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Title: ***Phases and phase transitions in disordered quantum systems***

Lectures:
1. Phase transitions and quantum phase transitions
a) review of basic concepts (first-order vs continous transitions, Landau theory, critical behavior, universality, scaling)
b) introduction to quantum phase transitions (experimental examples, quantum scaling, quantum-to-classical mapping)
2. Phase transitions in disordered systems
a) types of disorder (random mass and random fields)
b) Harris criterion and the stabilty of clean critical points
c) Imra-Ma argument and destruction of phase transitions by random fields
d) rounding of first-order phase transitions by disorder
3. Strong-disorder renormalization group
a) basic idea of the strong-disorder renormalization group
b) renormalizing the random transverse-field Ising chain
c) exotic infinite-randomness critical point
4. Griffiths phases
a) rare regions and large fluctuations
b) classical Griffiths singularities
c) quantum Griffiths singularities
5. Smeared phase transitions
a) rare regions in metallic systems (dissipation, freezing transition)
b) smearing of quantum phase transitions in metals
c) smeared transitions in system with correlated disorder

Training sessions:
Participants will explore the topics discussed in the lectures by working out specific examples
1. Quantum-to-classical mapping of the transverse-field Ising model
2. Harris criterion and Imry-Ma argument for correlated disorder
3. Random-singlet phase in disordered Heisenberg chain via strong-disorder RG
4. Percolation quantum phase transitions I
5. Percolation quantum phase transitions II
6. Replica trick

Lecture Notes Summer School Salerno 2012

Phases and phase transitions in disordered quantum systems

- **Lecture 1: Phase transitions and quantum phase transitions (Review)**

- discuss plan of lecture series
  1) review of PT and QPT
  2) phase transitions in disordered systems, overview + stability criteria
  3) strong-disorder renormalization group
  4) rare regions and Griffiths phases
  5) phase transitions smeared by disorder

- discuss plan for training sessions
  1) quantum-to-classical mapping of RTIM
  2) stability criteria for correlated disorder
  3) SDRG for Heisenberg chain

  4+5) Percolation QPT

  arXiv: 1301.7746 (Salerno)

- literature: review articles T.V. JPA 39, R143 (06)

  JLTP 161, 299 (10)

Who knows: Landau theory, Ginzburg criterion
scaling, critical exponent

1a Basic concepts of phase transitions

• What is a phase transition?

- every example : PTs of H_2O
  (⇒ show phase diagram)
- definition PT is singularity in free energy as function of external parameters $(T, p, B, x \ldots)$
- can only occur in thermodynamic (infinite system) limit
  (Why? sum of exponentials in partition function is analytic)

• Classification of PTs

- PTs can be divided into two qualitatively different classes
- to understand difference, consider again example of H_2O:
  start with piece of ice, $T < 0°C$,
  - heat it up ⇒ T rises
  - at $T = 0°C$, ice starts to melt while T stays constant

  ⇒ phase coexistence (latent heat to turn ice to liquid)
  - only after all ice has melted, T continues to rise

$\Rightarrow$ phase transitions following this scenario (phase coexistence + latent heat) = first-order PT

(name refers to 1st derivative of free energy being discontinuous, in general Ehrenfest 2nd, 3rd, ... order $\rightarrow$ not that useful)

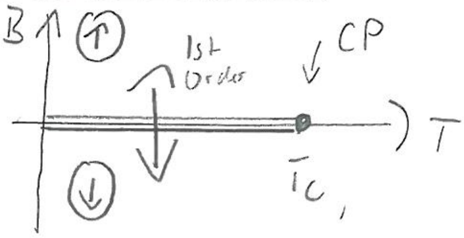
- PTs of H_2O are generically 1st order

• Are there other types?

- follow liquid-gas PB to higher T, p $\Rightarrow$ liquid and gas become more similar ($\Delta\rho \rightarrow 0$)
- at critical point, they become indistinguishable $\Rightarrow$ no latent heat, no phase coexistence

$\Rightarrow$ PTs going through such CPs are called continuous PTs

- further example ferromagnetic PT of iron



• Critical behavior

- CPs (continuous PTs) have many peculiar properties
- example: critical opalescence
  (⇒ show pictures of mixing CP)

  discuss origin: strong fluctuations on large length scales (λ of light)
- in general: CPs are associated with diverging fluctuations:

$$\underline{\text{fluid}} \quad G(\vec{r}-\vec{r}') = \langle \Delta\rho(\vec{r})\, \Delta\rho(\vec{r}') \rangle$$
$$\underline{\text{magnet}} \quad \langle \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r}') \rangle$$
} become long-ranged

- $G(\vec{r}-\vec{r}') \sim \exp(-|\vec{r}-\vec{r}'|/\xi)$

  correlation length ξ diverges at CP

(long-range order below T_c) $$\boxed{\xi \sim \left|\frac{T-T_c}{T_c}\right|^{-\nu}} \quad \text{or} \quad \left|\frac{\rho-\rho_c}{\rho_c}\right|^{-\nu}$$

  ν is example of a <u>critical exponent</u>

- fluctuations not only <u>large</u>, also <u>slow</u>

  correlation time

$$\boxed{\xi_t \sim \xi^z \sim \left|\frac{T-T_c}{T_c}\right|^{-\nu z}}$$ critical slowing down

  $z \mathrel{\hat{=}}$ dynamic critical exponent

- power laws in length and time scales lead to power laws in observables

examples : liquid gas CP : $\Delta\rho \sim |T-T_c|^{\beta}$

$\kappa \sim |T-T_c|^{-\gamma}$

FM CP : $m \sim |T-T_c|^{\beta}$

$\chi \sim |T-T_c|^{-\gamma}$

$m \sim B^{1/\delta}$ (at T_c

- list of exponents, see reviews
- Collection of power laws and exponents characterizes CP

$\Rightarrow$ critical behavior

- Universality

- critical exponents do not depend on system detail

$\Rightarrow$ all liquid-gas CP have precisely the same exponents

$\Rightarrow$ also identical to Ising FM

$\Rightarrow$ Critical behavior is universal, depends on dimensionality and symmetries only

(good for theory !)

• Critical dimensions

- fluctuations are strong close to CP
- How strong ??

  ⇒ depends on dimensionality d !

  - in low d, fluctuations are so strong that they destroy ordered phase at <u>all</u> T ⇒ no PT
  - this happens for $d \leq d_c^-$
    (d_c^- = lower critical dimension)
  - between d_c^- and d_c^+, ordered phase and PT exist, but exponents are influenced by fluctuations, <u>depend on d</u>
    (d_c^+ = upper critical dimension)
  - above d_c^+, fluctuations are unimportant for critical behavior
    ⇒ exponent values <u>independent</u> of d
    ⇒ mean-field theory valid

<u>Examples:</u> Ising magnet $d_c^- = 1$, $d_c^+ = 4$
Heisenberg magnet $d_c^- = 2$, $d_c^+ = 4$

• Scaling

- phenomenological description of CPs
- extremely powerful (analysis of exp and numerical data)
- can nowadays be derived within RG approach

Basic idea

- ξ is only relevant length scale close to (
- at CP, $\xi = \infty$ $\Rightarrow$ scale invariant

  off CP, ξ finite, large
- if ξ is only relevant length, rescaling all L and adjusting parameters such that ξ has same value $\Rightarrow$ system should be unchanged
- free energy density ($f = F/V$)

$$\boxed{f(t,B) = b^{-d} f(t b^{\frac{1}{\nu}}, B b^{y_B})}$$

b = arbitrary length scale factor

- scaling laws for other variables

  $\Rightarrow$ take derivatives of f
- how to use? set b to appropriate values (e.g. $b = t^{-\nu}$)

• Landau theory

- Landau provides framework for description of (conventional) PTs
- bulk phases are characterized by symmetries
  PT involve breaking of these symmetries (spontaneously)

  example   Ising FM
  - in high-T phase (PM), spins point up or down at random
    $\Rightarrow$ up-down symmetry not broken
  - in low T-phase (FM), spins pick preferred direction
    $\Rightarrow$ up-down symmetry spontaneously broken

- order parameter
  - characterizes degree of symmetry breaking
    (zero in one phase, nonzero + nonunique in other phase)
  - example FM: OP is magnetization m
    $m = 0$ in PM phase $(T > T_c)$
    $m \neq 0$, positive or negative in FM $(T < T_c)$

