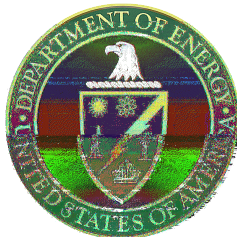


Phenomenology of High T_c Cuprates II

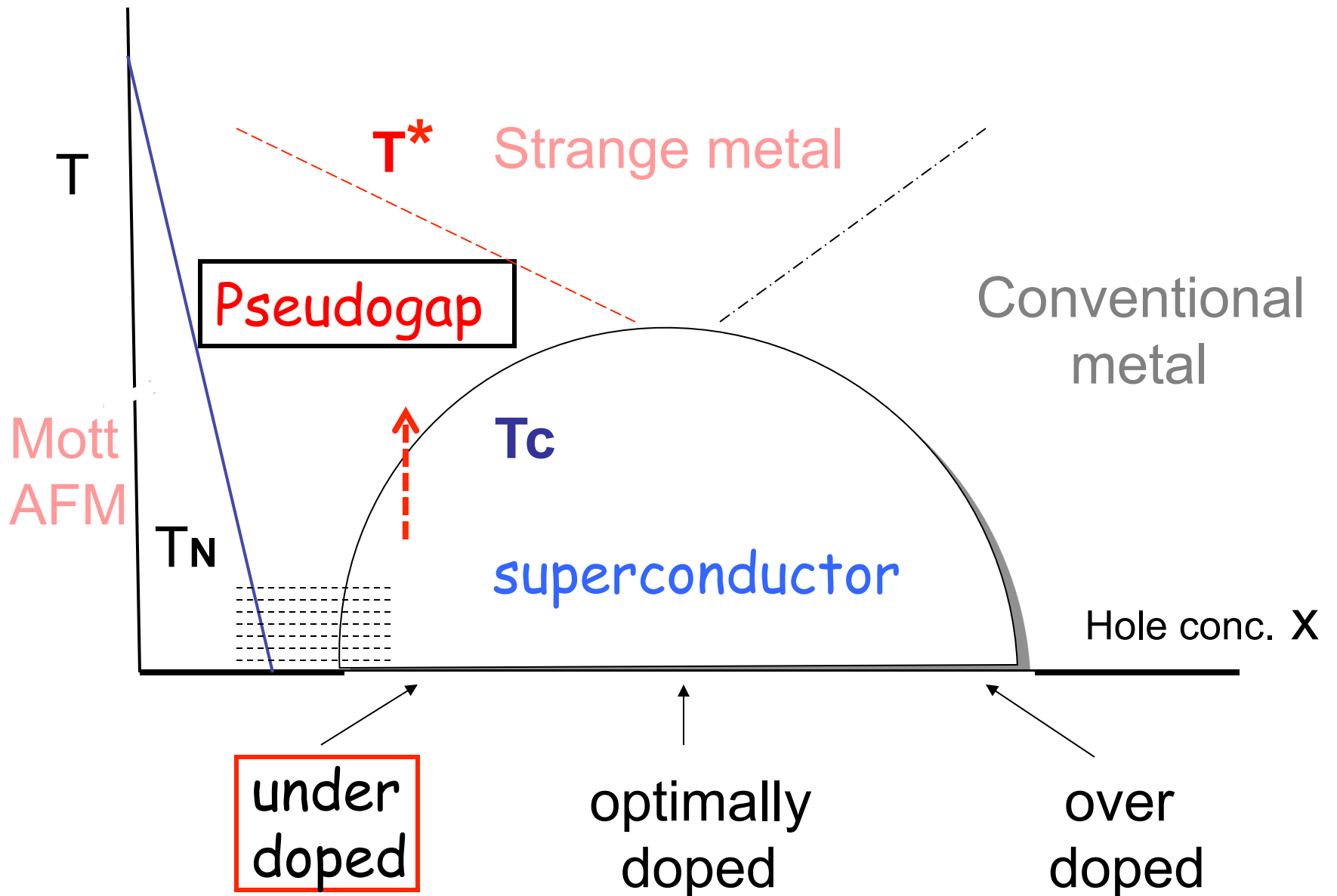
Pseudogap in Underdoped Cuprates

Mohit Randeria
Ohio State University



2014 Boulder School on
"Modern aspects of
Superconductivity"





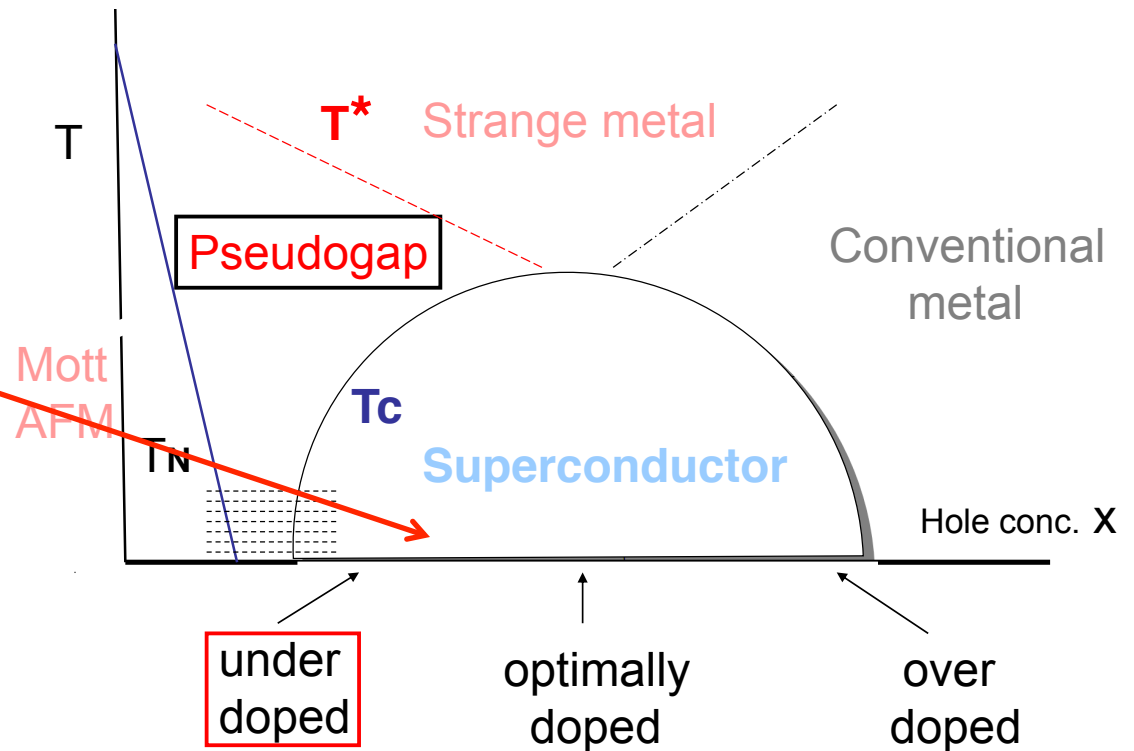
Pseudogap in Underdoped Cuprates

- Underdoped SC state: gap anisotropy
- What determines T_c ? Gap or superfluid density?
- Pseudogap above T_c
 - specific heat, NMR, optics, STM
- ARPES & Fermi arcs
- SC fluctuations above T_c
- Quantum Oscillations
- Competing orders



(Borrowed from P. A. Lee)

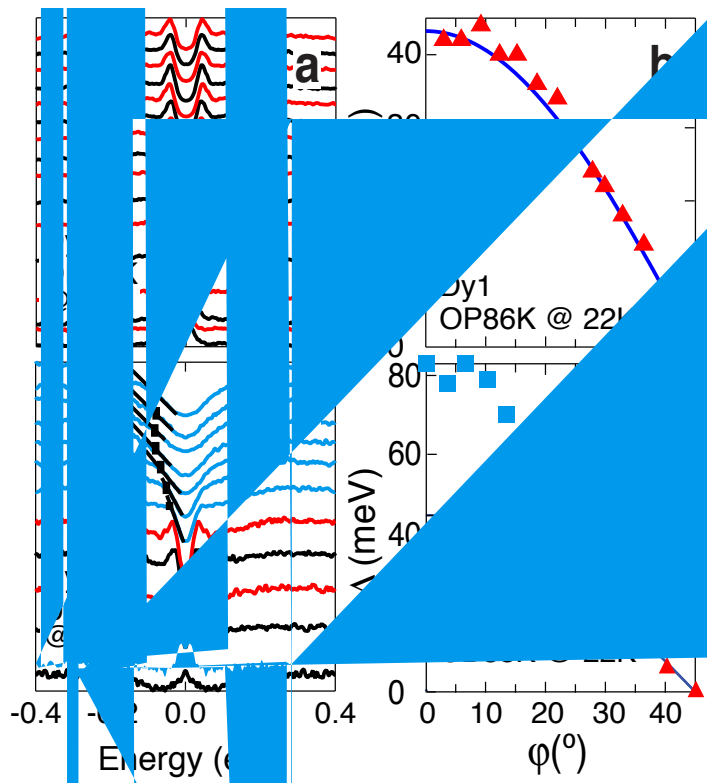
Start with
SC state as a
 function of
 underdoping
 $T < T_c$
 $x < \text{optimal}$



* ARPES gap $\Delta(\mathbf{k}) \rightarrow$ antinodal gap Δ_{\max}
 gap slope at node v_{Δ}

