Symmetry aspects of Unconventional Superconductivity

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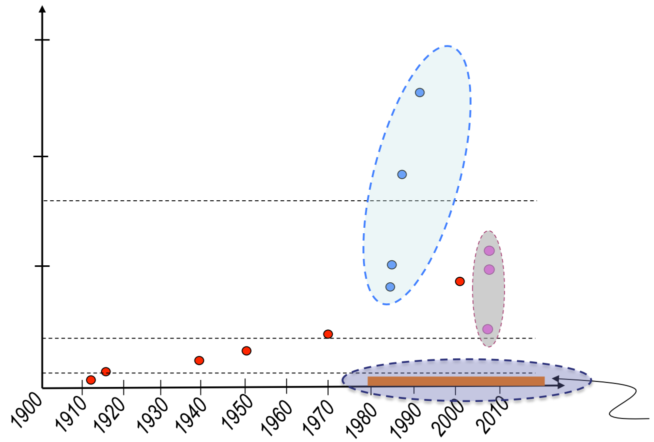
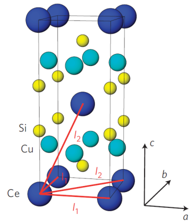
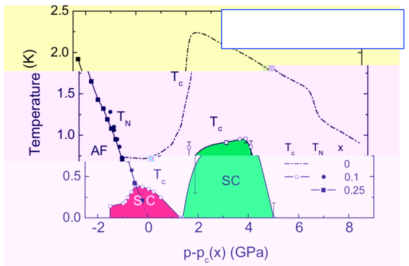
T ic : 1. C e i a e c e i a ec d ci i

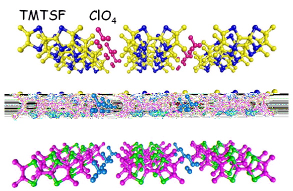
2. Ge e a i ed The f c e i a ec d ci i

3. Ca e : S $_2$R O$_4$
-ce e ic ec d c

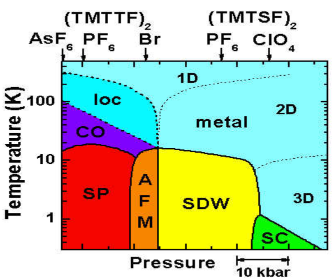




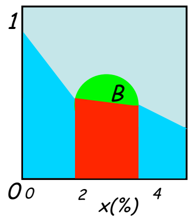




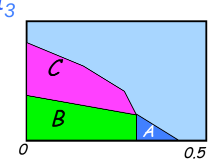
$(TMTSF)_2M$
($M=PF_6$, SbF_6, ReO_4,…)
$(BEDT\text{-}TTF)_2M$
…..

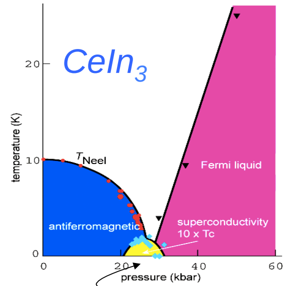


$U_{1-x}Th_xBe_{13}$



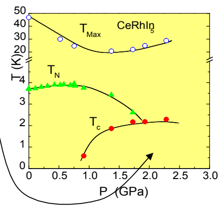
UPt_3

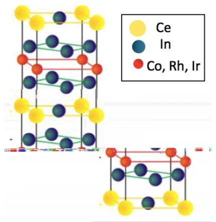


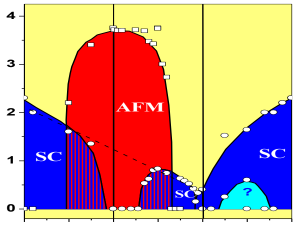


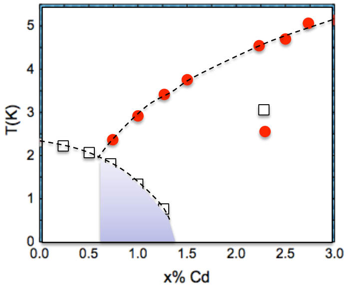
↔

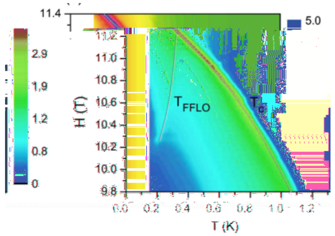
$CeRhIn_5$

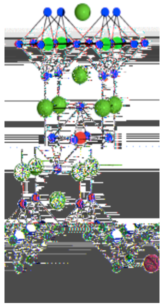


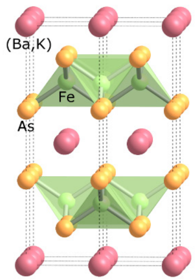

Ce
In
Co, Rh, Ir


AFM
SC
SC
SC
?

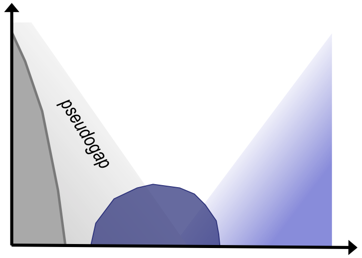

T(K)
x% Cd

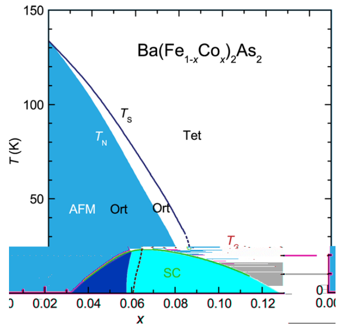

H (T)
T (K)
T_{FFLO}
5.0



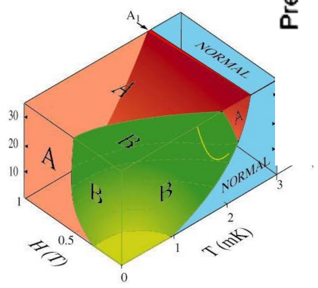


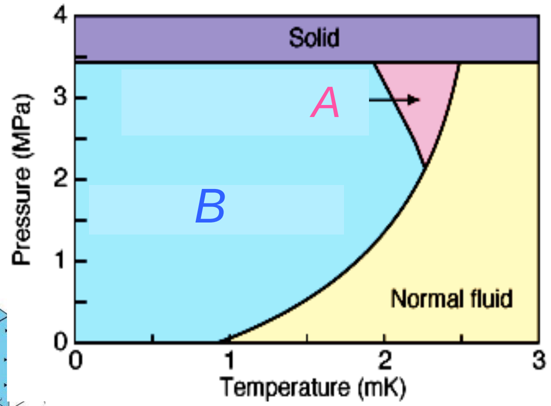
δ





δ



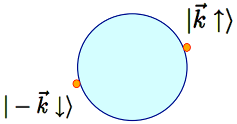


Ba dee -C e -Sch ieffe

Mic c ic he f e c d ci i

BCS c he e a e

B **C** **S**



$$\left.\begin{matrix}|\vec k\uparrow\rangle\\ |-\vec k\downarrow\rangle\end{matrix}\right\} \Rightarrow \quad \{$$

$$|\Psi_{BCS}\rangle = \prod_{\vec k}\left\{u_{\vec k} + v_{\vec k} c^\dagger_{\vec k\uparrow} c^\dagger_{-\vec k\downarrow}\right\}|0\rangle$$

$$\Psi_{\vec k} = \langle\Psi_{BCS}|c_{-\vec k\downarrow}c_{\vec k\uparrow}|\Psi_{BCS}\rangle = u_{\vec k}v_{\vec k}$$

$$c_{\vec k s} \rightarrow c_{\vec k s}e^{ie\chi/\hbar c} \quad\Rightarrow\quad \Psi_{\vec k} \rightarrow \Psi_{\vec k}e^{i2e\chi/\hbar c}$$

