Non-equilibrium phenomena in spinor Bose gases

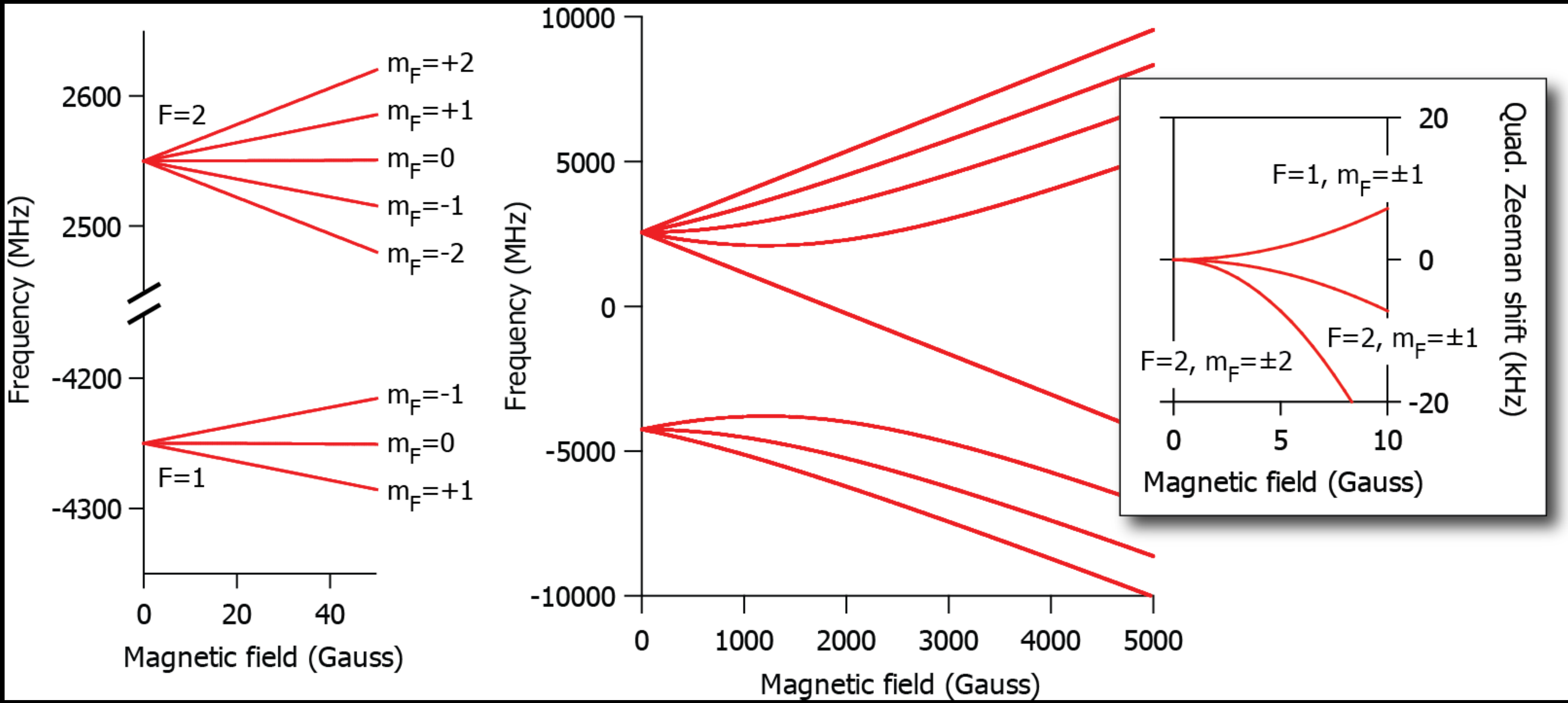
Dan Stamper-Kurn
University of California, Berkeley

outline

- Introductory material
- Interactions under rotational symmetry
- Energy scales
- Ground states
- Spin dynamics
  - microscopic spin mixing oscillations
  - single-mode mean-field dynamics
  - spin mixing instability
- More?

Breit-Rabi diagram

$$H_{hf} = ah\frac{1}{\hbar^2}\, I\cdot J\; - \mu\cdot B \qquad \mu = -g_J\mu_B\frac{1}{\hbar}J + g_I\mu_n\frac{1}{\hbar}I$$



$$g_F \simeq 2\,\frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} = \frac{\pm 1}{I+1/2}$$

TABLE I. Experimental candidates for the study of ultracold spinor Bose gases. Species are divided according to whether they are stable at zero magnetic field (information on thulium is lacking), and whether the dipolar relaxation rate is small enough to allow the longitudinal magnetization ($\langle F_z \rangle$) to be conserved in an experiment. The nature of the spin-dependent contact interactions is indicated in parentheses (f: ferromagnetic, af: antiferromagnetic, cyc: cyclic or tetrahedral, ?:,unknown). Stable pseudo-spin-1/2 gases of ^{87}Rb are indicated, with states labeled with quantum numbers $|F, m_F\rangle$ having the same low-field magnetic moment.

Stable: $\langle F_z \rangle$ conserved	Stable: $\langle F_z \rangle$ not conserved	Unstable
^{7}Li, $F = 1$ (f)	^{52}Cr, $F = 3$ (not f)	^{7}Li, $F = 2$
^{23}Na, $F = 1$ (af)	Dy, $F = 8$ (?)	^{23}Na, $F = 2$
^{41}K, $F = 1$ (f)	Er, $F = 6$ (?)	^{39}K
^{87}Rb, $F = 1$ (f)		^{85}Rb
^{87}Rb, $F = 2$ (af or cyc)		^{133}Cs
^{87}Rb pseudospin:	Tm, $F = 4$ (?)	
$\|1, 0\rangle$, $\|2, 0\rangle$		
$\|1, \pm 1\rangle$, $\|2, \mp 1\rangle$		

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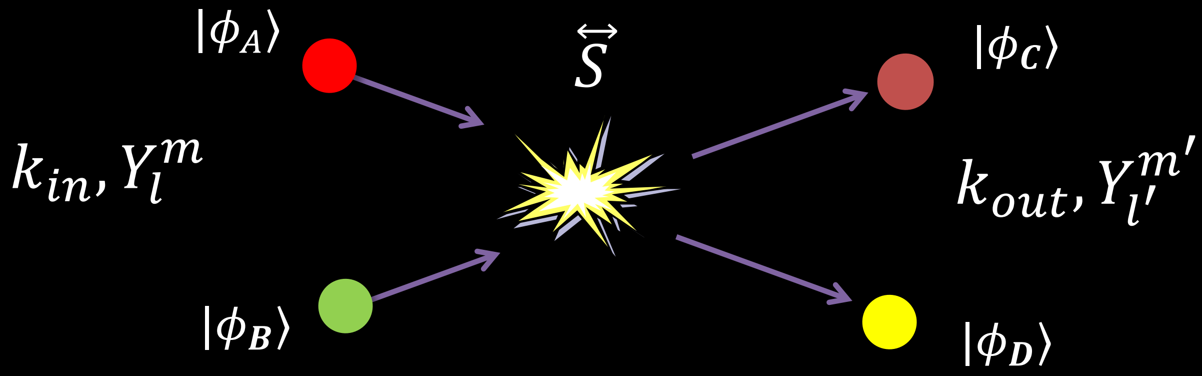
Interactions + rotational symmetry

central potential,
translation invariant

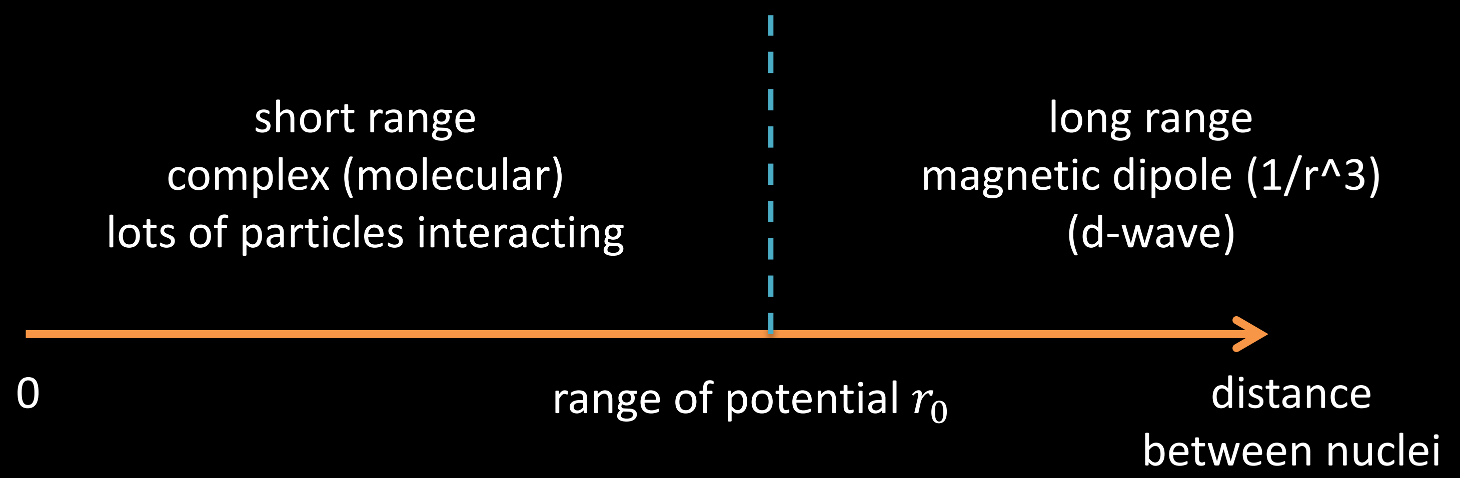


How complicated is the scattering matrix $\overleftrightarrow{S}$?

Make some approximations:

Interactions + rotational symmetry

typical molecular potential:



1. Low incident energy
   - only s-wave collisions occur (quantum collision regime), determined by short-range potential
   - long-range treated separately (depending on dimension)
   - still quite open problem

Interactions + rotational symmetry

2. Spinor gas approximation: interactions are rotationally symmetric
   - TOTAL angular momentum in = out
   - Note: imperfect approximation in case of…
     - applied B field (e.g. Feshbach resonance)
     - non spherical container

3. Weak dipolar approximation: Assume that dipolar interactions due to short-range potential are weak
   - no “spin-orbit” coupling
   - orbital angular momentum is separately conserved
   - $F_{tot}(in) = F_{tot}(out)$

4. Weak hyperfine relaxation
   - collisions keep atoms in the same hyperfine spin manifold

Interactions + rotational symmetry

After all these approximations:

$$V(\text{short range}) = \frac{4\,\pi\hbar^2}{m}\delta^3(\vec{r})\left[a_0\hat{P}_0 + a_1\hat{P}_1 + a_2\hat{P}_2 + \cdots\right]$$

Bose-Einstein statistics:
all terms with Ftot odd are zero

putting into more familiar form… (see blackboard)

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Energy scales in a spinor Bose-Einstein condensate

- spin-dependent contact interactions

$$E = -|c_2| n \langle \vec{F} \rangle^2$$

≈ 10 Hz, or 0.5 nK

- thermal energy

$$E = k_B T$$

≈ 1000 Hz, or 50 nK

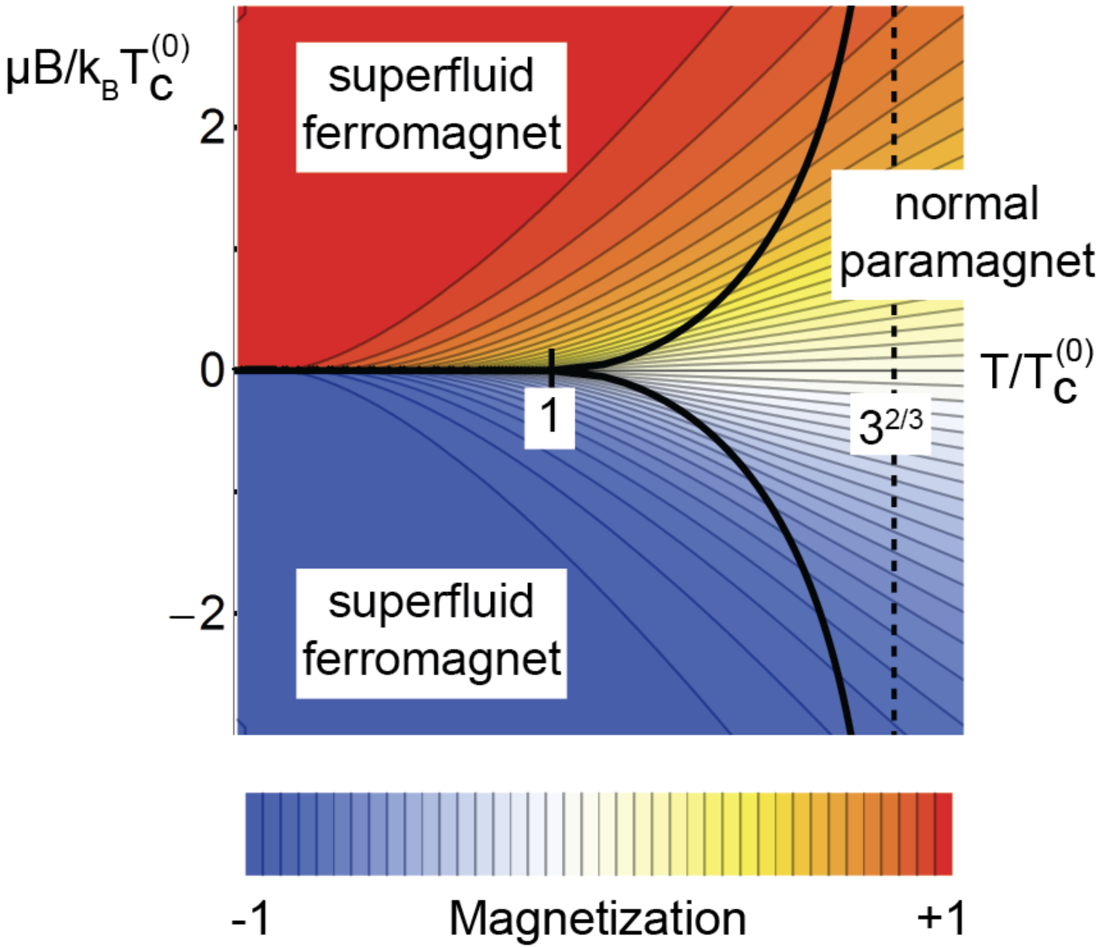
- linear Zeeman shift at typical magnetic fields

$$E = g_F \mu_B B$$

≈ 100,000 Hz, or 5000 nK

Bose-Einstein magnetism

magnetization of a non-interacting, spin-1 Bose gas in a magnetic field:



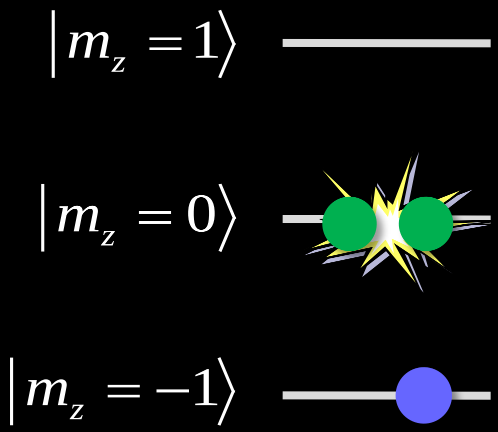
- Bose-Einstein condensation occurs at lower temperature at lower field (opening up spin states adds entropy)
- Magnetization jump at zero-field below Bose-Einstein condensation transition

Yamada, "Thermal Properties of the System of Magnetic Bosons," Prog. Theo. Phys. 67, 443 (1982)

Expt. with chromium:
Pasquiou, Laburthe-Tolra et al., PRL **106**, 255303 (2011).

magnetic ordering is "parasitic"

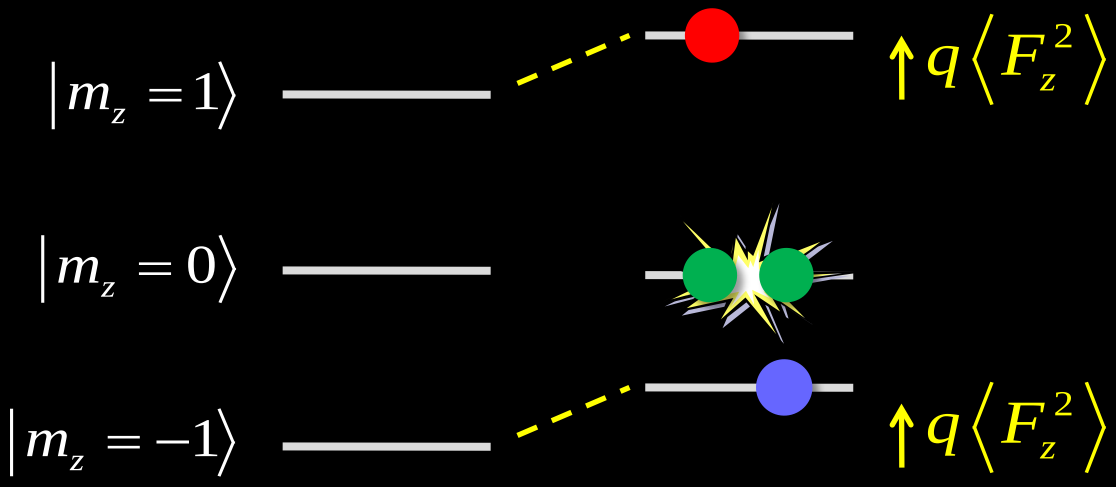
linear and quadratic Zeeman shifts



However, dipolar relaxation is extremely rare (for alkali atoms)

→ linear Zeeman shift is irrelevant!

linear and quadratic Zeeman shifts



However, dipolar relaxation is extremely rare (for alkali atoms)
$\rightarrow$ linear Zeeman shift is irrelevant!

spin-mixing collisions are allowed
q = quadratic Zeeman shift

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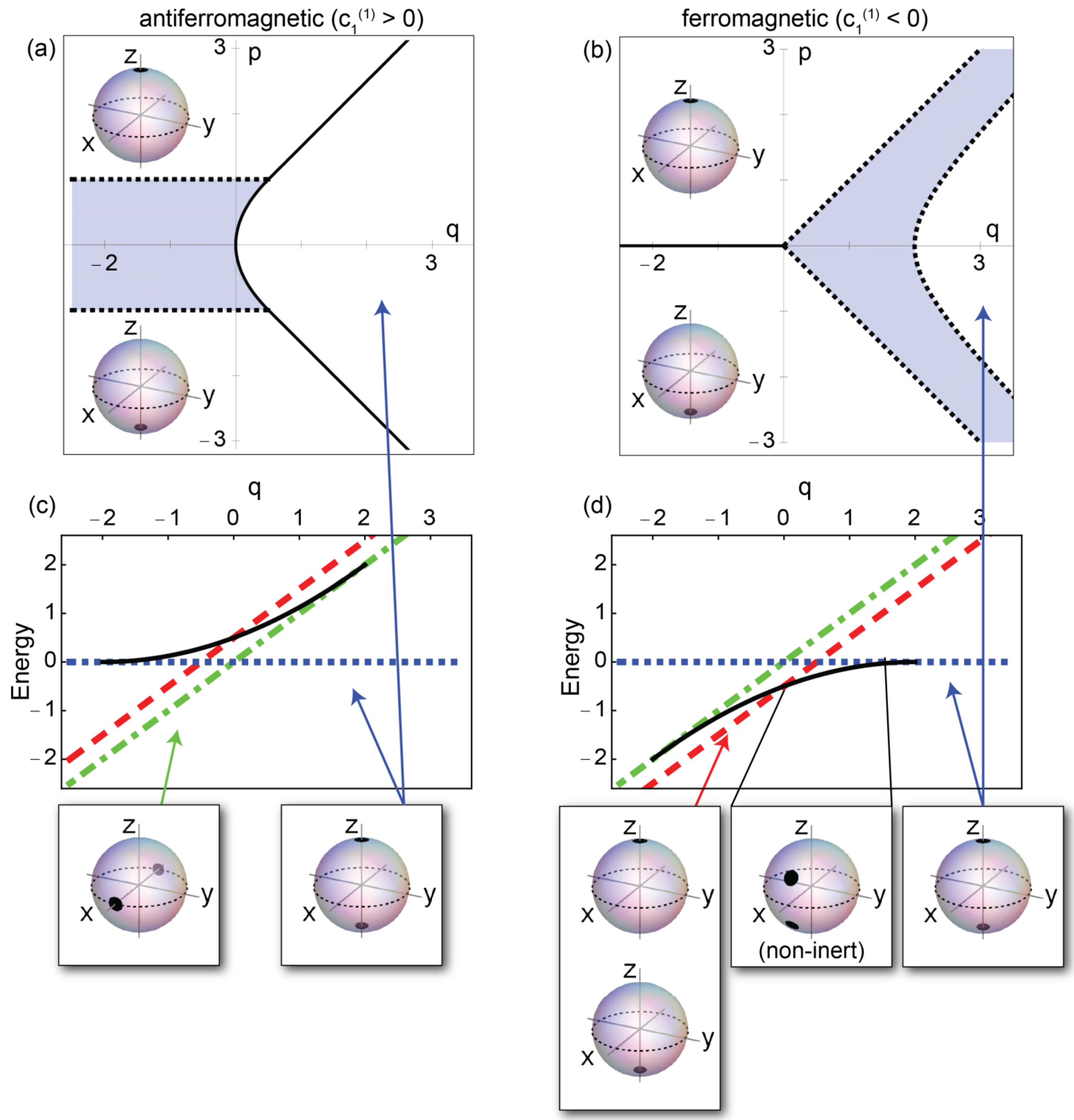
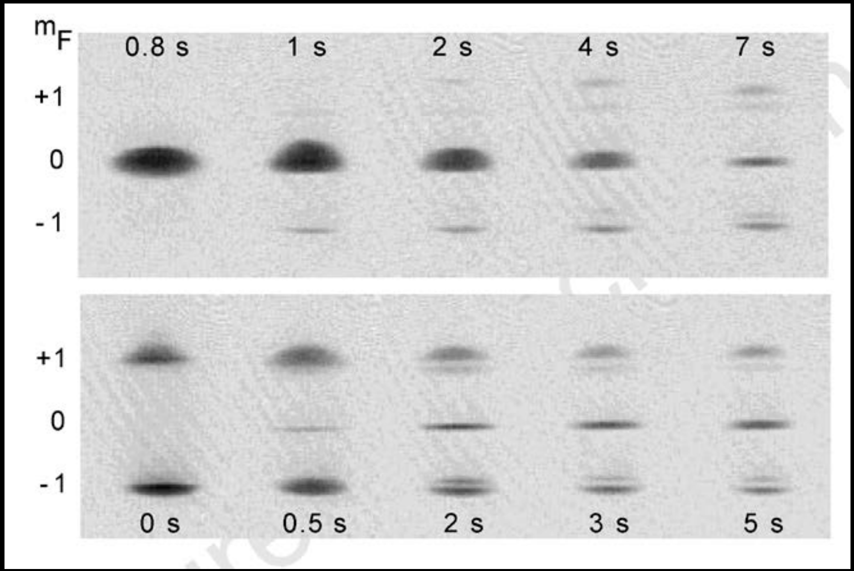
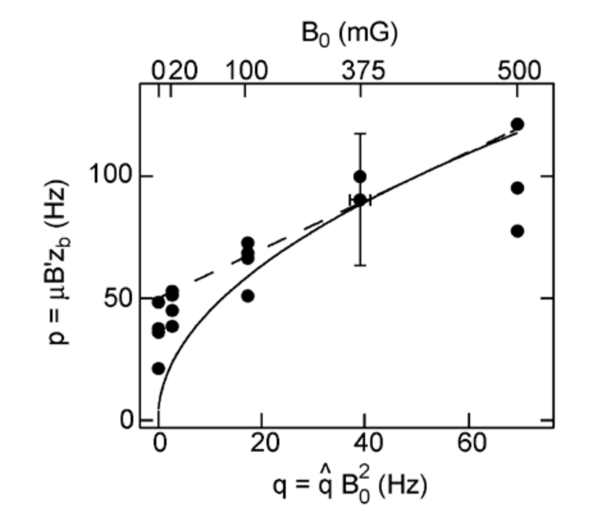
F=1 mean-field phase diagram