* Transport vs. ARPES v_F, v_{Δ}

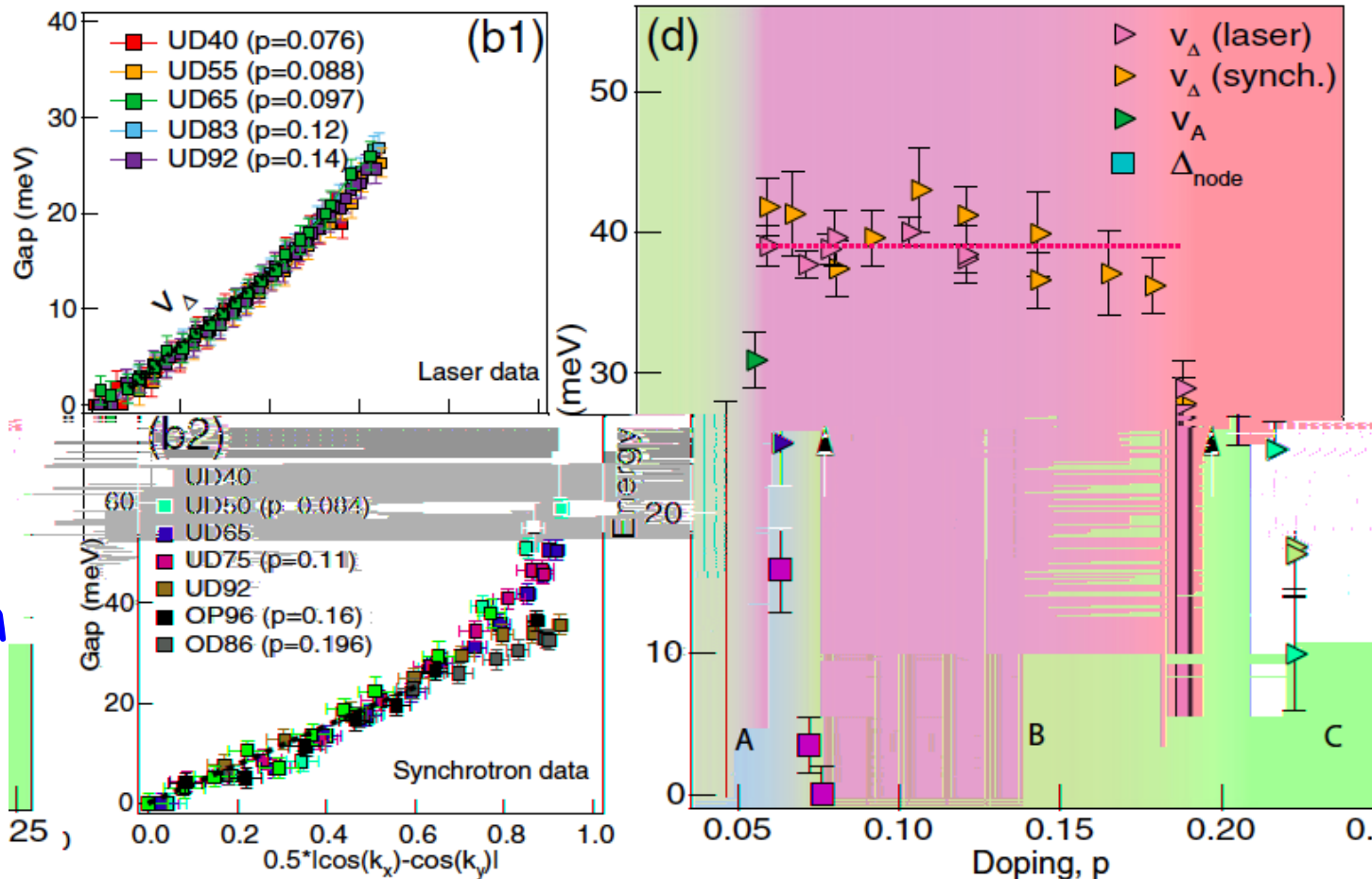
* Superfluid density n_s/m



Gap anisotropy & gap-slope v_{Δ} with underdoping

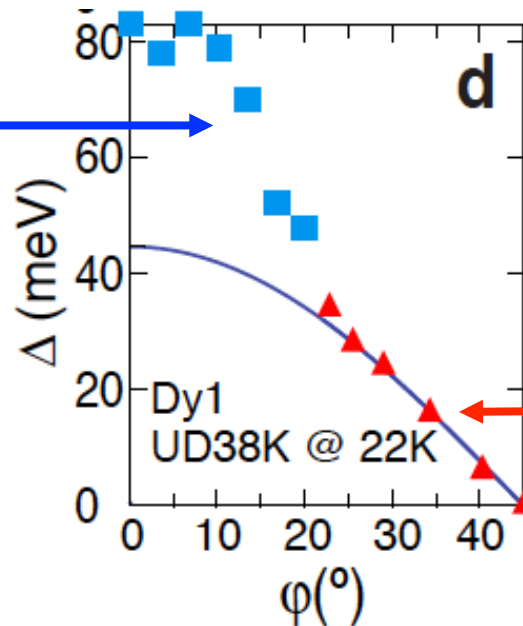
constant
gap-slope
at node
over broad
underdoping
range

Systematic
deviation from
simple d-wave
 $\Delta(k)$ with
underdoping



Terminology commonly used in the literature to describe the energy gap for $T \ll T_c$ (which may be potentially confusing)

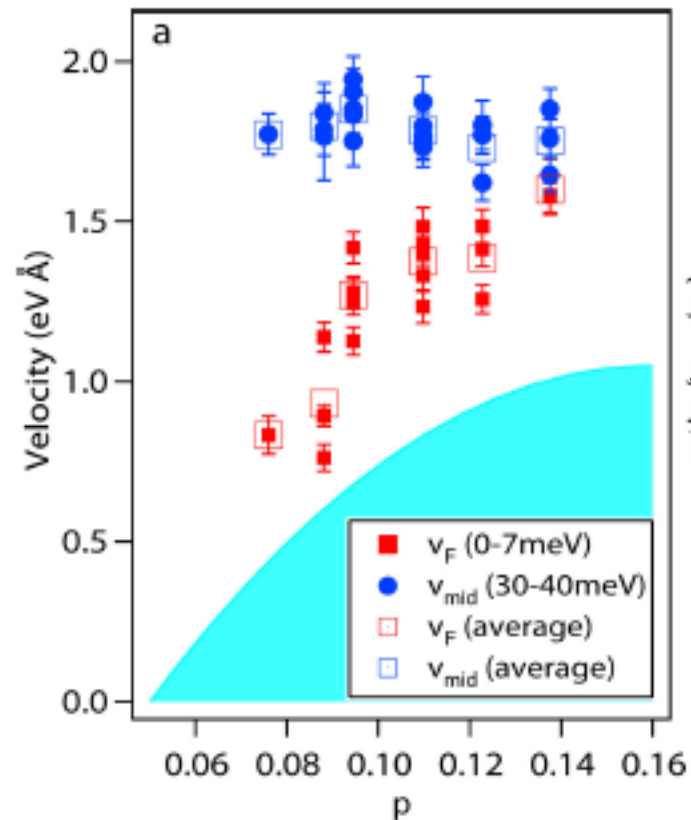
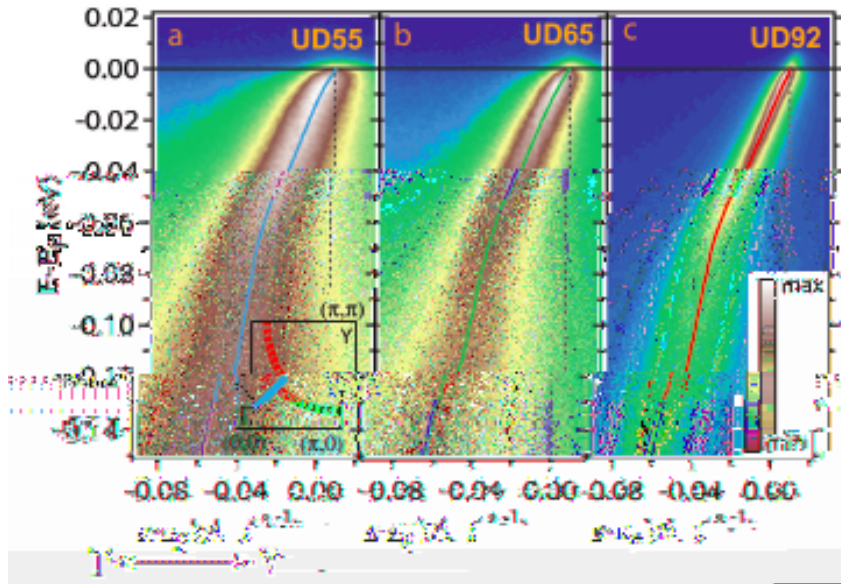
Antinodal gap is called the "Pseudogap" (because it persists above T_c in the Pseudogap Regime)



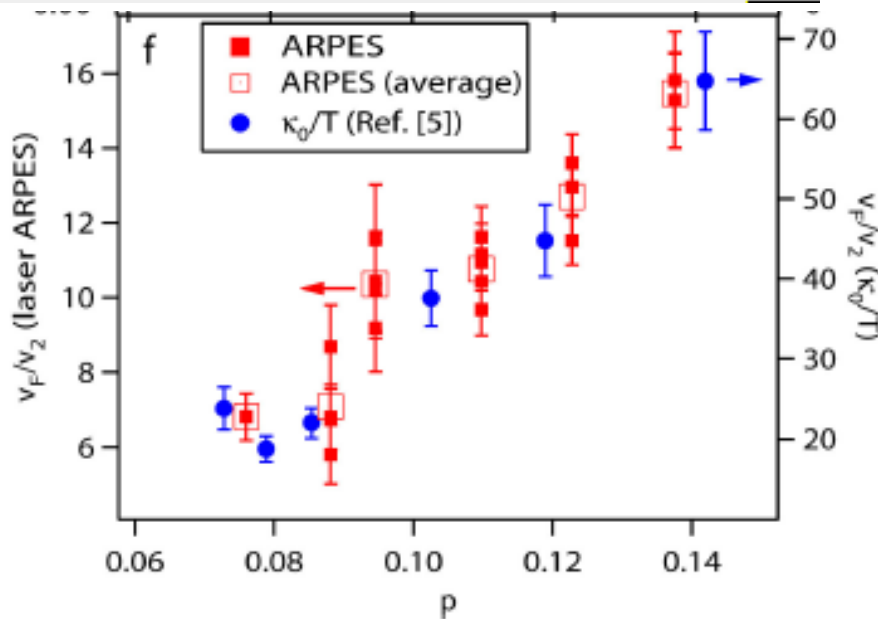
Near-nodal gap is called the "Superconducting Gap"

