by Zadov et al. and Mani et al. is discussed.
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

C. response to radiation: Theory of oscillations

Both theories deduce
a) Oscillatory resistivity with
   period $\sim 1/B$

b) negative local microscopic
   resistivity due to radiation

FIG. 3. Photoresistivity (normalized to the dark Drude value) for separated Landau levels vs $\omega_c/\omega$ at fixed $\omega \tau_\alpha = 16\pi$. The curves correspond to different levels of microwave power $P_\omega^{(0)} = \{0.004, 0.02, 0.04\}$.

Magneto-oscillations covered

Dmitriev, et al. cond-mat/0310668
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

Dynamical Symmetry Breaking as the Origin of the Zero-dc-Resistance State in an ac-Driven System

A. V. Andreev,1,2 I. L. Aleiner,3 and A. J. Millis3
1Physics Department, University of Colorado, Boulder, Colorado 80309, USA
2Bell Labs, Lucent Technologies, Room 1D-267, Murray Hill, New Jersey 07974, USA
3Physics Department, Columbia University, New York, New York 10027, USA
(Received 3 February 2003; published 1 August 2003)

Under a strong ac drive the zero-frequency linear response dissipative resistivity $\rho_d(j = 0)$ of a homogeneous state is allowed to become negative. We show that such a state is absolutely unstable. The only time-independent state of a system with $\rho_d(j = 0) < 0$ is characterized by a current which almost everywhere has a magnitude $j_0$ fixed by the condition that the nonlinear dissipative resistivity $\rho_d(j_0) = 0$. As a result, the dissipative component of the dc-electric field vanishes. The total current may be varied by rearranging the current pattern appropriately with the dissipative component of the dc-electric field remaining zero. This result, together with the calculation of Dunst et al., indicates the existence of regimes of applied ac microwave field and dc magnetic field where $\rho_d(j = 0) < 0$, explains the zero-resistance state observed by Mani et al. and Zudov et al.

Negative local resistivity explains the zeroes:

system unstable with $? < 0$, current circulations form

C. response to radiation: Theory of zeroes

any microscopic mechanism of nonequilibrium drive resulting in $\rho_d(j^2 = 0) < 0$ leads to the observed [1,2] zero dissipative differential resistance:

$$\frac{dV_x}{dl_{dc}} = 0.$$ (5)

FIG. 1. Assumed dependence of the dissipative (parallel to current) component of the local electric field $E_x$ on the current density $j_x$. Inset: dependence of the dissipative resistivity on the square of the current.

Andreev, et al. PRL ‘03
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. response to radiation: Theory of oscillations

Negative local resistivity explains the zeroes: system unstable with \( \omega < 0 \); current circulations form. These spontaneous current domains can result in the observed zero resistance.

Suggested to try to examine voltage drop from inner to outer contacts

FIG. 2. The simplest possible pattern of the current distribution—domain wall. The net current, \( I \), is accommodated by a shift of the position of the domain wall by the distance \( d \); see text. The electric field in the domain is \( E_0 = \rho_H j_0 \). The current pattern in the Corbino disc geometry is obtained by connecting the broken edges into a ring.

Andreev, et al. PRL ‘03
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. response to radiation:

![Graph showing resistivity and magnetic field](image-url)
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. Low magnetic fields: response to radiation

**Currents induced by radiation:**

No current driven through sample from external source, yet voltage pickup from internal to external contact

![Graph showing resistivity vs. magnetic field](image)
V. Other parts of the spectrum: from the electron solid to non-equilibrium effects

C. response to radiation: experiment and theory of zeroes

**Microscopic**

$\rho_{\text{eff}} > 0$

**Macroscopic currents**

Negative local resistivity explains the zeroes:

- system unstable with $\rho_{\text{eff}} < 0$
- current circulations form
- Vortices within the sample, induced by the radiation, and if large enough potentially detectable

These may be observed in experiments

**FIG. 2:** The simplest possible pattern of the current distribution—domain wall. The net current, $I$, is accommodated by a shift of the position of the domain wall by the distance $d$, see text.
V. Other parts of the spectrum:
from the electron solid to
non-equilibrium effects