Stenger et al., Nature **396**, 345 (1998)

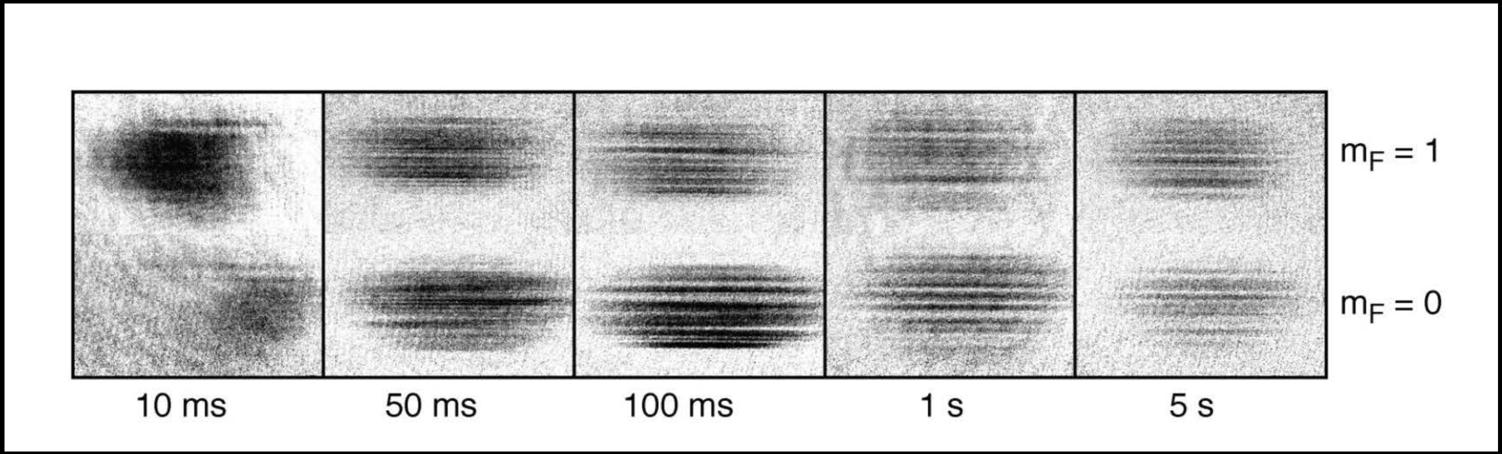


Evidence for antiferromagnetic interactions of F=1 Na





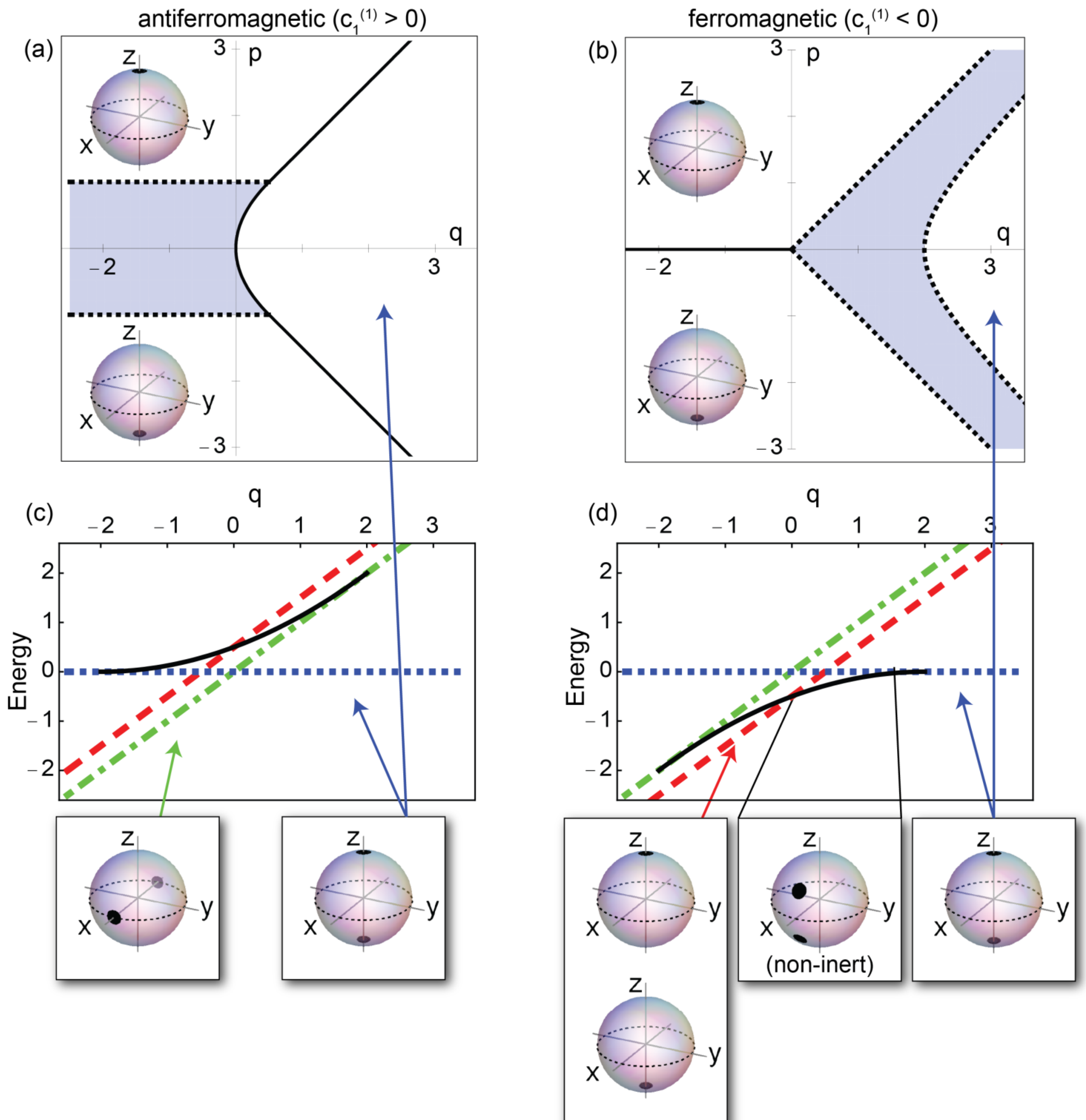
Stenger et al., Nature **396**, 345 (1998)



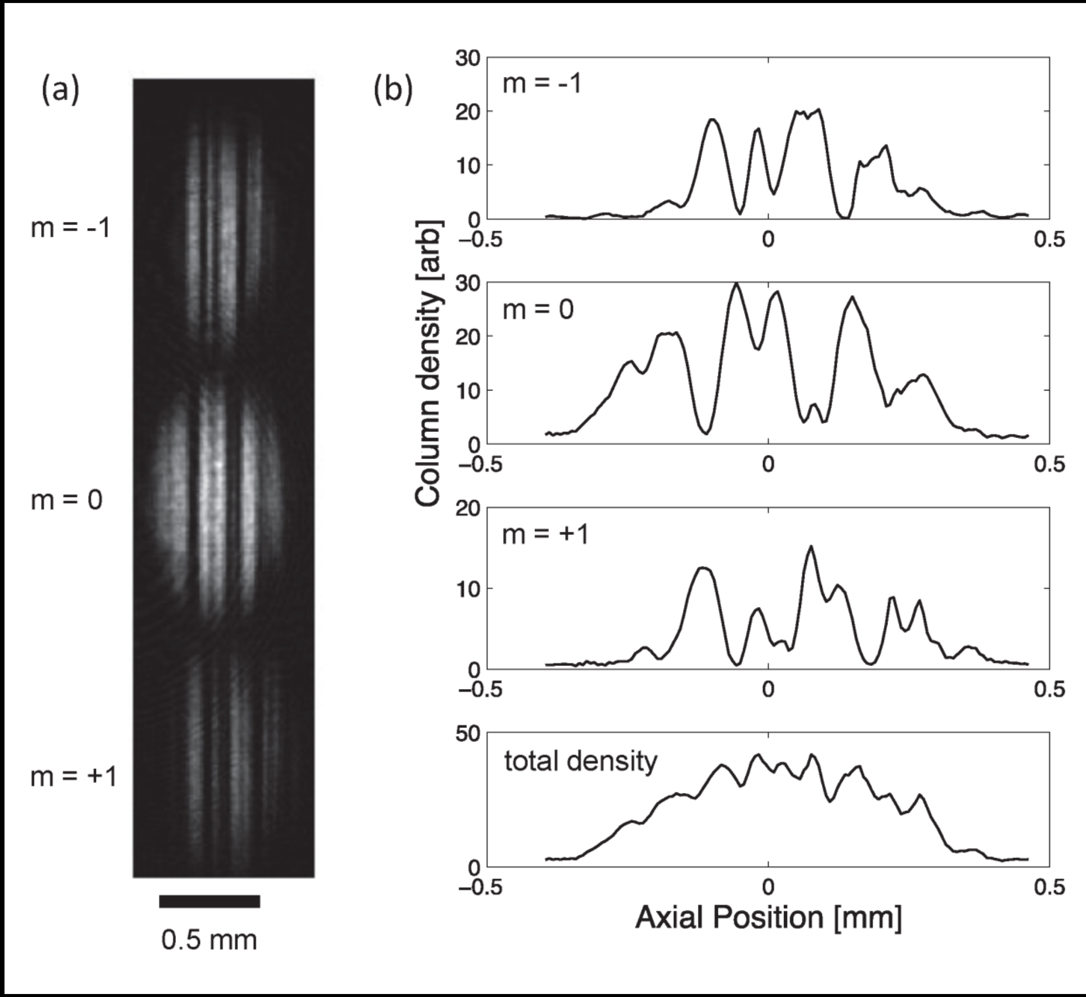
Miesner et al., PRL **82**, 2228 (1999).

F=1 mean-field phase diagram

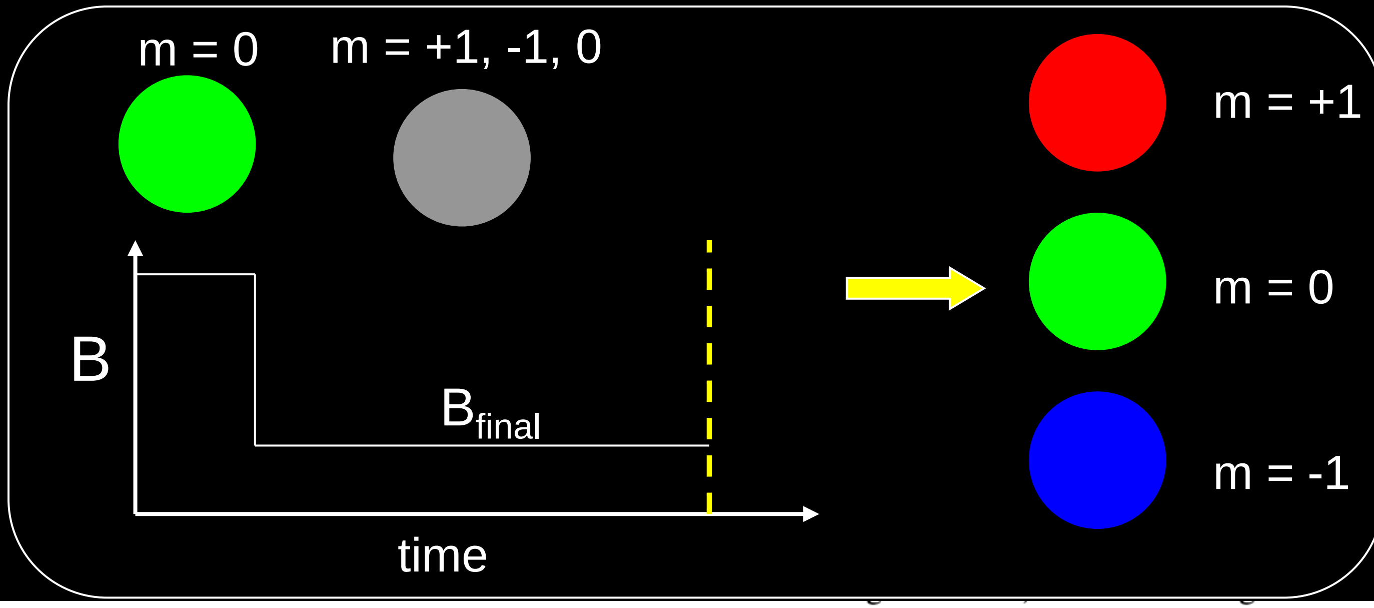
Stenger et al., Nature **396**, 345 (1998)



Evidence for antiferromagnetic interactions of F=1 Na



Bookjans, E.M., A. Vinit, and C. Raman, Quantum Phase Transition in an Antiferromagnetic Spinor Bose-Einstein Condensate. Physical Review Letters 107, 195306 (2011).



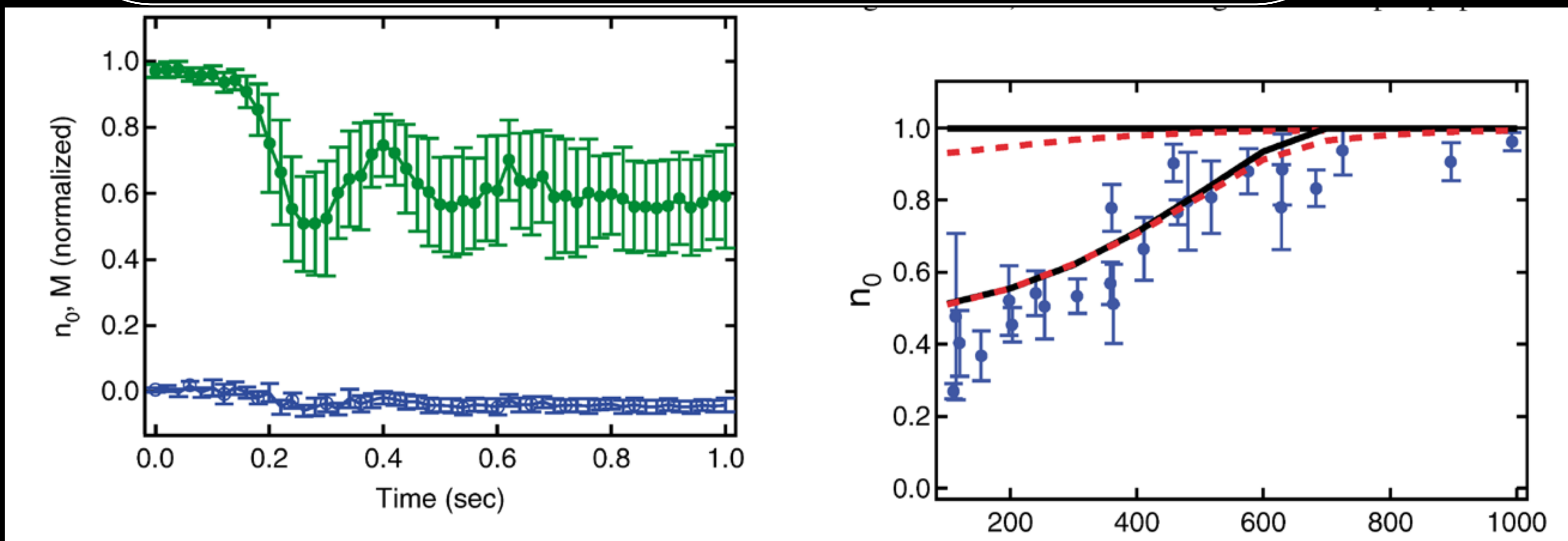
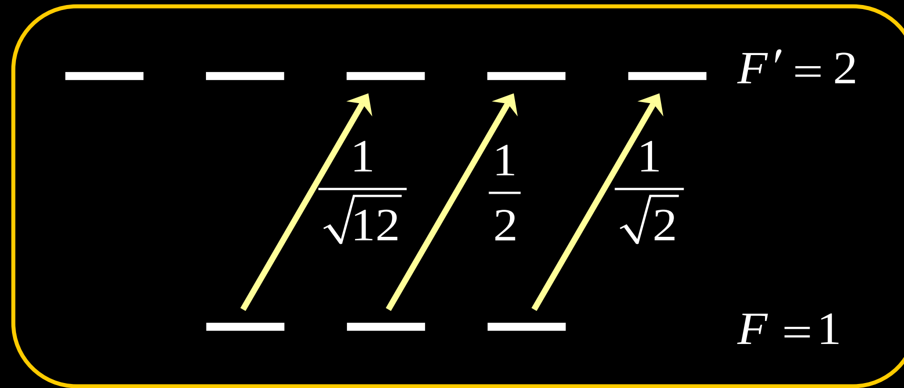


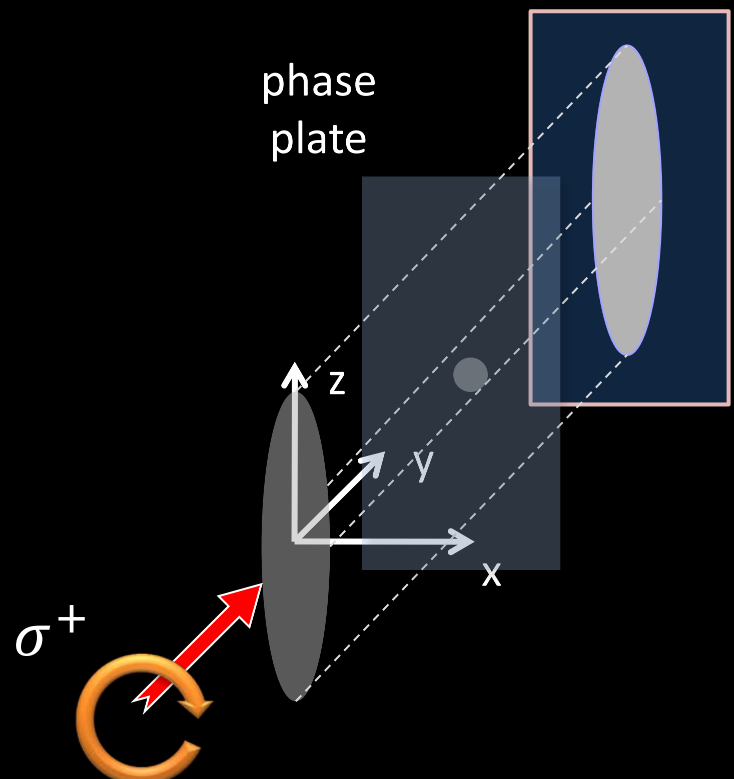
FIG. 3 (color online). Plot of the fraction in the $m_F = 0$ state

Chang, M.-S., et al., *Observation of spinor dynamics in optically trapped Rb Bose-Einstein condensates. PRL* ***92,*** *140403 (2004)*