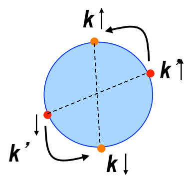
$$\vec A \rightarrow \vec A + \vec\nabla\chi$$

BCS ea fie d he ?

i e de: $\mathcal{H} = \underbrace{\sum_{\vec k,s}\xi_{\vec k}c^\dagger_{\vec k s}c_{\vec k s}}_{\text{ba de e g}} + \underbrace{g\sum_{\vec k,\vec k'}c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow}c_{-\vec k'\downarrow}c_{\vec k'\uparrow}}_{\text{ai i gi e aci}}$

ba d e e g : $\xi_{\vec k} = \epsilon_{\vec k} - \mu = \dfrac{\hbar^2}{2m}(\vec k^2 - k_F^2)$

ai i gi e aci : $U(\vec r - \vec r') = g\delta^{(3)}(\vec r - \vec r')$ a aci e c ac l e aci



$$V(\vec q = \vec k - \vec k') = \int d^3r\, U(\vec r)e^{i\vec q\cdot\vec r} = g = V_{\vec k,\vec k'}$$

c ide ca e i g be ee e - e e ec ai f ie i (i i ge)

BCS ea fie d he ?

i e de: $\mathcal{H} = \sum_{\vec k,s}\xi_{\vec k}c^\dagger_{\vec k s}c_{\vec k s} + g\sum_{\vec k,\vec k'}c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow}c_{-\vec k'\downarrow}c_{\vec k'\uparrow}$?

dec i g fi e aci e b ea f

ea fie d : $\rho_{\vec q} = \sum_{\vec k,s}\langle c^\dagger_{\vec k+\vec q s}c_{\vec k s}\rangle$ a ice de i

$\vec S_{\vec q} = \sum_{\vec k}\sum_{s,s'}\langle c^\dagger_{\vec k s}\vec\sigma_{ss'}c_{\vec k s'}\rangle$ i de i

$b_{\vec k} = \langle c_{-\vec k\downarrow}c_{\vec k\uparrow}\rangle$ BCS - " ff diag a "

BCS ea fie d he

i e de: $\mathcal{H} = \sum_{\vec k,s}\xi_{\vec k}c^\dagger_{\vec k s}c_{\vec k s} + g\sum_{\vec k,\vec k'}c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow}c_{-\vec k'\downarrow}c_{\vec k'\uparrow}$

e ace: $c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow} = b^*_{\vec k} + \{c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow} - b^*_{\vec k}\}$, $c_{-\vec k\downarrow}c_{\vec k\uparrow} = b_{\vec k} + \{c_{-\vec k\downarrow}c_{\vec k\uparrow} - b_{\vec k}\}$

ea fie d Ha i ia :

$$\mathcal{H}_{mf} = \sum_{\vec k,s}\xi_{\vec k}c^\dagger_{\vec k s}c_{\vec k s} + g\sum_{\vec k,\vec k'}\{b^*_{\vec k'}c_{-\vec k\downarrow}c_{\vec k\uparrow} + b_{\vec k'}c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow} - b^*_{\vec k}b_{\vec k'}\}$$

$$= \sum_{\vec k,s}\xi_{\vec k}c^\dagger_{\vec k s}c_{\vec k s} - \sum_{\vec k}\left\{\Delta^* c_{-\vec k\downarrow}c_{\vec k\uparrow} + \Delta c^\dagger_{\vec k\uparrow}c^\dagger_{-\vec k\downarrow} - \Delta^* b_{\vec k}\right\}$$

i h $\Delta^* = -g\sum_{\vec k'}b^*_{\vec k'}$, $\Delta = -g\sum_{\vec k'}b_{\vec k'}$

BCS ea fie d he

$$\mathcal{H}_{mf} = \sum_{\vec k,s}\xi_{\vec k} c^\dagger_{\vec k s} c_{\vec k s} + g\sum_{\vec k,\vec k'}\{b^*_{\vec k'} c_{-\vec k\downarrow} c_{\vec k\uparrow} + b_{\vec k'} c^\dagger_{\vec k\uparrow} c^\dagger_{-\vec k\downarrow} - b^*_{\vec k} b_{\vec k'}\}$$

?

$$= \sum_{\vec k,s}\xi_{\vec k} c^\dagger_{\vec k s} c_{\vec k s} - \sum_{\vec k}\{\Delta^* c_{-\vec k\downarrow} c_{\vec k\uparrow} + \Delta c^\dagger_{\vec k\uparrow} c^\dagger_{-\vec k\downarrow} - \Delta^* b_{\vec k}\}$$

fi d a i a ice a e i ?: $\frac{\partial}{\partial t}\gamma^\dagger_{\vec k} = i[\mathcal{H}_{mf}, \gamma^\dagger_{\vec k}] = E_{\vec k}\gamma^\dagger_{\vec k}$

B g b - a f ai: $c_{\vec k\uparrow} = u^*_{\vec k}\gamma_{\vec k1} + v_{\vec k}\gamma^\dagger_{\vec k2}$, $c^\dagger_{-\vec k\downarrow} = -v^*_{\vec k}\gamma_{\vec k1} + u_{\vec k}\gamma^\dagger_{\vec k2}$, $|u_{\vec k}|^2 + |v_{\vec k}|^2 = 1$

➡ a i a ice e e g $E_{\vec k} = \sqrt{\xi^2_{\vec k} + \Delta^2}$

➡ $\mathcal{H} = \sum_{\vec k}[\xi_{\vec k} - E_{\vec k} + \Delta b_{\vec k}] + \sum_{\vec k} E_{\vec k}(\gamma^\dagger_{\vec k1}\gamma_{\vec k1} + \gamma^\dagger_{\vec k2}\gamma_{\vec k2})$

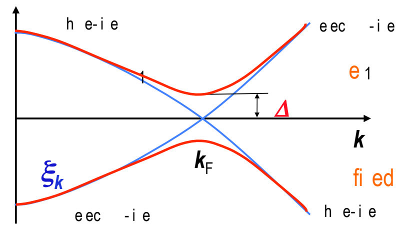
Q a i a ice S ec

1 1 ?