Is antinodal gap caused by some other Order parameter?

v_F , v_{Δ} from ARPES & thermal transport



“old”
“new”
laser
ARPES



Qualitative
agreement between
ARPES & transport

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- * Antinodal gap Δ_{\max} increases
- * gap slope at node v_{Δ} constant
- * T_c decreases with underdoping

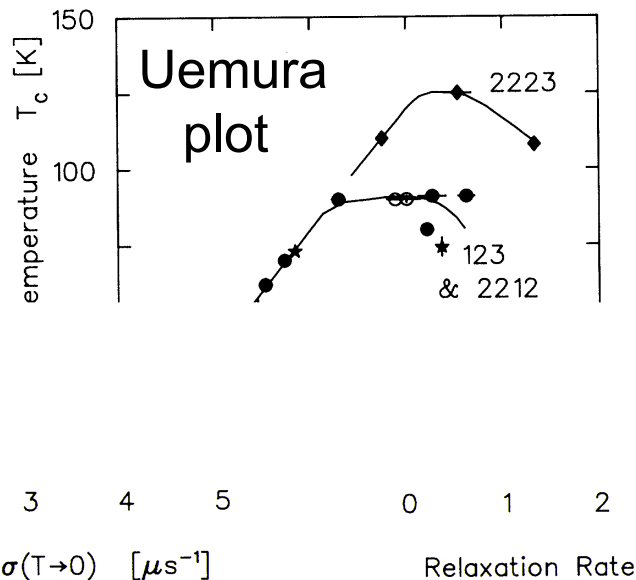
No part of the Energy gap correlates with doping dependent T_c

If not the gap, then what determines T_c ?

cf. BCS $2\Delta/T_c$

Phase fluctuations destroy superconductivity

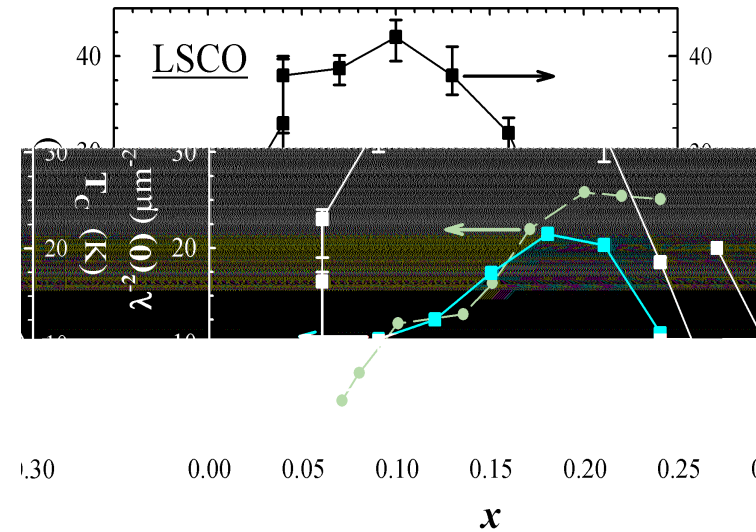
Emery & Kivelson, Nature (1995)



Superfluid stiffness ρ_s

$$T_c \sim \rho_s$$

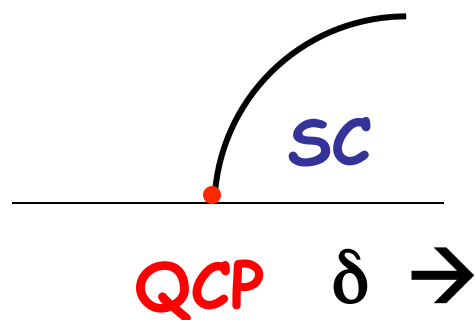
$$s = \frac{\hbar s}{m}$$



Uemura et al, PRL (1989)

Lemberger et al, PRB (2010)

Scaling of T_c and Superfluid Density near a QCP



$$T_c \propto \delta^{z\nu}$$

$$\rho_s \propto \delta^{(z+d-2)}$$

$$T_c \sim \rho_s^{z/(z+d-2)}$$

d = spatial dim.

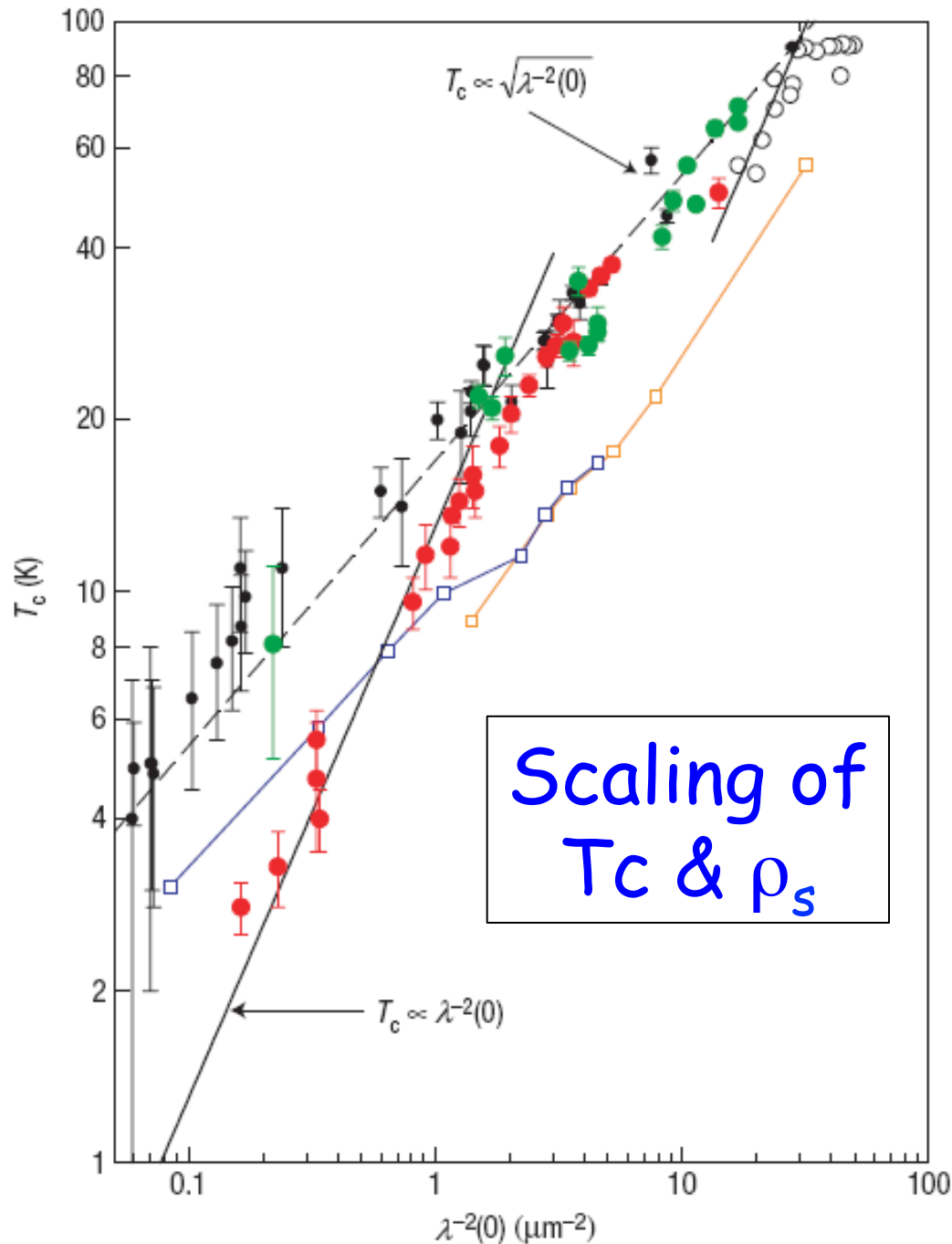
$$d = 3$$

$$T_c \sim \rho_s^{z/(z+1)}$$

$$z \geq 1$$

$$d = 2$$

$$T_c \sim \rho_s$$



Scaling of
 T_c & ρ_s

○ Uemura

□ Bonn & Hardy
□ single x'tals

● Lemberger
● 40 u.c. films

● Lemberger
● 2 u.c. films

$$T_c \sim \sqrt{\rho_s}$$

$$T_c \sim \rho_s$$

Hetel, Lemberger & Randeria,
Nature Phys. **3**, 700 (2007)
Broun et al., PRL (2007)

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What happens above T_c ?