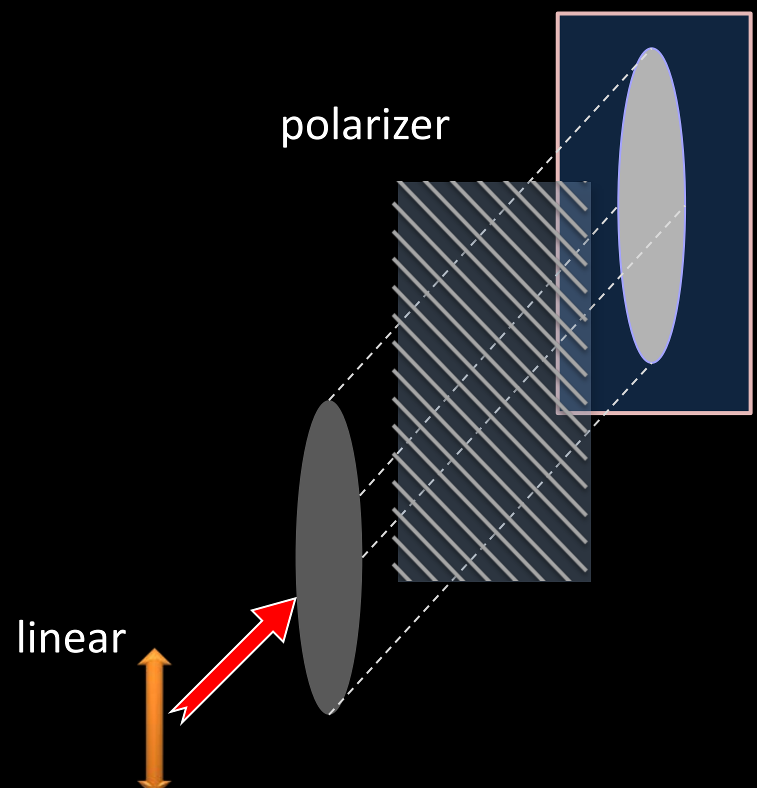
Dispersive birefringent imaging



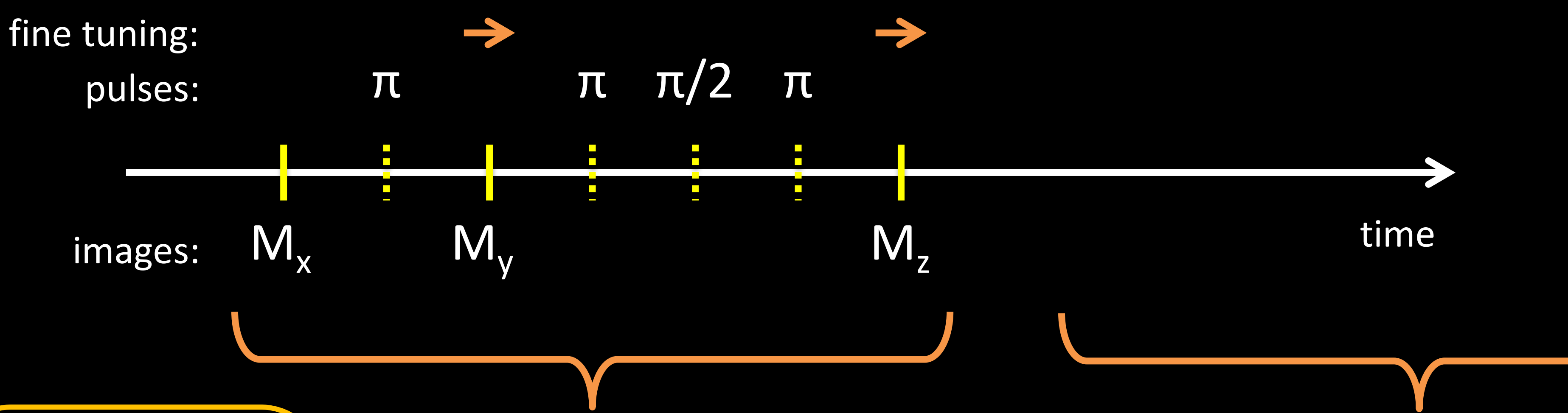
<u>phase-contrast imaging</u>



<u>polarization-contrast imaging</u>

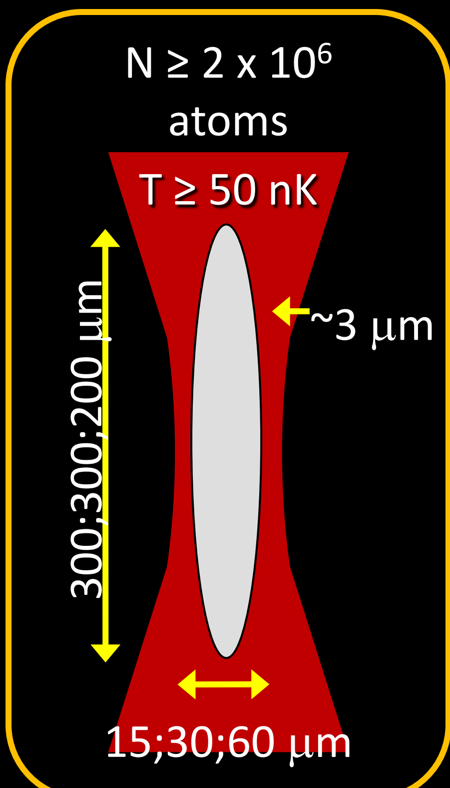


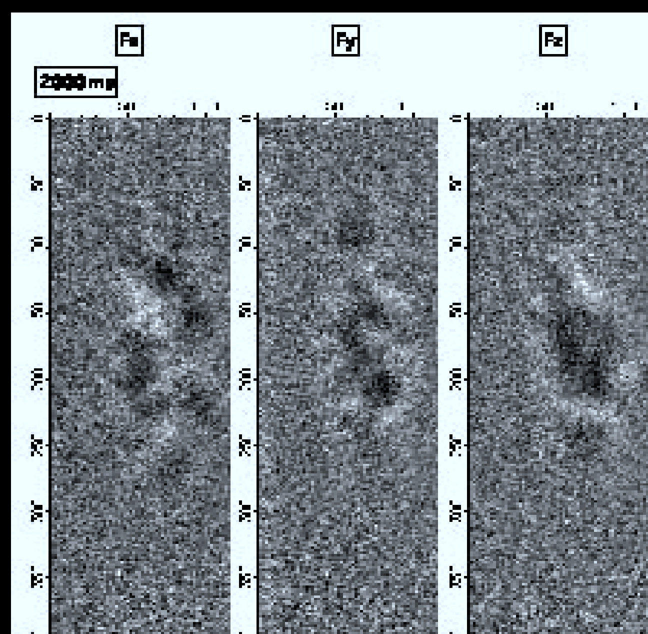
Spin echo imaging



vector imaging sequence

repeat?

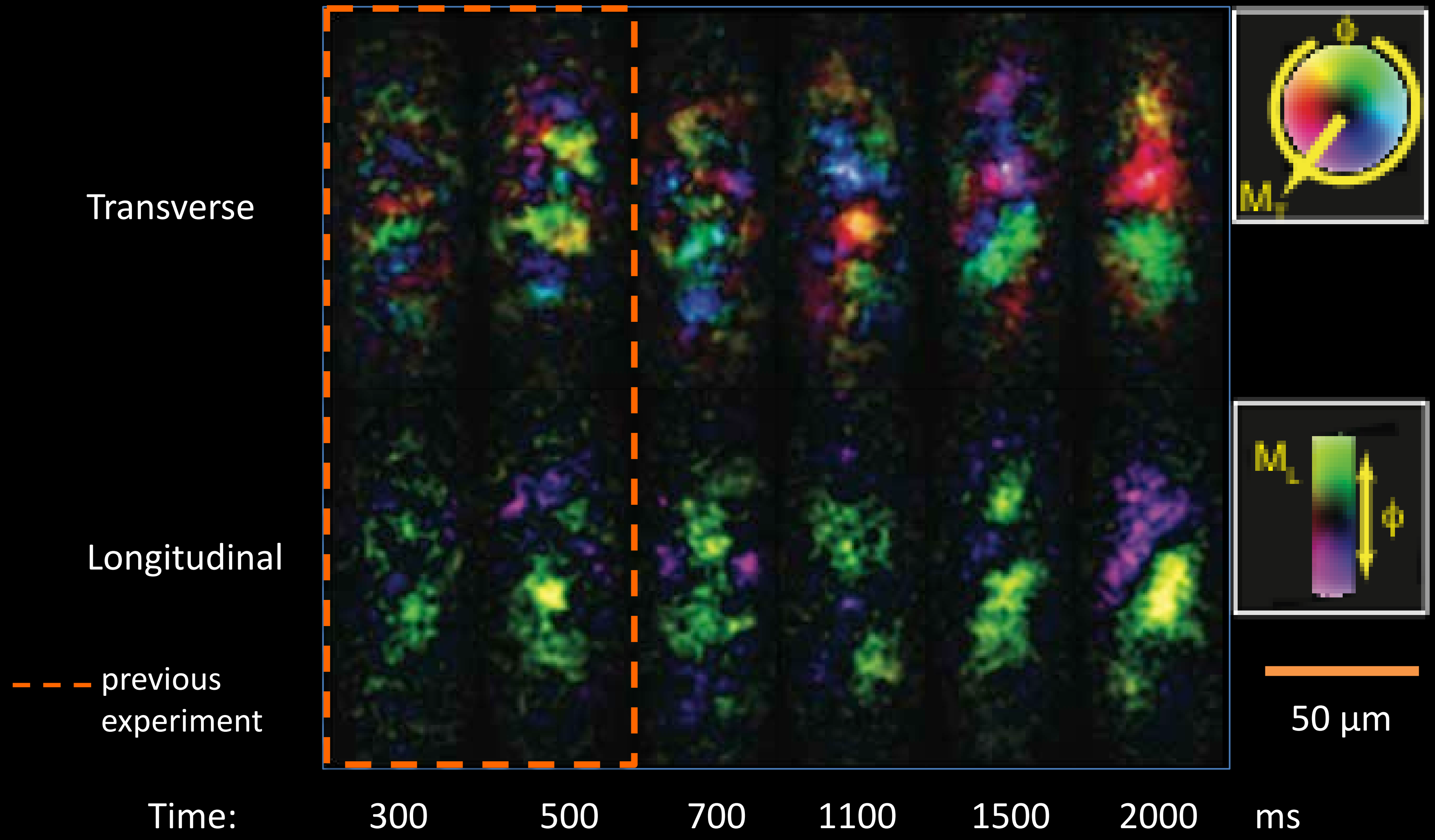




geometry
≈ surfboard

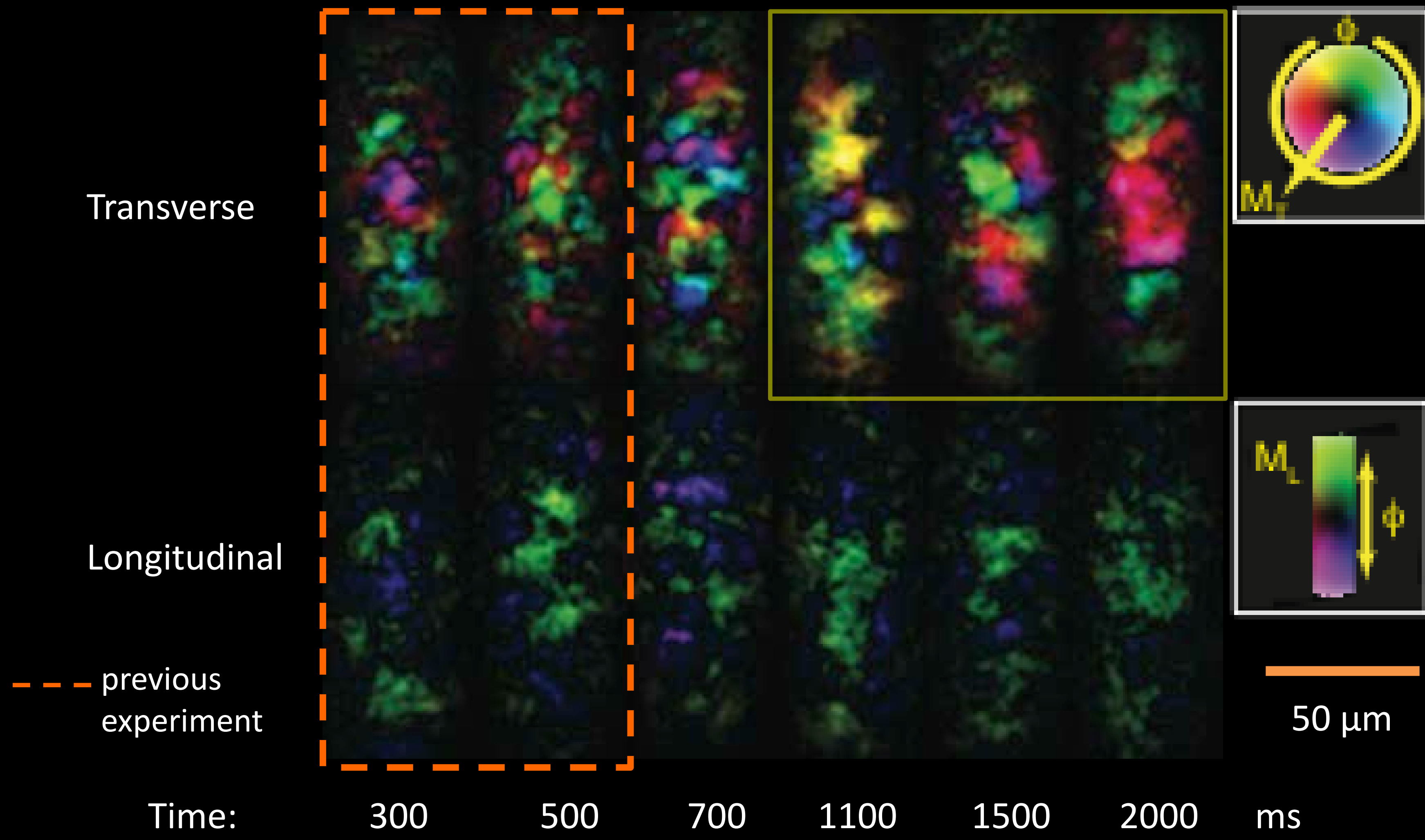
Development of spin texture

q/h = 0



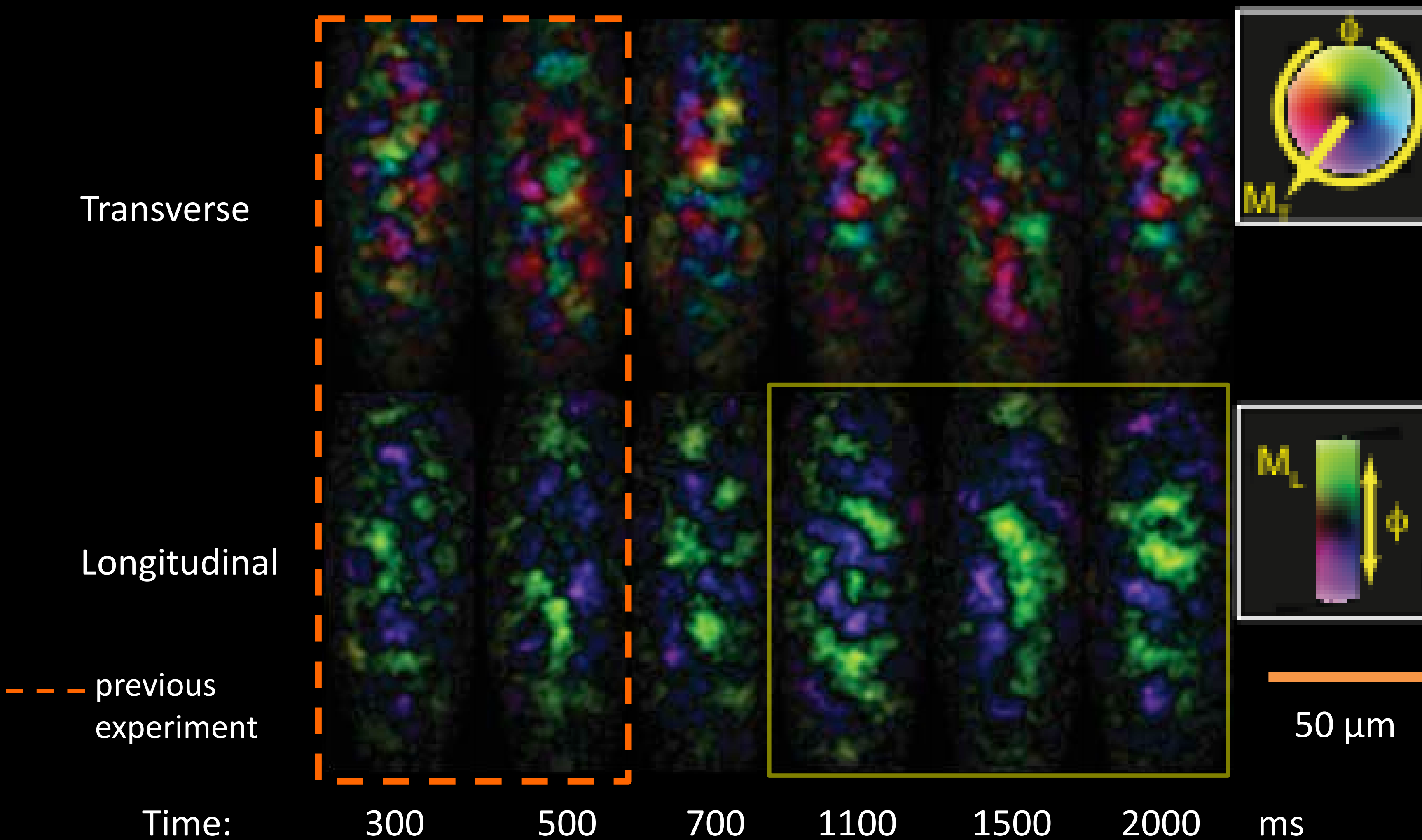
Development of spin texture

q/h = + 5 Hz

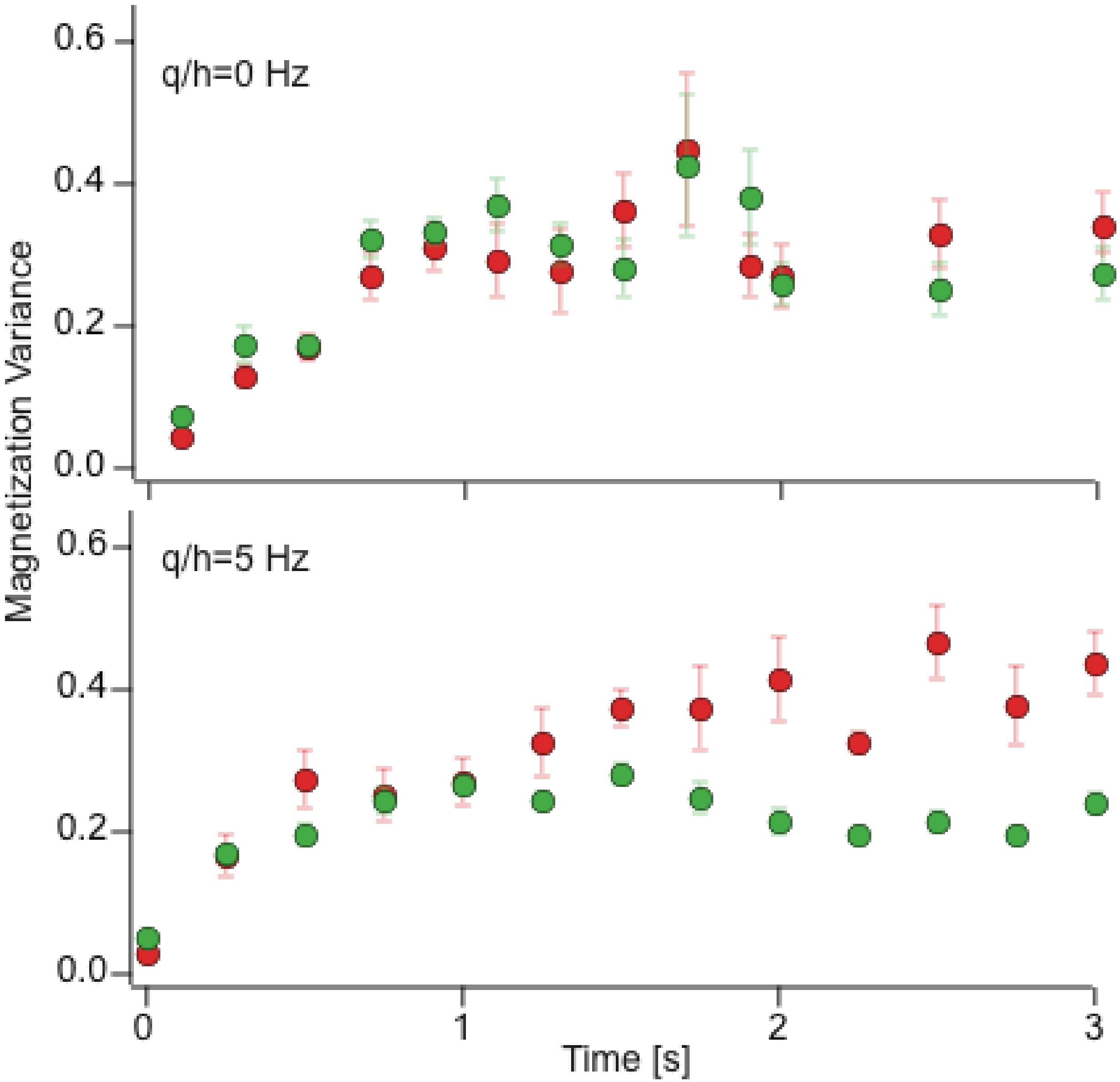


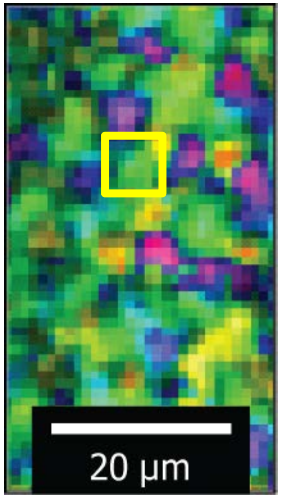
Development of spin texture

q/h = - 5 Hz



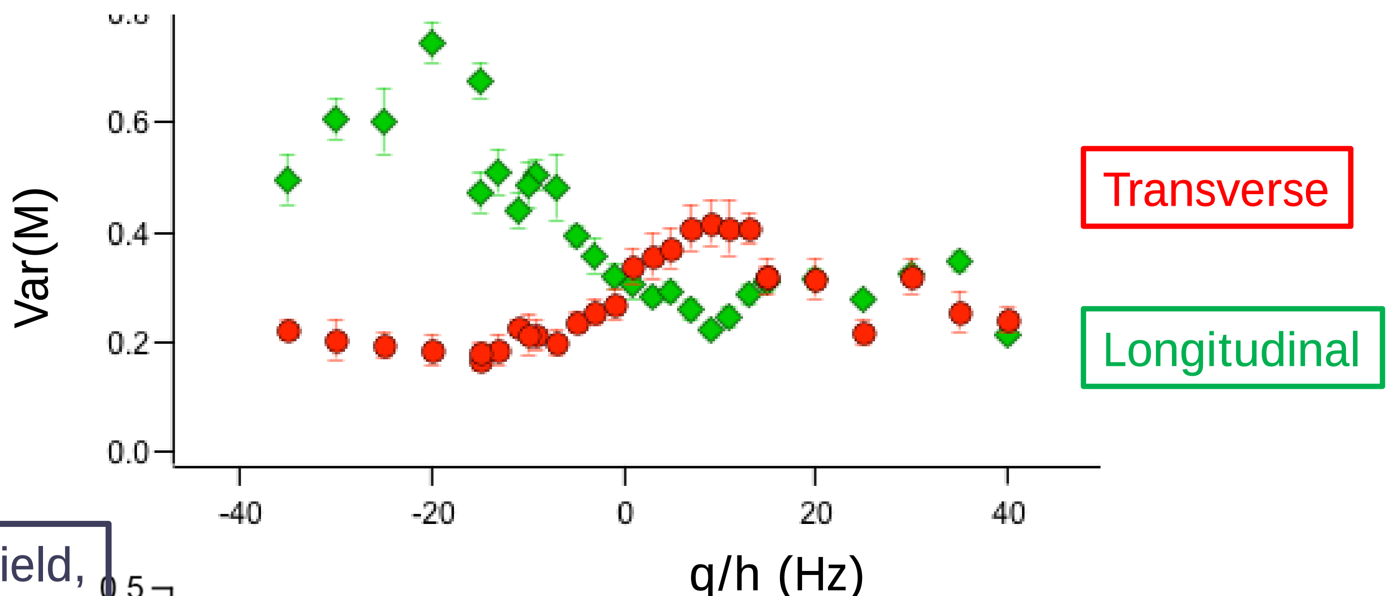
Growth of transverse/longitudinal magnetization