$$\mathcal{H} = \sum_{\vec k}[\xi_{\vec k} - E_{\vec k} + \Delta b_{\vec k}] + \sum_{\vec k} E_{\vec k}(\gamma^\dagger_{\vec k1}\gamma_{\vec k1} + \gamma^\dagger_{\vec k2}\gamma_{\vec k2})$$

$$E_{\vec k} = \sqrt{\xi^2_{\vec k} + \Delta^2}$$

quasiparticle excitation gap: Δ



Q a i a ice S ec

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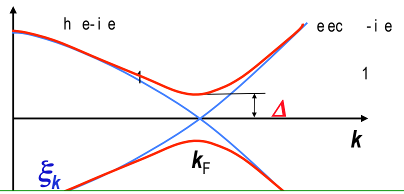
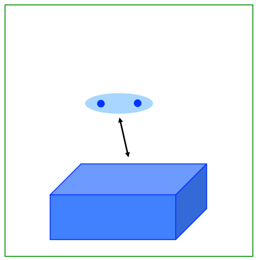
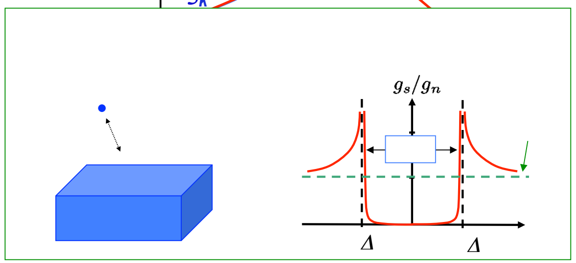
$$\mathcal{H} = \sum_{\vec k}[\xi_{\vec k} - E_{\vec k} + \Delta b_{\vec k}] + \sum_{\vec k} E_{\vec k}(\gamma^\dagger_{\vec k1}\gamma_{\vec k1} + \gamma^\dagger_{\vec k2}\gamma_{\vec k2})$$

$$E_{\vec k} = \sqrt{\xi^2_{\vec k} + \Delta^2}$$

quasiparticle excitation gap: Δ







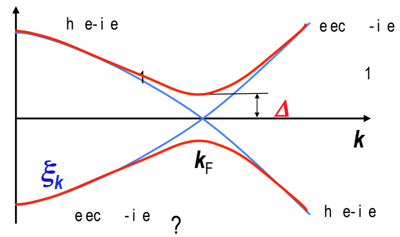
Q a i a ice S ec

1 1

$$\mathcal{H} = \sum_{\vec k}[\xi_{\vec k} - E_{\vec k} + \Delta b_{\vec k}] + \sum_{\vec k} E_{\vec k}(\gamma^\dagger_{\vec k1}\gamma_{\vec k1} + \gamma^\dagger_{\vec k2}\gamma_{\vec k2})$$

$$E_{\vec k} = \sqrt{\xi^2_{\vec k} + \Delta^2}$$

quasiparticle excitation gap: Δ



Se f-c i e cee ai :

$$\Delta = -g\sum_{\vec k} b_{\vec k} = -g\sum_{\vec k} u^*_{\vec k} v_{\vec k}[1 - f(E_{\vec k})]$$

$$= -g\sum_{\vec k}\frac{\Delta}{2E_{\vec k}}\tanh\left(\frac{E_{\vec k}}{k_B T}\right)$$

Fe i di ib i f ci

$$f(E) = \frac{1}{1 + e^{E/k_B T}}$$

i f < 0 a aci e

c i ica e e a e

c i a ii (2 d de) ➡ i ea i ed ga e a i

$T \to T_c \quad \Leftrightarrow \quad \Delta \to 0$?

$$\Delta = -g\Delta \sum_{\vec{k}} \frac{1}{2\xi_{\vec{k}}} \tanh\left(\frac{\xi_{\vec{k}}}{2k_BT}\right)$$

$$= -g \int d\xi \frac{N(\xi)}{2\xi} \tanh\left(\frac{\xi}{2k_BT_c}\right) \qquad 1 = -g \sum_{\vec{k}} \frac{1}{2\xi_{\vec{k}}} \tanh\left(\frac{\xi_{\vec{k}}}{2k_BT}\right)$$

ξ e ec de i f a e ? l e aci i h cha ac e i ic e e g ca e c ff

$$1 = -gN(0) \int_{-\epsilon_c}^{\epsilon_c} \frac{d\xi}{2\xi} \tanh\left(\frac{\xi}{2k_BT_c}\right) = -gN(0) \ln\left(\frac{1.14\epsilon_c}{k_BT_c}\right)$$

c a de i f a e be ee $-\varepsilon$ a d $+\varepsilon$

➡ $$k_BT_c = 1.14\epsilon_c e^{-1/|g|N(0)}$$

Ze - e e a e

Ga a

$$1 = -gN(0) \int_0^{\epsilon_c} \frac{d\xi}{\sqrt{\xi^2 + \Delta^2}} = -gN(0) \sinh^{-1}\frac{\epsilon_c}{\Delta}$$

?

➡ $$\Delta \approx 2\epsilon_c e^{-1/|g|N(0)} = 1.764k_BT_c$$

C de a i e e g a? e e g gai e a i e a ae

1

$$E_{cond} = \sum_{\vec{k}} [\xi_{\vec{k}} - E_{\vec{k}} + \Delta b_{\vec{k}}] = -\frac{1}{2}N(0)|\Delta|^2 \quad + \quad a$$

de e d de i f a e a he Fe i face a d he ga ag i de

ea -c i g a ach

Ze - e e a e

1

Ga a

$$1 = -gN(0) \int_0^{\epsilon_c} \frac{d\xi}{\sqrt{\xi^2 + \Delta^2}} = -gN(0) \sinh^{-1}\frac{\epsilon_c}{\Delta}$$

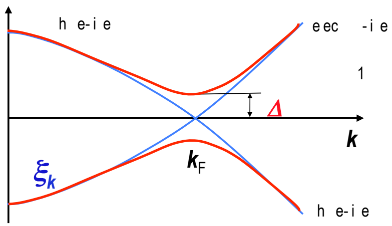
?

➡ $$\Delta \approx 2\epsilon_c e^{-1/|g|N(0)} = 1.764k_BT_c$$ ea c i g

C de a i e e g a e e g gai e a i e a ae

$$E_{cond} = -\frac{1}{2}N(0)|\Delta|^2$$

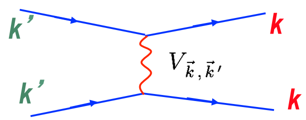
difica i f he a i a ice ec

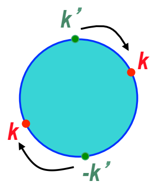


Pai i gi e aci

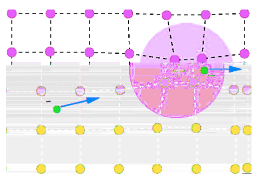
C e ai f ai (b d a e f2 e ec) eed a aci e i e aci

$$\mathcal{H}_{pair} = \frac{1}{2\Omega} \sum_{\vec{k},\vec{k}'} \sum_{s,s'} V_{\vec{k},\vec{k}'} c^\dagger_{\vec{k},s} c^\dagger_{-\vec{k},s'} c_{-\vec{k}',s'} c_{\vec{k}',s}$$





e ec h i e aci : e ec a i e hei e i e e a i ed C bi e aci



$$V_{\vec{k},\vec{k}'} = \frac{4\pi e^2}{q^2} \quad ➡ \quad V_{\vec{k},\vec{k}'} = \frac{4\pi e^2}{q^2 \varepsilon(\vec{q},\omega)}$$

e ec - h e C bi e aci

P ai ai effec : $$\frac{1}{\varepsilon(\vec{q},\omega)} \approx \frac{q^2}{q^2+k_{TF}^2} + \frac{q^2}{q^2+k_{TF}^2}\frac{\omega_{\vec{q}}^2}{\omega^2-\omega_{\vec{q}}^2}$$

i h $k_{TF}^2 = \frac{6\pi e^2 n_e}{\epsilon_F}$ Th a -Fe i cee i g e gh $\lambda_{TF} = k_{TF}^{-1} \sim 5-10\text{Å}$

$$V_{\vec{k},\vec{k}'} = \frac{4\pi e^2}{q^2\varepsilon(\vec{q},\omega)} = \underbrace{\frac{4\pi e^2}{q^2+k_{TF}^2}}_{\text{e .C b}} + \underbrace{\frac{4\pi e^2}{q^2+k_{TF}^2}\frac{\omega_q^2}{\omega^2-\omega_q^2}}_{\text{e ec - h}}$$ $\vec{q} = \vec{k} - \vec{k}'$