The **pseudogap** for underdoped cuprates

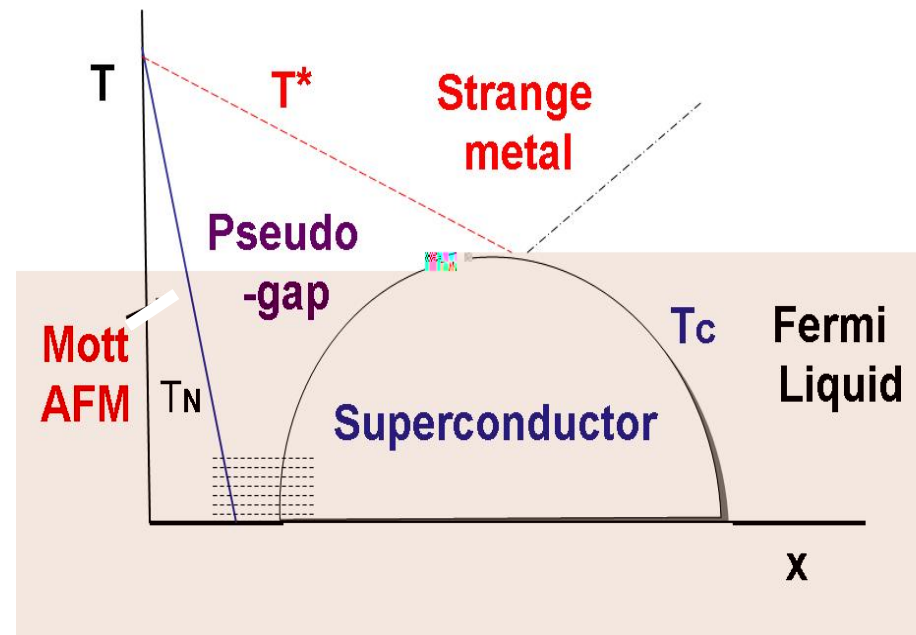


Loss of low-energy spectral weight



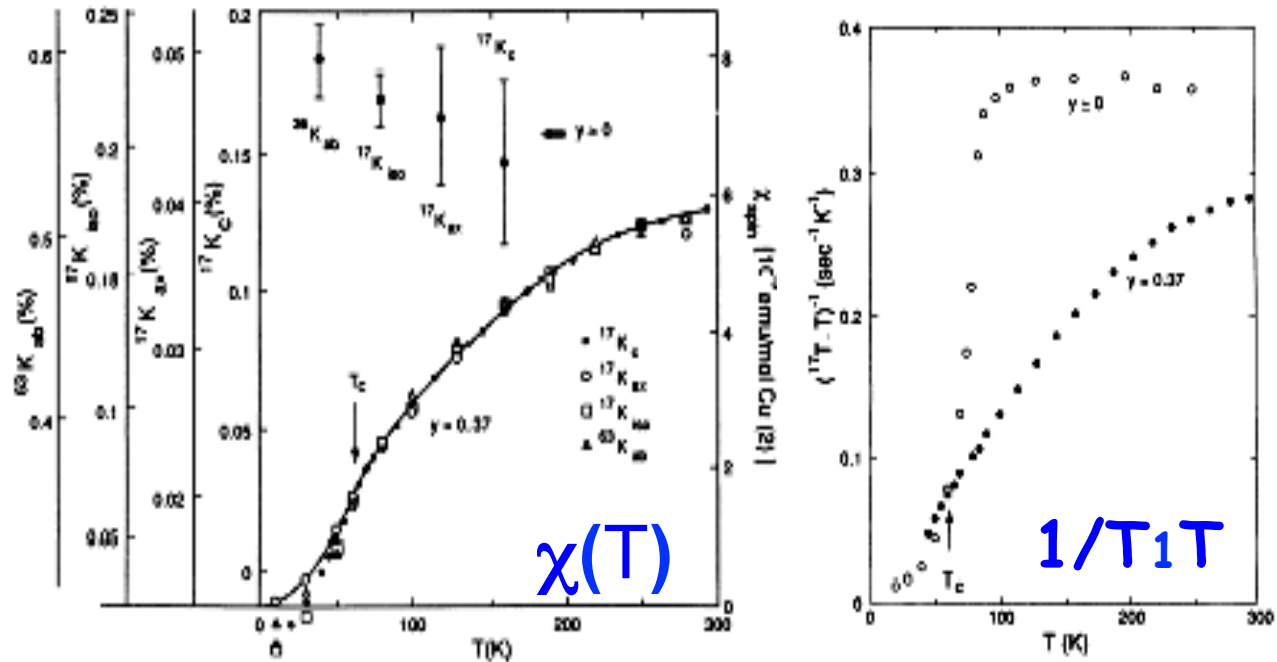
Highly anisotropic Gap persists up to T^* , well above T_c

- * Spin
- * Charge
- * Entropy
- * Single-particle spectral function



Cu and O NMR studies of the magnetic properties of $\text{YBa}_2\text{Cu}_3\text{O}_{6.63}$ ($T_c = 62$ K)

M. Takigawa,* A. P. Reyes,† P. C. Hammel, J. D. Thompson, R. H. Heffner, Z. Fisk, and K. C. Ott



Pseudogap:
Loss of
Spectral Weight
For low energy
Spin excitations
Above T_c

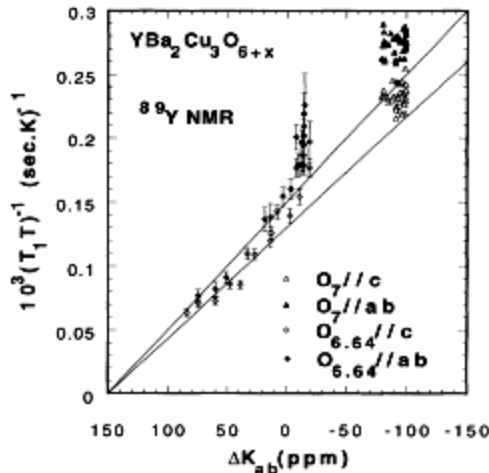


FIG. 4. The data for $(T_1T)^{-1}$ vs ΔK_{ab} for the two samples. Linear relations hold for the data for $T < 300$ K for the $x = 0.64$ sample and extend nearly through the results for $x = 1$. Those plotted here have been forced to extrapolate to zero for $\Delta K_{ab} = \delta_{ab} = 150$ ppm.

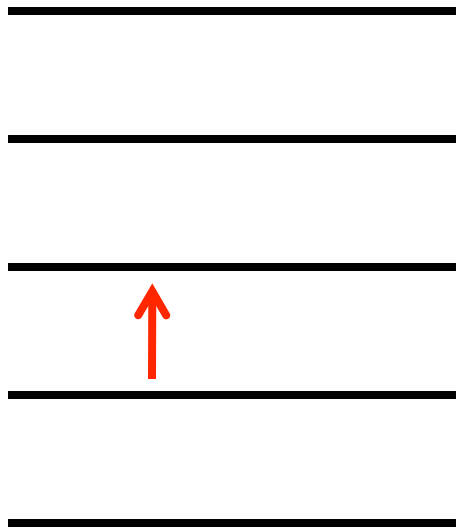
$$1/T_1T = \lim_{\omega \rightarrow 0} \sum_{\mathbf{q}} F(\mathbf{q}) \chi''(\mathbf{q}, \omega) / \omega$$

$$\leftarrow 1/T_1T \sim \chi(T)$$

^{89}Y NMR Study of the Anisotropy of the Static and Dynamic Susceptibilities in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

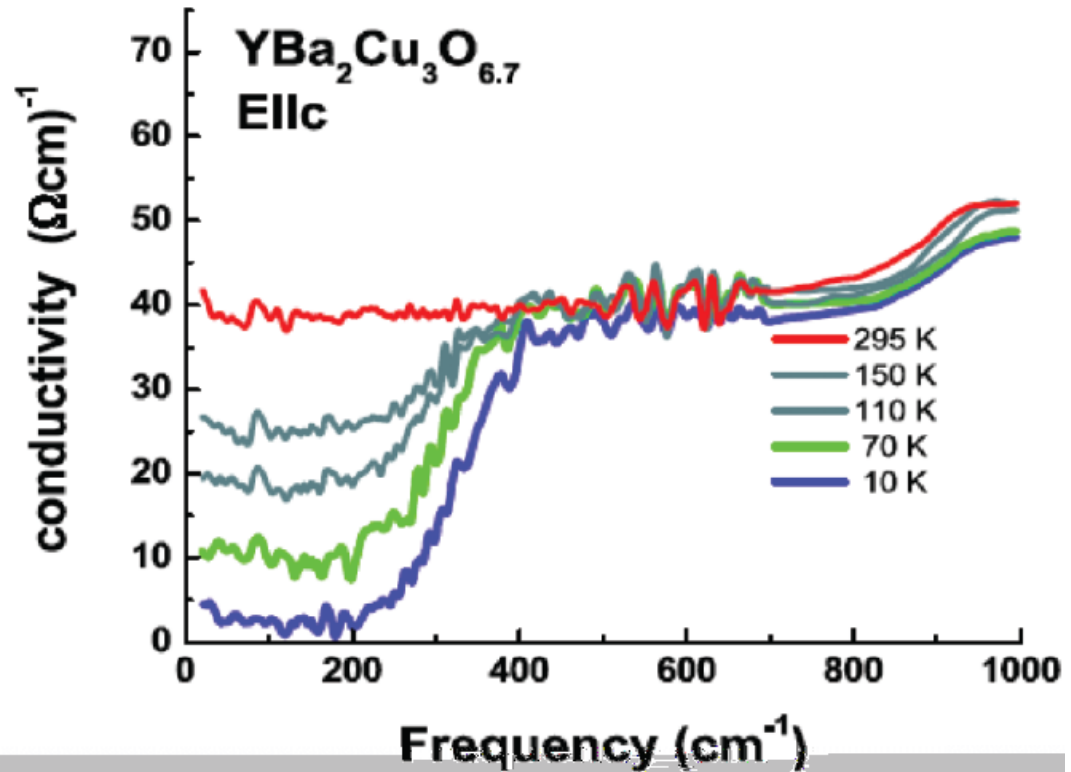
H. Alloul, A. Mahajan, H. Casalta, and O. Klein^(a)

Pseudogap in Transport: c-axis optical conductivity



c-axis transport dominated by tunneling near the antinode

$$t_{\perp}(\mathbf{k}) \sim (\cos k_x - \cos k_y)^2$$



Homes et al, PRL (1993)

* Much more subtle changes in ab-plane optical conductivity see Basov-Timusk RMP (2005)

Pseudogap \rightarrow Loss of entropy above T_c in Underdoped cuprates

Electronic Specific Heat of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ from 1.8 to 300 K

J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang

Specific heat \rightarrow Entropy

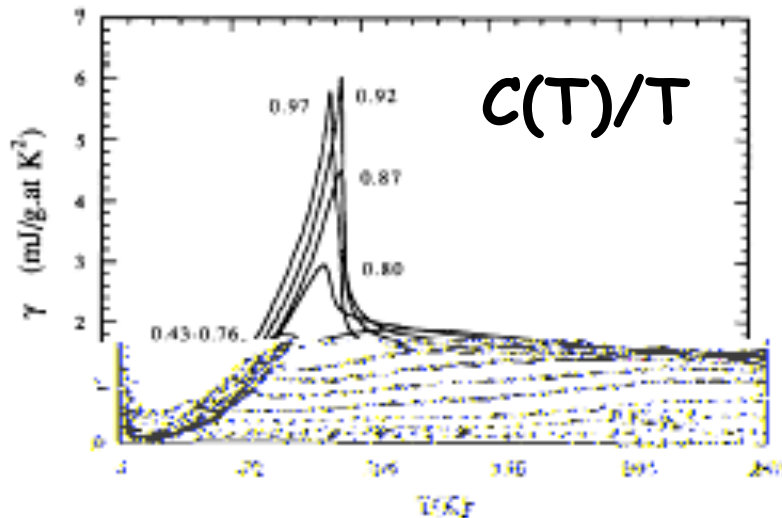


FIG. 4. Electronic specific heat coefficient $\rho(C, T) \equiv C/T$ for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, relative to $\text{YBa}_2\text{Cu}_3\text{O}_6$. Values of x are 0.43, 0.48, 0.53, 0.61, 0.68, 0.76, 0.80, 0.87, 0.92, and 0.97.