C. Low magnetic fields:
response to radiation

Summary:
1) Resistance oscillations of period $\sim \omega_c$ occur with GHz radiation
2) Higher mobility samples show zeroes in oscillation minima
3) Oscillations explained as due to non-equilibrium electron distribution function
4) Negative local resistivity can result, these induce current vortices
5) Current vortices observed experimentally as induced sample voltages
Outline:

I. Introduction: materials, transport, Hall effects

II. Composite particles – FQHE, statistical transformations

III. Quasiparticle charge and statistics

IV. Higher Landau levels

V. Other parts of spectrum: non-equilibrium effects, electron solid?

VI. Multicomponent systems: Bilayers

   A. Overview - materials
   B. Experiments – drag, tunneling, QHE
   C. Recent experiments and excitonic Bose-Einstein condensation

For review see also J.P. Eisenstein, Les Houches ‘04
VI. Multicomponent systems: bilayers

A. materials

To produce a bilayer system make two AlGaAs wells: the important parameters now are the well separation $d$ and the electron separation $l_o$.

Interesting physics when $d$ close to $l_o$, so that $d$ is < 20nm.
VI. Multicomponent systems: bilayers

A. materials

To produce a bilayer system make two AlGaAs wells: the important parameters now are the well separation $d$ and the electron separation $l_o$.

AlAs barrier is *dirty*. This limits mobility of the individual layers.

Typical GaAs layer thickness 20nm, AlAs thickness 3nm
VI. Multicomponent systems: bilayers

A. materials

In producing such a structure, the problem of the reverse interface is confronted: a smoother surface occurs when growing from GaAs to AlGaAs than from AlGaAs to GaAs.

This fact and the necessary doping scheme mean that the mobilities for these bilayer structures are not as high as for the single interface structures.
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Gramilla ‘91
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Gramilla '91
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Gramilla ‘91
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Backgating possible by allignment through sample

Gramilla ‘91
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Backgating possible by alignment through sample

Gramilla ‘91
VI. Multicomponent systems: bilayers

A. materials

Experimentally need to contact the layers independently to really study the interactions between the two layers.

Use a set of gates on the top and on the bottom of the sample to accomplish this.

Backgating possible by alignment through sample

Gramilla '91
VI. Multicomponent systems: bilayers

A. materials

Independent contacting facilitates a series of experiments:

1) Drag measurements: probing adjacent layer interactions

2) Tunneling measurements: response to injecting charge

3) Bilayer transport measurements showing QHE: collective state of two adjacent layers
VI. Multicomponent systems: bilayers

B. experiments: 1) drag

Examine Coulomb scattering between layers

\[ R_D \equiv \frac{V}{I} \]

“large” separation (>100Å)

Gramila, et al. 1991
VI. Multicomponent systems: bilayers

B. experiments: 1) drag

Guess that $R_D (\sim ? D^{-1}) \sim T^2$

From phase space argument

Gramila, et al. 1991
VI. Multicomponent systems: bilayers

B. experiments: 1) drag

Note!

Theory: \( \rho_D \propto \frac{T^2}{N^3 d^4} \)

MacDonald ‘91
Jauho & Smith ‘93

M. Kellogg, et al. 2002
VI. Multicomponent systems: bilayers

B. experiments: 1) drag and composite fermions
VI. Multicomponent systems: bilayers

B. experiments: 1) drag and composite fermions

Drag resistance greatly enhanced at 1/2 filling factor

\[ \rho_{xx} \]

\[ \rho_D \]

\[ B = 11.4T, \nu = \frac{1}{2} \]

\[ B = 0, \times 1000 \]

\[ R_D \sim \text{Not } T^2 \]

Lilly, PRL ‘98
VI. Multicomponent systems: bilayers

B. experiments: 1) drag and composite fermions

High B-field slow relaxation of charge density fluctuations

Theory gets $T^{4/3}$, not overall magnitude.

