Resonant Atomic Gases  [Rodzihovsky]

- Overview of atomic physics
  - degenerate Bose and Fermi atomic gas ('95)
  - optical lattices (potential from AC-Stark effect)
    - Superfluid (SF) — Mott Insulator (MI) transition
  - Feshbach resonance.
    - The two channels have different spin contents, & coupled by hyperfine interaction
    - The detuning & can be tuned by magnetic field
    - interactions can be weak/strong, and attractive/repulsive
    - paired superfluidity, and BCS-BEC crossover

- Experimental Probes
  - Time-of-flight measurement
    - momentum distribution
    - temperature (fitting tail to non-interacting gas)
  - noise → pair correlation
  - vortices
  - Thermodynamics (e.g. Ω-T curve)
  - Transport
    - Bragg spectroscopy (input two counter-propagating lasers)
    - RF spectroscopy
    - k-resolved photoemission
\[ \mathcal{H} = \hat{\psi}_t^\dagger \left( \frac{\hbar^2}{2m} \right) \hat{\psi}_t + \phi^\dagger \left( \frac{\hbar^2}{2m} \right) \phi - \frac{g}{\sqrt{2}} \phi\phi^\dagger \hat{\psi}_t \hat{\psi}_t + h.c. \]

\[ \mathcal{J}_\text{s} = \frac{-a'' + i\Phi}{\sqrt{2}} \]

Negative detuning

Resonance

Typical experimental range

Physical bound state

Virtual bound state

\[ a \sim -\frac{\hbar \sqrt{\omega}}{B_0} \lesssim \frac{\Delta B}{B_0} \sim -1/g^2 \]

Positive detuning

For many-body at finite density:

\[ \mathcal{H} = \hat{\psi}_t^\dagger \left( \frac{\hbar^2}{2m} - \mu \right) \hat{\psi}_t + \phi^\dagger \left( \frac{\hbar^2}{2m} - 3\mu + \varepsilon_0 \right) \phi - \frac{g}{\sqrt{2}} \phi\phi^\dagger \hat{\psi}_t \hat{\psi}_t + h.c. \]

\[ \mu \approx \frac{\varepsilon_f}{2} \]

\[ \mu = \varepsilon_f \]

\[ \mu \approx \varepsilon_f \]

\[ \mathcal{E}_\text{crossover} \times \mathcal{E}_\text{BS} \]

The dimensionless coupling is:

\[ \gamma \sim \left( \frac{\hbar}{\varepsilon_f} \right)^2 \sim \frac{1}{\mathcal{E}_\text{c}} \]

Narrow resonance when \( \gamma \ll 1 \):

\[ \Rightarrow \text{mean-field valid} \]

\[ \Rightarrow \text{can replace } \phi(\alpha) = B \]

Broad resonance when \( \gamma \gg 1 \):

\[ \Rightarrow \phi, \phi^\dagger \text{ strongly coupled} \]

\[ \Rightarrow \text{mean-field not valid} \]
- **P-wave Feshbach Resonance**

\[ H = \frac{1}{2m} \left( \frac{p^2}{2m} \right) + \phi^4 \left( \frac{p^2}{4m} - \varepsilon_0 \right) + ig \phi \delta \phi \nabla^2 \phi \]

\[ \phi \rightarrow \text{long-lived} \]

\[ f_p = \frac{-u^{-1} + (\alpha/2) q^3 - i q^5}{q^2} \]

\[ \tau \sim E^{-3/2} \]

- For narrow resonance \((\gamma < 1)\), can use mean-field,
  \[ \phi(\alpha) = \bar{B} = \bar{u} + i \bar{v} \]

- In isotropic resonance, in both BEC and BCS limit,
  the order parameter is at \( u = v \). And interaction is always ferromagnetic.

\[ \text{gapped} \quad \text{gapless (node)} \rightarrow \delta \]

- However, rotation invariance in Feshbach resonance turn out to be broken. Thus \( p_x \) may be preferred.

When anisotropy is moderate...