- quantitative description

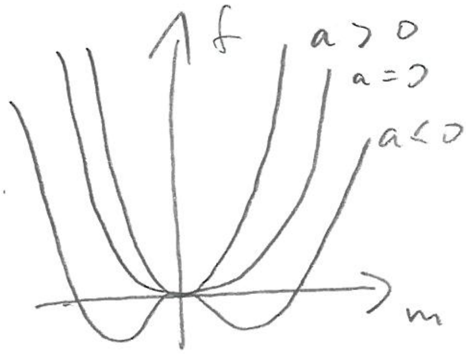
Landau: expand f in powers of OP m

$$f = r m^2 + \cancel{b m^3} + u m^4 - B m$$

Consider case where $b = 0$ by symmetry ($b \neq 0 \Rightarrow$ 1st order PT)

$B \hat{=}$ symmetry-breaking external field

- physical state: minimize f w.r.t. m



$r \sim T - T_c$

distance from criticality

Landau theory gives MF exponents

$m \sim |r|^{\frac{1}{2}}$ for $r<0, B=0 \quad \Rightarrow \beta = \frac{1}{2}$

$\chi = \left.\frac{\partial m}{\partial B}\right|_{B=0} \sim |r|^{-1} \quad \Rightarrow \gamma = 1$

$\Rightarrow$ Landau theory correct above d_c^+

$\Rightarrow$ Landau theory fails below d_c^+

Why? Does not contain fluctuations!

Generalize: Landau-Ginzburg-Wilson theory

domain-wall term

$$F = \int d^d x \left\{ r m^2(\vec{x}) + \left(\nabla m(\vec{x})\right)^2 + u m^4(\vec{x}) - B m(\vec{x}) \right\}$$

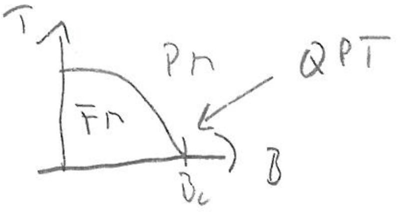
$$Z = \int \mathcal{D}[m(\vec{x})]\, e^{-F} \quad \Rightarrow \text{RG methods}$$

1b) Introduction to QPTs

- so far PTs at nonzero T, often tuned by T
- ordered phase destroyed by thermal fluctuations (ice melts due to thermal motion of H_2O molecules) ⇒ thermal PT (also classical PT, see later)

⇒ different type of PT, occurs at $T=0$ (in quantum ground state) as function of other parameter $(p, B, x, \ldots)$

Example: FM transition in $LiHoF_4$
(⇒ show PD of $LiHoF_4$)



FM order destroyed by transverse B. How!

$$H = -\sum_{\langle ij \rangle} J S_i^z S_j^z - \sum_i h S_i^x$$

$$S_i^x = S_i^+ + S_i^- \quad \text{flips spins}$$

⇒ order destroyed by quantum fluctuations, (zero point motion, uncertainty principle)

⇒ this type of PT ⇒ QPT
(1st order: simple level crossings, continuous involve diverging fluctuations)

⇒ show more examples: - $Sr_{1-x}Ca_xRuO_3$
- Mott transition in atomic gas
- MIT in metal

Question:

Can we generalize concepts of thermal PTs (scaling, ...) to QPTs?

important idea: quantum-to-classical mapping

- partition function of classical system

$$Z = \int dp\,dq\, e^{-\beta H(p,q)} = \int dp\, e^{-\beta T(p)} \int dq\, e^{-\beta V(q)} \sim \int dq\, e^{-\beta V(q)}$$

Gaussian, no singularities

(configuration integral only, explains why classical LGW theory only has fluctuations in space, not in time ($\Rightarrow$ see also Ising model)

- same factorization NOT possible in quantum system (T and V do not commute), but

Trotter decomposition

$$Z = \mathrm{Tr}\, e^{-\beta H} = \mathrm{Tr}\, e^{-\beta(T+V)} = \lim_{N\to\infty} \prod_{n=1}^{N} \left(e^{-\beta \frac{T}{N}} e^{-\beta \frac{V}{N}} \right)$$

$$Z = \int D[q(\tau)]\, e^{-S[q(\tau)]}$$

$\Rightarrow$ introduces imaginary time "slices"
time direction goes from 0 to $\beta = 1/T$
becomes infinite at $T = 0$

$\Rightarrow$ At QPT, imaginary time acts as extra dimension

$$\boxed{\text{A QPT in } d \text{ dimensions is equivalent to classical (thermal) PT in } d+1 \text{ dimensions}}$$

Caveats: - works for thermodynamics only, not applicaple to real-time dynamics

- resulting classical system can be unusual and anisotropic

- only works if resulting action is real (so that it can be interpreted as classical free energy)
(↯ Berry phases)

Explicit example: transverse-field Ising model (see training session)

Generalization of scaling to QPT

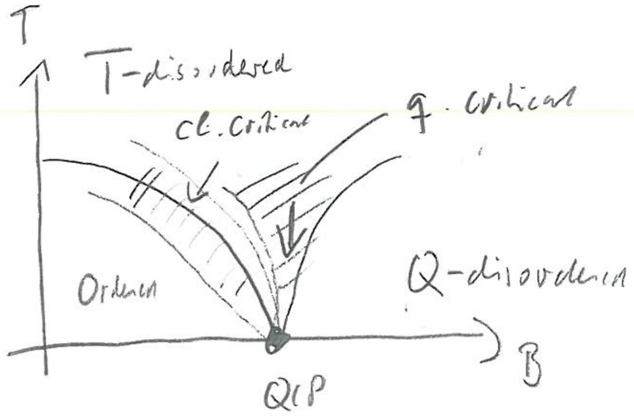
- include imaginary time τ as argument of scaling form
if $L \to bL \Rightarrow \tau \to b^z \tau$
- temperature T is independent parameter $T \to b^{-z} T$

$$f(r, B, T) = b^{-(d+z)} f(r b^{\frac{1}{\nu}}, B b^{y}, T b^{z})$$ free energy density

$$G(r, B, T, \vec{x}, \tau) = b^{-2\beta/\nu} G(r b^{\frac{1}{\nu}}, B b^{y}, T b^{z}, \vec{x} b^{-1}, \tau b^{-z})$$

- phase diagram close to QCP (B)

⇒ show picture



$\hbar\omega_c \lessgtr k_B T$

quantum vs thermal fluct.

- at any finite T ⇒ thermal fluctuations win because $\hbar\omega_c \sim \frac{1}{\xi_t} \to 0$

⇒ phase transition at any finite T is classical (extension in imaginary time finite = β if $\xi_t > \beta$, dimension drops out)

Lecture 2 : Phase transitions in disordered systems

2a) Types of disorder

- disorder or randomness can have many reasons
  - vacancies, impurity atoms in crystal
  - amorphous solid rather than crystal
  - larger defects: dislocations, grain boundaries
  - in cold atom systems: speckle light

  etc

- distinguish quenched and annealed disorder

  quenched: frozen in, does not change over experimental time scale

  annealed: fluctuations on time scales ≪ experimental scale

⇒ annealed disorder is conceptually easier: just include disorder d.o.f. (e.g. impurity position) into stat. mech partition function

⇒ quenched disorder is harder, each sample is different, averages are averages of F rather than Z (technically hard ⇒ Replica trick $\ln Z = \lim_{n\to 0} \frac{Z^n - 1}{n}$, works sometimes, see maybe training session)

From now on, we consider only quenched (frozen-in) disorder.