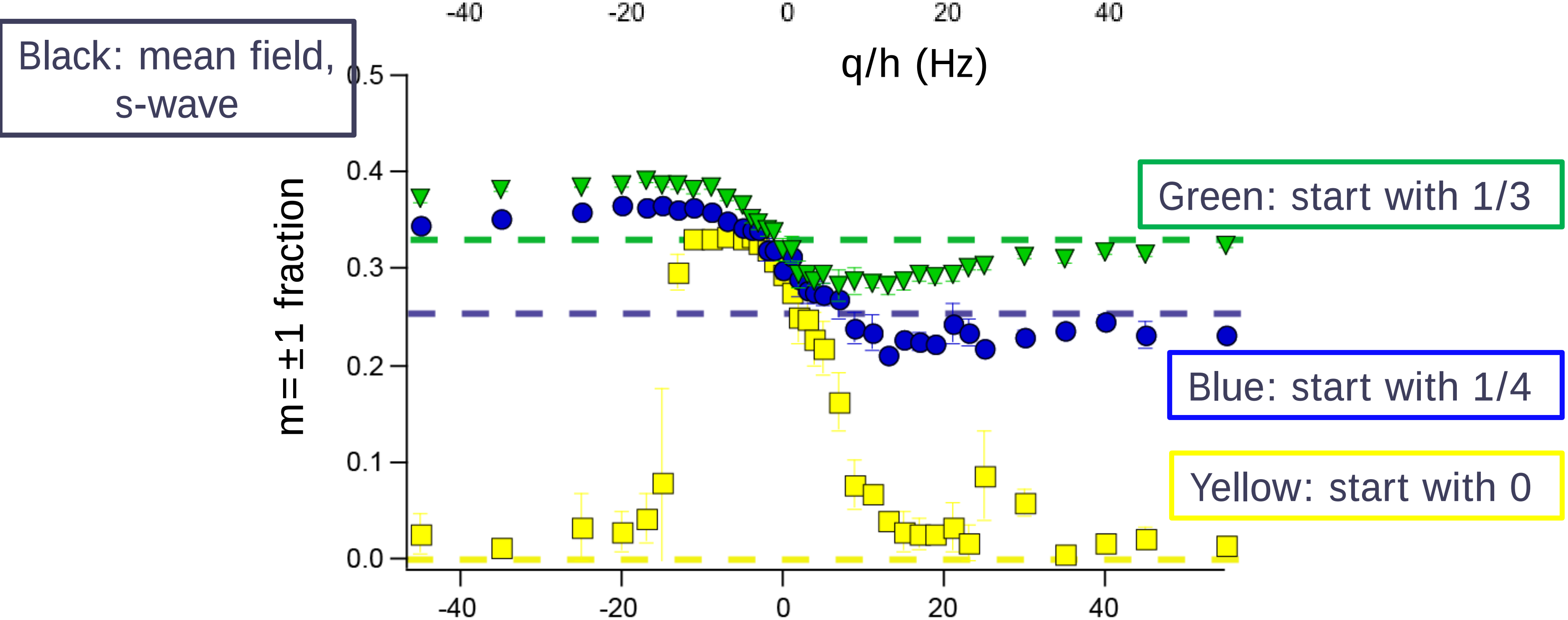




Var(M) = zero-range spatial correlation function

Easy axis/plane magnetic order: in-situ vs tof





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- More?

spin mixing of many atom pairs

Widera et al., PRL **95**, 190405 (2005)

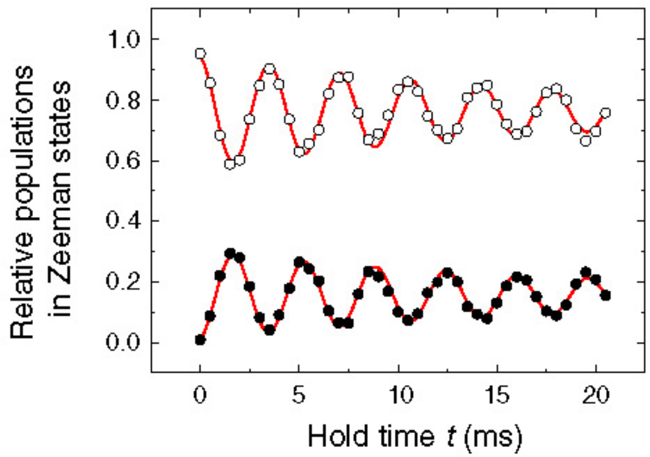
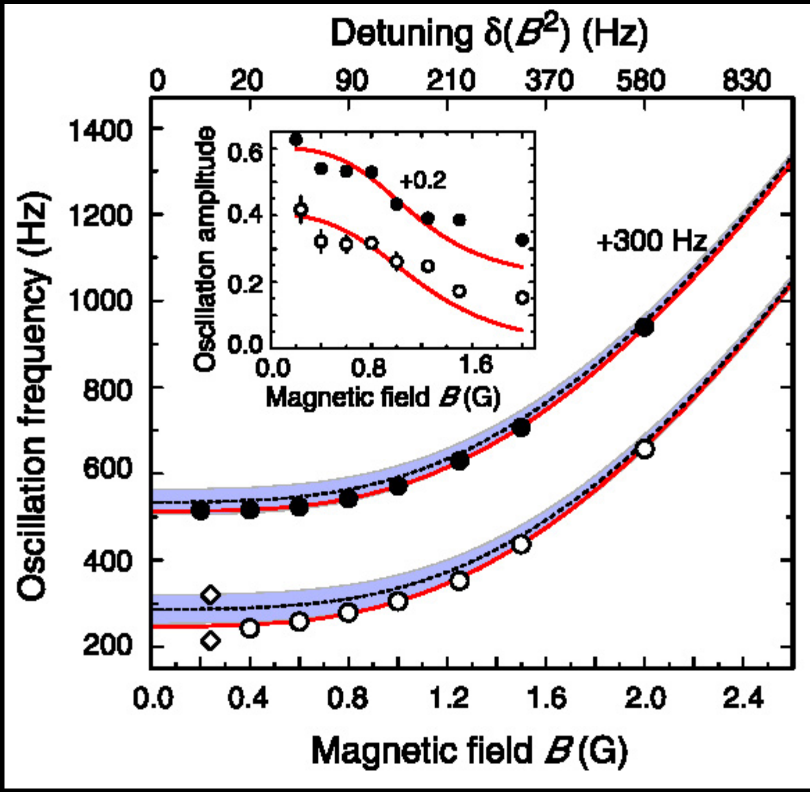
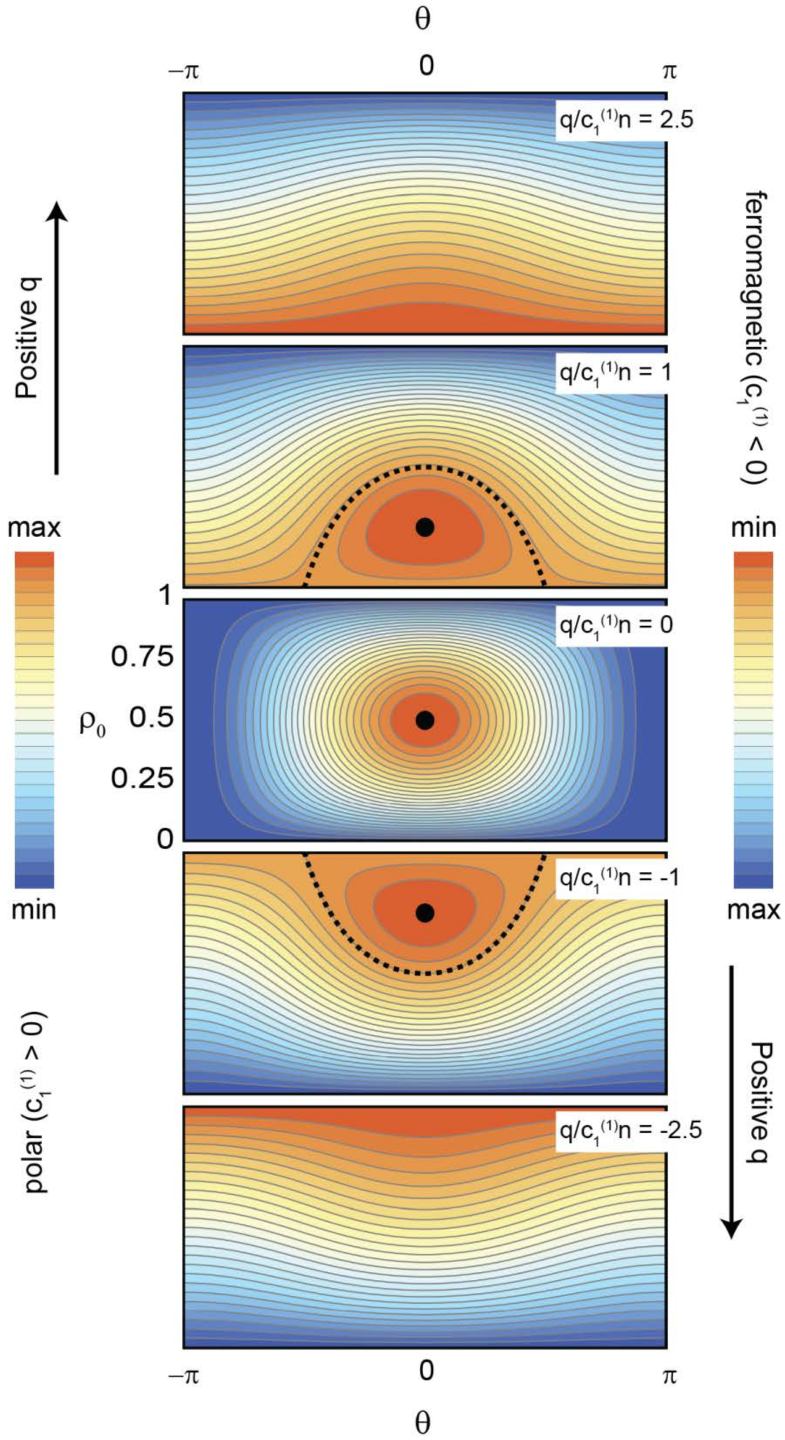
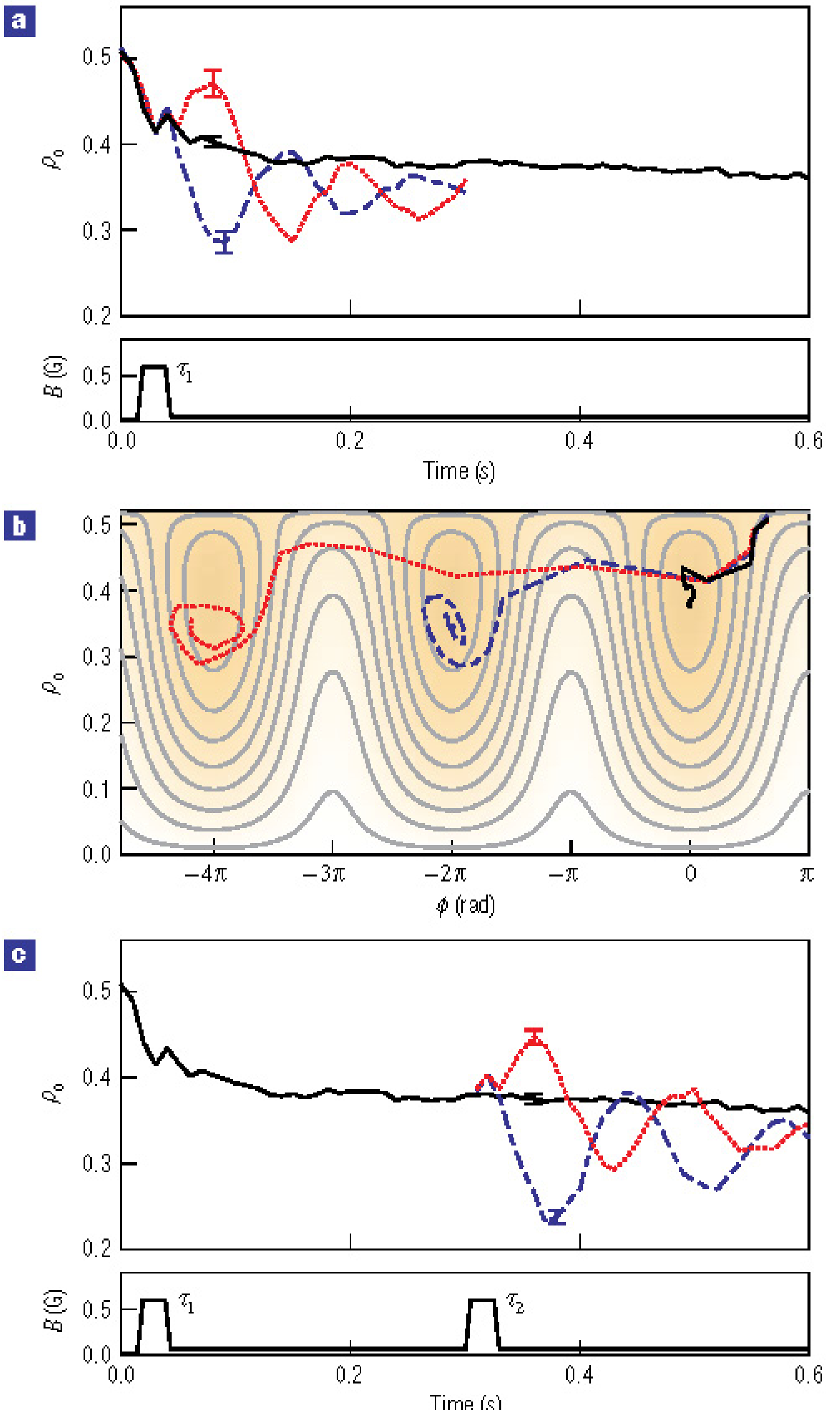


FIG. 2 (color online). Spin dynamics of atom pairs localized in an optical lattice at a magnetic field of $B = 0.8$ G. The atoms are initially prepared in $|0, 0\rangle$ and can evolve into $|+1, -1\rangle$. Shown are the populations in $m_f = 0$ (○) and $m_f = \pm 1$ (●) together with a fit to a damped sine yielding an oscillation frequency of $\Omega_{if}^{l} = 2\pi \times 278(3)$ Hz.





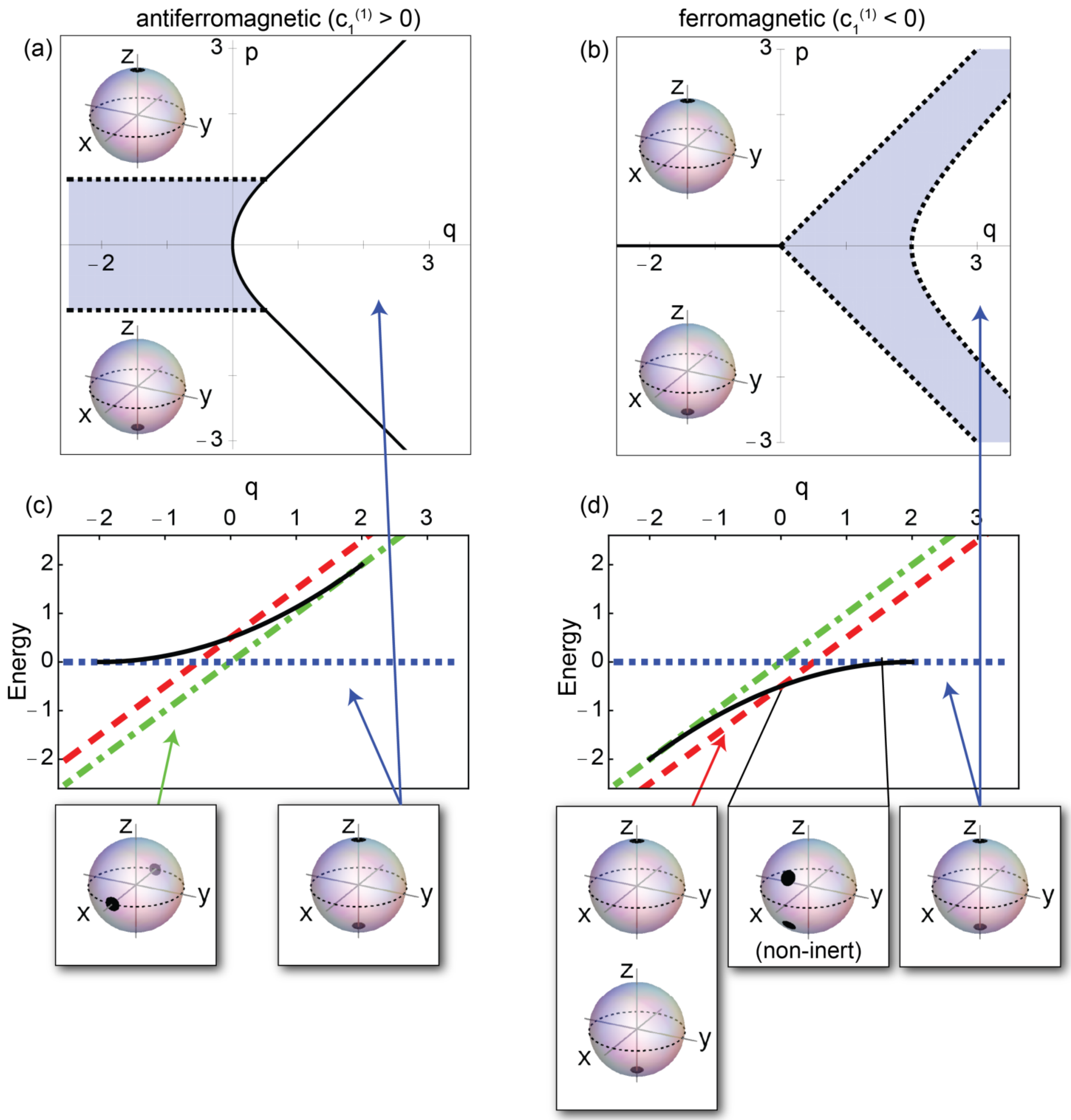
θ
−π
0
π
q/c1(1)n = 2.5
q/c1(1)n = 1
q/c1(1)n = 0
q/c1(1)n = -1
q/c1(1)n = -2.5
Positive q
ferromagnetic (c1(1) < 0)
max
min
1
0.75
ρ0
0.5
0.25
0
polar (c1(1) > 0)
Positive q

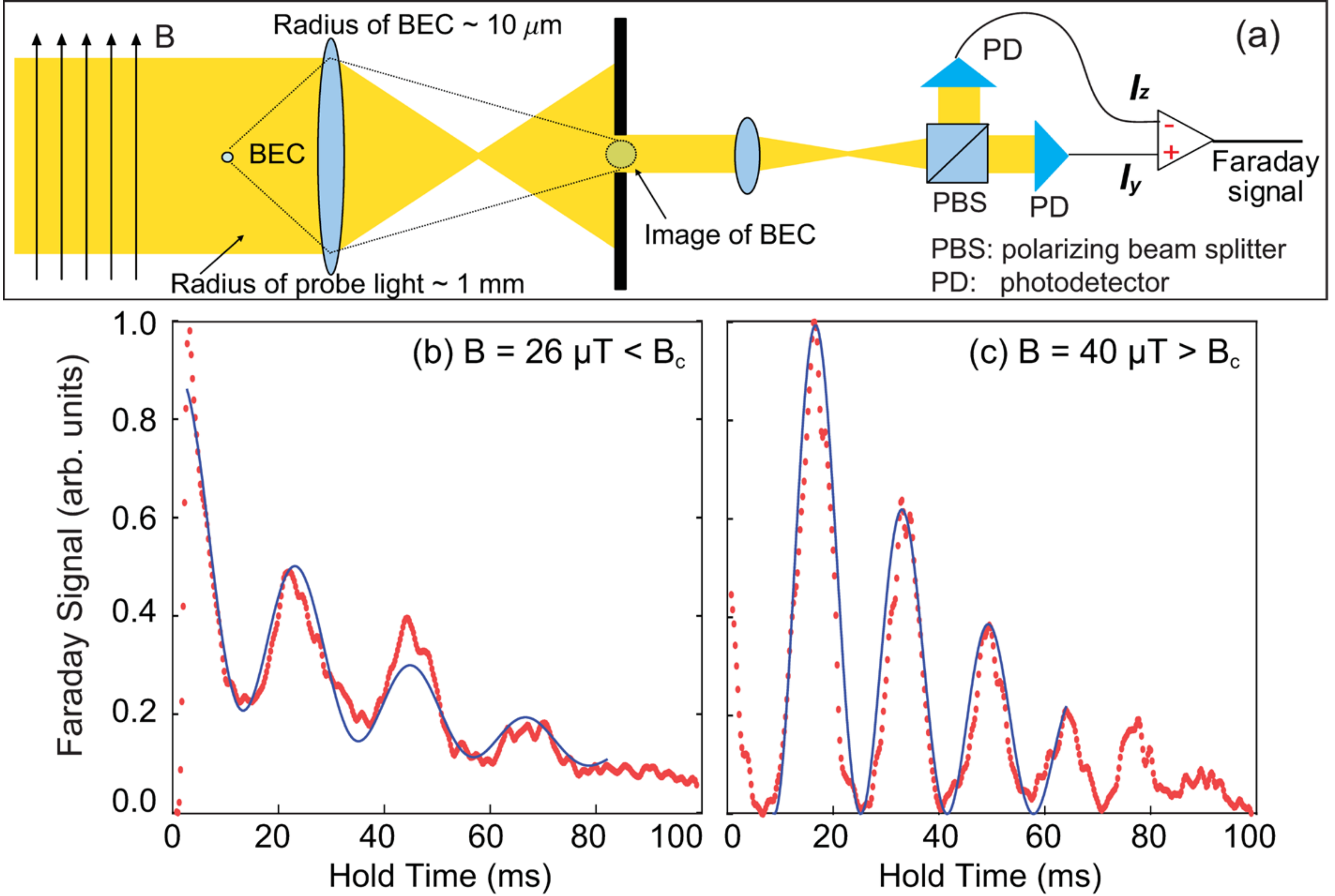


M. S. Chang et al, Nature Physics **1**, 111 (2005)

F=1 mean-field phase diagram

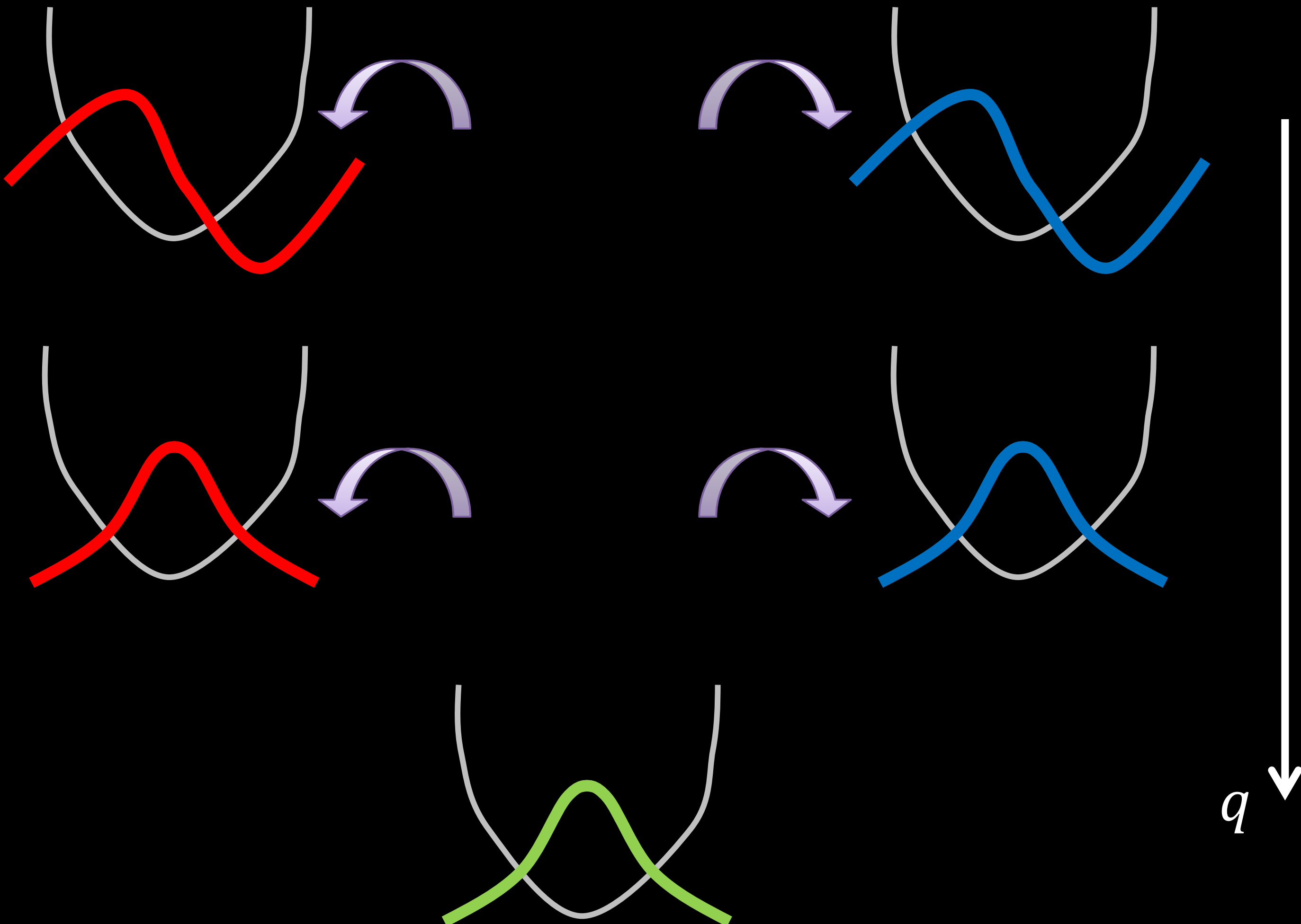
Stenger et al., Nature **396**, 345 (1998)





Liu, Y., S. Jung, S.E. Maxwell, L.D. Turner, E. Tiesinga, and P.D. Lett, Quantum Phase Transitions and Continuous Observation of Spinor Dynamics in an Antiferromagnetic Condensate. PRL **102**, 125301 (2009.