$\omega_q = sq$

Re a da i effec :

?

ea -c i g egi e $\mu^{\sqcup} < \lambda \ \square \ 1$ g-c i ? g egi e $\lambda > 1$

$$k_B T_c = 1.14 \epsilon_D \exp\left(-\frac{1}{\lambda - \mu^*}\right)$$

$$k_B T_c = 0.7 \epsilon_D \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right)$$

E ia hbe g, McMi a (68)

e ai ed C b e i

$$\mu^* = \frac{\mu}{1 + \mu \ln(W/\epsilon_D)}$$

I a : $\overline{\varepsilon}$ —

Me a ic g c e a ed e ec ? e

a e e g ? ca e :
a ba d id h :

g effec f C b e i

ha d -ca f e ec - h edia ed e c d c i i

When Coulomb repulsion is too strong for electron-phonon induced pairing

A'e a i e a s to supe e c d c i i y

I e ac i & C e ai e

C b a d e ec - h i e ac i e h - a ged (λ) " "

B d C e ai a ef c i :

$$\psi(\vec{r}, s; \vec{r}', s') = f(|\vec{r} - \vec{r}'|)\chi(s, s')$$

i h $f(r \to 0) \neq 0$

e a i e a g a e ***l*=0** i a f "c ac i e ac i "

H a id C b e i ?

highe -a g a e ai i g

***l* > 0** → $f(r \to 0) \propto r^l$

? "c ac i e ac i " effec i e

S e f ai f ide ica e ec : $\Psi_{ss'}(\vec{k}) = \langle \hat{c}_{\vec{k}s} \hat{c}_{-\vec{k}s'} \rangle = \Phi(\vec{k}) \chi(s,s')$ (orbital, spin)

a ef c i a a i e ic [6] de a ice e cha ge

$\vec{k}\, s$, $-\vec{k}\, s'$

$\vec{k} \to -\vec{k}$ $s \leftrightarrow s'$

e e ai : e e i ge dd

dd a i : dd i e e e

Key symmetries - Anderson's Theorems (1959,1984)

Cooper pairs with total momentum P_{tot}=0 form from degenerate quasiparticle states. *How to guarantee degenerate partners?*

- Spin singlet pairing: time reversal symmetry

$$|\vec{k}\uparrow\rangle \rightarrow \hat{T}|\vec{k}\uparrow\rangle = |-\vec{k}\downarrow\rangle \quad \Longleftrightarrow \quad \epsilon_{\vec{k}\uparrow} = \epsilon_{-\vec{k}\downarrow}$$

harmful: magnetic impurities, ferromagnetism, Zeemann fields (paramagnetic limiting)

- Spin triplet pairing: time reversal & inversion symmetry

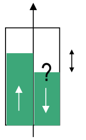
$$|\vec{k}\uparrow\rangle \rightarrow \begin{cases} \hat{I}|\vec{k}\uparrow\rangle = |-\vec{k}\uparrow\rangle \\ \hat{T}|\vec{k}\uparrow\rangle = |-\vec{k}\downarrow\rangle \\ \hat{T}\hat{I}|\vec{k}\uparrow\rangle = |\vec{k}\downarrow\rangle \end{cases} \Longleftrightarrow \quad \epsilon_{\vec{k}\uparrow} \begin{cases} = \epsilon_{-\vec{k}\uparrow} \\ = \epsilon_{-\vec{k}\downarrow} \\ = \epsilon_{\vec{k}\downarrow} \end{cases}$$

crystal structure without inversion center

Pa a ag eic i ii g: 6

Zee a i i g f Fe i face e ceed he ga ag i de

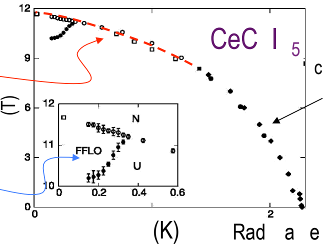
N i ge aii g ibe



1 1 Pa a ag eic e i

1 de a ii

d a ed F de-Fe e- La i -O chi i ha e



e ciica fied

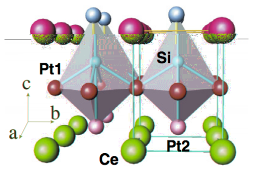
Rad a e a.

A i e ic i - bi c i g:

C a c e ih a i e i ce e

e.g. CeP $_3$Si

i a ef →



Ba e e a.

A e aie echa i f C e aii g

Pai i g f e e i ei eaci : K h & L i ge (1965)

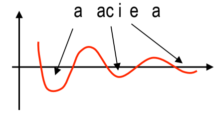
c ee ed C b e ia i ea ha g- a ged ci a ai (ha Fe i edge)

F iede ci a i : $V(r) \propto r^{13}\cos(2k_F r)$

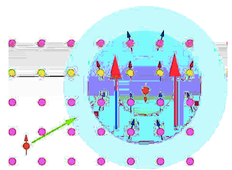
aii gi high-a g a e cha e

$T_c/T_F \sim \exp\{-(2l)^4\}$

e !

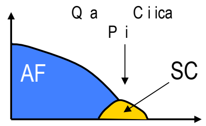


Pai i g b ag eicf c ai : Be & Sch ieffe (1966)



ea i i a i abe edi ge a gedi e aci

ea abe f highe a g a e aii g



S i f c a i e cha ge echa i

E cha ge i e aci :

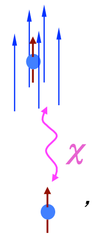
$$\mathcal{H}_{ex} = \int d^3r d^3r' U \delta(\vec{r} - \vec{r}')\rho_\uparrow(\vec{r})\rho_\downarrow(\vec{r}')$$

i -i d ced ca " ag e ic fie d" $\vec{H}(\vec{r},t) = \frac{I}{\mu_B \hbar}\vec{S}(\vec{r},t)$ Ω

i d ced i a i a i :

d a ica i ce ibii

$$\vec{S}(\vec{r}',t') = \mu_B \int d^3r \, dt \, \chi(\vec{r}' - \vec{r}, t' - t)\vec{H}(\vec{r},t)$$