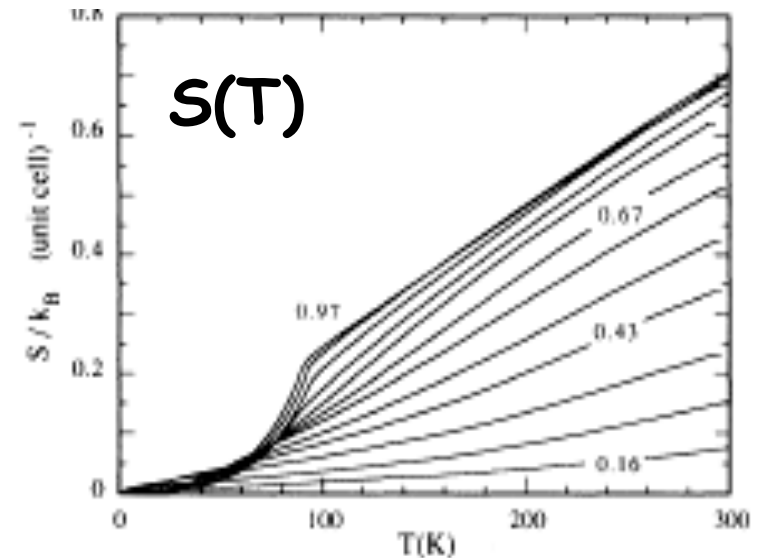


FIG. 5. Electronic entropy $S(x, T)$ for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$. Values of x as in Fig. 4.

Underdoping \downarrow

Pseudogap in Tunneling (STM):

Energy gap above T_c

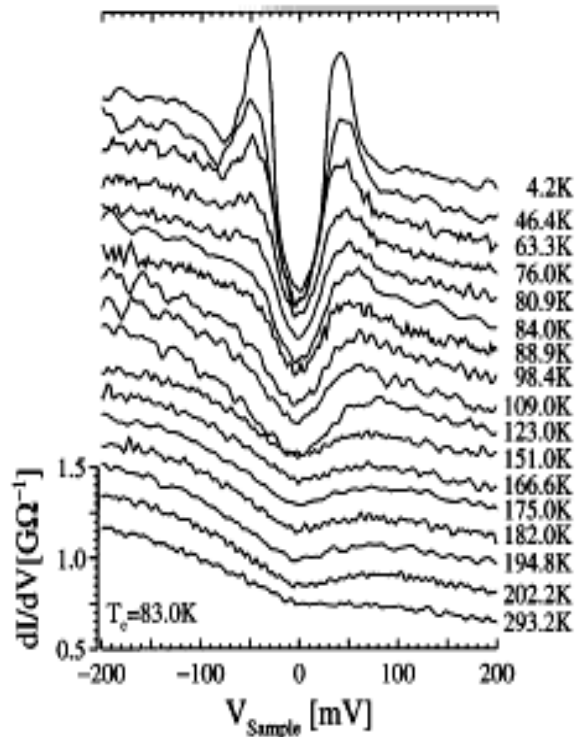
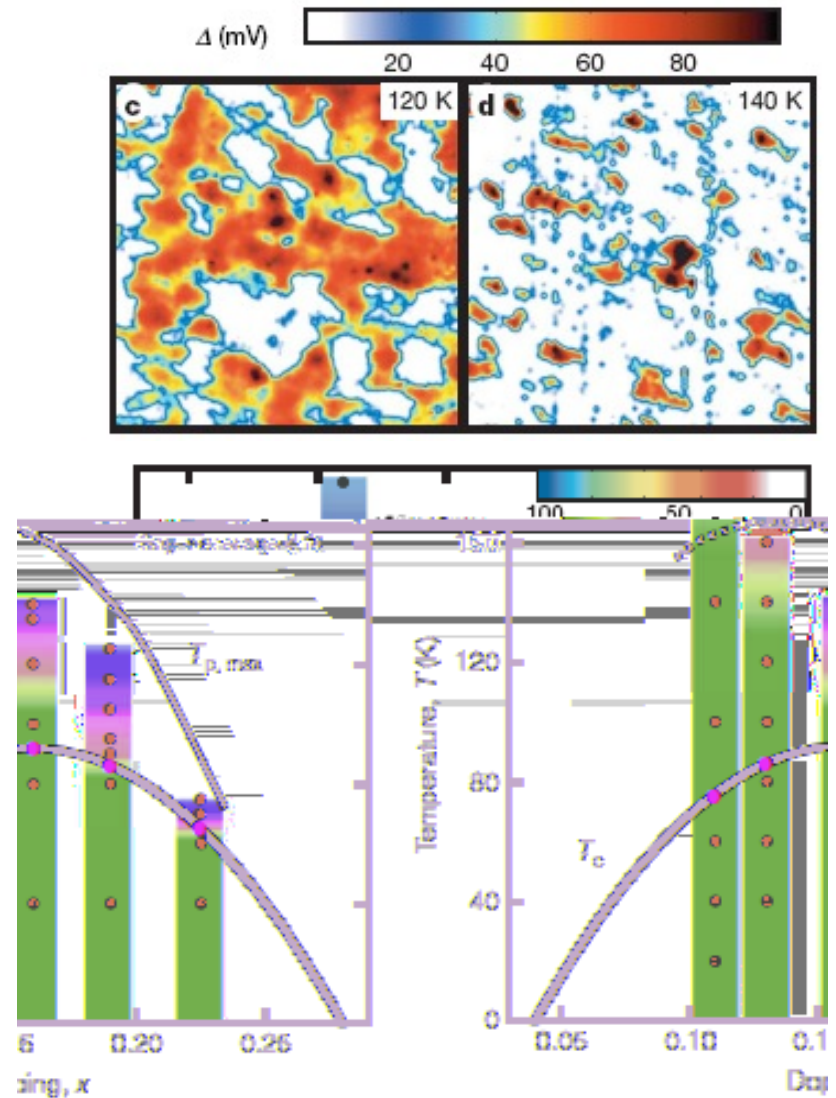


FIG. 2. Tunneling spectra measured as a function of temperature on underdoped Bi2212. The conductance scale corresponds to the 293 K spectrum, the other spectra are offset vertically for clarity.

Ch. Renner et al, PRL (1998)

