**Example 1:**

The spin-triplet superconductor \( \text{Sr}_2\text{RuO}_4 \)

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**Unconventional pairing - disorder effect**

sensitivity to non-magnetic impurities

- \( T_c \approx 1.5K \)

Maeno et al 1994

- layered crystal structure

quasi-2D metal

- \( \rho_{ab} \)

- \( \rho_c \)

at \( T = 2K \)

Maeno et al

- 3 quasi-2D bands derived from 4d-t\(^2\)g orbitals of Ru

most likely \( \gamma \)-band strongly dominant

---

**Sr\(_2\)Ru\(_4\) - quasi-2D-superconductor**

---

**Superconducting phases - analogy to \(^3\)He**

- basic symmetries:
  - \( G = G \times S \times K \times U(1) \)

- \(^3\)He phases

  - \( B \)-phase
    - \( \tilde{d}(k) = \hat{z}k_x + \hat{y}k_y + \hat{z}k_z \)

  - \( A \)-phase
    - \( \tilde{d}(k) = \hat{z}(k_x \pm i k_y) \)

\( \hat{\Psi}_k = (\tilde{d}(k) \cdot \hat{\sigma}) i \hat{\sigma}^y \)

- \( p \)-wave pairing states:
  - point group: \( D_{4h} \) tetragonal
  - spin-orbit coupling strong
  - inplane pairing

\( (k_z = (k_x, k_y, 0)) \)

---

**Textbook like suppression of \( T_c \)**

- \( \ell \leq \xi \approx 660 \AA \)

- \( T_c = 0 \)

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Mackenzie et al.

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Kikugawa et al.

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Bergemann, Mackenzie et al.
Experimental evidence for pairing symmetry

Spin-polarizability

\[ \vec{H} \parallel \hat{z} \]

In-plane equal-spin pairing \( \vec{d} \parallel \hat{z} \)

Ishida et al. (1998)

NMR-Knight shift

\[ T(K) \]

\[ 0 \rightarrow 0 \]

Yosida

Magnetic moment

\[ \mu_{SR} \text{ zero-field relaxation} \]

Luke et al. (1998)

Intrinsic magnetism

Field distribution

\[ \mu_{SR} \text{ field distribution in vortex phase} \]

Luke et al. (2000)

Ultrasound absorption

\[ T(K) \]

\[ 100 \rightarrow 100 \]

Lupien, Taillefer et al.

2-component order parameter

\[ \text{Sound velocity renormalization for transversal mode} \]
**Sr$_2$RuO$_4$ - chiral p-wave superconductor**

\[\tilde{d}(\vec{k}) = \hat{\tau}(k_x \pm ik_y) \quad \Rightarrow \quad \hat{\psi}_\vec{k} = \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix} \]

degeneracy: 2

topological phase

\[D_{4h} \times SU(2) \times \mathcal{K} \times U(1)_{\phi}\]

\[U(1)_{S_z} \times U(1)_{L_z+\phi}\]

broken time reversal symmetry $\mathcal{K}$

Edge states & Spontaneous currents

\[\text{Deguchi & Maeno}\]

---

**Sr$_2$RuO$_4$ - chiral p-wave superconductor**

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\[\text{Deguchi & Maeno}\]
Spontaneous currents
at inhomogeneities, surface and domain wall

Andreev bound state
at surface

subgap spectrum $E_{k_{\parallel}} = \Delta_0 \sin \theta_k = \Delta_0 k_{\parallel} k_F$
penetration depth $\xi = \hbar v_F / \Delta_0$

surface current

analogous at domain walls and around impurities

direct observation
negative
- scanning Hall probe
Tamegai et al
- scanning SQUID probe
Kirtley, Moler et al

screening current
**Quasiparticle states**

at inhomogeneities, surface and domain wall

Andreev bound state

at surface

Electron hole

$k_\perp$ $\theta$

Surface $x$ $y$

Quasiparticle tunneling in NS junctions

Tunneling conductance

$g(eV) = \frac{dI}{dV}(eV)$

conventional superconductor

chiral p-wave superconductor

$g_s/g_{1\Delta}$

$g_s/g_{1\Delta}$

Honerkamp, Matsumoto & MS

Yamashiro, Tanaka et al.

**Quasiparticle states**

at inhomogeneities, surface and domain wall

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Yamashiro, Tanaka et al.
Quasiparticle tunneling in NS junctions

Tunneling conductance

\[ g_s / g_n \]

\[ + \Delta \]

\[ - \Delta \]

\[ (eV) \]

NS-tunneling

\[ \text{Sr}_2\text{RuO}_4 \]

Mao, Liu et al.

chiral p-wave superconductor

Point contact spectroscopy

\[ \text{Sr}_2\text{RuO}_4 \]

Laube, Goll, von Lohneysen et al.