S i de i - i de i i e aci :

$$\mathcal{H}_{sf} = -\frac{I^2}{\hbar^2}\int d^3r \, d^3r' \int dt \, dt' \, \chi(\vec{r} - \vec{r}'; t - t')\, \vec{S}(\vec{r},t) \cdot \vec{S}(\vec{r}',t')$$

6

i ified i f c a i e cha ge de

S i f c a i e cha ge echa i

effeci e aii gi e aci :

$$\mathcal{H}'_{sf} = \sum_{\vec{k},\vec{k}'} \sum_{s_1,s_2,s_3,s_4} V_{\vec{k},\vec{k}';s_1s_2s_3s_4} c^\dagger_{\vec{k},s_1} c^\dagger_{-\vec{k},s_2} c_{-\vec{k}',s_3} c_{\vec{k}',s_4}$$

$$V_{\vec{k},\vec{k}';s_1s_2s_3s_4} = -\frac{I^2}{4} Re\chi(\vec{k} - \vec{k}', \omega = \varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'})\vec{\sigma}_{s_1s_4} \cdot \vec{\sigma}_{s_2s_3}$$

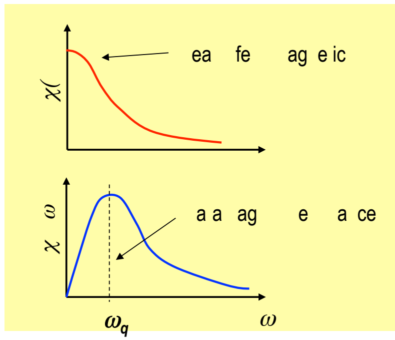
d a ica i ce ibii :

$$\chi(\vec{q},\omega) = \frac{\chi_0(\vec{q},\omega)}{1 - I\chi_0(\vec{q},\omega)} \quad \text{RPA}$$

f i ic e ec ga :

$$\chi_0(\vec{q},\omega) \approx N(0)\left(1 - \frac{\vec{q}^2}{12k_F^2} + i\frac{\pi}{2}\frac{\omega}{v_F|\vec{q}|}\right)$$

ω ε



S i f c ai e cha ge echa i

effeci e aii gi e aci :

$$\mathcal{H}'_{sf} = \sum_{\vec{k},\vec{k}'} \sum_{s_1,s_2,s_3,s_4} V_{\vec{k},\vec{k}';s_1s_2s_3s_4} c^\dagger_{\vec{k},s_1} c^\dagger_{-\vec{k},s_2} c_{-\vec{k}',s_3} c_{\vec{k}',s_4}$$

$$V_{\vec{k},\vec{k}';s_1s_2s_3s_4} = -\frac{I^2}{4} Re\chi(\vec{k}-\vec{k}',\omega=\varepsilon_{\vec{k}}-\varepsilon_{\vec{k}'})\vec{\sigma}_{s_1s_4}\cdot\vec{\sigma}_{s_2s_3}$$

C e i cha e :

$$V^s_{\vec{k},\vec{k}'} = \frac{3I^2}{4} Re\chi(\vec{k}-\vec{k}',\omega=\varepsilon_{\vec{k}}-\varepsilon_{\vec{k}'})$$ **S=0** i i ge

$$V^t_{\vec{k},\vec{k}'} = -\frac{I^2}{4} Re\chi(\vec{k}-\vec{k}',\omega=\varepsilon_{\vec{k}}-\varepsilon_{\vec{k}'})$$ **S=1** i i e

, : e i e : a aci e

S i f c ai e cha ge echa i

Pai i g f i i e

a g a c e f ga f ci

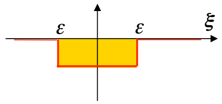
$$\Delta_{\vec{k}} = \Delta g^m_{\vec{k}}$$

$$g^m_{\vec{k}} = \begin{cases} \frac{k_x+ik_y}{\sqrt{2}k_F} & m=+1 \\ \frac{k_z}{k_F} & m=0 \\ \frac{k_x-ik_y}{\sqrt{2}k_F} & m=-1 \end{cases}$$

P ec ed effeci e i e aci :

$$V(\xi,\xi') = -\frac{I^2}{4\Omega}\sum_{,} g \quad \chi(\vec{k}-\vec{k}',\omega=0)g \quad ,\delta(\xi-\xi\ )\delta(\xi'-\xi^?,) \approx \begin{cases} V_1 & |\xi|,|\xi'|<\epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$V_1 = -\frac{I}{12}\frac{IN(0)}{(1-IN(0))^2}$$



$$k_B T_c = \hbar\epsilon_c e^{-1/\lambda_s} \qquad \lambda$$

cha ac e i ice e g : a a ag ec

$$\epsilon_c = \frac{8}{\pi IN(0)}(1-IN(0))E_F$$

S i f c ai e cha ge echa i

S e i abii c ie i :

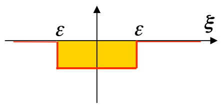
Q a ha e a ii

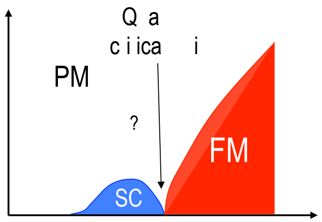
$\to \infty$

$\varepsilon_c \to 0$

$\xi \to \infty$ FM c e a i e g h

$$V_1 = -\frac{I}{12}\frac{IN(0)}{(1-IN(0))^2}$$





? 6
e de aied a a i : M h & L a ich (1999-)

$$k_B T_c = 1.14\epsilon_c e^{-1/\lambda_s} \qquad \lambda$$

cha ac e i ice e g : a a ag ec

$$\epsilon_c = \frac{8}{\pi IN(0)}(1-IN(0))E_F$$

Generalize BCS theory
New aspects

Ge e a i ed f a i f he BCS ea fie d he

BCS Ha i ia :

$$\mathcal{H} = \sum_{\vec{k},s} \xi_{\vec{k}} c^\dagger_{\vec{k}s} c_{\vec{k}s} + \frac{1}{2} \sum_{\vec{k},\vec{k}'} \sum_{s_1,s_2,s_3,s_4} V_{\vec{k},\vec{k}';s_1s_2s_3s_4} c^\dagger_{\vec{k}s_1} c^\dagger_{-\vec{k}s_2} c_{-\vec{k}'s_3} c_{\vec{k}'s_4}$$

Mea fie d Ha i ia :

$$\mathcal{H}_{mf} = \sum_{\vec{k},s} \xi_{\vec{k}} c^\dagger_{\vec{k}s} c_{\vec{k}s} - \frac{1}{2} \sum_{\vec{k},s_1,s_2} \left[\Delta_{\vec{k},s_1s_2} c^\dagger_{\vec{k}s_1} c^\dagger_{-\vec{k}s_2} + \Delta^*_{\vec{k},s_1s_2} c_{\vec{k}s_1} c_{-\vec{k}s_2} \right]$$

$$-\frac{1}{2} \sum_{\vec{k},\vec{k}'} \sum_{s_1,s_2,s_3,s_4} V_{\vec{k},\vec{k}';s_1s_2s_3s_4} \langle c^\dagger_{\vec{k}s_1} c^\dagger_{-\vec{k}s_2} \rangle \langle c_{-\vec{k}'s_3} c_{\vec{k}'s_4} \rangle$$