- in Lecture 1:
  qualitative properties of PT depend on dimensionality and symmetries, but not on details

$\Rightarrow$ Classify disorder according to symmetries

Example ferromagnet clean Hamiltonian

$$H = -J \sum_{\langle ij \rangle} S_i S_j \qquad (J>0, \; S=\pm 1)$$

- randomness may lead to local fluctuations of J (but all $J_{ij} > 0$) maybe because nonmagnetic impurity atoms modulate the n.n. distance $\Rightarrow$ $H = -\sum_{\langle ij \rangle} J_{ij} S_i S_j$

- Can also be realized by dilution (nonmagnetic, isoelectronic impurities)

$\Rightarrow$ randomness changes local tendency towards FM $\Rightarrow$ local "T_c" changes

$\Rightarrow$ random-T_c disorder

- does not break symmetry),
- most benign disorder no change in bulk pha[se]

- in LGW theory

$$F = \int d^d x \left\{ \left(r + \underbrace{\delta r(\vec{x})}_{\text{randomness}}\right) m^2(\vec{x}) + \left(\nabla m(\vec{x})\right)^2 + u m^4(\vec{x}) \right\}$$

disorder couples to m^2 (mass term in QFT)

$\Rightarrow$ random mass disorder

- If disorder is added to clean system undergoing PT, the following questions appear:

  - Are the bulk phases changed?
  - Is transition still sharp or is it smeared because different parts of sample order independently?
  - Does the order of the PT change (1st order vs. continuous)
  - Does the critical behavior (exponents) of a CP change?

2.5) Harris criterion and stability of clean CPs

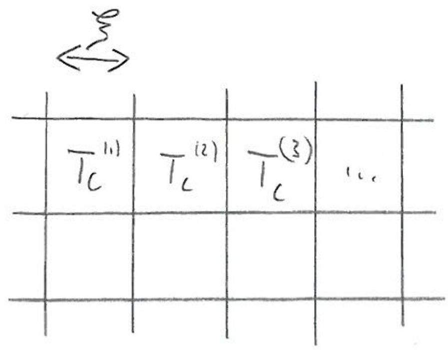
- consider <u>clean</u> system having critical point (for example clean FM)

Add random-T_c quenched disorder!
– we know, bulk phases will <u>not</u> change

$\boxed{\text{Question: Will character of PT change?}}$

Harris (1974) devised criterion for stability of clean critical point

- <u>Derivation</u>



– divide system into blocks of size ξ (spins in such a block fluctuate together)
– each block has its own average $T_c^{(i)}$ determined by local impurity conf.

– compare variation of T_c from block to block with global distance from criticality

– central limit theorem $\Delta T_c \sim \xi^{-d/2}$
– global distance $T - T_c \sim \xi^{-\frac{1}{\nu}}$

uncorrelated disorder, see train

– for stable CP, we must have $\Delta T_c < T - T_c$ for $\xi \to \infty$ $\Rightarrow$ $d/2 > \frac{1}{\nu}$

$\Rightarrow$ $\boxed{\text{Harris criterion} \quad d\nu > 2}$

Interpretation of the Harris criterion

- if $d\nu > 2$, $\Delta T_c \ll T - T_c$ as you approach CP
- system looks less and less disordered on large length scales $\Rightarrow$ effective disorder strength vanishes at CP

$\Rightarrow$ clean critical behavior stable
(disordered system has same exponents as clean one,

observables self-averaging

- example: 3D classical Heisenberg model
$\nu \approx 0.69 > 2/d$

In contrast

- if $d\nu < 2$, $\Delta T_c \gg T - T_c$ as one approaches CP
- some blocks are in one phase, some in the other

$\Rightarrow$ uniform sharp transition impossible

$\Rightarrow$ if $d\nu < 2$, clean critical point is unstable

$\Rightarrow$ character of transition must change!

NOTE:

Harris criterion holds in same form, $d\nu > 2$, for QCPs (replace T by quantum control parameter in derivation) $\Rightarrow$ d is NOT replaced by $d+z$

Reason: d enters via central limit theorem, counts dimensions with randomness; quenched disorder, no randomness in τ - direction

- alternatively

- disorder could couple linearly to OP m

$$F = \int d^dx \left\{ r\, m^2(\vec{x}) + \left(\nabla m(\vec{x})\right)^2 + u\, m^4(\vec{x}) - B(\vec{x})\, m(\vec{x}) \right\}$$

- in our toy FM, this could be realized by a magnetic field that varies randomly from site to site $\Rightarrow$ random-field disorder

- random-field disorder locally breaks up-down symmetry
  ($\Rightarrow$ generally stronger than random-T_c disorder)

- many other types of disorder are possible
  - random anisotropy in Heisenberg magnet (breaks symmetry of $u\, m^4(\vec{x})$ term)
  - random signs in interaction $\Rightarrow$ frustration, spin glass
  - random phase of OP
  - many more

In this lecture series, we only consider random-mass and random-field disorder!

(for now values of random coupling at different sites uncorrelated)

- What happens if Harris criterion is violated, $d\nu < 2$?

$\Rightarrow$ Harris criterion itself cannot tell!

We will explore some possibilities in this lecture series!

Simplest possibility

- transition still sharp,
- new CP with exponents that fulfill $d\nu > 2$

$\Rightarrow$ disorder strength remains finite at large length scales

$\Rightarrow$ no self-averaging

Example: 3D classical Ising magnet
clean $\nu \approx 0.63 \Rightarrow$ dirty $\nu \approx 0.68$

($\Rightarrow$ show data of Wiseman/Domany)

- Many classical PTs follow this scenario, QPTs often display more exotic behavior (see next sections)

NOTE: - Harris criterion does NOT pose a bound for dirty ν
- However, Chayes et al showed (under mild assumptions) that the dirty ν must fulfill same inequality $d\nu > 2$. (Exceptions

2c) Random-field disorder and Imry-Ma argument

- Consider clean system undergoing PT, add weak random-field disorder

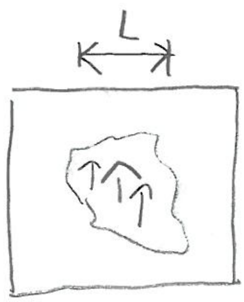
(example: Ising ferromagnet with random magnetic field
$$H = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i h_i S_i$$
$$\langle h_i \rangle = 0, \quad \langle h_i h_j \rangle = W \delta_{ij}, \quad S_i = \pm 1)$$

- random locally breaks up-down symmetry
  spins with $h_i > 0$ want to point $\uparrow$ $(S_i = 1)$
  $h_i < 0$ ~ $\downarrow$ $(S_i = -1)$

Question: Do we still get ordered (FM) phase in the presence of weak ($W \ll J^2$) random fields?

- What prevents all spins from simply following their field? $\Rightarrow$ neighboring spins want to be $\upuparrows$

$\Rightarrow$ Imry + Ma: Compare energy gain due to RF with loss due to domain walls



- Consider domain of linear size L
- energy gain from aligning domain with average local RF
  $\Delta E_{RF} \sim \sqrt{W}\, L^{d/2}$ (central limit theorem)
- domain wall energy $\sim$ area of DW
  $\Delta E_{DW} \sim J L^{d-1}$

- if $\Delta E_{RF} < \Delta E_{DW}$, uniform FM is stable

$$\sqrt{W}\, L^{d/2} \overset{!}{<} J L^{d-1}$$

$$\sqrt{W} < J L^{d/2-1}$$

$\Rightarrow$ if $d > 2$, ordered state is stable against weak RF (strong enough RF still destroy order)

if $d < 2$, ordered state becomes unstable against domain formation if L is large enough

if $d = 2$, marginal case more sophisticated methods necessary

- argument can be made rigorous $\Rightarrow$

Aizenman - Wehr theorem

> Random-fields destroy long-range order (and prevent spontaneous symmetry breaking) for all $d \leq 2$ for discrete symmetry (Ising) or $d \leq 4$ for continuous symmetry (Heisenberg)

for continuous symmetry DW gets spread out over L

$$|\xleftarrow{\quad L \quad}\!\!\xrightarrow{\qquad}|$$

$$\uparrow\uparrow\nearrow\nearrow\rightarrow\rightarrow\searrow\searrow\downarrow\downarrow$$

$$\Delta E_{DW} \sim L^d (\nabla m)^2$$
$$\sim L^d \left(\frac{1}{L}\right)^2 \sim L^{d-2}$$

rather than L^{d-1} for discrete symmetry

2d) Rounding of 1st-order PT by random-T_c disorder

- a very similar argument can be made to attack a different problem

- consider clean system undergoing 1st-order PT
- add weak random-T_c disorder

Question: Will 1st-order PT survive?