Hannover experiments: single-mode quench



m = -1 m = 0 m = +1

Instability to non-uniform mode (less likely to contain technical noise)

Instability to nearly uniform mode (more likely to contain technical noise)

stable

Hannover experiments: single-mode quench

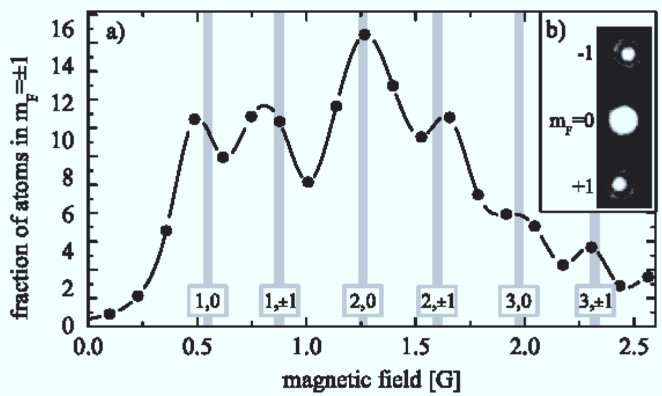


FIG. 1. (a) The fraction of atoms transferred into the $|\pm 1\rangle$ state within 18.5 ms as a function of the applied magnetic field. Each data point is an average over 30 realizations. The vertical gray lines indicate the resonance positions obtained from a 2D circular box model, and the labels indicate the corresponding Bessel modes. (b) Absorption image of a $|0\rangle$ BEC and the $|\pm 1\rangle$ clouds recorded at 1.29 G.

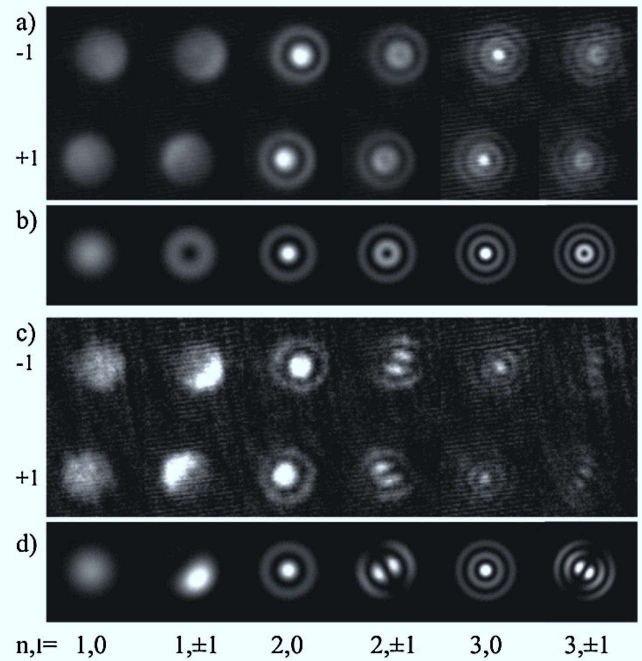
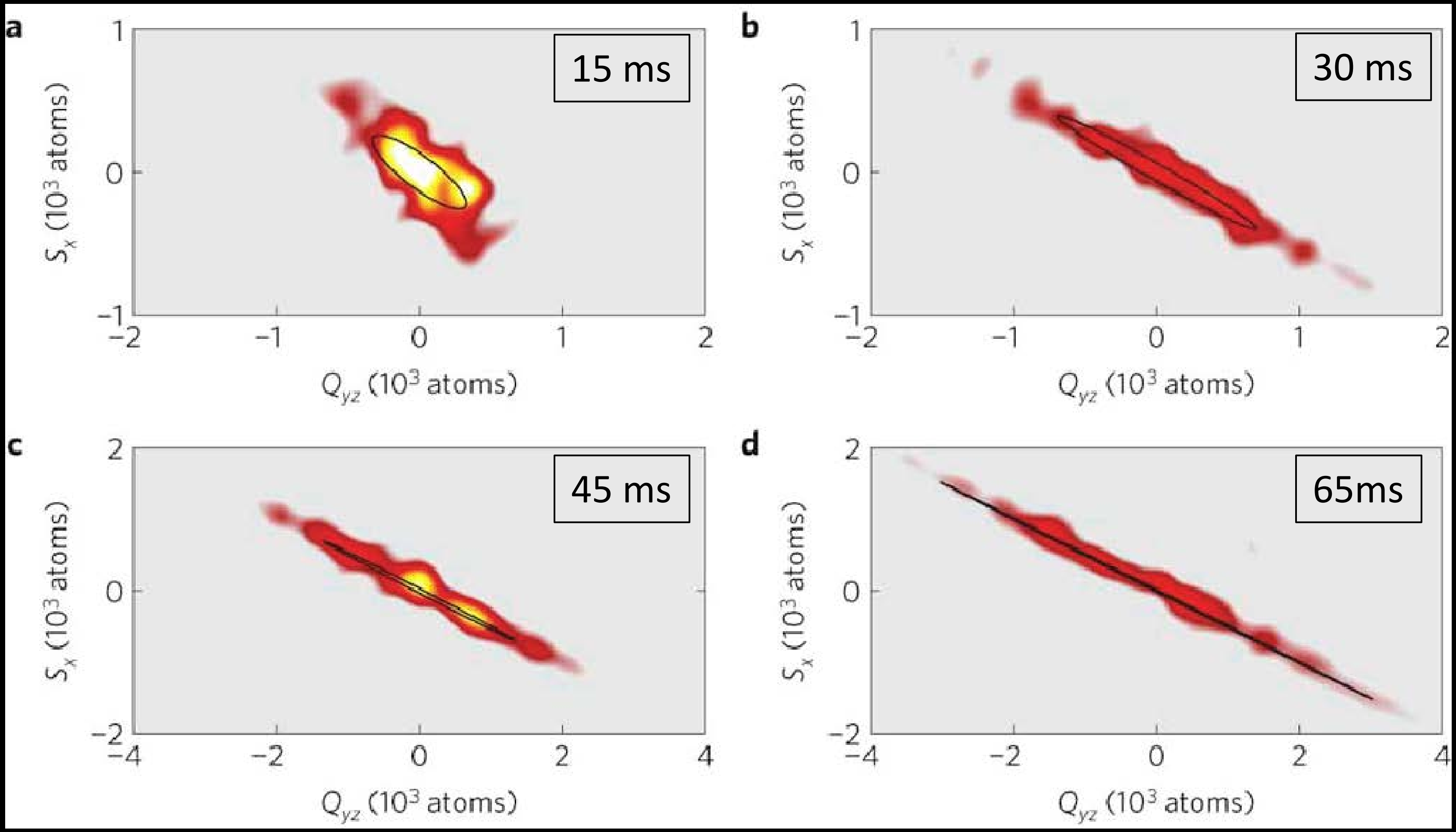


FIG. 2. The experimental and theoretical density distributions on the resonance positions after time-of-flight expansion. (a) Averaged experimental density profiles. (b) Calculated pure Bessel distributions corresponding to the experimental situation. (c) Individual experimental density profiles. (d) Calculated superpositions of Bessel distributions (see text). The $|0\rangle$ BEC was omitted in (a) and (c) for clarity.

PRL **103**, 195302 (2009)
PRL **104**, 195303 (2010)
PRL **105**, 135302 (2010)

Quantum spin-nematicity squeezing
(Chapman group, Georgia Tech)



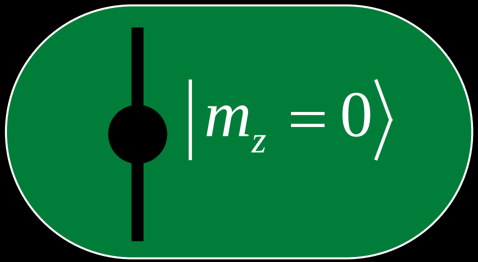
observed squeezing 8.6 dB below standard quantum limit!

Hamley et al., Nature Physics **8**, 305 (2012).

see also Gross et al., Nature 480, 219 (2011) [Oberthaler group], and

Lücke, et al., Science 334, 773 (2011) [Klempt group]

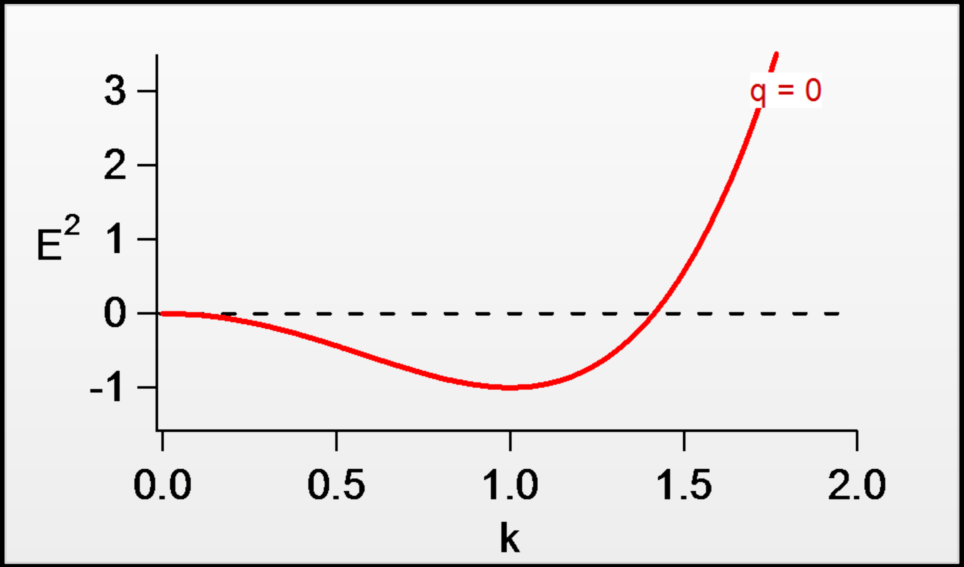
Spectrum of stable and unstable modes



- Bogoliubov spectrum
  - Gapless phonon (m=0 phase/density excitation)
  - Spin excitations

$$E_S^2 = (k^2 + q)(k^2 + q - 2)$$

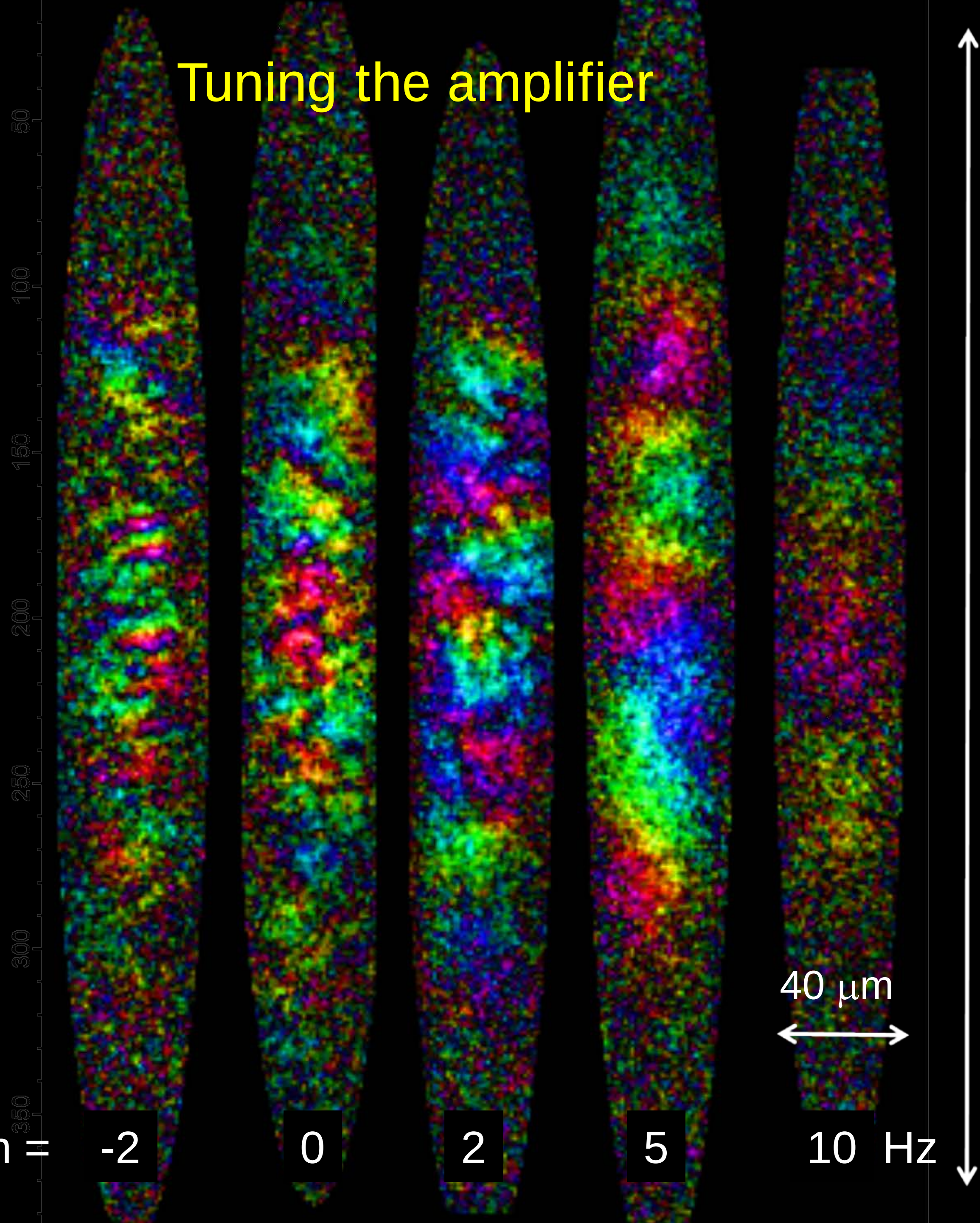
Energies scaled by $c_2 n$



q>2:	spin excitations are gapped by $\sqrt{q(q-2)}$
1>q>2:	broad, “white” instability
0>q>1:	broad, “colored” instability
q<0:	sharp instability at specific q≠0

Tuning the amplifier

T = 170 ms



Quench end point: q / h = -2 0 2 5 10 Hz

Topology of cosmic domains and strings

T W B Kibble

Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

Received 11 March 1976

Abstract. The possible domain structures which can arise in the universe in a spontaneously broken gauge theory are studied. It is shown that the formation of domain walls, strings or monopoles depends on the homotopy groups of the manifold of degenerate vacua. The subsequent evolution of these structures is investigated. It is argued that while theories generating domain walls can probably be eliminated (because of their unacceptable gravitational effects), a cosmic network of strings may well have been formed and may have had important cosmological effects.

J. Phys A: Math. Gen. 9, 1397 (1976)

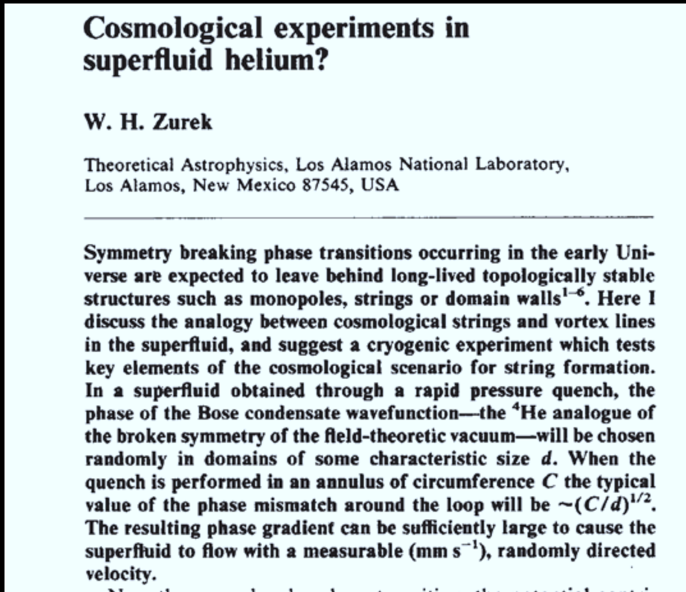
Big bang

Time, temperature

$\langle \phi \rangle = 0$, no broken symmetry

$\langle \phi \rangle \neq 0$, broken symmetry

- What defects can form?
- How many?
- Stability?
- Size? Mass?
- Coarsening


Cosmological experiments in superfluid helium?

W. H. Zurek

Theoretical Astrophysics, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

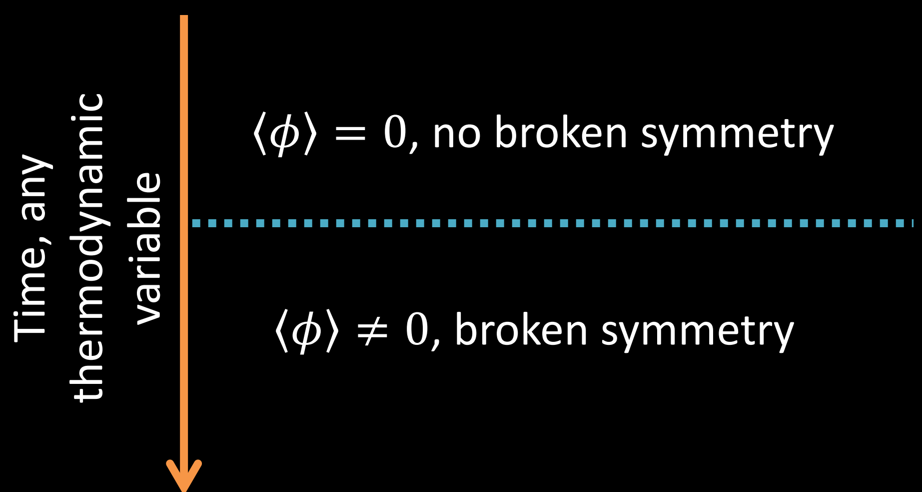
Symmetry breaking phase transitions occurring in the early Universe are expected to leave behind long-lived topologically stable structures such as monopoles, strings or domain walls[1–6]. Here I discuss the analogy between cosmological strings and vortex lines in the superfluid, and suggest a cryogenic experiment which tests key elements of the cosmological scenario for string formation. In a superfluid obtained through a rapid pressure quench, the phase of the Bose condensate wavefunction—the ^{4}He analogue of the broken symmetry of the field-theoretic vacuum—will be chosen randomly in domains of some characteristic size d. When the quench is performed in an annulus of circumference C the typical value of the phase mismatch around the loop will be $\sim(C/d)^{1/2}$. The resulting phase gradient can be sufficiently large to cause the superfluid to flow with a measurable (mm s^{-1}), randomly directed velocity.