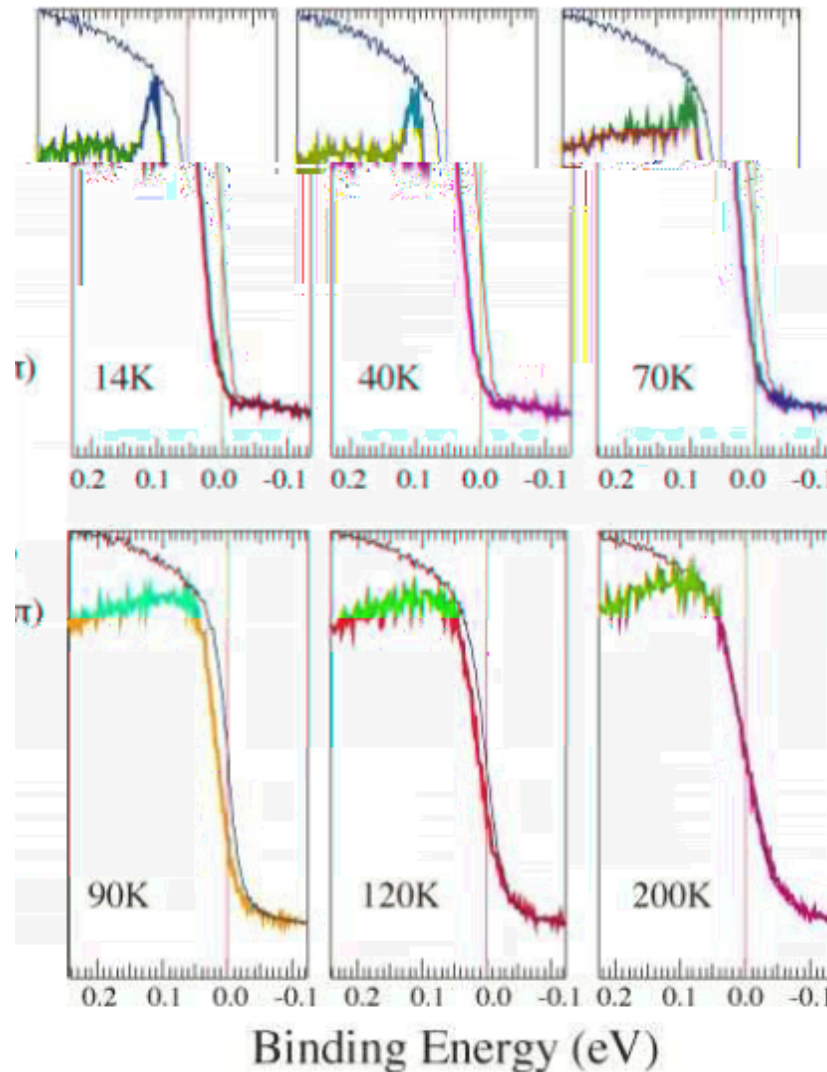
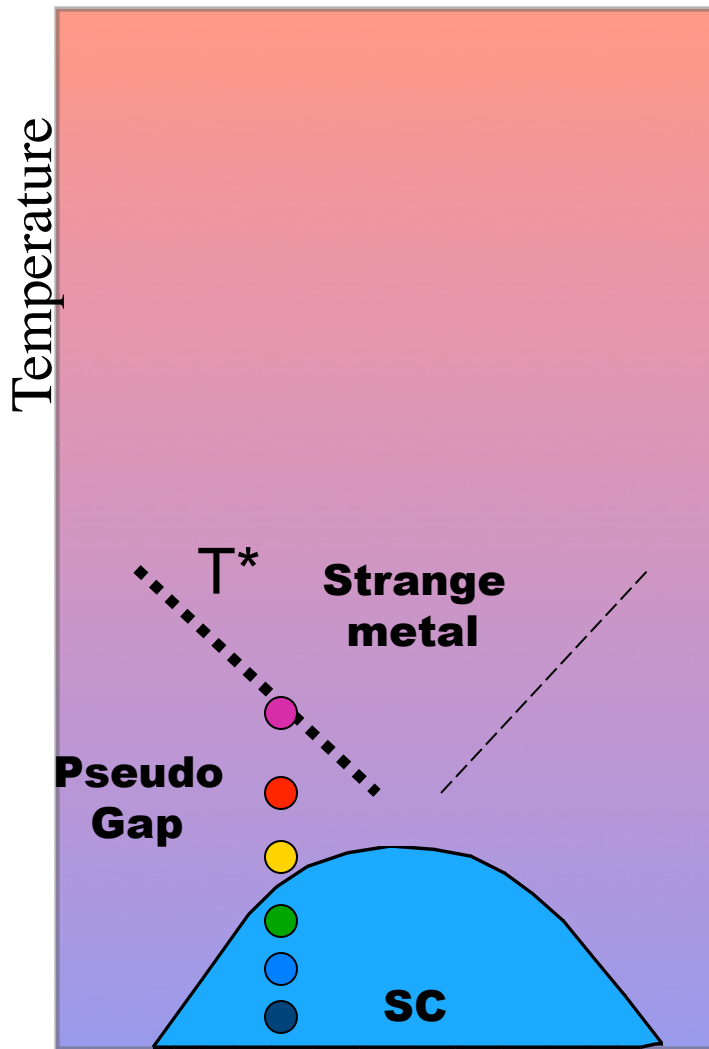


K. Gomes et al, Nature (2007)

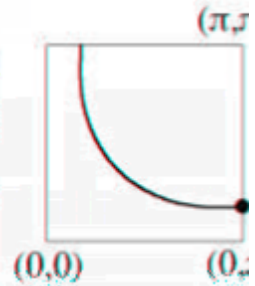
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ARPES Pseudogap in UD Cuprates: $T_c < T < T^*$



Sharp QPs below T_c



Gap below T^*

$T_c = 83K$
UD Bi2212

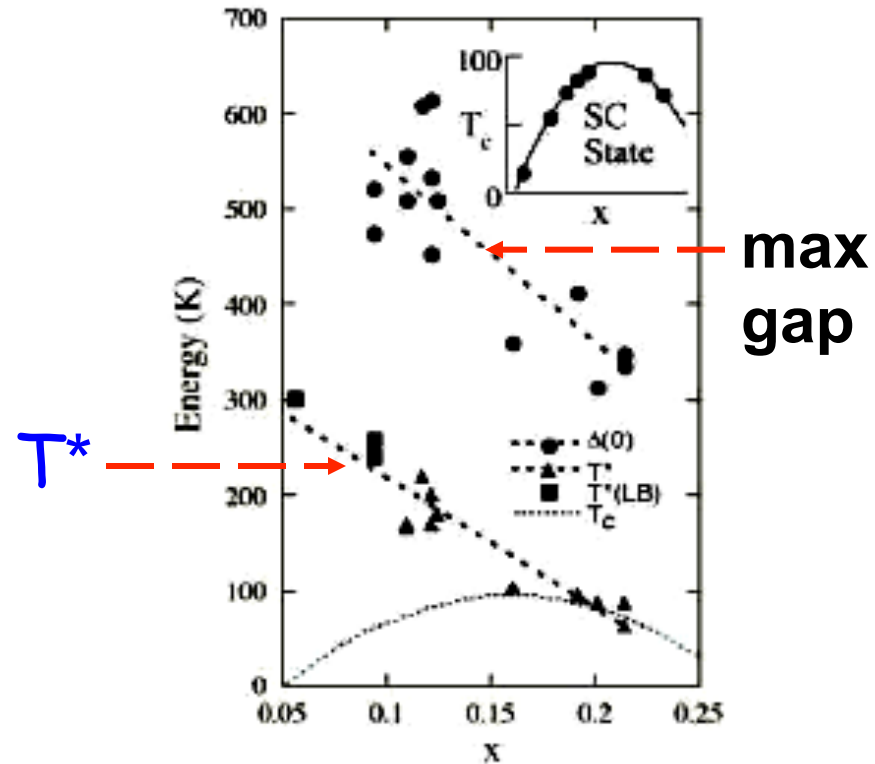
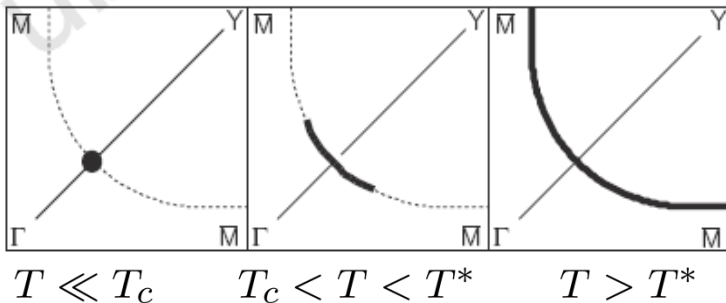
Ding et al, Nature **382**, 51 (1996)
See also: Loesser et al, Science (1996)

Pseudogap:

Suppression of low-energy spectral weight above T_c

- PG near antinode $\sim T$ -indep.
- PG appears below a temp. T^* which scales with gap
- PG is strongly anisotropic. Anisotropy is T -dependent \rightarrow "Fermi arcs"

Norman et al, Nature 392, 157 (1998)



Campuzano et al, PRL 83, 3709 (1999)

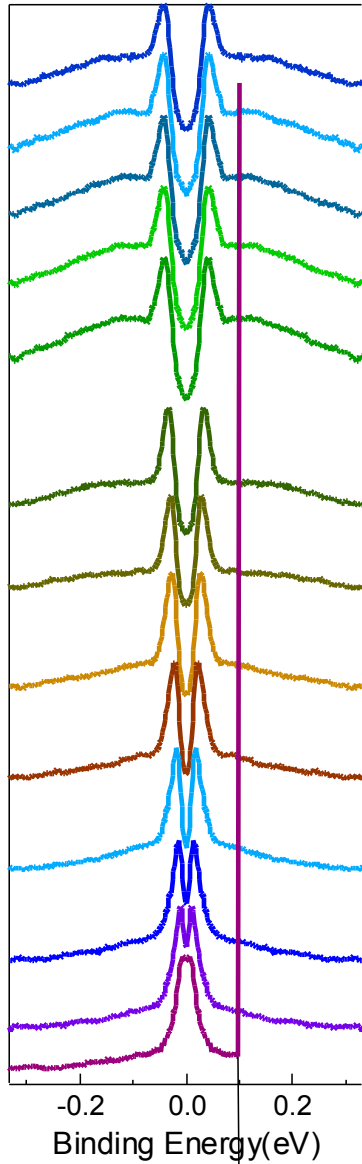
$$I(\omega) = A(\mathbf{k}, \omega) f(\omega)$$

$$I(-\omega) = A(\mathbf{k}, -\omega) f(-\omega)$$

$$I(\omega) = [1 - f(\omega)]$$

energy (meV)