- Remember, 1st-order PT characterized by macroscopic phase coexistence at T_c

- however, random-T_c disorder locally favors one phase over the other
- will domains form?
  compare free energies for domain of size L

$$\Delta F_{dis} \sim \sqrt{W}\, L^{d/2}$$

$$\Delta F_{DW} \sim \sigma \cdot L^{d-1}$$

(in general, the two phases are not connected by continuous transformation)

if $\Delta F_{dis} < \Delta F_{DW}$, macroscopic phases stable

$$\sqrt{W}\, L^{d/2} < \sigma L^{d-1}$$

$$\frac{\sqrt{W}}{\sigma} < L^{d/2-1} \Rightarrow$$ 1st-order PT destroyed for $d \leq 2$

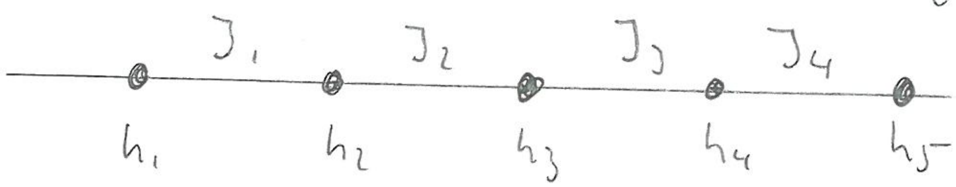
Lecture 3: Strong-disorder renormalization group

- clever method to study disordered systems
- invented by Ma, Dasgupta and Hu in 79, greatly developed by DSFisher 92–95

Here: SDRG for 1D random transverse-field Ising model (will get exact critical behaviour)

$$\boxed{H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x}$$

FM GS for $J_i \gg h_i$
PM GS for $h_i \gg J_i$
CP for $\prod J_i = \prod h_i$



J_i, h_i are independent random variables with probability distributions $P_I(J)$, $R_I(h)$

- traditional approach: first solve clean problem $J_i \equiv J$, $h_i \equiv h$, then treat disorder as perturbation

  $\Rightarrow$ works very poorly (will see later why – infinite randomness)

- Here instead: make use of the disorder from the outset

– basic idea: (Ma-Dasgupta-Hu 79)
identify largest local energy ($\hat{=}$ highest local excited state)
use perturbation theory for the neighboring couplings (good if P_I, R_I are broad)

3b) Renormalization group transformation

identify maximum energy $\Omega = \max(h_i, J_i)$

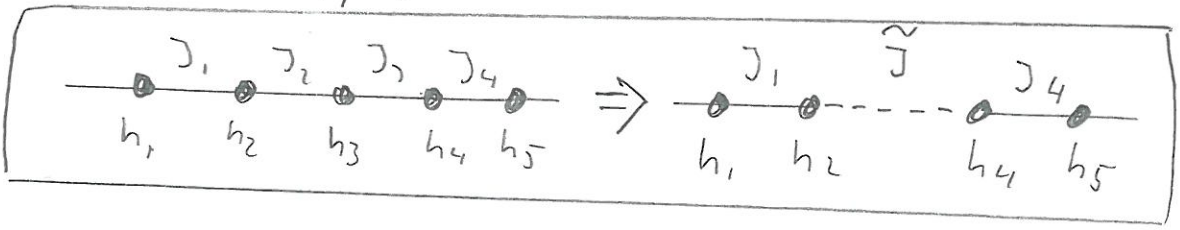
① <u>if largest energy is field, say $h_3 \gg J_2, J_3$</u>

- $\vec{\sigma}_3$ is pinned in x-direction, does not contribute to z-magnetization
- $\vec{\sigma}_3$ can be eliminated, but virtual excitations of $\vec{\sigma}_3$ from $|\rightarrow\rangle$ to $|\leftarrow\rangle$ generate a coupling $\tilde{J}$ between $\vec{\sigma}_2^z$ and $\vec{\sigma}_4^z$
- to calculate $\tilde{J}$, consider 3-site system $(\vec{\sigma}_2, \vec{\sigma}_3, \vec{\sigma}_4)$ with $H = H_0 + H_1$

$$H_0 = -h_3 \sigma_3^x \quad , \quad H_1 = -J_2 \sigma_2^z \sigma_3^z - J_3 \sigma_3^z \sigma_4^z$$

- 2nd order perturbation theory in H_1:

$$\tilde{J} = J_2 J_3 / h_3$$

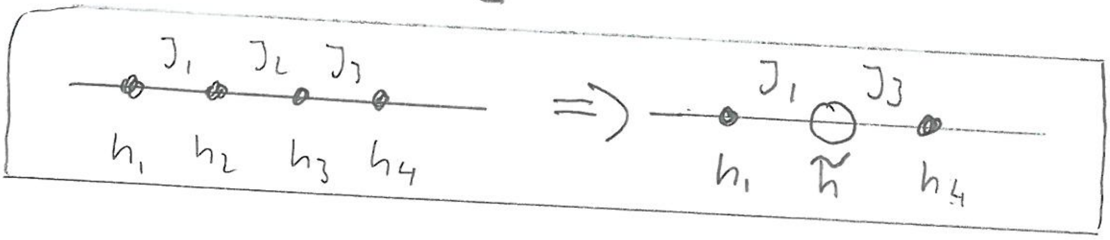


(2) <u>if largest energy is an interaction, say $J_2 \gg h_2, h_3$</u> (3.

- spins $\vec{\sigma}_2$ and $\vec{\sigma}_3$ want to be parallel,
  $\Rightarrow$ can be treated as cluster or "superspin" $\tilde{\sigma}$ of magnetic moment $\tilde{\mu} = \mu_2 + \mu_3$
- effective transverse field acting on $\tilde{\sigma}$ can be calculated in 2nd order perturbation theory using

$$H_0 = -J_2 \sigma_2^z \sigma_3^z \,, \quad H_1 = -h_2 \sigma_2^x - h_3 \sigma_3^x$$

$$\tilde{h} = h_2 h_3 / J_2$$



<u>Result of RG step</u>

- in each case, one spin gets removed
- maximum energy decreases because $\tilde{J} \ll \Omega$, $\tilde{h} \ll \Omega$

$$\boxed{\tilde{J} = \frac{J_{i-1} J_i}{h_i} \,, \quad \tilde{h} = \frac{h_i h_{i+1}}{J_i} \,, \quad \tilde{\mu} = \mu_i + \mu_{i+1}}$$

<u>Note 1</u> symmetry between J and h (duality in problem)

<u>Note 2</u> $\tilde{h} \sim h_i h_{i+1}$ multiplicative, crucial !!

$\mu \sim \mu_i + \mu_{i+1}$ additive

suggests $h \sim \exp(c\mu)$ exponential relation between energy and length (volume) ?!

Flow equations

- we now iterate the RG steps, slowly decreasing the max energy Ω
- Question: How do distributions $P(J)$ and $R(h)$ change under this RG?

$\Rightarrow$ derive flow equations for P and R

- reduce max energy from Ω to $\Omega - d\Omega$ by decimating all J, h in $[\Omega - d\Omega, \Omega]$

$$-\frac{\partial P(J,\Omega)}{\partial \Omega} = R(\Omega)\left[-2P(J) + \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right)\right] + \Big(R(\Omega) + P(\Omega)\Big) P(J)$$

$[\ \ ]$: decimation, remove 2 bonds, add one bond

$(\ \ )$: keep P normalized

$$\boxed{\begin{aligned} -\frac{\partial P}{\partial \Omega} &= \left[P(\Omega) - R(\Omega)\right] P(J) + R(\Omega) \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right) \\ -\frac{\partial R}{\partial \Omega} &= \left[R(\Omega) - P(\Omega)\right] R(h) + P(\Omega) \int dh_1 dh_2\, R(h_1) R(h_2)\, \delta\left(h - \frac{h_1 h_2}{\Omega}\right) \end{aligned}}$$

Solve flow equations, look for fixed points, i.e. for distributions invariant under the RG transformation

Logarithmic variables

multiplicative structure of the recursions suggests using logarithmic variables

$\Gamma = \ln\left(\frac{\Omega_I}{\Omega}\right)$. Ω_I initial value of Ω

$\zeta = \ln\left(\Omega/J\right) \geq 0$ interactions $P(J) = \frac{1}{J}\bar{P}(\zeta)$

$\beta = \ln\left(\Omega/h\right) \geq 0$ fields $R(h) = \frac{1}{h}\bar{R}(\beta)$

(bar on P, R will be dropped if it is clear which distribution is meant)

transform flow equations

$$\boxed{\begin{aligned}\frac{\partial P}{\partial \Gamma} &= \frac{\partial P}{\partial \zeta} + (P_0 - R_0)\,P + R_0 \int_0^{\zeta} d\zeta_1\, P(\zeta_1)\,P(\zeta-\zeta_1) \\ \frac{\partial R}{\partial \Gamma} &= \frac{\partial R}{\partial \beta} + (R_0 - P_0)\,R + P_0 \int_0^{\beta} d\beta_1\, R(\beta_1)\,R(\beta-\beta_1)\end{aligned}}$$

$P_0 = P(0)$

How to solve?