Nature 317, 505 (1985)

Translates ideas to non-equilibrium condensed-matter systems

- Condensed-matter (and atomic, optical, etc) systems are test-beds for cosmolgy theory
- Family of generic phenomena in materials

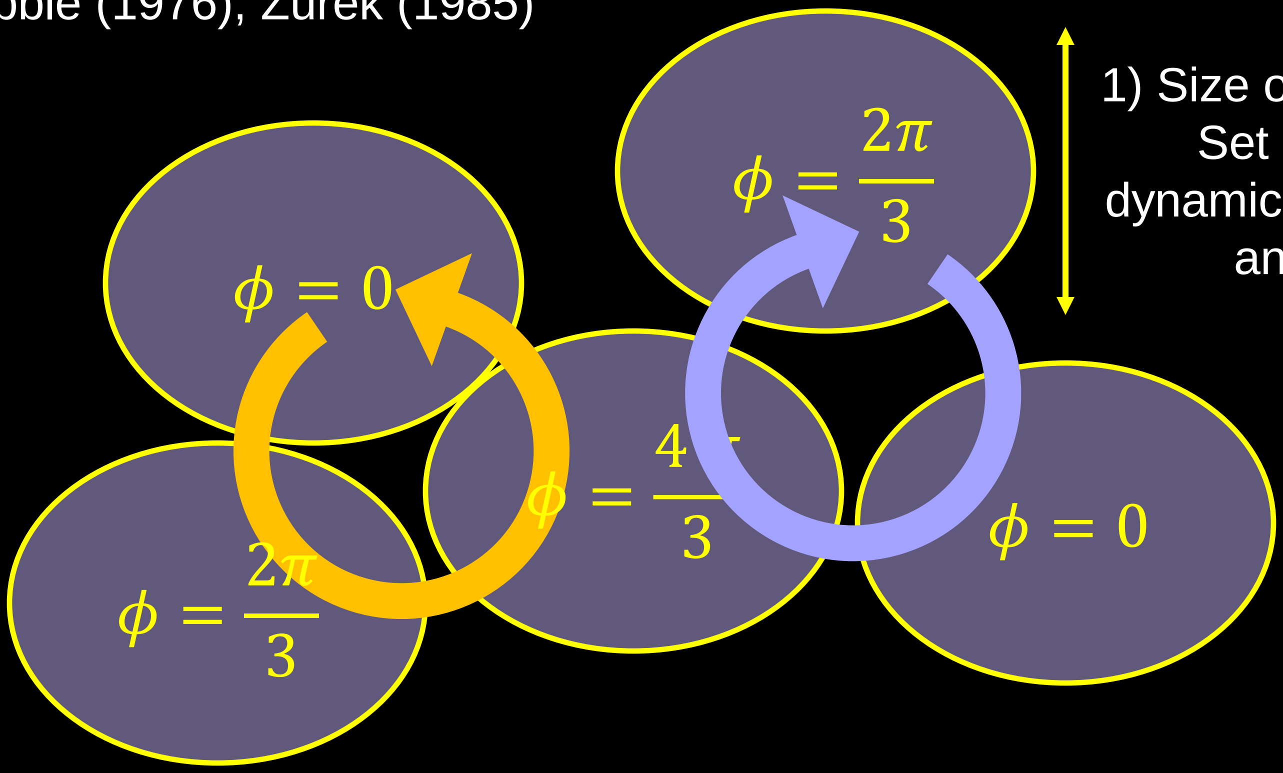
Hot experiment



- What defects can form?
- How many?
- Stability?
- Size? Mass?
- Coarsening

Topological defect formation across a symmetry-breaking phase transition

Kibble (1976), Zurek (1985)



1) Size of thermal fluctuation
Set by correlation +
dynamical critical exponents
and sweep rate\

2) Discordant regions heal into various defects (homotopy group)

3) Defects evolve, interact, persist or annihilate each other, etc.

"Thermal" Kibble-Zurek mechanism: first experiments

- Liquid crystals: quench of nematic order parameter
  - Mostly confirm predictions 2) and 3)

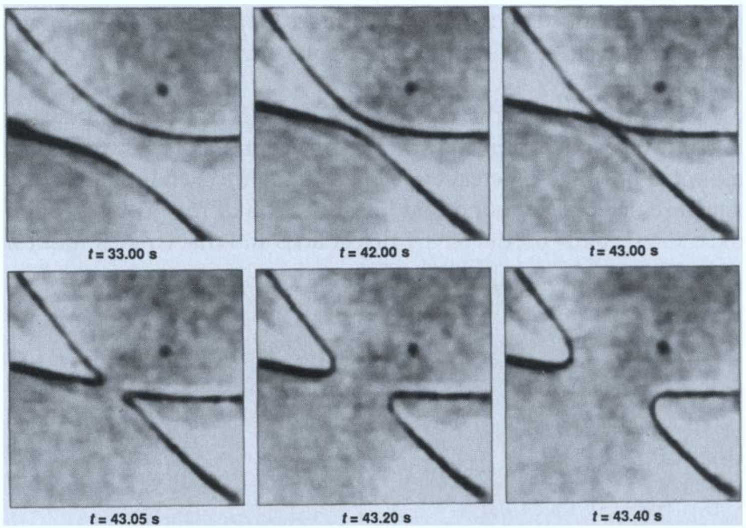


Fig. 1. String intercommutation sequence, showing two type-$\frac{1}{2}$ strings crossing each other and reconnecting the other way. Each picture shows a region 140 µm in width. Note that the two strings lie almost in the same plane—the intercommutation occurs after the strings move toward each other under their mutual attraction.

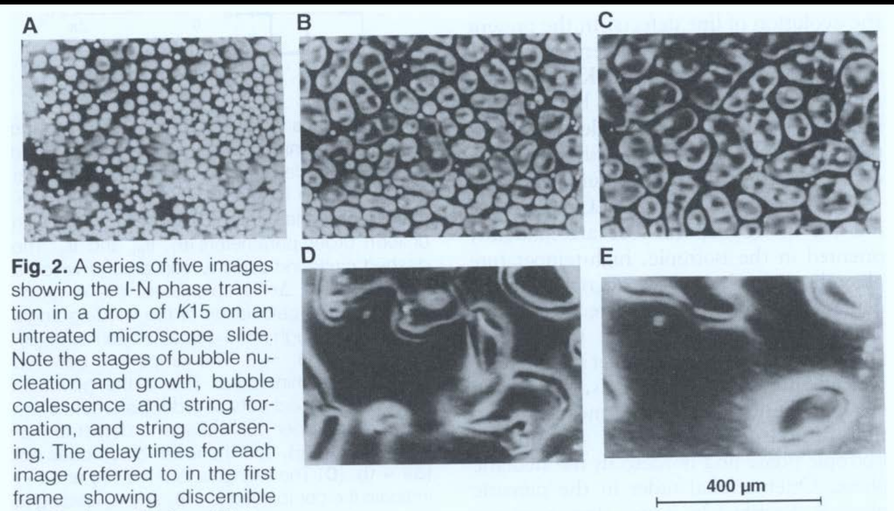


Fig. 2. A series of five images showing the I-N phase transition in a drop of *K*15 on an untreated microscope slide. Note the stages of bubble nucleation and growth, bubble coalescence and string formation, and string coarsening. The delay times for each image (referred to in the first frame showing discernible bubbles) are (**A**) 2 s, (**B**) 3 s, (**C**) 5 s, (**D**) 11 s, and (**E**) 23 s. The scale is for all five pictures.

944 SCIENCE • VOL. 263 • 18 FEBRUARY 1994

Chuang et al, Science 251, 1336 (1991)

Bowich et al, Science 263, 943 (1994)

"Thermal" Kibble-Zurek mechanism: first experiments

- Liquid helium 4 (pressure quench) and helium 3 (local re-cooled bubbles)
  - Lots of vortices form, but experiments are messy

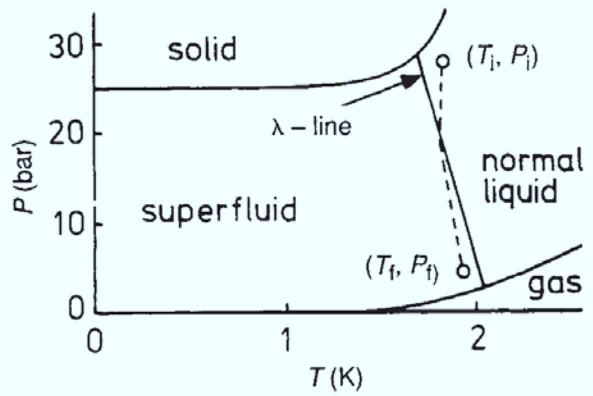


FIG. 1 Sketch of expansion trajectory (dashed) through the λ-transition on the ^{4}He pressure–temperature (P–T) phase diagram, from initial values (T_i, P_i) to final values (T_f, P_f).

315

Publishing Group

Helium 4: Hendry et al., Nature **368**, 315 (1994)

Helium 3: Bauerle et al, Nature **382**, 332 (1994);
Ruutu et al, ibid, p. 334.

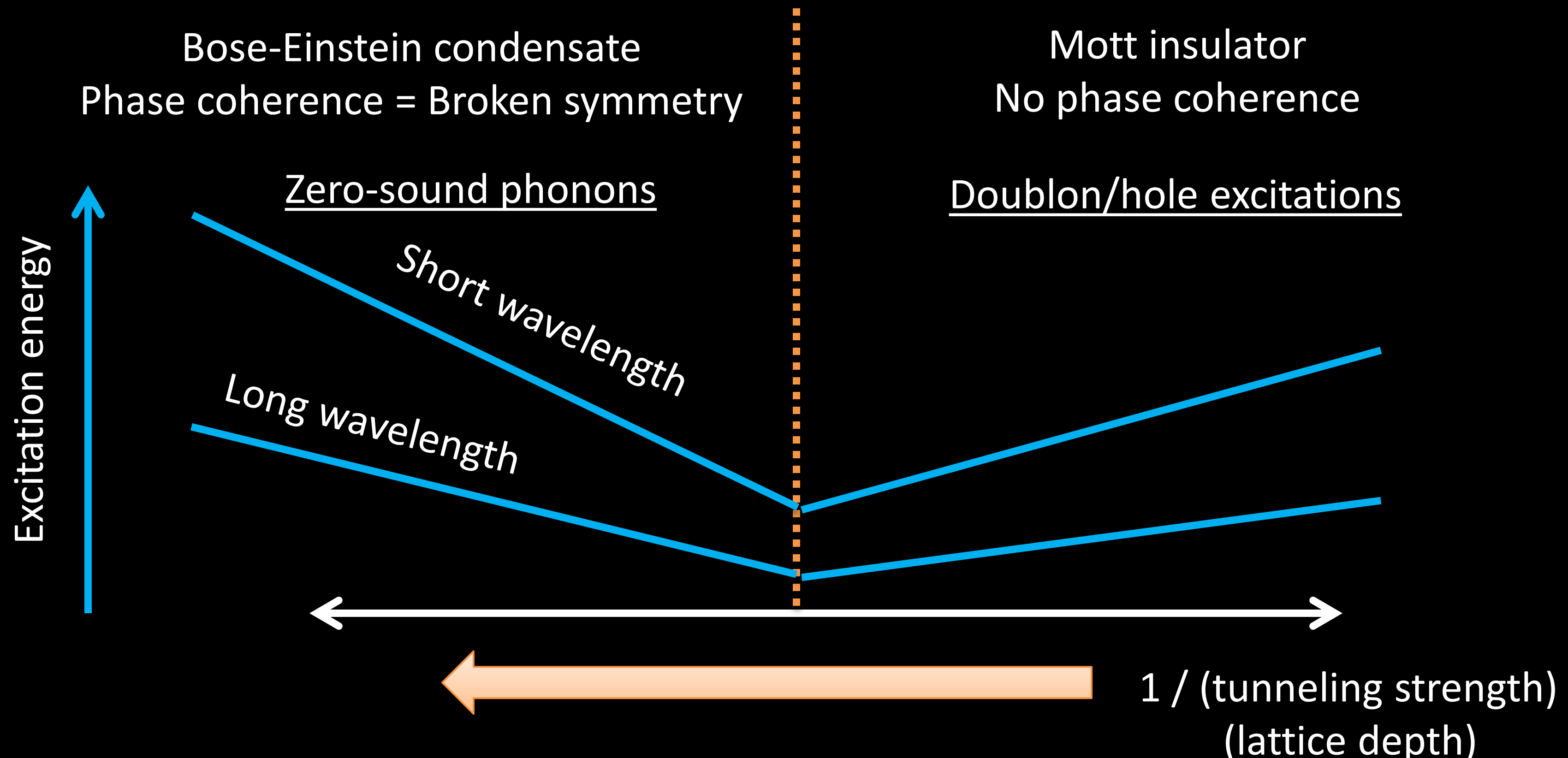
“Quantum” Kibble-Zurek mechanism; Quantum quenches

- Fluctuations are quantum mechanical
- Growth of order parameter from initial seed is quantum mechanical
- Sweeps of the Hamiltonian across a symmetry breaking transition:
  - Landau-Zener crossing/avoided crossing determines length scales
- Subsequent growth/evolution *may be* quantum mechanical

Some theoretical foundations (but this was a natural idea)

Zurek, Dorner, Zoller; Dziarmaga; Polkovnikov

Crossing the scalar-boson MI -> SF transition

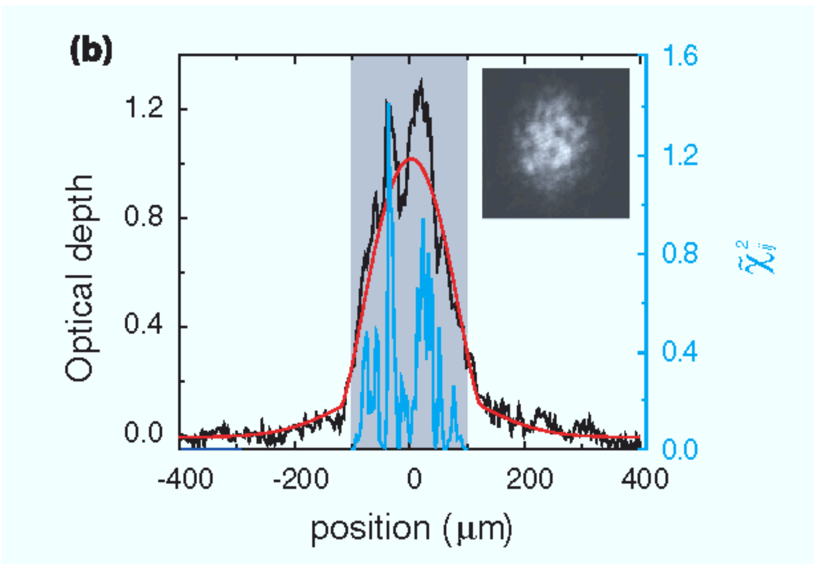


How much energy/entropy/defect
is generated by sweep?

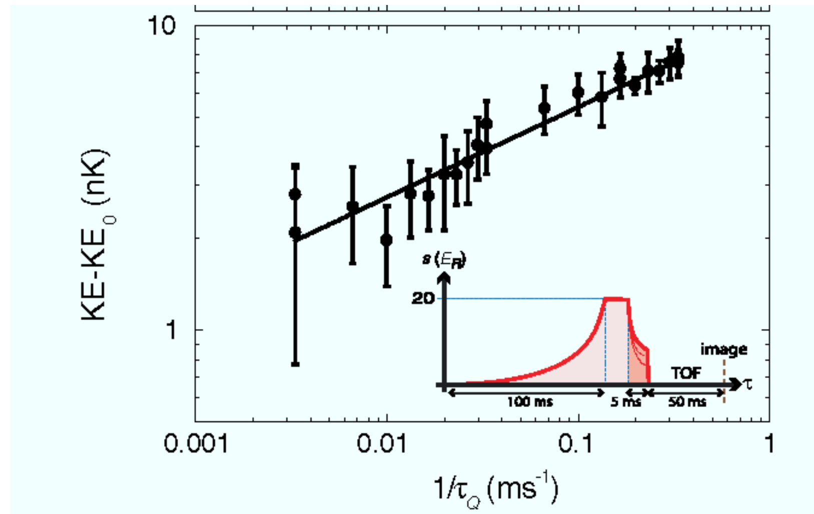
PRL 106, 235304 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

Quantum Quench of an Atomic Mott Insulator

David Chen,[1] Matthew White,[1,*] Cecilia Borries,[1,†] and Brian DeMarco[1]
[1]Department of Physics, University of Illinois, 1110 West Green Street, Urbana, Illinois 61801, USA
(Received 4 April 2011; revised manuscript received 13 May 2011; published 10 June 2011)



"lumpiness" (chi^2) of time-of-flight distribution measures quasiparticle number / kinetic energy / defects…

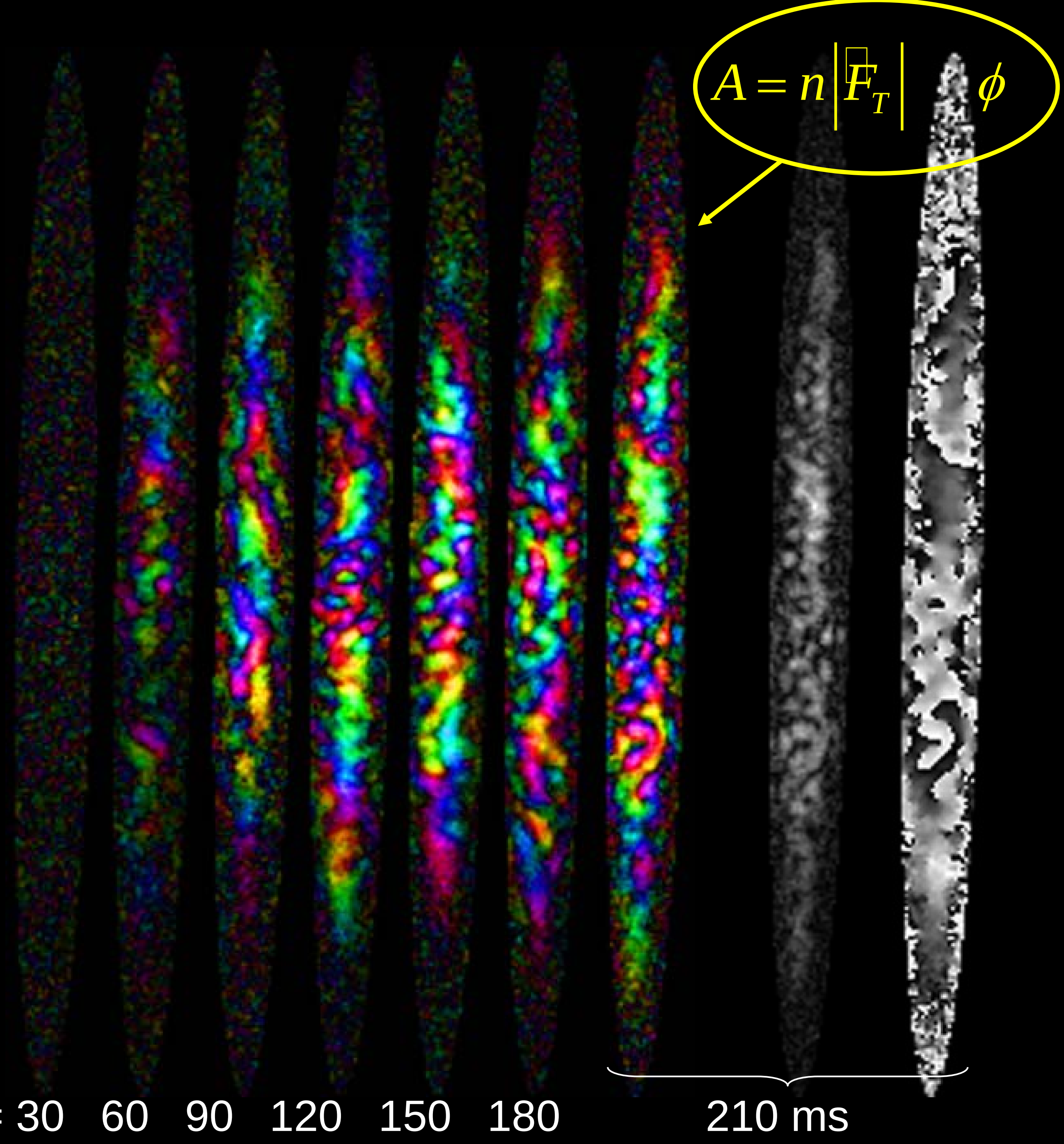


Power law?
Exponents don't match "theory"
But: start from multiple Mott shells (n=1, 2, 3); "phase front" in inhomogeneous sample; sweep varies other quantities…

See also Bakr et al, Science **329**, 547 (2010): Effects of sweeps in microscopic samples

Spontaneously formed ferromagnetism

- inhomogeneously broken symmetry
- ferromagnetic domains, large and small
- unmagnetized domain walls marking rapid reorientation



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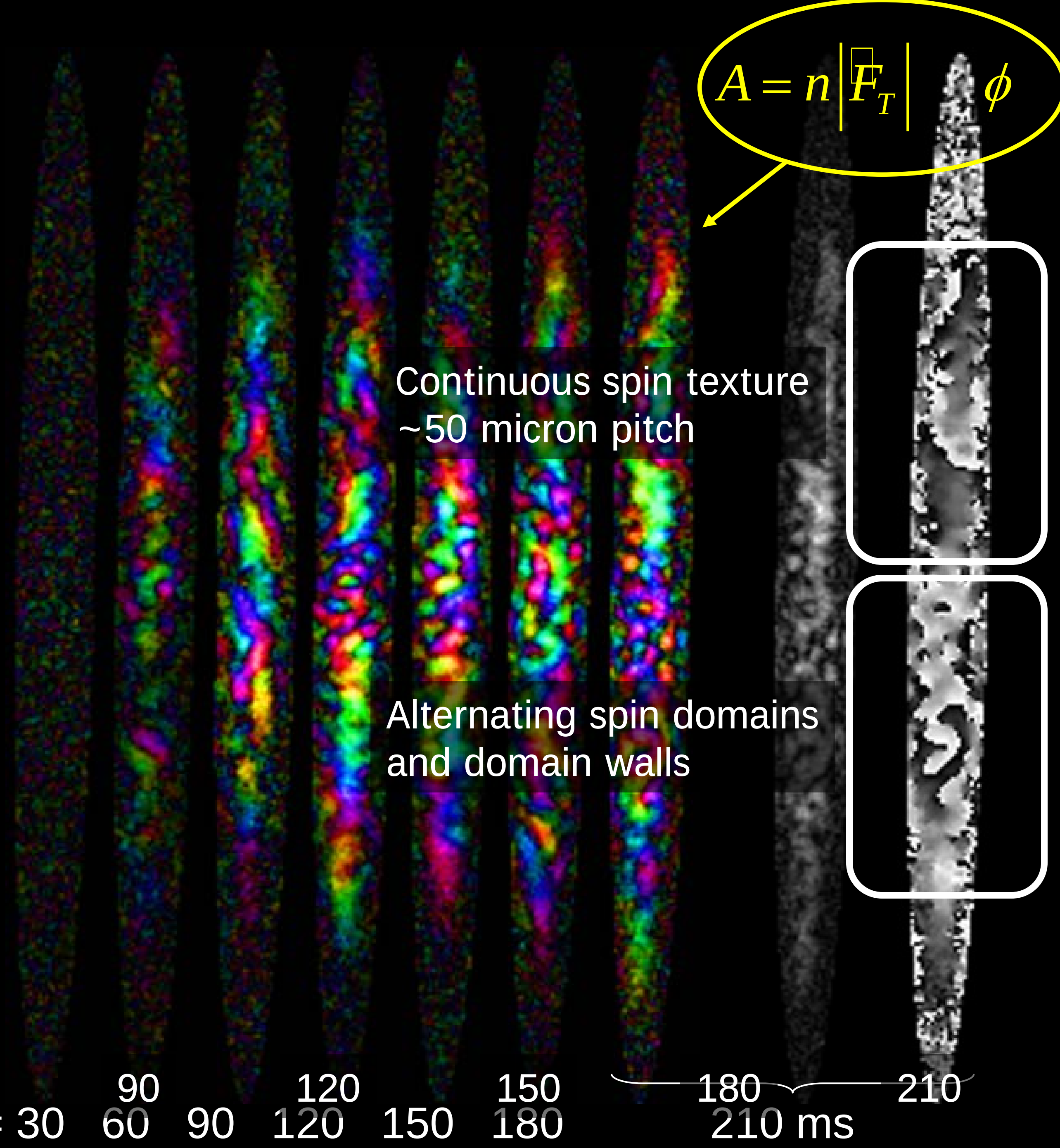
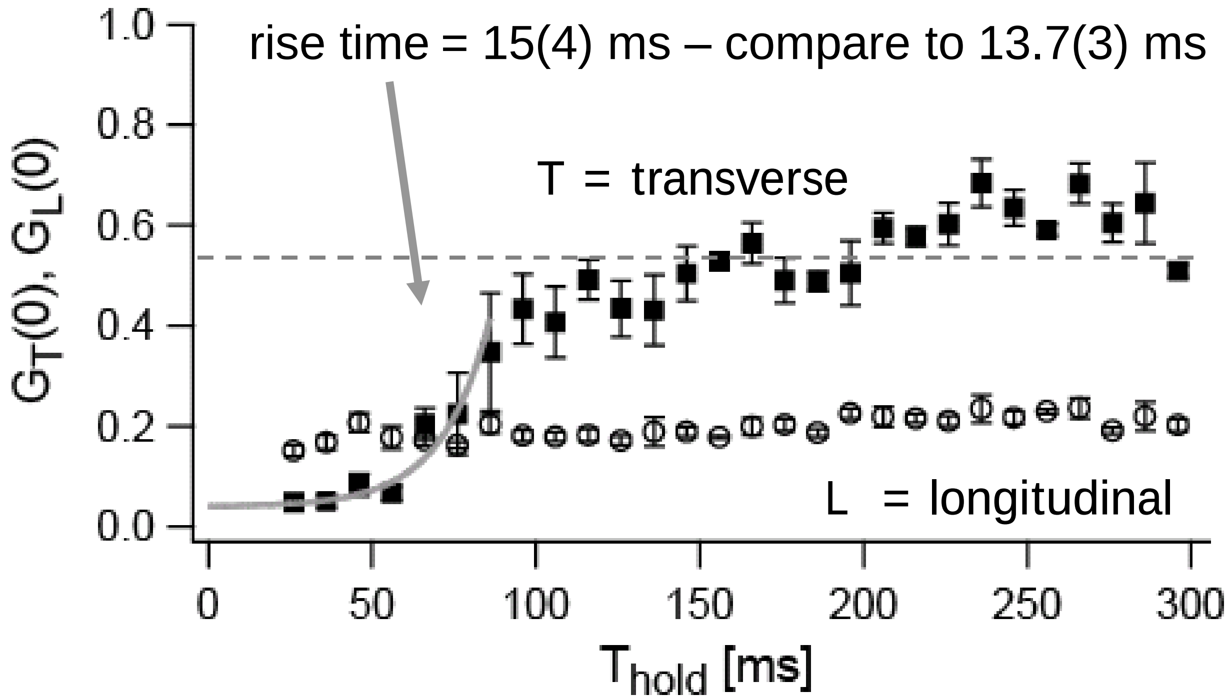


$$G(\delta r) = \frac{\sum_r n\vec{F}(r+\delta r) \cdot n\vec{F} n(r)}{\sum_r n(r+\delta r) n(r)}$$