Se f-c i e ce e ai :

$$\Delta_{\vec{k},ss'} = -\sum_{\vec{k}',s_3s_4} V_{\vec{k},\vec{k}';ss's_3s_4} \langle c_{\vec{k}'s_3} c_{-\vec{k}'s_4} \rangle$$

$$\Delta^*_{\vec{k},ss'} = -\sum_{\vec{k}'s_1s_2} V_{\vec{k}',\vec{k};s_1s_2s's} \langle c^\dagger_{\vec{k}'s_1} c^\dagger_{-\vec{k}'s_2} \rangle$$

ga : 2 2- a i

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} \Delta_{\vec{k}\uparrow\uparrow} & \Delta_{\vec{k}\uparrow\downarrow} \\ \Delta_{\vec{k}\downarrow\uparrow} & \Delta_{\vec{k}\downarrow\downarrow} \end{pmatrix}$$

Se f-c i e ga e ai

B g b a f ai ⟶ Q a i a ice ec

$$E_{\vec{k}} = \sqrt{\xi^2_{\vec{k}} + |\Delta_{\vec{k}}|^2} \qquad |\Delta_{\vec{k}}|^2 = \frac{1}{2} \mathrm{tr}\left(\hat{\Delta}^\dagger_{\vec{k}} \hat{\Delta}_{\vec{k}} \right)$$

? ? ? ? ?

N e: a i a ice ga i **k**-de e de $\hat{\Delta}_{\vec{k}} = \begin{pmatrix} \Delta_{\vec{k}\uparrow\uparrow} & \Delta_{\vec{k}\uparrow\downarrow} \\ \Delta_{\vec{k}\downarrow\uparrow} & \Delta_{\vec{k}\downarrow\downarrow} \end{pmatrix}$

A i :

$$\hat{\Delta}^\dagger_{\vec{k}} \hat{\Delta}_{\vec{k}} = |\Delta_{\vec{k}}|^2 \hat{\sigma}_0$$

i dege e ac : SU(2) e
i e e e a&i e i e

Se f-c i e cee ai :

$$\Delta_{\vec{k},s_1s_2} = -\sum_{\vec{k}',s_3s_4} V_{\vec{k},\vec{k}';s_1s_2s_3s_4} \frac{\Delta_{\vec{k}',s_4s_3}}{2E_{\vec{k}}} \tanh\left(\frac{E_{\vec{k}}}{2k_BT} \right)$$

S c e f he ga f c i

Ga f ci : 2 2 a i i ace

$$\Delta_{\vec{k},ss'} = -\sum_{\vec{k}',s_3s_4} V_{\vec{k},\vec{k}';ss's_3s_4} \langle c_{\vec{k}'s_3} c_{-\vec{k}'s_4} \rangle$$

$$\Delta^*_{\vec{k},ss'} = -\sum_{\vec{k}'s_1s_2} V_{\vec{k}',\vec{k};s_1s_2s's} \langle c^\dagger_{\vec{k}'s_1} c^\dagger_{-\vec{k}'s_2} \rangle$$

$$\langle c_{-\vec{k}s_1} c_{\vec{k}s_2} \rangle = \phi(\vec{k}) \chi_{s_1s_2}$$

bi a i

$\phi(\vec{k}) = \phi(-\vec{k}) \Leftrightarrow \chi_{s_1s_2} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ e e a i , i i ge

$\phi(\vec{k}) = -\phi(-\vec{k}) \Leftrightarrow \chi_{s_1s_2} = \begin{cases} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |\downarrow\downarrow\rangle \end{cases}$ dd a i , i i e

$$\Delta_{\vec{k},s_1s_2} = -\Delta_{-\vec{k},s_2s_1} = \begin{cases} \Delta_{-\vec{k},s_1s_2} = -\Delta_{\vec{k},s_2s_1} & \text{even} \\ -\Delta_{-\vec{k},s_1s_2} = \Delta_{\vec{k},s_2s_1} & \text{odd} \end{cases}$$

S c e f he ga f c i

Ga f ci : 2 2 a i i ace

$$\Delta_{\vec{k},ss'} = -\sum_{\vec{k}',s_3s_4} V_{\vec{k},\vec{k}';ss's_3s_4} \langle c_{\vec{k}'s_3} c_{-\vec{k}'s_4} \rangle$$

$$\Delta^*_{\vec{k},ss'} = -\sum_{\vec{k}'s_1s_2} V_{\vec{k}',\vec{k};s_1s_2s's} \langle c^\dagger_{\vec{k}'s_1} c^\dagger_{-\vec{k}'s_2} \rangle$$

E e ai i i ge

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} \Delta_{\vec{k},\uparrow\uparrow} & \Delta_{\vec{k},\uparrow\downarrow} \\ \Delta_{\vec{k},\downarrow\uparrow} & \Delta_{\vec{k},\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix} = i\hat{\sigma}_y \psi(\vec{k})$$

e e e ed b caa f ci ψ ψ e e $E_{\vec{k}} = \sqrt{\xi^2_{\vec{k}} + |\psi(\vec{k})|^2}$

Odd a i i e

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} -d_x(\vec{k}) + i d_y(\vec{k}) & d_z(\vec{k}) \\ d_z(\vec{k}) & d_x(\vec{k}) + i d_y(\vec{k}) \end{pmatrix} = i\left(\vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right) \hat{\sigma}_y$$

e e e ed b ec f ci dd $E_{\vec{k}} = \sqrt{\xi^2_{\vec{k}} + |\vec{d}(\vec{k})|^2}$

S c ẽ f he ga f ci

Ga f ci : 2 2 a i i i ace

$$\Delta_{\vec{k},ss'} = -\sum_{\vec{k}',s_3s_4} V_{\vec{k},\vec{k}';ss's_3s_4}\langle c_{\vec{k}'s_3} c_{-\vec{k}'s_4}\rangle$$

$$\Delta^*_{\vec{k},ss'} = -\sum_{\vec{k}'s_1s_2} V_{\vec{k}',\vec{k};s_1s_2s's}\langle c^\dagger_{\vec{k}'s_1} c^\dagger_{-\vec{k}'s_2}\rangle$$

E e ai i i ge

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} \Delta_{\vec{k},\uparrow\uparrow} & \Delta_{\vec{k},\uparrow\downarrow} \\ \Delta_{\vec{k},\downarrow\uparrow} & \Delta_{\vec{k},\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix} = i\hat{\sigma}_y \psi(\vec{k})$$

e e e ed b ca a f ci ψ ψ e e $E_{\vec{k}} = \sqrt{\xi_{\vec{k}}^2 + |\psi(\vec{k})|^2}$

Odd ai i i e

i c fig ai

$$d_x \leftrightarrow |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle$$

$$d_y \leftrightarrow -i|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle$$

$$d_z \leftrightarrow |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

⇔ " $\vec{d} \perp \vec{S}$ "

