Shot Noise and the Non-Equilibrium FDT

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Outline

• Shot noise is quantum noise

• Shot noise of a tunnel junction

• Measurements of shot noise – testing the non-eq. FDT

• “Quantum shot noise”
  – measuring the frequency dependence of shot noise

• Experiments on the zero point noise in circuits

• Shot noise and the nonequilibrium FDT (time permitting)
Fundamental Noise Sources

**Johnson-Nyquist Noise**

$$S_I(f) = \frac{4k_B T}{R} \left[ \frac{A^2}{Hz} \right]$$

- Frequency-independent
- Temperature-dependent
- Used for thermometry

**Shot Noise**

$$S_I(f) = 2eI \left[ \frac{A^2}{Hz} \right]$$

- Frequency-independent
- Temperature independent
Shot Noise – “Classically”

Incident “current” of particles

Barrier w/ finite trans. probability

what’s up here?

\[ I \sim qDn \]

Poisson-distributed fluctuations

“white” noise with

\[ S_I = 2qI \]
Shot Noise is Quantum Noise

Einstein, 1909: Energy fluctuations of thermal radiation

\[
\langle (\Delta E)^2 \rangle = \left[ \hbar \omega \rho(\omega) + \frac{\pi^2 c^3}{\omega^2} \rho^2(\omega) \right] V d\omega
\]

particle term = shot noise! wave term
first appearance of wave-particle complementarity?

Can show that “particle term” is a consequence of \([a, a^\dagger] = 1\)

\[
\langle \Delta n^2 \rangle = \langle a^\dagger a a^\dagger a \rangle - \langle a^\dagger a \rangle^2
\]

\[
= \langle a^\dagger (a^\dagger a + 1) a \rangle - \langle a^\dagger a \rangle^2
\]

\[
= \langle a^\dagger a^\dagger a a \rangle + \langle a^\dagger a \rangle - \langle a^\dagger a \rangle^2
\]

\[
\langle \Delta n^2 \rangle = \bar{n}^2 + \bar{n}
\]

\[
\bar{n} = \langle n \rangle = \left( e^{\hbar \omega / kT} - 1 \right)^{-1}
\]

\[
P_n = \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}}
\]

\[
\langle a^\dagger a^\dagger a a \rangle = \sum n(n-1) P_n = 2\bar{n}^2
\]

Noise and Quantum Measurement
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Conduction in Tunnel Junctions

Assume: Tunneling amplitudes and D.O.S. independent of energy
Fermi distribution of electrons

\[ I = I_{L \rightarrow R} - I_{R \rightarrow L} = GV \]

\[ I_{L \rightarrow R} = \frac{G}{e} \int f_L (1 - f_R) dE \]
\[ I_{R \rightarrow L} = \frac{G}{e} \int f_R (1 - f_L) dE \]

Fermi functions

Conductance (G) is constant
Non-Equilibrium Noise of a Tunnel Junction

(Zero-frequency limit)

**Sum** gives noise:

\[ S_I(f) = 2e(I_{L\rightarrow R} + I_{R\rightarrow L}) \]

\[ S_I(f) = 2eI \coth \left( \frac{eV}{2k_B T} \right) \]

\[ I = \frac{V}{R} \]

Non-Equilibrium Fluctuation Dissipation Theorem

Transition Region

\[ eV \sim k_B T \]

\[ T_2 = 2T_1 \]

\[ T = 0 \]

\[ \frac{4k_B T}{R} \]

\[ 2eI \]

\[ \text{Shot Noise} \]

\[ \text{Johnson Noise} \]

\[ S_I(f) = 2eV / R \coth \left( \frac{eV}{2k_B T} \right) \]
Noise Measurement of a Tunnel Junction

Measure symmetrized noise spectrum at $\hbar \omega < kT$
Seeing is Believing

\[ \frac{\delta P}{P} = \frac{1}{\sqrt{B\tau}} \]