Momentum dependence of the gap

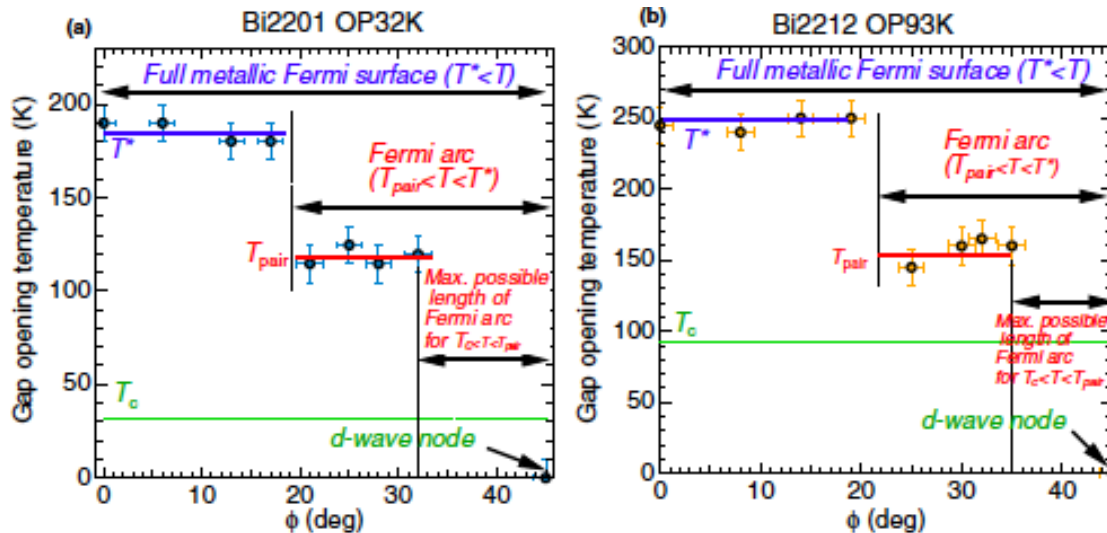
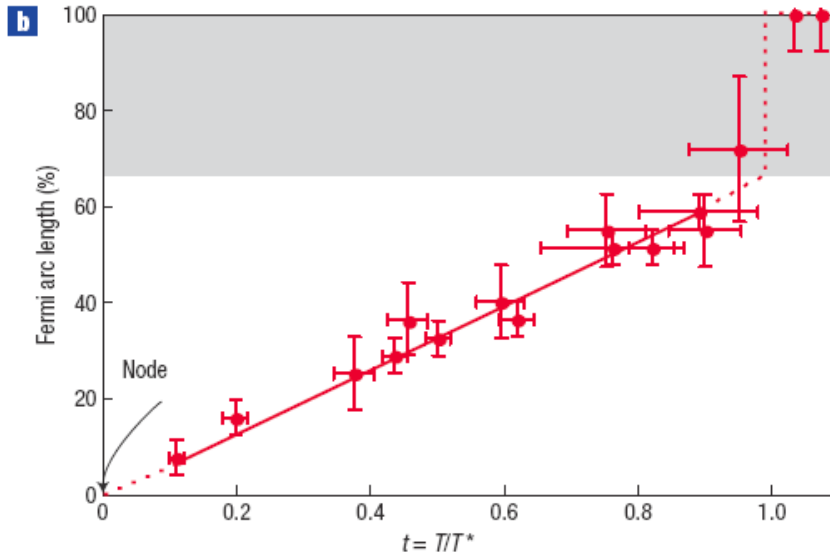


T-dependence of the "Fermi arcs"

Scaling with



Kanigel *et al*,
Nature Phys. (2006)

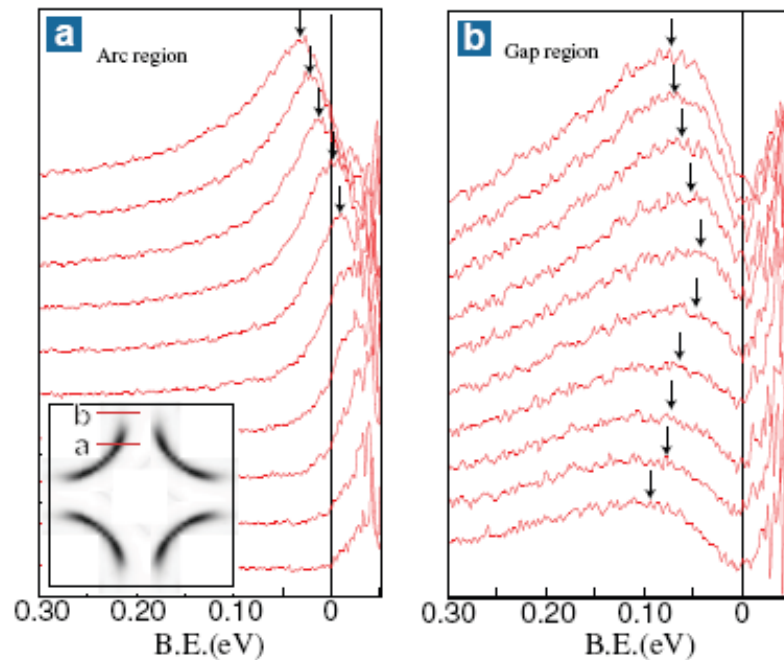
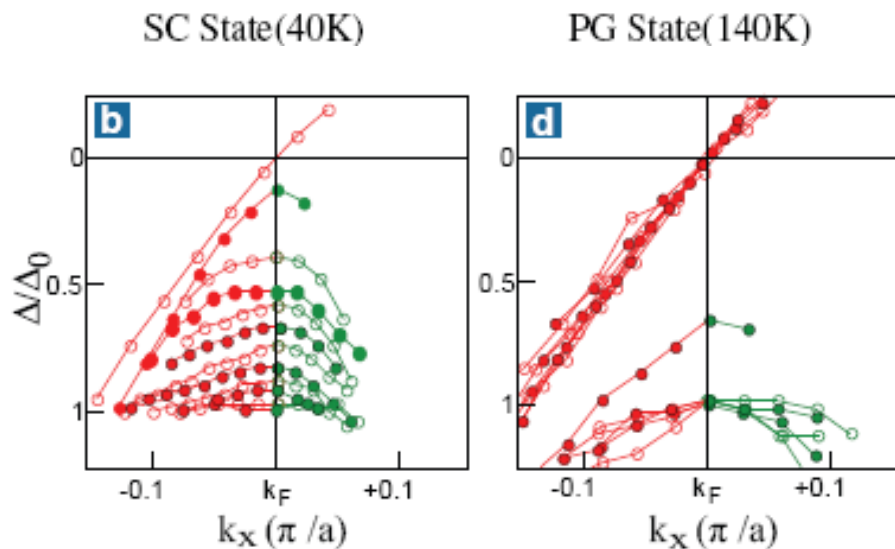


Different conclusions
From a different way
of analyzing the data

Kondo *et al*, PRL 111,
157003 (2013)

Bogoliubov-like dispersion in Pseudogap

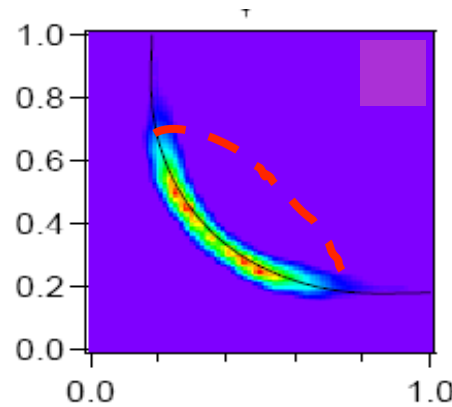
Kanigel, et al, PRL (2008)



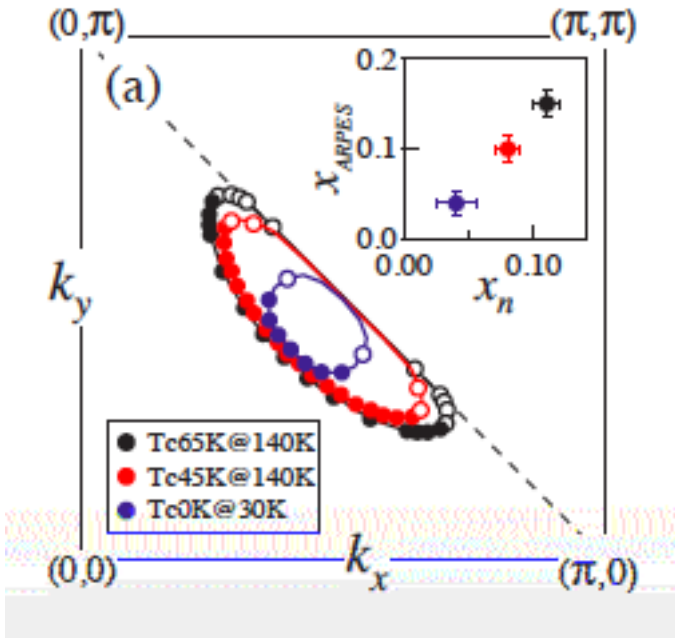
Dispersion
through E_f
on the arc

Bogoliubov-like
bending back
near AN

Fermi Arcs or hole pockets?

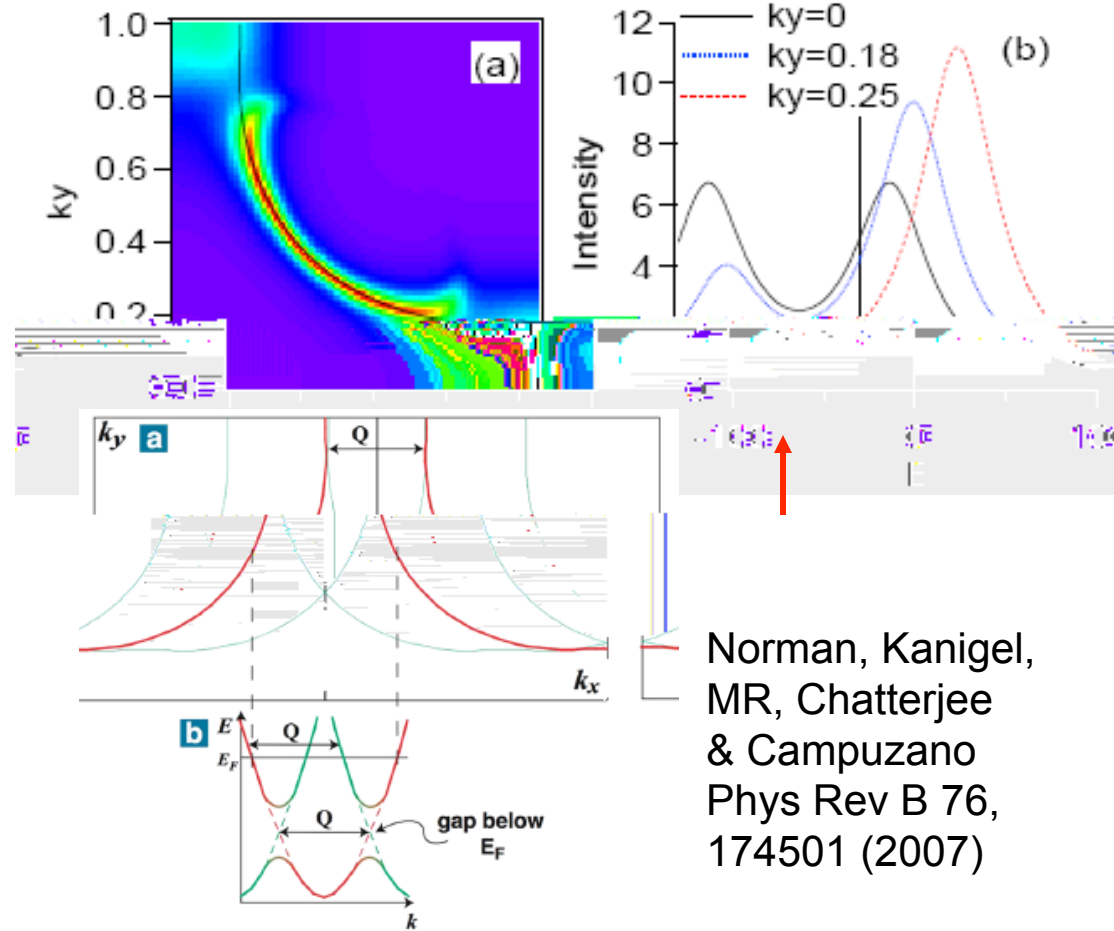


No evidence for Pockets or umklapp bands associated with a finite Q Order parameter