Ta ii e ea e

Pai i gi e aci :

$$V_{\vec{k},\vec{k}';s_1s_2s_3s_4} = J^0_{\vec{k},\vec{k}'}\hat{\sigma}^0_{s_1s_4}\hat{\sigma}^0_{s_2s_3} + J_{\vec{k},\vec{k}'}\hat{\vec{\sigma}}_{s_1s_4}\cdot\hat{\vec{\sigma}}_{s_2s_3}$$

de i -de i i - i

Se f-c i e cee ai :

e e ai i i ge

$$\psi(\vec{k}) = -\sum_{\vec{k}'} \underbrace{(J^0_{\vec{k},\vec{k}'} - 3J_{\vec{k},\vec{k}'})}_{=v^s_{\vec{k},\vec{k}'}} \frac{\psi(\vec{k}')}{2E_{\vec{k}'}}\tanh\left(\frac{E_{\vec{k}'}}{2k_BT}\right)$$

→

$$-\lambda\psi(\vec{k}) = -N(0)\langle v^s_{\vec{k},\vec{k}'}\psi(\vec{k}')\rangle_{\vec{k}',FS}$$

dd ai i i e

$$\vec{d}(\vec{k}) = -\sum_{\vec{k}'} \underbrace{(J^0_{\vec{k},\vec{k}'} + J_{\vec{k},\vec{k}'})}_{=v^t_{\vec{k},\vec{k}'}} \frac{\vec{d}(\vec{k}')}{2E_{\vec{k}'}}\tanh\left(\frac{E_{\vec{k}'}}{2k_BT}\right)$$

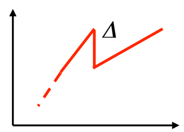
→

$$-\lambda\vec{d}(\vec{k}) = -N(0)\langle v^t_{\vec{k},\vec{k}'}\vec{d}(\vec{k}')\rangle_{\vec{k}',FS}$$

eige a e λ ➡ $k_BT_c = 1.14\epsilon_c e^{-1/\lambda}$

Thermodynamic properties
and
Low energy spectrum

S ecific hea di c i i a



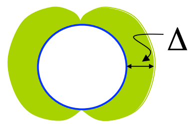
2 d de ha e a ii ⟶ di c i i f ecific hea

E a d ecific hea :

$$S = -\frac{2k_B}{\Omega}\sum_{\vec{k}}\left\{f(E_{\vec{k}})\ln(f(E_{\vec{k}})) + (1-f(E_{\vec{k}}))\ln(1-f(E_{\vec{k}}))\right\} \Rightarrow$$

$$C = T\frac{dS}{dT} = -\frac{2}{\Omega}\sum E_{\vec{k}}\frac{df(E_{\vec{k}})}{dT} = -\frac{2N(0)}{T}\int_{-\infty}^{+\infty} d\xi \left\langle \frac{\partial f(E_{\vec{k}})}{\partial E_{\vec{k}}}E_{\vec{k}}^2 - \frac{T}{2}\frac{\partial|\Delta_m(T)|^2}{\partial T}|g_{\vec{k}}|^2\right\rangle$$

Ga a i : $|\Delta_{\vec{k}}|^2 = \Delta_m^2|g_{\vec{k}}|^2$



a i a ga

S ecific hea di c i i :

" ie a a e"

$$\left.\frac{\Delta C}{C_n}\right|_{T=T_c} = \left.\frac{C-C_n}{C_n}\right|_{T=T_c} = 1.43\frac{\langle|g_{\vec{k}}|^2\rangle^2_{\vec{k},FS}}{\langle|g_{\vec{k}}|^4\rangle_{\vec{k},FS}} \leq 1.43$$

1 ? ?

L -e e a e e ie 6

he d a ic i d i a ed b he e ci ed a i a ice

$$E_{\vec{k}} = \sqrt{\xi_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2} \qquad |\Delta_{\vec{k}}|^2 = \frac{1}{2}\mathrm{tr}\left(\hat{\Delta}_{\vec{k}}^{\dagger}\hat{\Delta}_{\vec{k}}\right) \qquad \Delta_k = \Delta \quad k$$

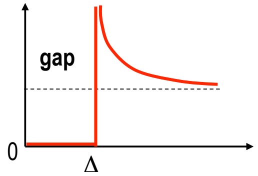
e a i :

$$N(E) = \frac{2}{\Omega}\sum_{\vec{k}} \delta(E_{\vec{k}} - E)$$

$$N(E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\int d\xi\,\delta(\sqrt{\xi^2 + |\Delta_m g_{\vec{k}}|^2} - E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\frac{E}{\sqrt{E^2 - |\Delta_m g_{\vec{k}}|^2}}$$

I ic ga f ci : $\Delta_k = \Delta \;\; = c$.

$$N(E) = N(0)\begin{cases} 0 & |E| < \Delta_m \\ \dfrac{E}{\sqrt{E^2 - |\Delta_m|^2}} & \Delta_m \geq |E|^2 \end{cases}$$



1 ? ?

L -e e a e e ie 6

he d a ic i d i a ed b he e ci ed a i a ice

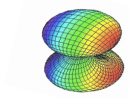
$$E_{\vec{k}} = \sqrt{\xi_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2} \qquad |\Delta_{\vec{k}}|^2 = \frac{1}{2}\mathrm{tr}\left(\hat{\Delta}_{\vec{k}}^{\dagger}\hat{\Delta}_{\vec{k}}\right) \qquad \Delta_k = \Delta \quad k$$

e a i :

$$N(E) = \frac{2}{\Omega}\sum_{\vec{k}} \delta(E_{\vec{k}} - E)$$

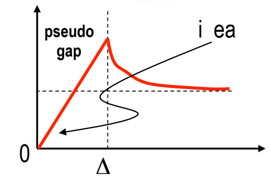
$$N(E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\int d\xi\,\delta(\sqrt{\xi^2 + |\Delta_m g_{\vec{k}}|^2} - E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\frac{E}{\sqrt{E^2 - |\Delta_m g_{\vec{k}}|^2}}$$

A i ic ga f ci : $\Delta_{\vec{k}} = \Delta_m \cos\theta$



i e
de

$$N(E) = N(0)\frac{E}{\Delta_m}\begin{cases} \dfrac{\pi}{2} & |E| < \Delta_m \\ \arcsin\left(\dfrac{\Delta_m}{E}\right) & \Delta_m \geq |E| \end{cases}$$



1 ? ?

L -e e a e e ie 6

he d a ic i d i a ed b he e ci ed a i a ice

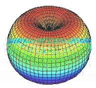
$$E_{\vec{k}} = \sqrt{\xi_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2} \qquad |\Delta_{\vec{k}}|^2 = \frac{1}{2}\mathrm{tr}\left(\hat{\Delta}_{\vec{k}}^{\dagger}\hat{\Delta}_{\vec{k}}\right) \qquad \Delta_k = \Delta \quad k$$

e a i :

$$N(E) = \frac{2}{\Omega}\sum_{\vec{k}} \delta(E_{\vec{k}} - E)$$

$$N(E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\int d\xi\,\delta(\sqrt{\xi^2 + |\Delta_m g_{\vec{k}}|^2} - E) = N(0)\int \frac{d\Omega_{\vec{k}}}{4\pi}\frac{E}{\sqrt{E^2 - |\Delta_m g_{\vec{k}}|^2}}$$