High bandwidth measurements of noise

\[ B \sim 10^8 \text{ Hz}, \tau = 1 \text{ second} \quad \Rightarrow \quad \frac{\delta P}{P} = 10^{-4} \]
Test of Nonequilibrium FDT

Agreement over four decades in temperature to 4 digits of precision

Self-Calibration Technique

\[ P(V) = \text{Gain} \left( S_{I}^{\text{Amp}} + S_{I}(V,T) \right) \]
Comparison to Secondary Thermometers

- **Oxford ROX**
- **Lakeshore ROX comparison**
- **RhFe Comparison**
- **$T_{EGT} = T_{secondary}$**

- **Deviation from Oxford ROX**
- **Deviation from Lakeshore ROX**
- **Deviation from RhFe**
- **Deviation from RhFe(/1000)**

Graph showing the comparison of $T_{EGT}$ and $T_{secondary}$ with various deviation markers for different thermometers.
Two-sided Shot Noise Spectrum

(Quantum, non-equilibrium FDT)

\[
S_I(\omega) = \frac{(\hbar \omega + eV)/R}{1 - e^{-(\hbar \omega + eV)/kT}} + \frac{(\hbar \omega - eV)/R}{1 - e^{-(\hbar \omega - eV)/kT}}
\]


Noise and Quantum Measurement
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Finite Frequency Shot Noise

Symmetrized Noise: \[ S_{\text{sym}} = S(+\omega) + S(-\omega) \]

- Shot noise
- Quantum noise

Don’t add powers!
Measurement of Shot Noise Spectrum

Theory

Expt.

Schoelkopf et al., PRL 78, 3370 (1998)
Shot Noise at 10 mK and 450 MHz

\[ \frac{\hbar v}{kT} = 2 \]

L. Spietz, in prep.
With An Ideal Amplifier and $T=0$

$S_I = 2\hbar \nu G$

Quantum noise from source

Quantum noise added by amplifier

$eV = -\hbar \nu$

$eV = \hbar \nu$
Summary – Lecture 1

• Quantizing an oscillator leads to quantum fluctuations present even at zero temperature.

• This noise has built in correlations that make it very different from any type of classical fluctuations, and these cannot be represented by a traditional spectral density- requires a “two-sided” spectral density.

• Quantum systems coupled to a non-classical noise source can distinguish classical and quantum noise, and allow us to measure the full density – next lecture!
Additional material on Johnson noise follows
Nyquist’s Derivation of Johnson’s Noise
Connection Between Johnson Noise and Blackbody Radiation*

*R. H. Dicke, Rev. of Sci. Instrum. 17, 268 (1946)

\[ T_R = T_{bb} \]

Johnson Noise Power \( \equiv \) Blackbody Radiation Power
Connection Between Johnson Noise and Blackbody Radiation in Rayleigh-Jeans Limit

Resistor Temperature \( T_R \)

\[ T_R \gg h\nu/k_B \]

\[ S_I = 4k_B T / R \]

\[ P_{\text{Johnson}} \sim k_B T_R B \]

\[ P_{\text{bb-Rayleigh-Jeans}} \sim k_B T_{bb} B \]
Radiative Cooling of a Resistor?

Total radiated power: \[ P = kTB \]

Total conductance: \[ G_{\text{photon}} = \frac{dP}{dT} = \frac{k^2}{h} T \]

More correct: \[ P = \int_0^\infty h\nu n(\nu) d\nu = \int_0^\infty \frac{h\nu}{e^{h\nu/kT} - 1} d\nu \]

One quantum of thermal conductance per electromagnetic mode

Schmidt, Cleland, and Schoelkopf, PRL 93, 045901 (2004)
Resistor as Ideal Square Law Detector

Single photon heats one resistor to \( T_H = \frac{E}{C_e} \)

If no other thermal conductances, cools entirely by radiation!

Photon number gain is large!:
\[
    n_\gamma \sim \frac{E_\gamma}{kT_H}
\]

Where’s the nonlinearity?

Noise and Quantum Measurement
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