Further evidence for chiral p-wave pairing

Phase sensitive experiments

Josephson effect and domain walls

two domains

\[ \bar{d}_\pm (k) = \eta_0 \hat{z} (k_x \pm i k_y) \]

interference effect in magnetic field

Josephson contact

\[ k = \frac{2\pi}{\Phi_0} dH_{c2} \]

\[ \Phi_0 = \frac{hc}{2e} \]

critical current

\[ I = \max_\alpha \left| \int dy I_c(y) \sin(kx + \phi(y)) + \alpha \right| \]

intrinsic phase shift associated with domain

Phase sensitive experiments

SQUID type of measurements

corner „SQUID“ geometry

\[ J_y \propto i(\psi^* \eta_x - \psi_x \eta_y) \propto \sin(\alpha) \]

\[ \pm \pi / 2 \]

phase difference

Interference pattern

off-centered

broken time reversal chiral p-wave
**Kerr effect**

Rotation of polarization axis for reflected light

\[
\theta_K \approx \frac{4\pi \sigma_{xy}}{n(n^2-1)\omega} \left( \frac{\Delta^2}{\hbar \omega^3} \right)
\]

Kapitulnik et al. (2006)

Kerr effect strongly reduced for \(\hbar \omega \gg \Delta\)

debate:
- effects of gauge-invariance
- Yakovenko, Kallin, Mineev, ...
- Buhmann & MS in 90s: Joynt; Yip & Sauls, ...

Case 2:

**Superconductors without inversion center**

Key symmetries for Cooper pairing

Heavy Fermion superconductor \(\text{CePt}_3\text{Si}\)

**Status of evidence for chiral p-wave**

<table>
<thead>
<tr>
<th>Evidence</th>
<th>Consistent</th>
<th>Unexplained</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMR Knight shift (\vec{H} \parallel \hat{z})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu)SR ZF relaxation rate</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Flux distribution/ultrasound (2-comp OP)</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Phase sensitive test / Josephson effect</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Edge states in quasiparticle tunneling</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Direct observation of spontaneous currents</td>
<td>negative</td>
<td></td>
</tr>
<tr>
<td>Phenomenology of 3K-phase</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Limiting behavior for inplane field</td>
<td>unexplained</td>
<td></td>
</tr>
<tr>
<td>Kerr effect</td>
<td>consistent</td>
<td></td>
</tr>
<tr>
<td>Disorder effects non-magnetic impurities</td>
<td>consistent</td>
<td></td>
</tr>
</tbody>
</table>

**Superconductivity and magnetism**

\(T_c = 0.45 \text{K}\) \quad \(T_N = 2.2 \text{K}\)

\(\text{CePt}_3\text{Si}\)

discovered by Ernst Bauer et al (2003)
Heavy Fermion superconductor $\text{CePt}_3\text{Si}$

Superconductivity and magnetism

$\text{CePt}_3\text{Si}$

$\text{LaPt}_3\text{Si}$

$T_c = 0.45 \, \text{K}$

$T_N = 2.2 \, \text{K}$

non-centrosymmetric crystal

$P4mm \rightarrow C_{4v}$

tetragonal

Onuki et al.,

Bauer et al.

Pressure (GPa)

Temperature (K)

$T_N = 2.2 \, \text{K}$

$T_c = 0.45 \, \text{K}$

non-centrosymmetric crystal

$P4mm \rightarrow C_{4v}$

tetragonal

Other Ce-based compounds

$\text{CeRhSi}_3$, $\text{CeIrSi}_3$, $\text{CeCoGe}_3$

antiferromagnets with superconductivity under pressure at QCP

non-centrosymmetric point group $C_{4v}$

Doniach's phase diagram

Onuki et al.

Kimura et al. (2005)

Onuki et al. (2005)

Onuki et al. (2007)
Electronic states of non-centrosymmetric metal

Non-centrosymmetric superconductors

\[ \hat{\Delta}_k = \left( \begin{array}{ccc} \Delta_{k,11} & \Delta_{k,12} & \Delta_{k,14} \\ \Delta_{k,21} & \Delta_{k,22} & \Delta_{k,24} \\ \Delta_{k,41} & \Delta_{k,42} & \Delta_{k,44} \end{array} \right) \]

\[ \hat{\Delta}_k = i \sum_{\mu=0}^{3} d_{\mu}(\vec{k}) \hat{\sigma}^\mu \hat{\sigma}^\nu \]

gap function

\[ \hat{\mathcal{H}} = \sum_{\vec{k},s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + \sum_{\vec{k},s,s'} \tilde{g}_{\vec{k}} \cdot \hat{\sigma}^s \hat{c}_{\vec{k}s}^\dagger c_{\vec{k}s'} \]