- Complete solution given by DSF (PRB 1995) long, tedious math

- here instead: use ansatz

$$\boxed{P = P_0\, e^{-P_0 \zeta}, \quad R = R_0\, e^{-R_0 \beta}}$$

where P_0, R_0 are functions of Γ

(distributions are exponential, width changes with R_0)

Insert ansatz into flow equations

$$\frac{dP_0}{d\Gamma} \equiv \dot{P}_0$$

$$\dot{P}_0 e^{-P_0 \varrho} - P_0 \varrho \dot{P}_0 e^{-P_0 \varrho} = -P_0^2 e^{-P_0 \varrho} + (P_0 - R_0) P_0 e^{-P_0 \varrho} + R_0 \underbrace{\int_0^{\varrho} d\varrho_1 \, P_0^2 e^{-P_0 \varrho_1} e^{-P_0(\varrho - \varrho_1)}}_{P_0^2 e^{-P_0 \varrho} \varrho}$$

$$\dot{P}_0 (1 - P_0 \varrho) = - R_0 P_0 (1 - P_0 \varrho)$$

$$\boxed{\begin{aligned} \frac{dP_0}{d\Gamma} &= -R_0 P_0 \\ \frac{dR_0}{d\Gamma} &= -P_0 R_0 \end{aligned}}$$

flow equations for parameters $P_0(\Gamma)$, $R_0(\Gamma)$

now look for (fixed point) solutions!

3c) Infinite-randomness critical point

- criticality $\prod J_i = \prod h_i$, (duality!)
- CP for $P_0 = R_0$

$$\boxed{\frac{dP_0}{d\Gamma} = -P_0^2}$$

$$-\frac{dP_0}{P_0^2} = d\Gamma$$

$$\frac{1}{P_0} = \Gamma - \Gamma_0$$ (set $\Gamma_0 = 0$, redefinition of Ω)

$$P_0(\Gamma) = \frac{1}{\Gamma}$$

$$\boxed{P(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}}$$

$$R(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}$$

$$P(J) = \frac{1}{J} P(\zeta) = \frac{1}{J}\frac{1}{\Gamma} e^{-\zeta/\Gamma}$$

$$= \frac{1}{J}\frac{1}{\Gamma} e^{-\ln\left(\frac{\Omega}{J}\right)/\Gamma}$$

$$\boxed{P(J) = \frac{1}{J}\frac{1}{\Gamma}\left(\frac{J}{\Omega}\right)^{\frac{1}{\Gamma}}}$$

- distribution becomes arbitrarily broad for $\Gamma \to \infty$ ($\Omega \to 0$) $\Rightarrow$ <u>infinite-randomness CP</u>

  (CP at Harris criterion is fulfilled, disorder goes to 0)

- <u>number of surviving clusters</u>

$$\frac{dn_\Gamma}{d\Gamma} = -(P_0 + R_0)\, n_\Gamma = -\frac{2}{\Gamma} n_\Gamma$$

(number of decimations when $\Gamma \to \Gamma + d\Gamma$! $(P_0 + R_0)\, d\Gamma$)

$$-2\frac{d\Gamma}{\Gamma} = \frac{dn}{n} \Rightarrow \boxed{n(\Gamma) \sim \frac{1}{\Gamma^2}}$$

3.

$$n(\Omega) = \left(\ln \frac{\Omega_I}{\Omega}\right)^{-2}$$

distance between surviving clusters

$$\ell(\Omega) \sim \frac{1}{n(\Omega)} \sim \left(\ln \frac{\Omega_I}{\Omega}\right)^{2}$$

$\Rightarrow$ relation between length and time scales

$$\boxed{\ln\left(\frac{\Omega_I}{\Omega}\right) \sim \ell^{\psi} \qquad \psi = \frac{1}{2}}$$

- exponential relation between length and time (rather than power law)

$\Rightarrow$ activated scaling, z formally ∞

- <u>magnetic moment of surviving cluster</u>
  (lengthy calculation)

$$\boxed{\mu(\Omega) \sim \left(\ln \frac{\Omega_I}{\Omega}\right)^{\phi}} \qquad \phi = \frac{\sqrt{5}+1}{2} < 2$$

holes in cluster

$\Rightarrow$ unusual properties, will be reflected in behaviour of observables!!

Off-critical solutions

- $P_0 \neq R_0$

- focus on paramagnetic side $\langle \ln h_i \rangle > \langle \ln J_i \rangle$
- under RG, building of clusters stops at some point because most h are larger than most J $\Rightarrow$ only h are decimated

$\Rightarrow$ $R(\beta)$ should become stationary
$P(\zeta) \sim$ scale to smaller ζ rapidly

$$\left.\begin{aligned} \frac{dP_0}{d\Gamma} &= -R_0 P_0 \\ \frac{dR_0}{d\Gamma} &= -R_0 P_0 \end{aligned}\right\} \frac{d}{d\Gamma}(R_0 - P_0) = 0$$

$$R_0 = P_0 + 2\delta \qquad \delta \equiv \text{const}$$

Meaning of δ

$$\langle \ln \frac{J}{\Omega} \rangle = \langle -\zeta \rangle = -\frac{1}{P_0}$$

$$\langle \ln \frac{h}{\Omega} \rangle = \langle -\beta \rangle = -\frac{1}{R_0}$$

$$\langle \ln h \rangle - \ln \langle J \rangle = -\frac{1}{R_0} + \frac{1}{P_0} \sim R_0 - P_0$$

$\delta \equiv$ measure for distance from criticality

Solution for $\Gamma \to \infty$ ($\Omega \to 0$)

$$R_0 = 2\delta$$
$$P_0 = \frac{2\delta}{e^{2\delta\Gamma} - 1}$$

$$\boxed{\begin{aligned} R(\beta) &= 2\delta\, e^{-2\delta\beta} \\ P(\zeta) &= e^{-2\delta\Gamma}\, e^{-(e^{-2\delta\Gamma})\beta} \end{aligned}}$$

independent of Γ

becomes extremely broad

$\Rightarrow$ J extremely small $\Rightarrow$ clusters decouple

$$\Rightarrow \boxed{R(h) = 2\delta\,\frac{1}{h}\left(\frac{h}{\Omega}\right)^{2\delta}}$$

power-law with non-universal exponent

- number clusters

$$\frac{dn}{d\Gamma} = -(R_0 + P_0)\,n = -2\delta n$$

$$n(\Gamma) = n_0 e^{-2\delta\Gamma} = n_0\, e^{-2\delta \ln\left(\frac{\Omega_I}{\Omega}\right)} \sim \left(\frac{\Omega}{\Omega_I}\right)^{2\delta}$$

distance between clusters

$$\ell(\Omega) \sim \frac{1}{n(\Omega)} \sim \left(\frac{\Omega}{\Omega_I}\right)^{-2\delta}$$

power law scaling with $z = \frac{1}{2\delta}$

non universal

- Correlation length

P_0 deviates from critical flow $\frac{1}{\Gamma}$ for $\Gamma_x > \frac{1}{2\delta}$

$$\ell(\Gamma_x) \sim \Gamma_x^2 \sim \left(\frac{1}{2\delta}\right)^2$$

$\nu = 2$ saturating Chayes inequality

Thermodynamics at IRCP

- general strategy for finding T-dependence of observables:
    - run RG to energy scale $\Omega = T$
      all d.o.f. decimated have energy $\gg T$, do not contribute
    - all remaining clusters have $J, h \ll T$, can be considered free

at criticality

$$n(\Omega) \sim \left[\ln(\Omega_I/\Omega)\right]^{-\frac{1}{\psi}} \qquad \psi = \frac{1}{2}$$