Functional dependence due to composite particle conductivity wavevector dependence:

\[ \rho_D \approx 0.8 \frac{h}{e^2} \left( \frac{T}{T_0} \right)^{4/3} \]

\[ T_0 \approx \pi e^2 N_s d / \varepsilon \approx 190 \text{ K} \]

\[ \rho_D \approx 10 \Omega/\square \text{ @ } T = 0.5 \text{ K} \]
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

Sharp resonance at $B=0$ due to momentum and energy conservation in 2D

(Tunneling through oxide from and to normal metals)

Murphy, PRB ‘95
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

\[ \mathbf{A} = -B \mathbf{z} \hat{y} \]
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

Tunneling in a Parallel Magnetic Field

\[ \mathbf{A} = -Bz\mathbf{\hat{y}} \]
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

**Tunneling in a Large Perpendicular Magnetic Field**

![Graph showing tunnel current as a function of interlayer voltage](image)

**Zero bias suppression suggests Coulomb gap**
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

*Tunneling in a Large Perpendicular Magnetic Field*

![Diagram of tunneling in a large perpendicular magnetic field](image)

- B=0
- B=13T, d/\ell=5.3

![Graph showing tunneling](image)
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

Field Dependence: $B = 8 → 14T$

Suppression is generic
VI. Multicomponent systems: bilayers

B. experiments: 2) tunneling

**Lowest Landau Level is a Strongly Correlated System**

**Tunneling is fast compared to relaxation time of charge defects.**

**Result:** Tunneling is suppressed at voltages below mean Coulomb energy.

\[ e \Delta V_{\text{gap}} \approx 0.3 \frac{e^2}{\varepsilon \ell} \]
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

Quantum Hall effect observed at total filling factor $1 = \frac{1}{2}$ in each layer and at $\frac{1}{4}$ in each layer
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

As $d/l_o$ increases, the QHE is lost at total filling factor $\frac{1}{2}$, $\frac{1}{4} + \frac{1}{4}$:

The interlayer Coulomb interactions are necessary for the QHE at these bilayer filling factors.
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

Experiments show
1) Phase boundary
2) Phase boundary intercepts vertical axis at finite $d/l_o$ for $\nu = 1$ state

Murphy et al '90
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

\[ \Psi_{l,m,n}(z_1, \ldots; w_1, \ldots) \sim \prod_{i<j} (z_i - z_j)^l \prod_{i<j} (w_i - w_j)^m \prod_{i<j} (z_i - w_j)^n \]

At \( \nu_T = 1/2 \):

\[ \Psi_{331} \sim \prod_{i<j} (z_i - z_j)^3 \prod_{i<j} (w_i - w_j)^3 \prod_{i<j} (z_i - w_j)^3 \]

Similar to a 1/3 FQHE state plus one hole/electron
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

At $\nu_T = 1/2$:

$$\Psi_{331} \sim \prod_{i<j} (z_i - z_j)^3 \prod_{i<j} (w_i - w_j)^3 \prod_{i<j} (z_i - w_j)$$

Bilayer QHE states related to Halperin generalized Laughlin states?
VI. Multicomponent systems: bilayers

B. experiments: 3) quantum Hall effect

QHE has two possible origins:

Single particle & Many-body effect

Reduce tunneling to get rid of the single particle origin
VI. Multicomponent systems: bilayers

C. Experiments – $\psi_{\text{total}} = 1$ excitonic Bose condensate

For low tunneling regime, predicted that the condensed state at $\psi_{\text{total}} = 1$ may be an excitonic Bose Condensate?