spin-orbit coupling

\[ \tilde{g}_{\vec{k}} = \alpha (\hat{z} \times \vec{k}) \]

spin-splitting

symmetry classification

inversion is not a symmetry

mixed-parity pairing

parity

even-spin-singlet odd-spin-triplet

d\(0(\vec{k})\) = \Delta_e \quad d\(\vec{k}\) = \Delta_o \tilde{g}_{\vec{k}}

full-symmetry pairing state

non-unitary \( \hat{\Delta} \hat{\Delta} \neq \hat{\sigma}^0 \)

full-symmetry pairing state

even-parity component

odd-parity component

pairing symmetry

\[ d_0(\vec{k}) = \Delta_e \quad d(\vec{k}) = \Delta_o \tilde{g}_{\vec{k}} \]

parity

full-symmetry pairing state

mixed-parity pairing

even-parity component

odd-parity component

\[ \Gamma^+ \otimes \Gamma^- \]

\[ \Gamma \]

\[ \begin{array}{c|c|c} \Gamma & d_0(\vec{k}) & d(\vec{k}) \\ \hline \Gamma_1 & \Gamma^- & \Gamma^- \\ \Gamma_2 & k_x k_y (k_x^2 - k_y^2) & k_x k_y (k_x^2 - k_y^2) \tilde{g}_{\vec{k}} \\ \Gamma_3 & k_x^2 - k_y^2 & (k_x^2 - k_y^2) \tilde{g}_{\vec{k}} \\ \Gamma_4 & k_x k_y & k_x k_y \tilde{g}_{\vec{k}} \\ \Gamma_5 & \{k_x k_z, k_y k_z\} & \{k_x k_z \tilde{g}_{\vec{k}}, k_y k_z \tilde{g}_{\vec{k}}\} \end{array} \]
Non-centrosymmetric superconductors

Pairing symmetry

\[ D_{4h} \]

\[ C_{4v} \]

Even-parity component

\( \Gamma^+ \)

Odd-parity component

\( \Gamma^+ \otimes \Gamma^- \)

\[ \Gamma \]

\[ d_0(\mathbf{k}) = \Delta_e \]

\[ \bar{d}(\mathbf{k}) = \Delta_0 \mathbf{g}_k \]

\[ \begin{array}{c|c}
\Gamma & d_0(\mathbf{k}) \\
\hline
\Gamma_1 & 1 \\
\Gamma_2 & k_z k_y (k^2_x - k^2_y) \\
\Gamma_3 & k^2_x - k^2_y \\
\Gamma_4 & k_z k_y (k^2_x - k^2_y) \mathbf{g}_k \\
\Gamma_5 & \{k_z k_x, k_y k_z\} \\
\end{array} \]

\[ \hat{\Delta}_k = (\Delta_e + \Delta_0 \mathbf{g}_k \cdot \hat{\sigma}) \hat{i} \hat{\sigma}_y \]

Mixed-parity states are non-unitary

Unitary superconducting states:

\[ \hat{\Delta} \Delta^\dagger = |\Delta|^2 \delta_{\mathbf{0}} \propto 2\times2 \text{ unit matrix} \]

\[ \hat{\Delta} = \{\psi(\mathbf{k})\hat{\sigma}_0 + \bar{d}(\mathbf{k}) \cdot \hat{\sigma}\} i \hat{\sigma}_y \]

\[ \hat{\Delta} \Delta^\dagger = \{\psi^* \bar{d} + \psi \bar{d}^*\} \cdot \hat{\sigma} + i \{\bar{d} \times \bar{d}^*\} \cdot \hat{\sigma} \]

\[ \chi = \text{const. for } \bar{d}(\mathbf{k}) \cdot \mathbf{H} = 0 \]

Note: \( \hat{\mathbf{S}} \| \mathbf{H} \) with \( \bar{d} \perp \hat{\mathbf{S}} \)

\[ \chi = \text{Yosida behavior of spin susceptibility} \]

Spin singlet pairing

\[ \chi \uparrow \]

Pair breaking by spin polarization

Spin triplet pairing

\[ \chi \downarrow \]

No pair breaking for equal-spin pairing

\[ \chi = \text{const. for } \bar{d}(\mathbf{k}) \cdot \mathbf{H} = 0 \]
Spin susceptibility & Rashba spin-orbit coupling

Spin susceptibility of non-centrosymmetric SC

$$\vec{g}_k = \vec{z} \times \vec{k} = \left( \begin{array}{c} k_y \\ -k_x \\ 0 \end{array} \right)$$
Gorkov & Rashba
Frigeri et al., Samokhin et al.

van Vleck type of spin polarization
not pair breaking

as in CePt$_3$Si, CeRhSi$_3$, CeIrSi$_3$

van Vleck type of spin polarization
not pair breaking

"transverse field" interband spin polarization
partially "transverse field" interband spin polarization limited