$$\mu(\Omega) \sim \left[\ln(\Omega_I/\Omega)\right]^{\varphi} \qquad \varphi = (\sqrt{5}+1)/2$$

a) Susceptibility

each free cluster contributes $\frac{\mu^2}{T}$

$$\chi(T) \sim \frac{1}{T}\, n(T)\, \mu^2(T) \sim \frac{1}{T}\left[\ln\left(\frac{\Omega_I}{T}\right)\right]^{-\frac{1}{\psi}+2\varphi}$$

b) entropy

each free cluster has entropy $\ln 2$

$$S(T) = (\ln 2)\, n(T) \sim \left[\ln\left(\frac{\Omega_I}{T}\right)\right]^{-\frac{1}{\psi}}$$

c) specific heat

$$C_V = T\left(\frac{\partial S}{\partial T}\right) = \frac{\partial S}{\partial \ln T} \sim \left[\ln\left(\frac{\Omega_I}{T}\right)\right]^{-\frac{1}{\psi}-1}$$

off criticality (PM phase)

$$n(\Omega) \sim \Omega^{2\delta} = \Omega^{\frac{1}{z}}$$

entropy:

$$S = (\ln 2)\, T^{\frac{1}{z}}$$

specific heat

$$C_v = T\left(\frac{\partial S}{\partial T}\right) \sim T^{\frac{1}{z}} = T^{2\delta}$$ non-universal exponent

$\Rightarrow$ singular thermodynamics even off criticality

analogously $\chi \sim T^{\frac{1}{z}-1}$

$\Rightarrow$ example of quantum Griffiths singularities

$\Rightarrow$ see Lecture 4

Lecture 4 Griffiths singularities and Griffiths phase

Wie APCTP lecture

Lecture 1 : Introduction to the quantum Griffiths phase

Overview :

1. What is the Griffiths phase?
2. Classical Griffiths singularities — diluted Ising model
3. Quantum Griffiths singularities - diluted transverse-field Ising model
4. Classification of Griffiths (rare region) effects

Literature T. Vojta, J. Phys A 39, R143 (2006) (review)

(1) What is the Griffiths phase?

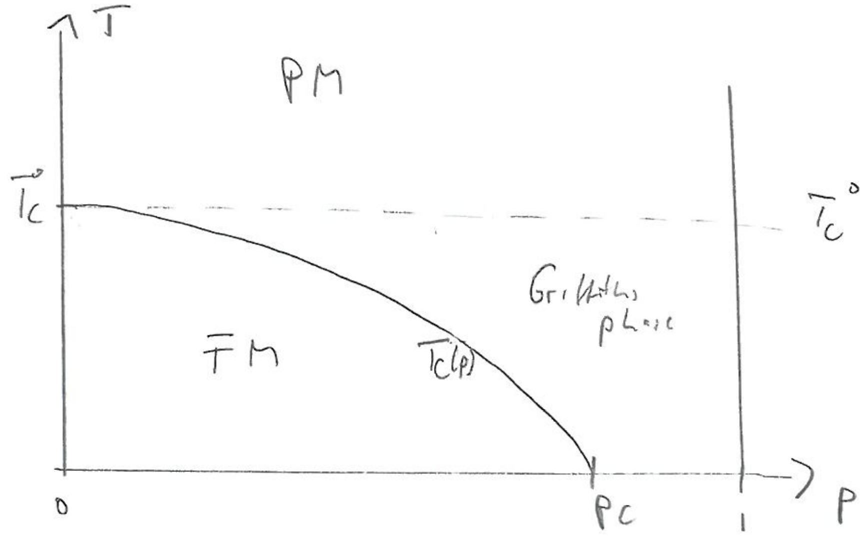
- Consider clean ferromagnet, for definiteness Ising model $\boxed{H = -J \sum_{\langle ij \rangle} S_i S_j}$ $\quad S_i = \pm 1$ on square or cubic lattice

$\Rightarrow$ magnetic phase transition at some T_c which is known exactly in 2D and numerically in 3D

- now: site dilution $\boxed{H = -J \sum_{\langle ij \rangle} \varepsilon_i \varepsilon_j S_i S_j}$

$$\varepsilon_i = \begin{cases} 0 & \text{vacancy with probability } p \\ 1 & \text{spin with } \sim \quad 1-p \end{cases}$$

- dilution reduces T_c, phase diagram



- rare regions: small, but non-zero probability that large spatial region is free of impurities

$\Rightarrow$ forms a finite-size piece of the clean system $\Rightarrow$ rare region locally ordered below T_c^0 (clean system T_c)

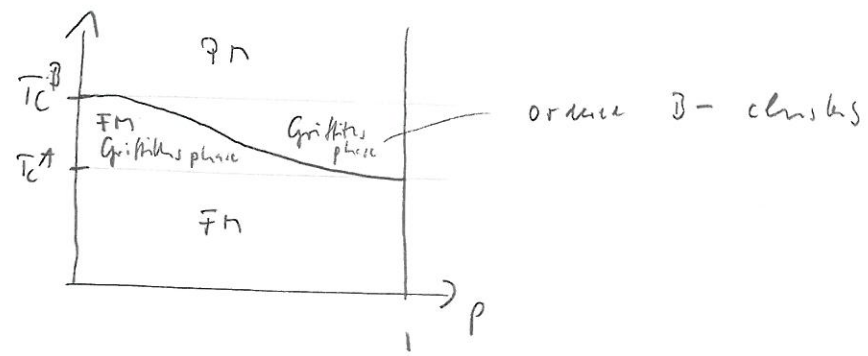
$\Rightarrow$ acts as superspin

$\Rightarrow$ parameter region, where these locally ordered regions exist, but no long-range-order, is called "Griffiths phase" (better Griffiths region)

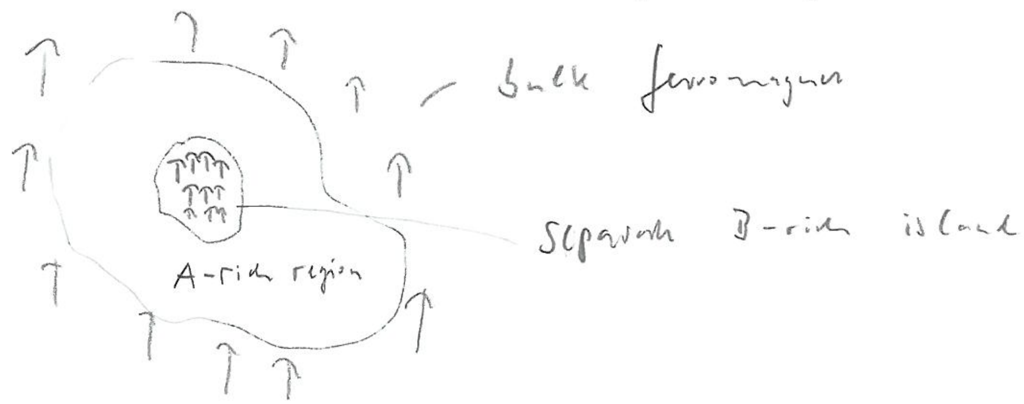
Here entire region below T_c^0 but above $T_c(p)$

Remark 1. extension of the Griffiths phase depends on model

for $H = -\sum_{\langle ij \rangle} J_{ij} S_i S_j$ ($J_{ij} = J_A, J_B$ with probabilities $p, 1-p$)



Remark 2: One can define Griffiths phase also on the ferromagnetic side



Question: Why is the Griffiths phase interesting?