Charge neutral excitons

J.P. Eisenstein, ‘03
VI. Multicomponent systems: bilayers

C. Experiments – $\nu_\text{total} = 1$ excitonic Bose condensate

Zero Tunneling Limit

$\nu_T = 1$

NO QHE

QHE

$N_1 \sim N_2 \sim 5 \times 10^{10} \text{ cm}^{-2}$

$\text{Al}_{0.5}\text{Ga}_{0.1}\text{As}$

10nm

18nm

Quantum critical point

$\nu_T = 1$

$\nu_T = 1/2 + 1/2$

layer spacing

J.P.E. ‘04
VI. Multicomponent systems: bilayers

C. Experiments – \( \nu_{\text{total}} = 1 \) excitonic Bose condensate

\( \nu_{\text{total}} = 1 \) QHE clearly not due to simple single particle spectrum

\[ \text{estimated } \Delta_{\text{SAS}} \approx 90\mu K \approx 1.2 \times 10^{-6} \left( \frac{e^2}{\varepsilon \ell} \right) \]

very low tunneling & large \( \Delta = 0.34K \) energy gap in these samples
VI. Multicomponent systems: bilayers

C. Experiments – $n_{\text{total}} = 1$ excitonic Bose condensate

Again, examine tunneling near the excitonic phase

Remember:

$B=0$ Narrow resonance, created by momentum and energy conservation; a single particle effect. $\Gamma \sim 200\mu\text{eV}$

$B >> 0$ Strongly suppressed tunneling around zero bias, broad high energy response; a many-body-effect.
VI. Multicomponent systems: bilayers

C. Experiments – $n_{\text{total}} = 1$ excitonic Bose condensate

However, near condensate........

Tunneling is fast compared to relaxation time of charge defects.

Result: Tunneling is suppressed at voltages below mean Coulomb energy.

\[ e \Delta V_{\text{gap}} \approx 0.3 \frac{e^2}{\varepsilon \ell} \]
VI. Multicomponent systems: bilayers

C. Experiments – $\nu_{\text{total}} = 1$ excitonic Bose condensate

As phase boundary is crossed into the QHE range, tunneling peak occurs similar to $B=0$
VI. Multicomponent systems: bilayers

C. Experiments – $\gamma_{\text{total}} = 1$ excitonic Bose condensate

Counter-flow superfluidity rapidly relaxes charge defects created by tunneling.
VI. Multicomponent systems: bilayers

C. Experiments – $\nu_{\text{total}} = 1$ excitonic Bose condensate

What is expected in transport when the layers can be probed independently? *Counter-flow experiment*

Charge neutral excitons should feel no Lorentz force!
VI. Multicomponent systems: bilayers

C. Experiments – \( \nu_{\text{total}} = 1 \) excitonic Bose condensate

What is expected in transport when the layers can be probed independently?

**Counter-flow experiment – Hall resistance goes to zero**

At \( \nu_T = 1 \), \( R_{xy}^{CF} \to 0 \) as \( T \to 0 \)

exciton transport dominates counterflow

Kellogg, PRL ’04
VI. Multicomponent systems: bilayers

C. Experiments – $\nu_{\text{total}} = 1$ excitonic Bose condensate

What is expected in transport when the layers can be probed independently?

**Counter-flow experiment – longitudinal resistance goes to zero**

At $\nu_T = 1$: $R_{xy}^{CF} \to 0$ and $R_{xx}^{CF} \to 0$

*Excitonic superfluidity?*

See also results on drag at total filling factor 1

Kellogg, PRL '04
VI. Multicomponent systems: bilayers

C. Experiments – $\phi_{\text{total}} = 1$ excitonic Bose condensate

Conductivities

$d/\ell = 1.5$

Counterflow dissipation small but non-zero at all finite $T$.

See also results on drag at total filling factor 1
VI. Multicomponent systems: bilayers

C. Experiments – $\phi_{\text{total}} = 1$ excitonic Bose condensate

Results

In closely-spaced bilayer 2D electron systems at $\nu_T = 1$:

Start with a double layer 2D electron gas

Add a magnetic field

Presto! A BCS-like superfluid comprised of interlayer excitons.
Summary:
1) Multilayer systems of high mobility (correlations) possible
2) Drag experiments between layers expose Coulomb interaction properties
3) Tunneling measurements show the effects of correlations in the layers through their relaxation
4) Bilayer total filling factor 1 state shows ensemble of effects that are interpreted as a BEC of excitons
The end