Spin susceptibility & Rashba spin-orbit coupling

Spin susceptibility of non-centrosymmetric SC

$$\chi / \chi_p$$

as in CePt$_3$Si, CeRhSi$_3$, CeIrSi$_3$

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partially "transverse field" interband spin polarization limited

"transverse field" interband spin polarization
partially "transverse field" interband spin polarization limited

"irrespective of pairing symmetry"
Paramagnetic limiting

Spin susceptibility of non-centrosymmetric SC

\[ \chi_{\mu\nu}(T = 0) \approx \chi_p \left\{ \delta_{\mu\nu} - \frac{g_\mu^2 g_\nu^2}{|\mathbf{g}|^2} \right\} \]

Gorkov & Rashba, Frigeri et al., Samokhin et al.

“Irrespective of pairing symmetry”

Paramagnetic limiting field

\[ H_p = \frac{H_c(0)}{\sqrt{4\pi(\chi_p - \chi(0))}} \]

\[ \mathbf{H} \perp \hat{z} \quad \text{paramagnetic limiting} \]
\[ \mathbf{H} \parallel \hat{z} \quad \text{no paramagn. limiting} \]

Upper critical field and paramagnetic limiting

Comparison of different heavy Fermion superconductors

Paramagnetic limit

(BCS weak coupling)

\[ H_P = 1.85k_B T_c / g\mu_B \]

Non-centrosymmetric superconductors

\[ \text{CeIrSi}_3 \quad \text{CeRhSi}_3 \quad \text{CeCoGe}_3 \]

Highest \( H_{c2} \) among heavy Fermion materials
CeIrSi₃

Upper critical field and paramagnetic limiting

29Si-Knight shift

fits very well to theoretical expectations of paramagnetic limiting

Mukuda et al (2010)

Phase diagram - QPT

quantum critical fluctuations

unusual T-dependence of $H_{c2}$ for $\vec{H} \parallel \hat{z}$

Tada, Fujimoto and Kawakami (2009)

Onuki et al.

Upper critical field and paramagnetic limiting

Comparison with the non-heavy Fermion variant

100 x smaller $H_{c2}$

orbital depairing relevant light electrons $\xi$

LalIrSi₃

Upper critical field and paramagnetic limiting

Onuki et al.

Upper critical field and paramagnetic limiting

$H_{c2}(T = 0) \parallel \hat{z}$

Onuki et al.

2.6 GPa

quantum critical fluctuations

unusual T-dependence of $H_{c2}$ for $\vec{H} \parallel \hat{z}$

Tada, Fujimoto and Kawakami (2009)

Upper critical field and paramagnetic limiting

Upper critical field and paramagnetic limiting

Comparison with the non-heavy Fermion variant

100 x smaller $H_{c2}$

orbital depairing relevant light electrons $\xi$

LalIrSi₃

Upper critical field and paramagnetic limiting

Onuki et al.
Conclusion & Remarks

inversion key symmetry for Cooper pairing
without inversion symmetry → mixed-parity pairing

dominant unconventional component
rich phenomena and complex phenomenology

Intriguing novel features:

magnetolectric phenomena connection to spintronics and multiferroics Edelstein, Mineev, Samokhin, Eschrig, Aoyama, …

Josephson effect phase sensitive probes Hayashi, Linder, Subdo, Borkje, Klam, …

Coexistence of magnetism and superconductivity at quantum critical points Yanase, Fujimoto, …

Order parameter symmetry of unconventional superconductors

Identified order parameters

• High-$T_c$ superconductors
  \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4, \text{YBa}_2\text{Cu}_3\text{O}_7, \ldots \)

• Ruthenate \( \text{Sr}_2\text{RuO}_4 \)
  spin singlet, \( d \)-wave pairing
  \( \psi(\mathbf{k}) = k_x^2 - k_y^2 \) 1D rep.

  \( \mathbf{d}(\mathbf{k}) = \hat{z}(k_x \pm ik_y) \) 2D rep.

Conclusions and final remarks

Superconductivity in strongly correlated electron systems likely unconventional
strong Coulomb repulsion favors angular momentum \( l > 0 \)
exotic pairing mechanisms in particular close to quantum critical points

Unconventional order parameters give rise to new phenomena
  quasi-particle properties, tunneling and Josephson effect
mixed phase, vortex matter, flux dynamics
superconducting multi-phase diagrams
magnetism and connection to competing phases
disorder effects

higher dimensional order parameters (\( \text{Sr}_2\text{RuO}_4, \text{U,Th} \text{Be}_13 \), \( \text{UPt}_3 \), …) are more interesting than one-dimensional ones (high-$T_c$ superconductors, …)

Many chapters on unconventional superconductivity are still unwritten and new materials are discovered at an accelerating pace (sample purity is mandatory ! ?)