$\Rightarrow$ peculiar properties of large locally-ordered clusters / droplets

② Classical Griffiths singularities

a) Thermodynamics of the Griffiths phase

- Griffiths (1969) : free energy is singular everywhere in the Griffiths phase
  (but only existence proof, no explanation

- estimate contribution of large droplets to $m(H)$, 1χ

$\Rightarrow$ locally ordered droplet of volume V acts as superspin with moment proportional to $\mu_B V$

in a field H, the energy is

$E \sim - H \mu_B V$

$\Rightarrow$ if $|E| > k_B T$ $\Rightarrow$ superspin is fully polarized

if $|E| < k_B T$ $\Rightarrow$ magnetization is small

$$m_{RR}(H) \sim \sum_{\text{droplet}}^{(|E| > k_B T)} w(V) \, \mu_B V \qquad \boxed{w(V) \sim e^{-pV}}$$

$$\sim \int_{\frac{k_B T}{\mu_B H}}^{\infty} dV \; e^{-pV} \; \mu_B V$$

to exponential accuracy

$$m_{RR}(H) \sim e^{-\frac{k_B T}{\mu_B H}}$$

Close to transition $\boxed{m_{RR}(H) \sim e^{-\frac{k_B T_c}{\mu_B H}}}$

essential singularity

$$\chi_{RR} \sim \int dV \; w(V) \; \frac{(\mu_B V)^2}{T}$$

large droplets make small contribution

$\Rightarrow$ rare regions indeed lead to singular free energy, but singularity is weak (essential singularity), likely unobservable in experiment

Classical dynamics

auto correlation function $\quad C(t) = \frac{1}{N} \sum_i \langle S_i(t) S_i(0) \rangle$

rare region contribution:

$$C_{RR}(t) \sim \int dV \; w(V) \; e^{-t/\xi_t(V)}$$

life time $\xi_t(V)$

create domain wall of free energy $\sigma L^{d-1} = \sigma V^{\frac{d-1}{d}}$

$$\xi_t \sim \tau_0 \, e^{\sigma V^{\frac{d-1}{d}}}$$

$$C_{RR}(t) \sim \int dV \, e^{-pV} \, e^{-t/\tau_0 \, e^{-\sigma V^{\frac{d-1}{d}}}}$$

saddle point method $\quad 0 = p - \frac{t}{\tau_0} c V^{-\frac{1}{d}} \, e^{-\sigma V^{\frac{d-1}{d}}}$

$$\Rightarrow t \sim \tau_0 e^{\sigma V^{\frac{d-1}{d}}} \qquad \ln(t/\tau_0)^{\frac{d}{d-1}} \sim V$$

$$\boxed{\ln C_{RR}(t) \sim -(\ln t)^{\frac{d}{d-1}}}$$

slow dynamics dominated by RR

(3) Quantum Griffiths singularities

- transverse-field Ising model

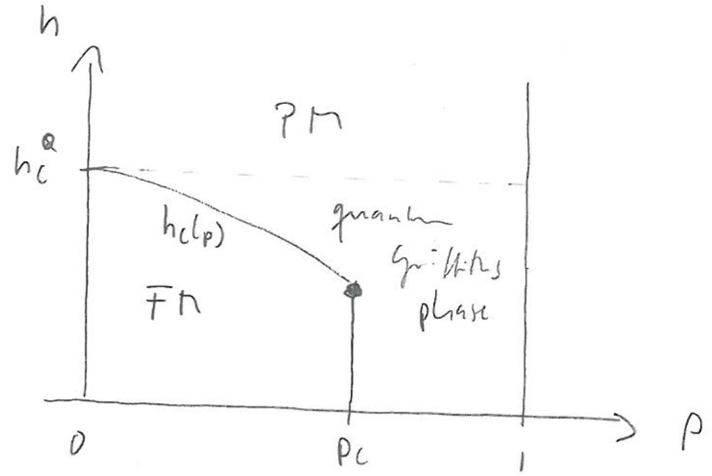
$$H = -J \sum_{\langle ij \rangle} \hat{S}_i^z \hat{S}_j^z - h^x \sum_i \hat{S}_i^x$$

$\hat{\vec{S}}_i$ spin operators

$S_i^x \sim S_i^+ + S_i^-$

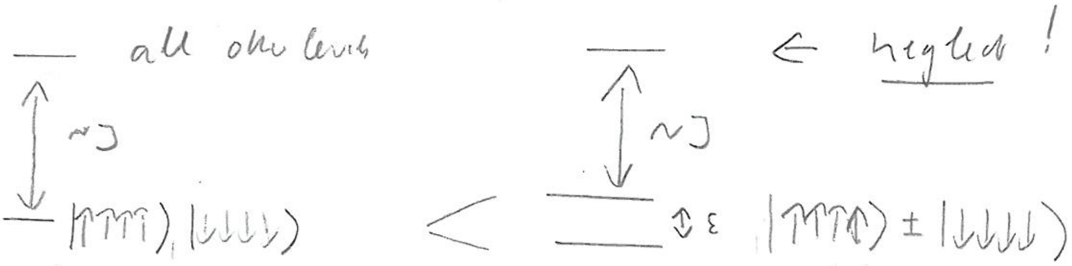
⇒ magnetic quantum phase transition (zero temperature) at h_c between PM and FM with magnetization in z-direction

- add site dilution



- energy spectrum of locally ordered droplet

$h^x = 0$ $\qquad\qquad$ $0 < h^x \ll J$



$\varepsilon \hat{=}$ tunnel splitting

estimate ε in perturbation theory in h^x

$$\varepsilon \sim \langle \uparrow\uparrow\uparrow\uparrow | \left(h^x \sum_i \hat{S}_i^x\right)^m | \downarrow\downarrow\downarrow\downarrow \rangle$$

lowest nonzero contribution for $m = N$

number of spins in droplet

$$\varepsilon \sim (h^x)^N \sim e^{-aV} \qquad a \sim -\ln(h_x/J)$$

$$\boxed{\varepsilon(V) \sim e^{-aV}}$$ gap depends exponentially on V

$$\boxed{w(V) \sim e^{-pV}}$$ droplet probability depends ~

low energy density of states

$$\rho(\varepsilon) \sim \int dV\, w(V)\, \delta\left(\varepsilon - e^{-aV}\right) \qquad e^{-aV} = x$$

$$-ae^{-aV} dV = dx$$

$$\sim \int \frac{dx}{x}\, x^{p/a}\, \delta(\varepsilon - x)$$

$$\rho(\varepsilon) \sim \varepsilon^{\frac{p}{a}-1}$$ power law with continuously varying exponent

$$\boxed{\rho(\varepsilon) \sim \varepsilon^{\frac{d}{z'}-1}}$$ defines z'

$\Rightarrow$ quantum Griffiths phase is gapless, power law DOS

---

distance between excitations with energy below ε

$$N(\varepsilon) \sim \varepsilon^{d/z'} \qquad r_{typ} \sim \left(\frac{1}{N}\right)^{\frac{1}{d}} \sim \varepsilon^{-\frac{1}{z'}}$$

$$\boxed{\varepsilon \sim r_{typ}^{-z'}}$$ explains notion of dynamical exponent

Observables

- local susceptibility of a droplet with gap ε

$$\chi_{loc}(\tau) \sim e^{-\varepsilon\tau}$$

average

$$\chi_{av}(\tau) \sim \int d\varepsilon\, \rho(\varepsilon)\, e^{-\varepsilon\tau} \sim \tau^{-\frac{d}{z'}}$$

Scale our

Fourier transformation

$$\chi_{av}(T) \sim \int_0^{1/T} d\tau\, \chi_{av}(\tau) \sim T^{\frac{d}{z'}-1}$$

Alternatively
$\chi_{av}(T) \sim \frac{N(T)}{T}$
(each free spin gives $1/T$)

- specific heat

$$\Delta E = \int d\varepsilon\, \rho(\varepsilon)\, \varepsilon \frac{e^{-\varepsilon/T}}{1+e^{-\varepsilon/T}} \sim T^{\frac{d}{z'}+1}$$

$$C_V \sim T^{d/z'}$$

- $m(H)$ $\quad H$ ordering field in z-direction

$\varepsilon < H$ droplets fully polarized

$\varepsilon > H$ m small

$$m(H) \sim \int_0^H d\varepsilon\, \rho(\varepsilon) \sim H^{d/z'}$$

power-law singularities in entire quantum Griffiths region

(4) Classification of Griffiths (rare region) effects

- crucial for phenomenology of rare region effects:

  How does energy gap of locally ordered cluster depend on size?

$\varepsilon(V) \sim V^{-x}$ Griffiths effects weak, exponential

$\varepsilon(V) \sim \exp(-aV)$ strong power-law Griffiths effects

$\varepsilon(V)$ vanishes at finite V (phase transition of independent cluster)

$\Rightarrow$ transition is smeared, see lecture 4

can be related to dimensionality of RR, d_{RR}

d_{RR}	$\varepsilon(V)$	Griffiths	CP	Examples
$d_{RR} < d_c^-$	power	weak exp	conventional	class magnets QH magnets
$d_{RR} = d_c^-$	exp	strong power	IRFP	random quantum Ising
$d_{RR} > d_c^-$	finite-V divergence	RR static	smeared	dissipative random q Ising

Lecture 5: Smeared Phase Transitions

- in all cases we have looked at so far, PT remained sharp (sharp onset of nonzero OP) in the presence of disorder

Reason: no spatial region (not even a strongly coupled RR) can order independently before the bulk system

This suggests:

If rare regions can develop true static order, global PT is smeared because OP develops gradually on one RR after the other (not a collective effect of the entire system)

How can RR order independently?