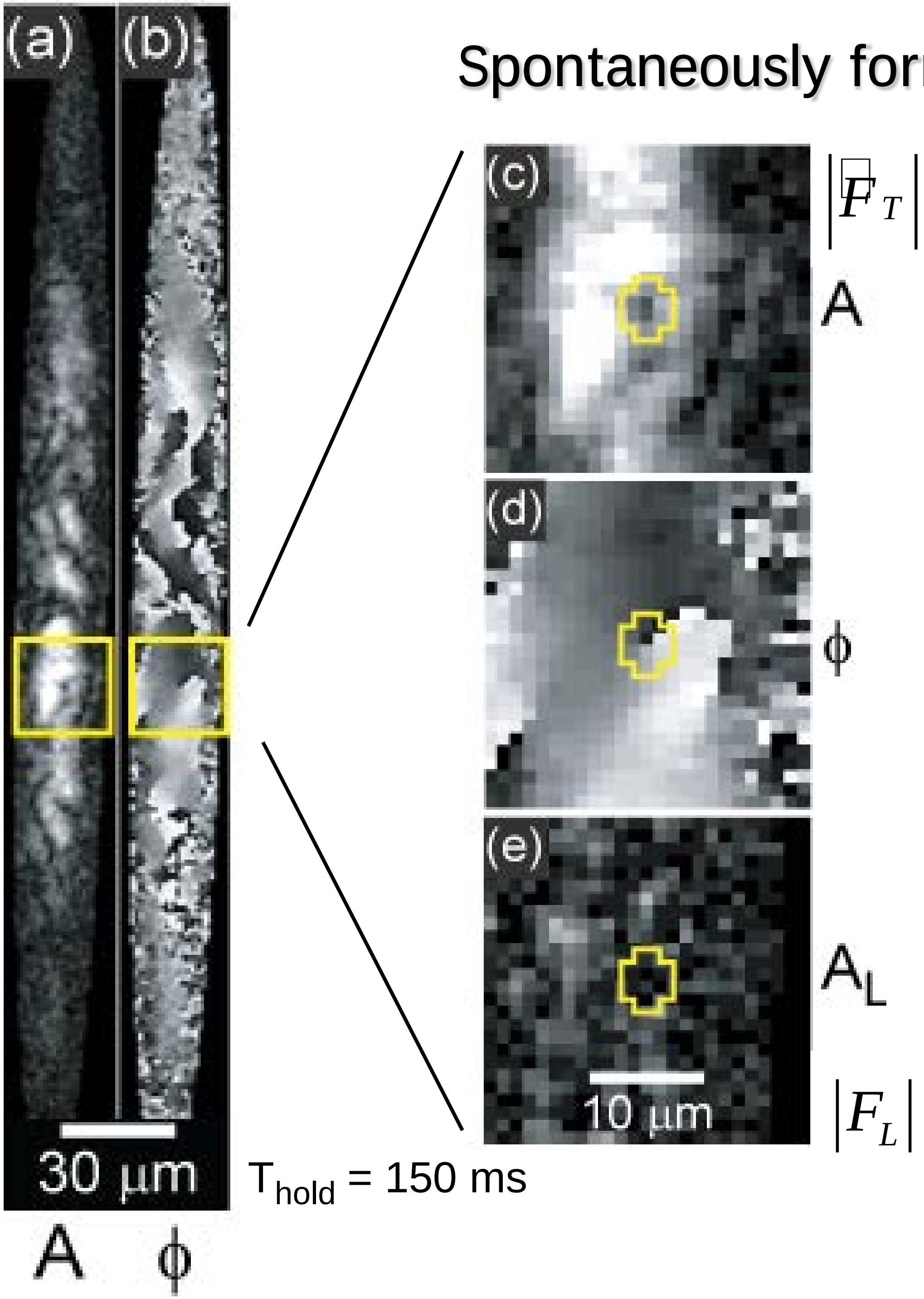
spin-spin correlation function



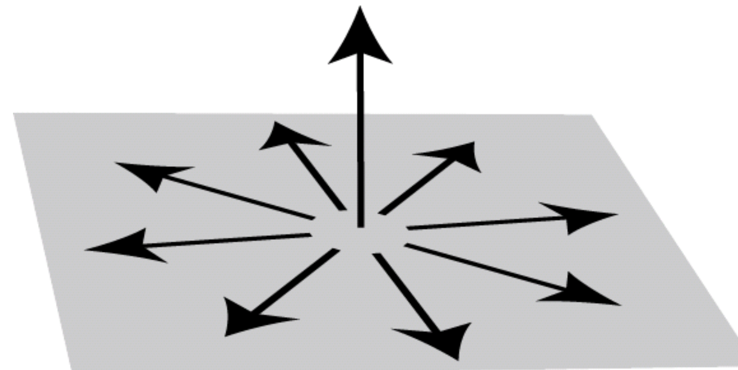
measure of area of domain walls

“Spontaneous symmetry breaking in a quenched ferromagnetic spinor BEC,” Nature 443,312 (2006)

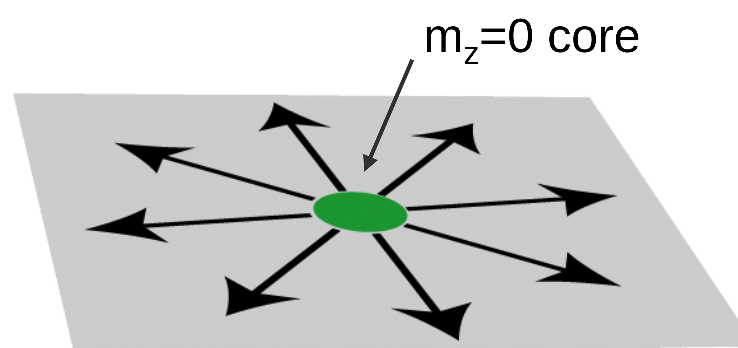
Spontaneously formed spin vortices



candidates:



Mermin-Ho vortex (meron)

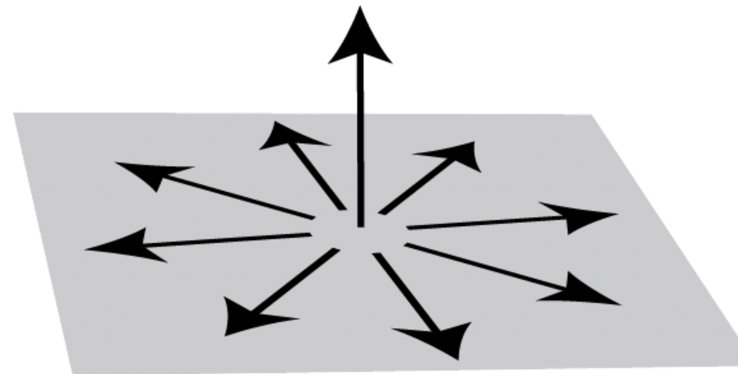


“Polar core” spin vortex

Spontaneously formed spin vortices

candidates:

$$\vec{\Psi} = \begin{pmatrix} a(r) & \times & 1 \\ b(r) & \times & e^{-i\phi} \\ c(r) & \times & e^{-2i\phi} \end{pmatrix}$$



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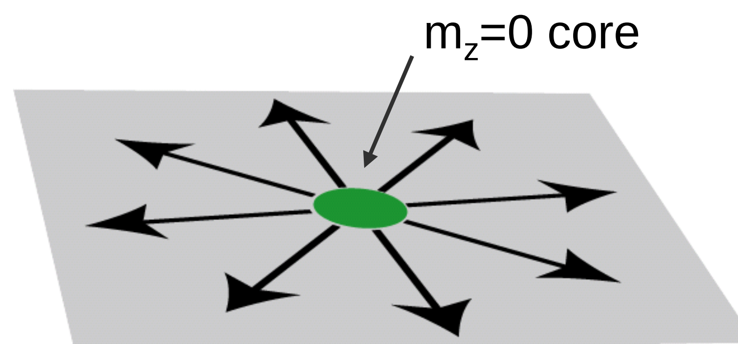


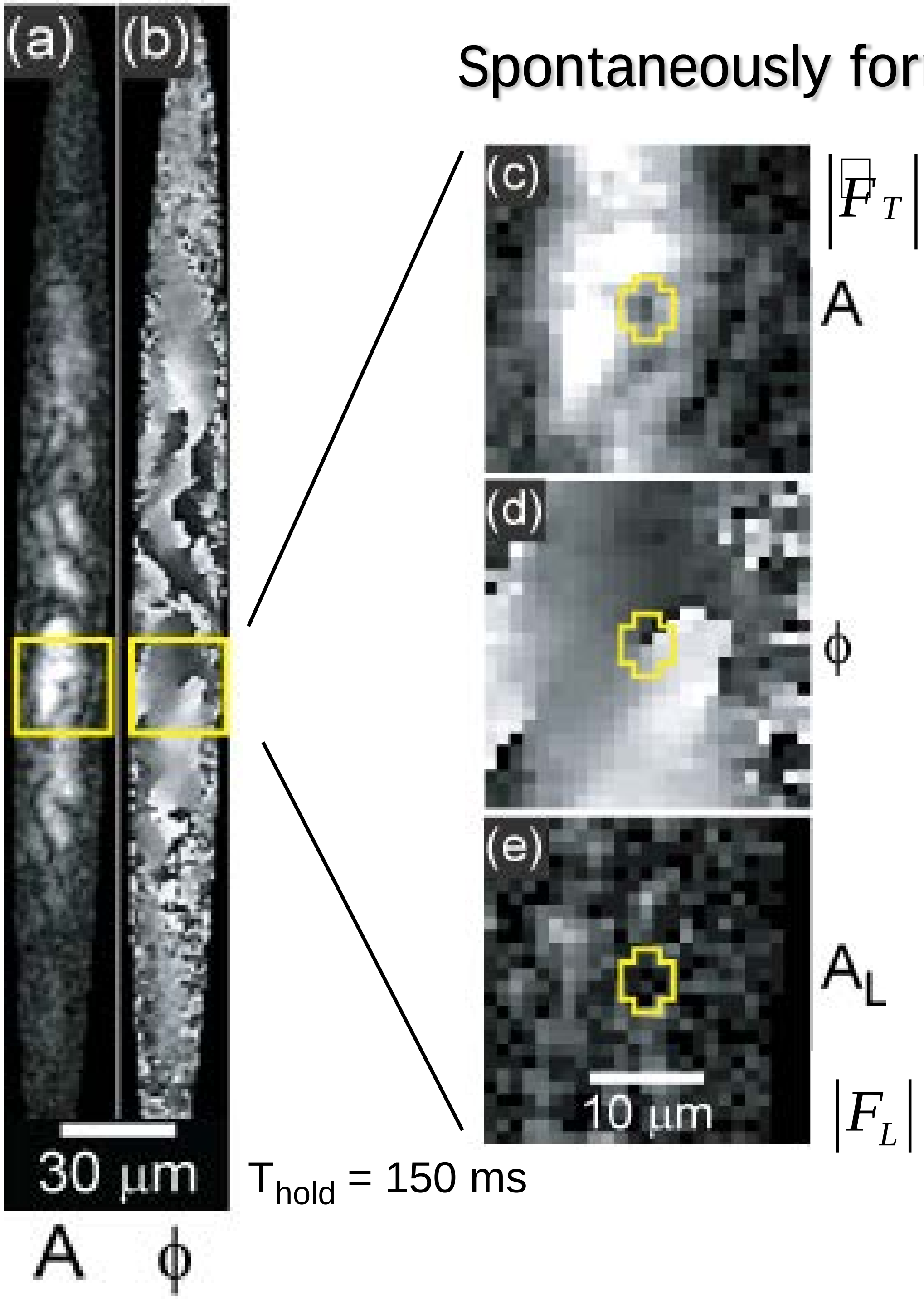
"Polar core" spin vortex

Broken chiral symmetry;
Saito, Kawaguchi, Ueda, PRL **96**, 065302 (2006)

outline

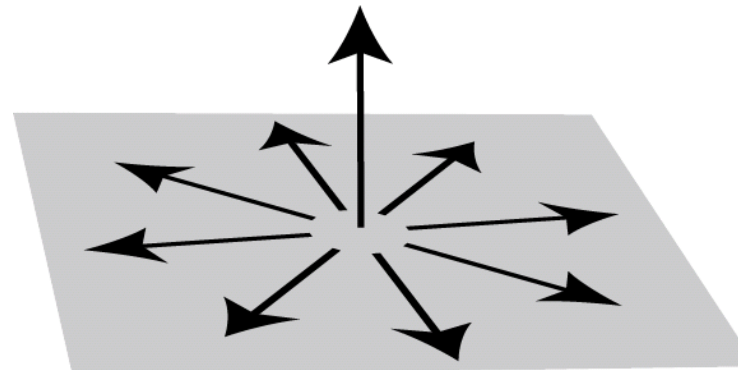
- Introductory material
- Interactions under rotational symmetry
- Energy scales
- Ground states
- Spin dynamics
  - microscopic spin mixing oscillations
  - single-mode mean-field dynamics
  - spin mixing instability
- More?

Spontaneously formed spin vortices



T_{hold} = 150 ms

candidates:



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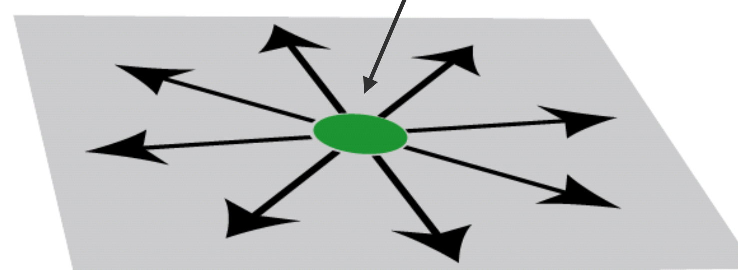

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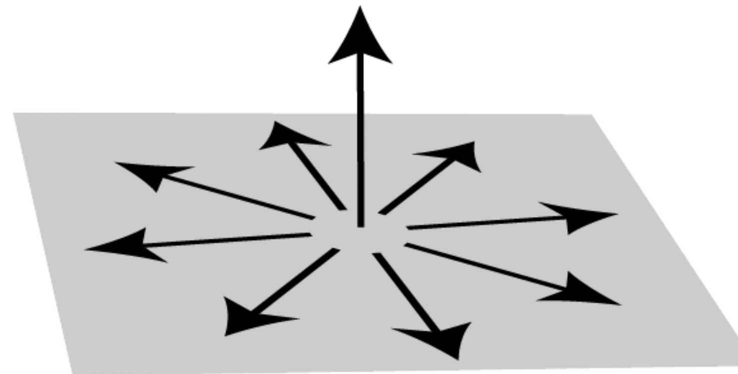

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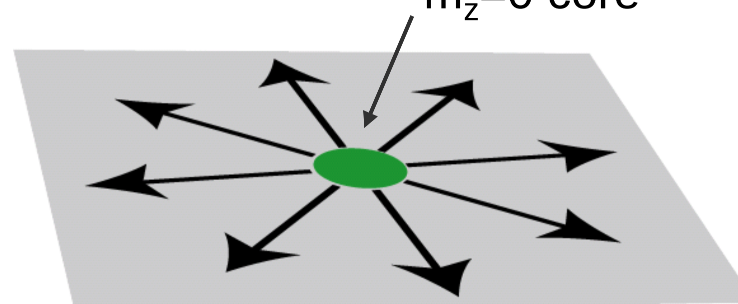
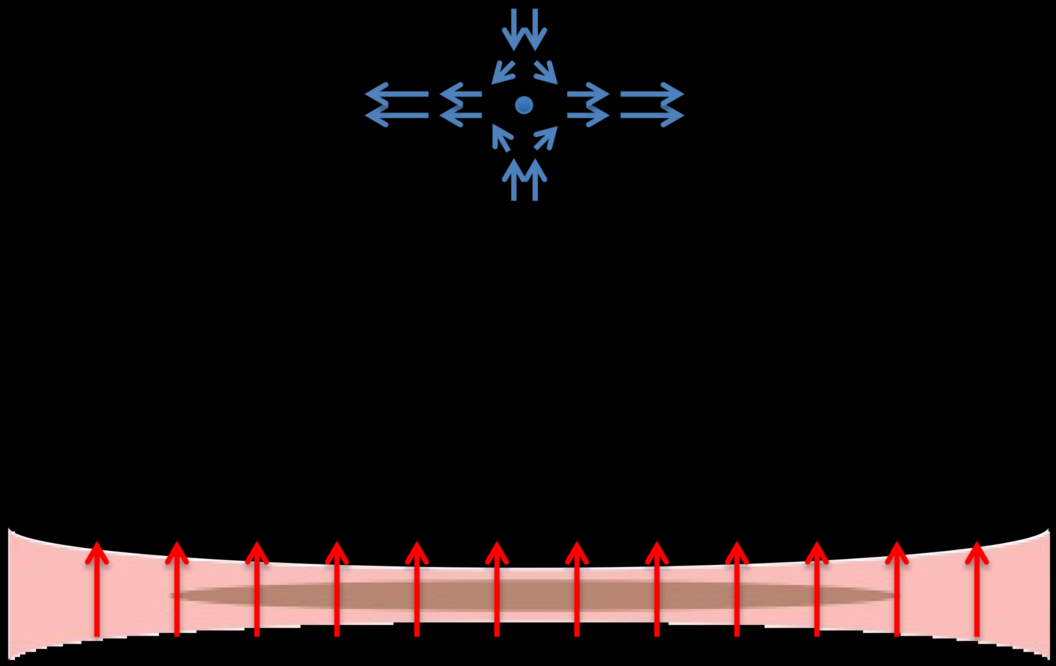


“Polar core” spin vortex

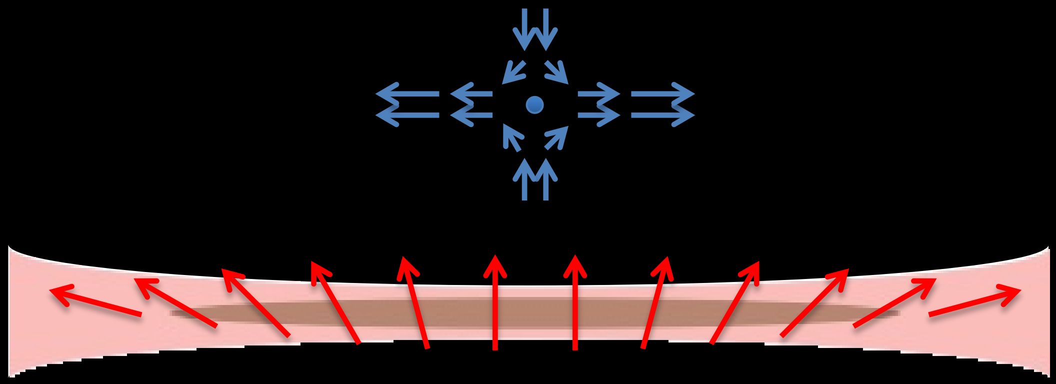
Broken chiral symmetry;
Saito, Kawaguchi, Ueda, PRL **96**, 065302 (2006)