A i ic ga f ci : $\Delta_{\vec{k}} = \Delta_m \sin\theta$



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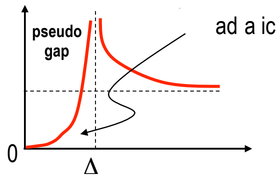
de

$$N(E) = N(0)\frac{E}{\Delta_m}\ln\left|\frac{1 + \frac{E}{\Delta_m}}{1 - \frac{E}{\Delta_m}}\right|$$

f Δ



L -e e a e e ie

1

S ecific hea : e ic ed a i a ice c ib i

$$C(T) = \frac{2}{\Omega}\sum_{\vec{k}} E_{\vec{k}}\frac{df(E_{\vec{k}})}{dT} = \int dE\, N(E)\, E\,\frac{df(E)}{dT} = \int dE\, N(E)\,\frac{E^2}{k_B T^2}\frac{1}{4\cosh^2(E/2k_BT)}$$

- I ic ga f ci : ac i a ed beha i i h a ea ga (e ic d c -i e)

$$C(T) \approx N(0)k_B\left(\frac{\Delta_m}{k_BT}\right)^2\sqrt{2\pi k_BT\Delta_m}\,e^{-\Delta_m/k_BT}$$

- A i ic ga f ci : c ib i f “ bga a e ” ➡ e a

$$C(T) = \int dE\,\underbrace{N(E)}_{\propto E^n}\frac{E^2}{k_BT^2}\frac{1}{4\cosh^2(E/2k_BT)} \propto T^{n+1}\Bigg\{$$

L -e ea e e ie

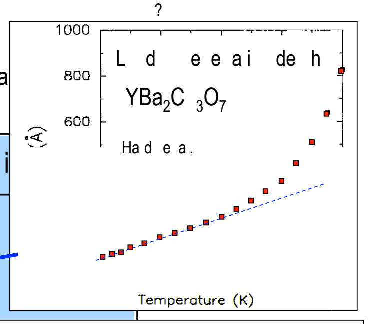
e a i he a iie de e di g ga g

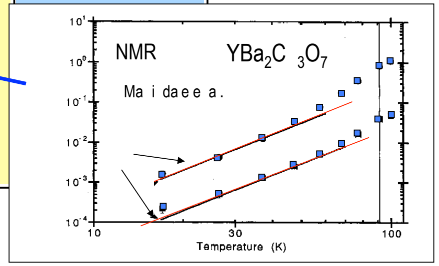
a i	i e de	i de
ecific hea C(T) ? L d e e ai de h λ NMR hea c d ci i κ		

L -e ea e e ie

e a i he a iie de e di g ga

a i	i e de	i
ecific hea C(T) ? L d e e ai de h λ NMR hea c d ci i κ		





high- e ea e e c d c i h i e de i he ga

Other characteristic Properties

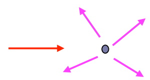
I ? i ca e i g - A de ' he e (1959)

P e ? e a :

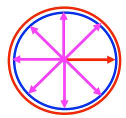
e ec e e defi ed

Di e a :

i i ca e i g (- ag e ic)



e a e agi g e he Fe i face



I e fe e ce effec f C e ai

[illegible]

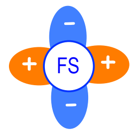
• c e i a aii g: i ic

$\Psi_0 \neq 0$

e a e age ha e A de ' he e f - ag e ici iie

• c e i a aii g: a i ic

$\Psi_0 = 0$



M e a e age de ci ei e fe e ce

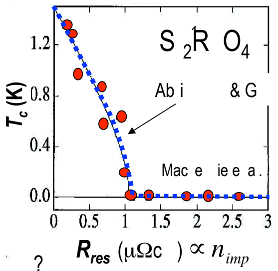
Suppression of superconductivity

?

I i ca e i g - A de ’ he e (1959)

S e i f [1] [1]

ihi cea i g i i c ce a i



?

ea f ee a h: $l = v_F \tau$

ife i e: $\tau \propto 1/n_{imp}$

$\rightarrow \quad k_B T_{c0} \sim \frac{\hbar}{\tau}$

cea a e a e ec d ci g

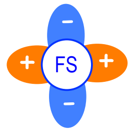
I e fe e ce effec f C e ai

• c e i a ai i g: i ic

$\Psi_0 \neq 0$

e a e age ha e
A de ’ he e
f - ag e ic i ie

• c e i a ai i g: a i ic



$\Psi_0 = 0$

M e a e age
de ci e i e fe e ce

→ **Suppression of superconductivity**

S i ce ibi i

S i i ge ai i g: S i a i a i i ai -b ea i g

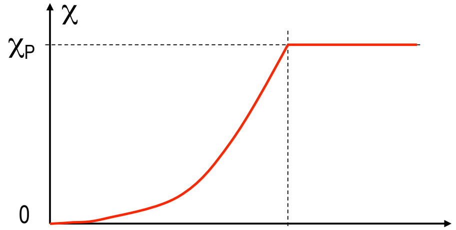
$$\chi(T) = \frac{M(T)}{H} = 2\mu_B^2 N(0) \int \frac{d\Omega_{\vec{k}}}{4\pi} \int \frac{d\xi}{4k_B T \cosh^2(E_{\vec{k}}/2k_B T)}$$

?

$$= \chi_P \int \frac{d\Omega_{\vec{k}}}{4\pi} Y(\hat{k};T) = \chi_P Y(T)$$

Y ida f c i

?



$\chi_P = 2\mu_B N(0)$

Pa i i ce ibi i

e i f i ce ibi i
d e he ga ed a i a ice
ec

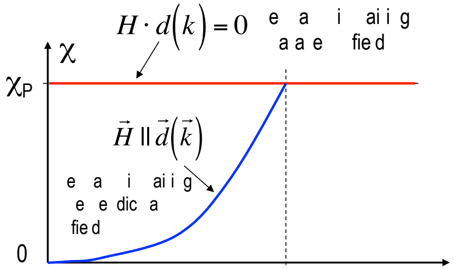
S i ce ibi i

S i i e ai i g: S i a i a i i a a ai -b ea i g

$$\chi_{\mu\nu}(T) = \chi_P \int \frac{d\Omega_{\vec{k}}}{4\pi} \left\{ \delta_{\mu\nu} - Re \frac{d_\mu(\vec{k})^* d_\nu(\vec{k})}{|\vec{d}(\vec{k})|^2} (1 - Y(\hat{k};T)) \right\}$$

?

Y ida f c i



$\chi_P = 2\mu_B N(0)$

Pa i i ce ibi i

Equal spin pairing:
ai i g ih a a e i
i he a e di ec i f
a di ec i f ***k***

S i ce ibi i

S i i e ai i g: S i a i a i i a a ai -b ea i g

$$\chi_{\mu\nu}(T) = \chi_P \int \frac{d\Omega_{\vec{k}}}{4\pi} \left\{ \delta_{\mu\nu} - Re \frac{d_\mu(\vec{k})^* d_\nu(\vec{k})}{|\vec{d}(\vec{k})|^2} (1 - Y(\hat{k};T)) \right\}$$

?

Y ida f c i

Odd a i i i e

i c fig a i

$d_x \leftrightarrow |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle$

$d_y \leftrightarrow -i|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle$

$d_z \leftrightarrow |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

⟺ " $\vec{d} \perp \vec{S}$ "