(1) RR need to be infinitely large
$\Rightarrow$ extended defects (perfectly correlated disorder)

(2) RR need to be coupled to infinite bath
$\Rightarrow$ dissipative quantum magnets

Classical Example: Ising model with planar disorder

(⇒ show layered magnet figure)

$$H = -\sum_{ijk} J_i^{\parallel} \left[S_{ijk} S_{i+1,jk} + S_{ijk} S_{ij+1k} \right] - \sum_{ijk} J_i^{\perp} S_{ijk} S_{ijk+1}$$

in-plane perpendicular

- planar defects:

  J depends on i only, each plane is clean, random stack of planes

  (artificial nanostructures, 1D random optical lattice)

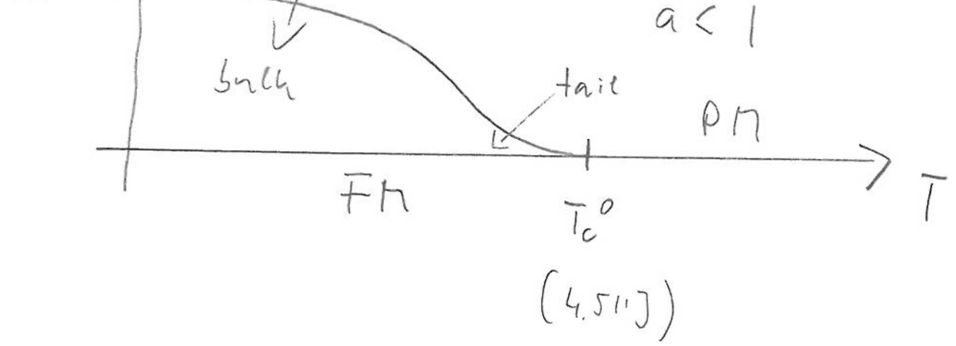
- RR are slabs of consecutive strong planes
- RR are 2D Ising models, have PT

⇒ RR order independently, PT smeared

to be specific

$$J_i^{\perp} \equiv J \qquad J_i^{\parallel} = \begin{cases} J & (\text{with prob } 1-p) \\ aJ & (\quad \sim \quad p) \end{cases}$$

$a < 1$



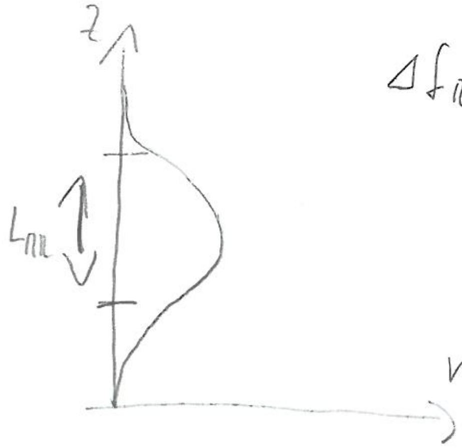
Behavior of m in tail

- probability for finding slabs of L_{RR} strong layers:

$$\boxed{W(L_{RR}) \sim e^{-\tilde{p} L_{RR}}}$$

- RR has PT at some $T_c(L_{RR})$ <u>below</u> clean bulk T_c^0 because of surface effects

<u>estimate from Landau theory</u>



$$\Delta f_{RR} \sim t m^2 + (\nabla m)^2 \approx t m^2 + \frac{m^2}{L_{RR}^2} \overset{!}{=} 0$$

$(t<0)$

$$t_c \sim T_c - T_c^0 \sim \frac{-1}{L_{RR}^2}$$

- can be refined by FSS: $\boxed{T_c(L_{RR}) - T_c^0 \sim -L_{RR}^{-\phi}}$

$\phi = \frac{1}{\nu}$ = FSS shift exponent

at given $T < T_c^0$, all rare regions larger than some $L_c \sim (T_c^0 - T)^{-\frac{1}{\phi}}$ are in FM phase

$$m \sim \int_{L_c}^{\infty} dL_{RR}\; m(L_{RR})\; W(L_{RR})$$

power law

exponential, dominates

$$\boxed{m(T) \sim e^{-\tilde{p} L_c} = e^{-A(T_c^0 - T)^{-1/\phi}}}$$

- exponential magnetization tail
- local magnetization very inhomogeneous (zero in bulk, large on RR)

(⇒ show MC results)

Example : Metallic quantum magnet

(see David's talk on Monday)

- in metallic magnetic, magnetization fluctuations are damped due to coupling to conduction electrons (see Chubukov lecture)

- LGW free energy functional

QPT tuning

$$F = \sum_q \sum_{\omega_n} m(\vec{q},\omega_n)\left[r + q^2 + |\omega_n|\right] m(-\vec{q},-\omega_n) + \text{higher order terms}$$

damping $\hat{=} \dfrac{1}{|\tau-\tau'|^2}$ interaction in "time" direction

- Ising symmetry

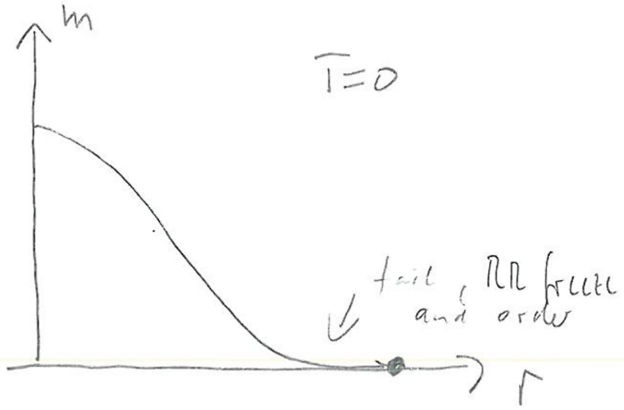
  - each RR corresponds to two-level system ($\uparrow\uparrow\uparrow$, $\downarrow\downarrow\downarrow$) coupled to Ohmic dissipative bath!

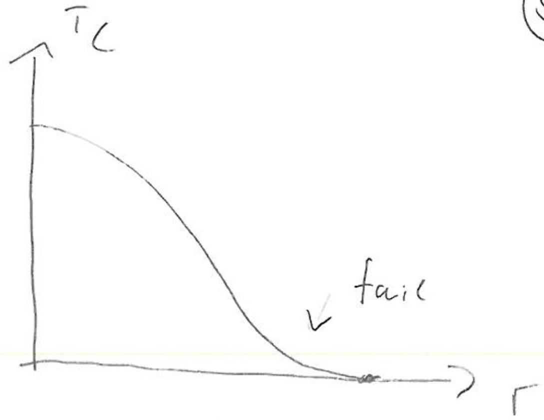
    $\hat{=}$ famous dissipative two-level system (large literature)

  - important: PT from fluctuating to localized with increasing dissipation (at $T=0$)

$\Rightarrow$ large islands freeze, small fluctuate

$\Rightarrow$ global QPT smeared





(⇒ show $Sr_{1-x}Ca_xRuO_3$ data)

Summary

- analyzed effects of randomness on classical and quantum PT
- focused on CP (stability for 1st order, — less is known)
- <u>two types of classifications emerge</u>

a) by behavior of average disorder strength for large length scales (under RG, coarse graining)

Harris fulfilled : disorder $\to 0$ clean CP

violated: disorder $\to$ finite — finite disorder CP (example classical diluted magnet, new exponents, same scaling)

disorder $\to \infty$ — infinite-disorder CP (example RTIM) exotic activated scaling

- smeared transition do not quite fit, because length scale does <u>not</u> go ∞ freezing transition at finite size of RR

b) <u>by RR effect</u>

- Griffiths exponentially weak
- power law Griffiths
- smearing

depends on d_c^- vs d_{RR}

($\Rightarrow$ show classification)

two classifications are related !!