Making a spin texture



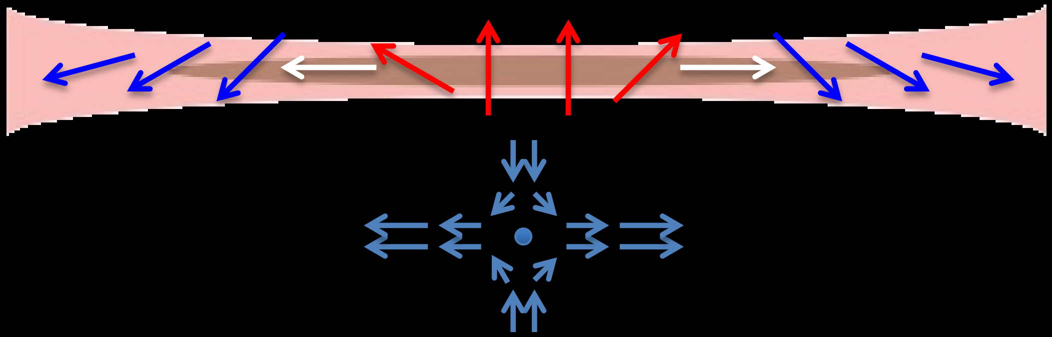
A. Leanhardt, *et. al.* PRL 90.140403 (2003)

Making a spin texture



A. Leanhardt, *et. al.* PRL 90.140403 (2003)

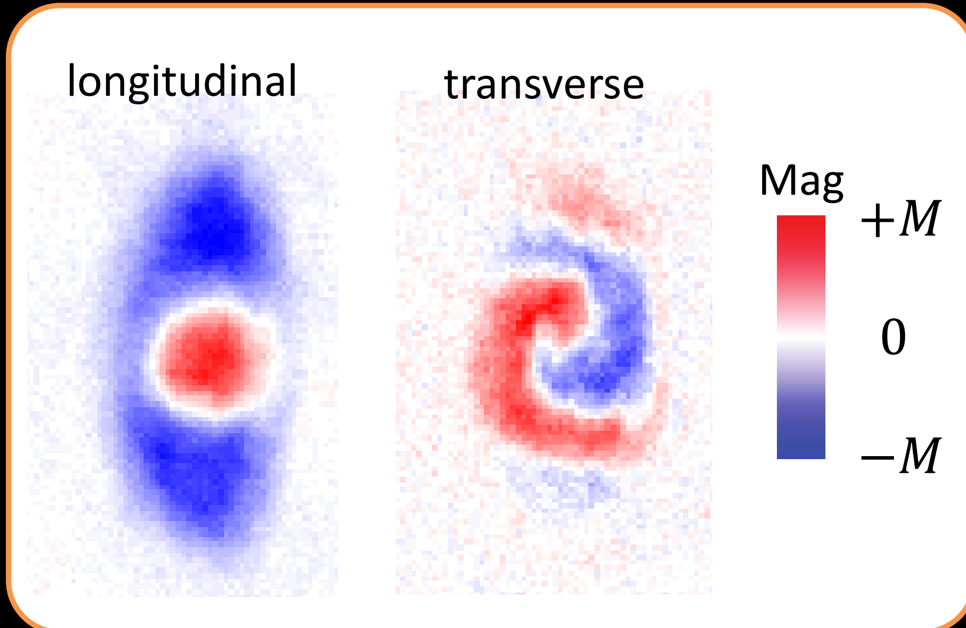
Making a spin texture

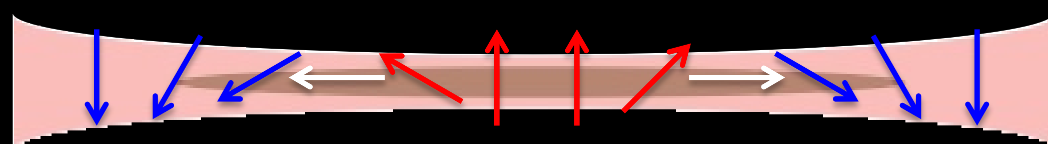


A. Leanhardt, *et. al.* PRL 90.140403 (2003)

Making a spin texture

Direct image of magnetization texture:



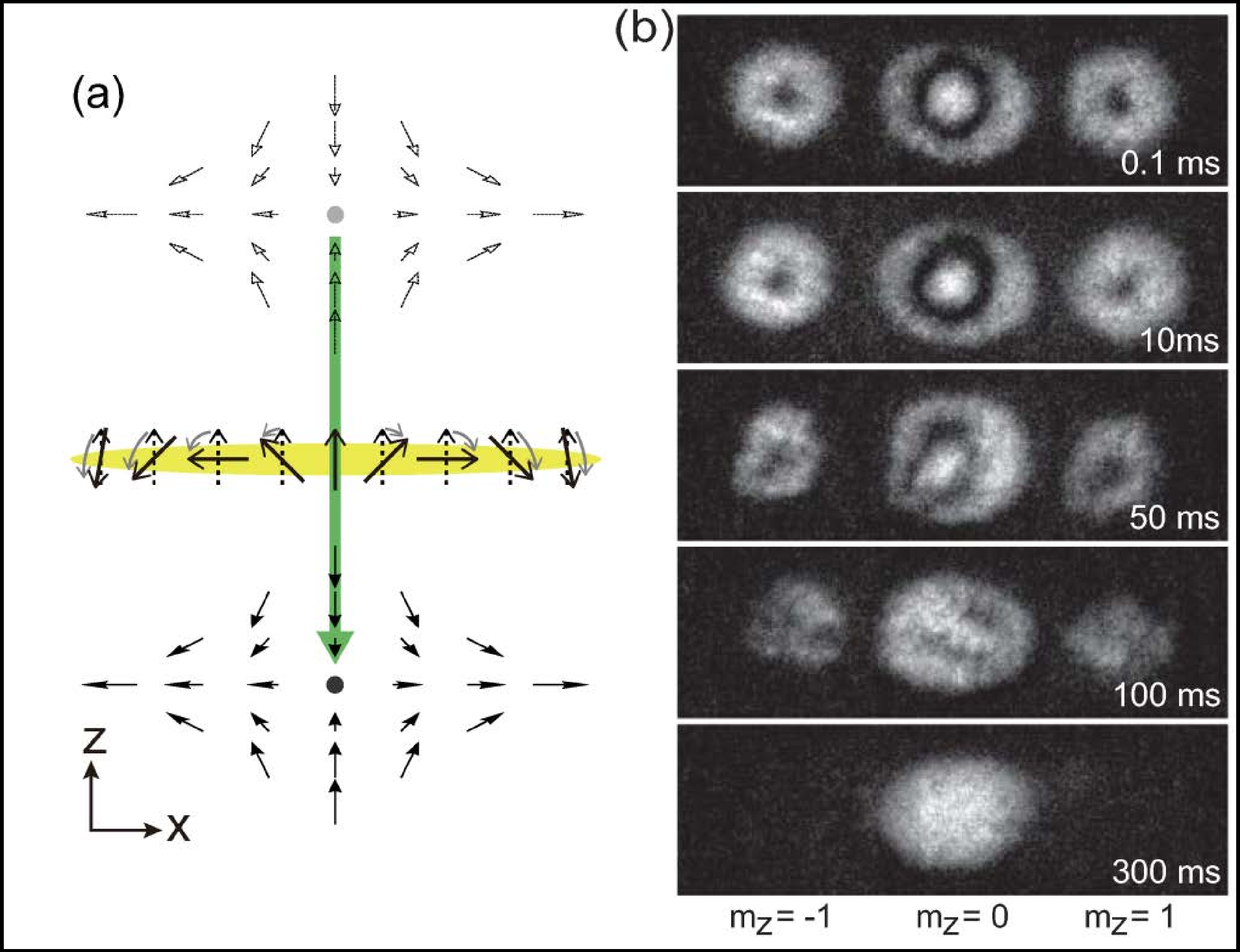


Between: magnetized radially (sideways)

Core: magnetized up

Periphery: magnetized down

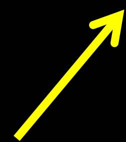
A. Leanhardt, *et. al.* PRL 90.140403 (2003)

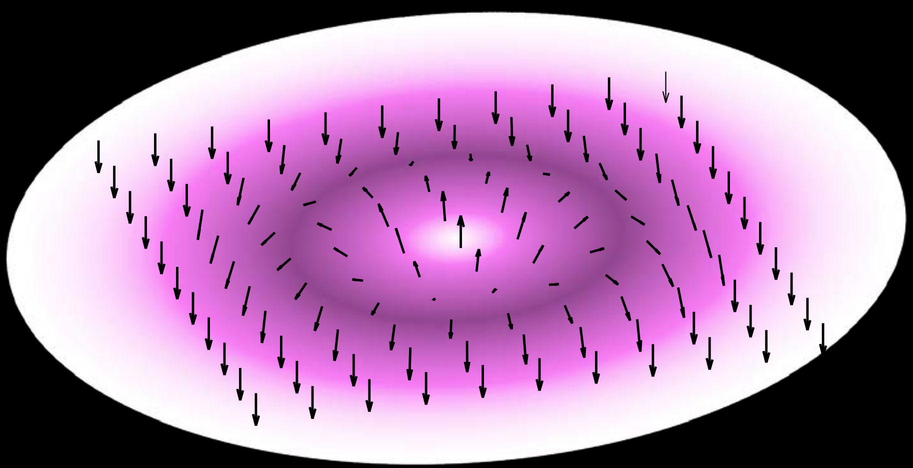


Choi, J.Y., W.J. Kwon, and Y.I. Shin, Observation of Topologically Stable 2D Skyrmions in an Antiferromagnetic Spinor Bose-Einstein Condensate. PRL **108**, 035301 (2012)

skrymion, or not skyrmion?

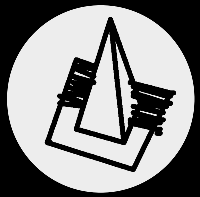
Ferromagnet or polar spinor condensate:

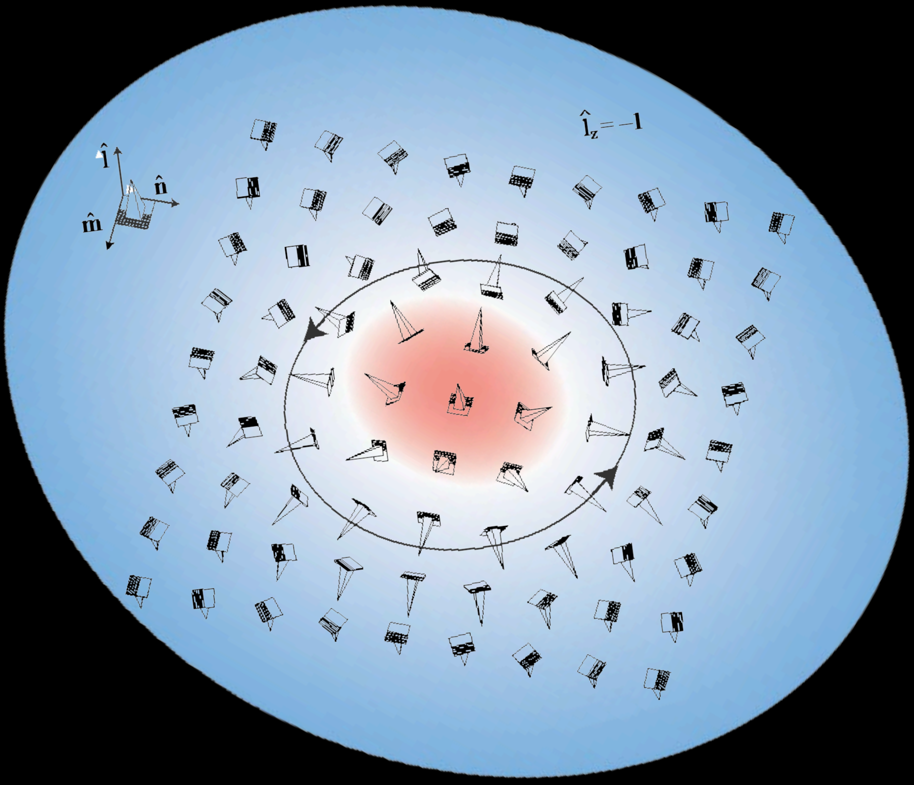
Order parameter = 



= skyrmion (topological)

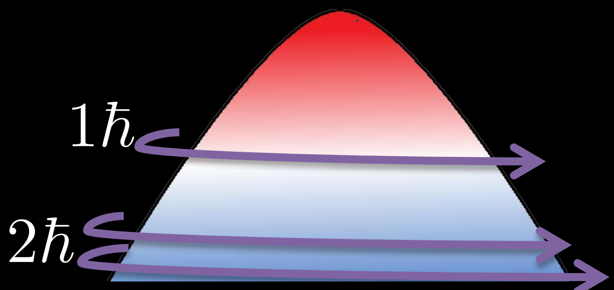
Ferromagnetic spinor condensate:

Order parameter = 



≠ skyrmion (not topological)

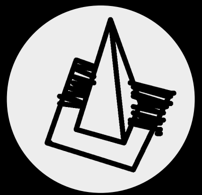
skrymion, or not skyrmion?

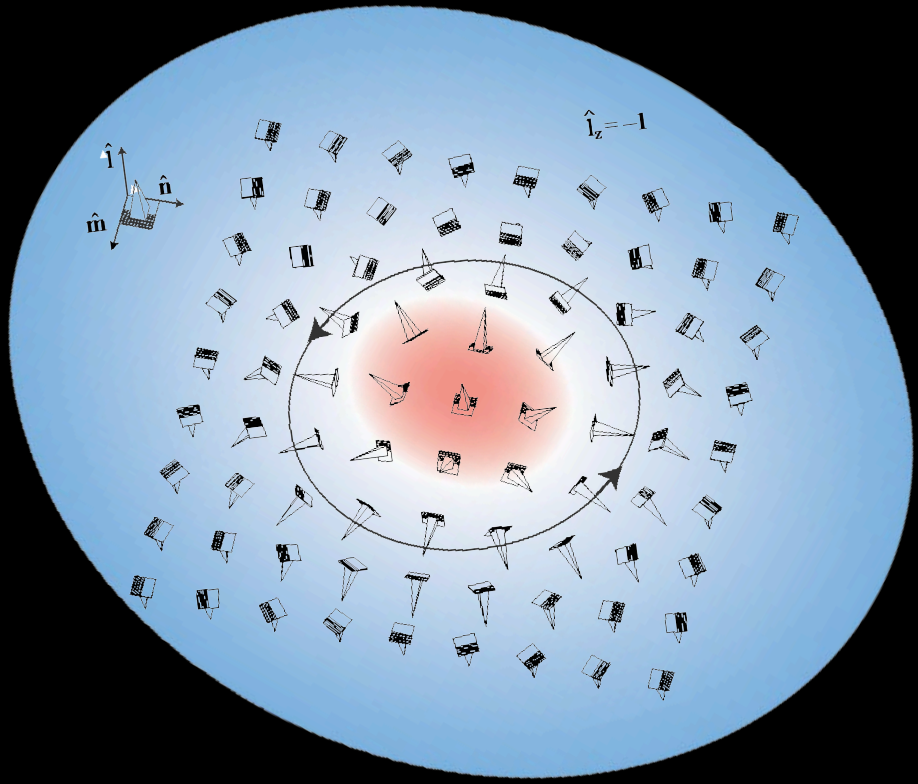


$$\pi_2 = \{0\}$$

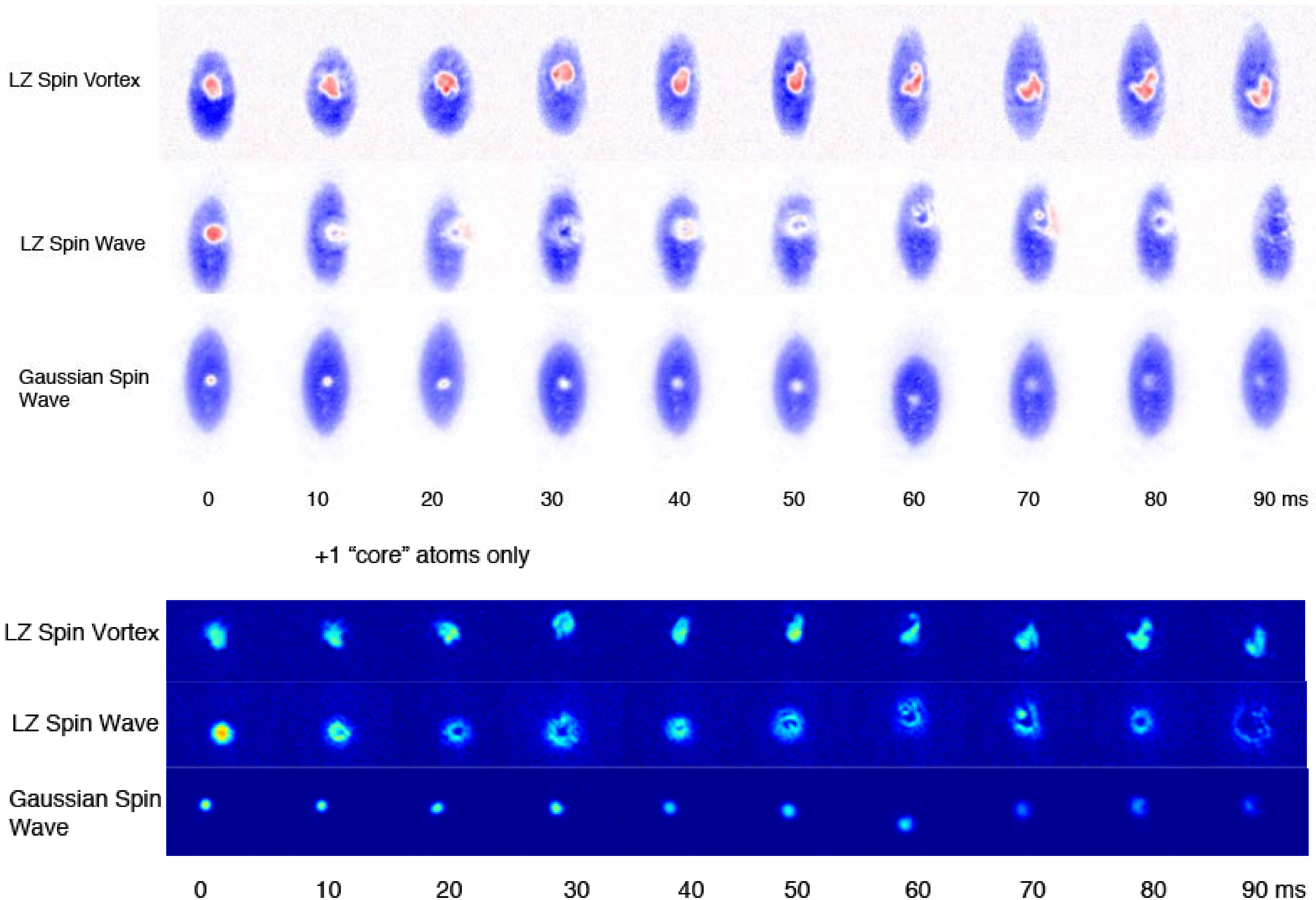
but… is it stabilized by rotation?

Ferromagnetic spinor condensate:

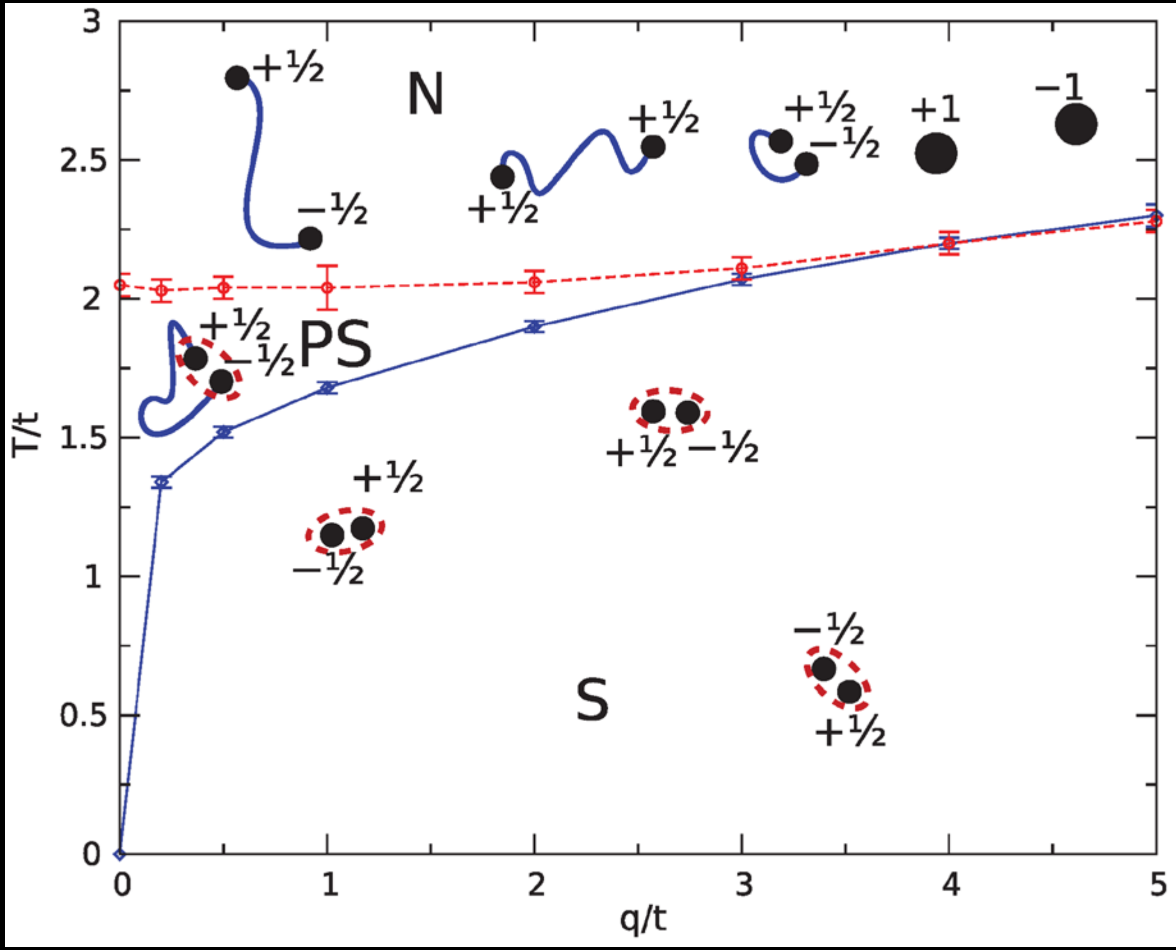
Order parameter = 



≠ skyrmion (not topological)


LZ Spin Vortex
LZ Spin Wave
Gaussian Spin Wave
0
10
20
30
40
50
60
70
80
90 ms
+1 “core” atoms only
LZ Spin Vortex
LZ Spin Wave
Gaussian Spin Wave
0
10
20
30
40
50
60
70
80
90 ms

antiferromagnetic F=1 condensate in 2D



Mukerjee, S., C. Xu, and J.E. Moore, *Topological Defects and the Superfluid Transition of the s = 1 Spinor Condensate in Two Dimensions*. PRL **97**, 120406 (2006).

James, A.J.A. and A. Lamacraft, *Phase Diagram of Two-Dimensional Polar Condensates in a Magnetic Field*. PRL **106**, 140402 (2011).