BNL group: H-B Yang et al, Nature (2008) & PRL (2011)

Non-zero q model

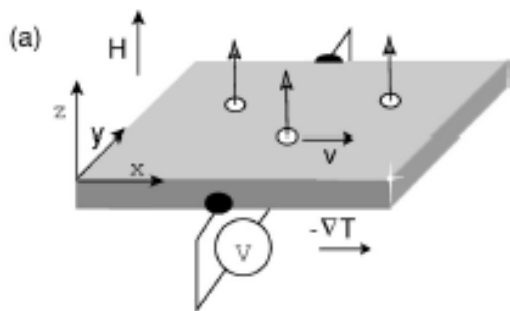


Norman, Kanigel, MR, Chatterjee & Campuzano Phys Rev B 76, 174501 (2007)

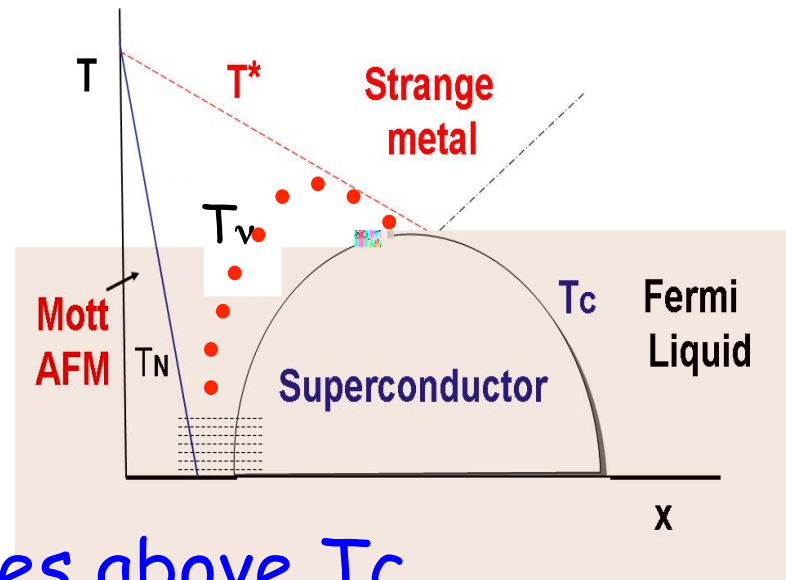
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Nernst effect above T_c



$$v_N = \frac{E_y}{|\partial_x T| B}$$



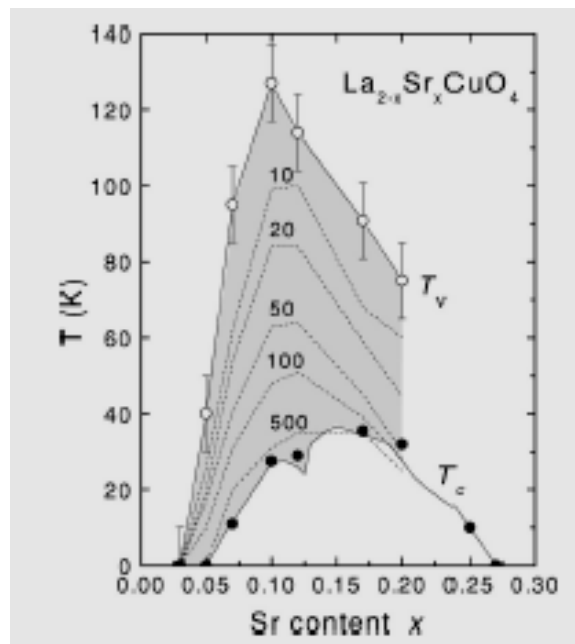
Vortices above T_c

SC short range order

Temperature gradient \rightarrow Vortices motion
Phase slip \rightarrow Josephson voltage (transverse)

Note: Nernst region

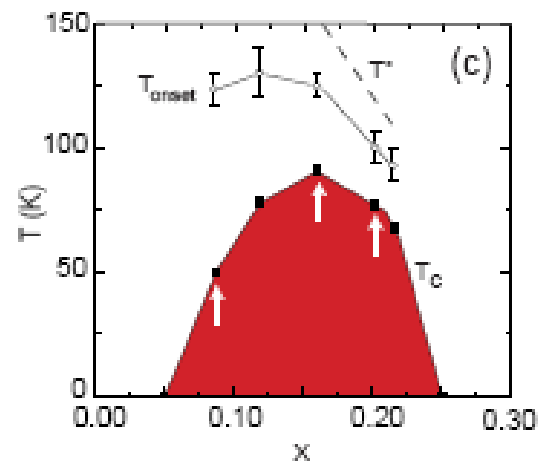
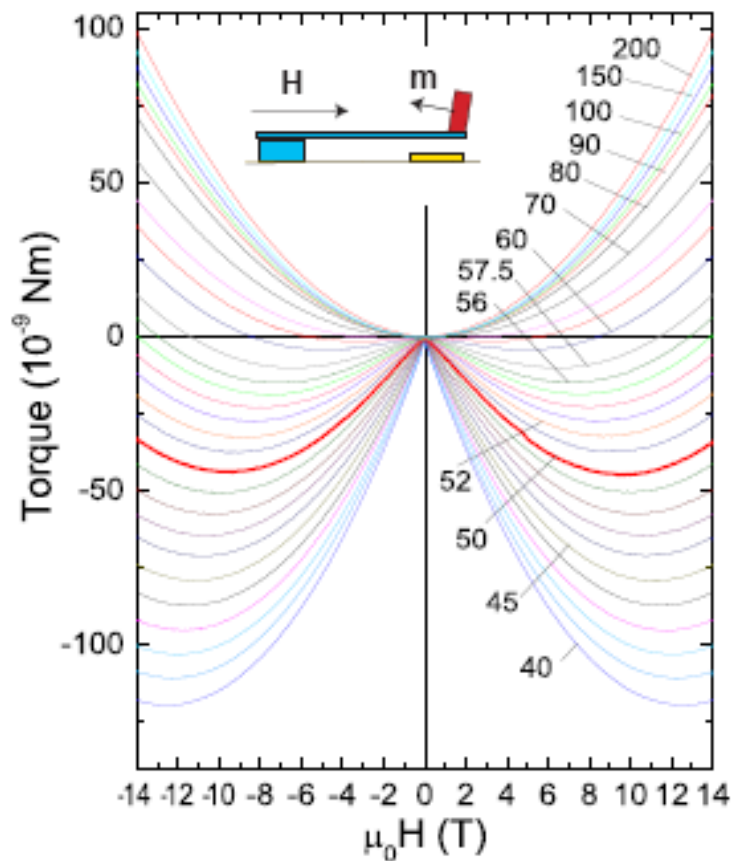
- (1) does not go up to T^*
- (2) onset has x-dep. similar to T_c



N. P. Ong's group
(2000 - 04)

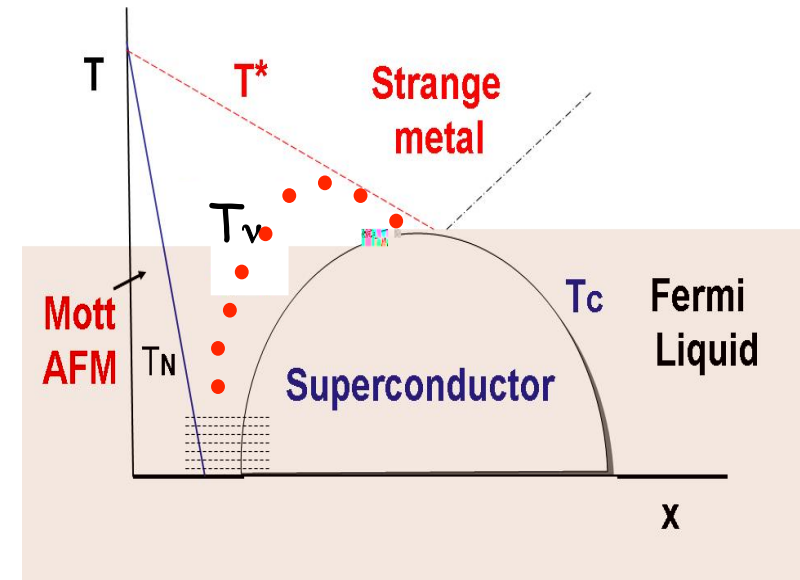
Fluctuation Diamagnetism above T_c

L. Li & N. P. Ong (2006)



Pseudogap summary (thus far)

- * Anisotropic gap above T_c
- * Pseudogap temp. scale T^*
- * T-dep Fermi arcs
- * No sharp QP's
- * Spin pairing up to T^*
- * SC fluctuations
(but not all the way to T^*)



Next:

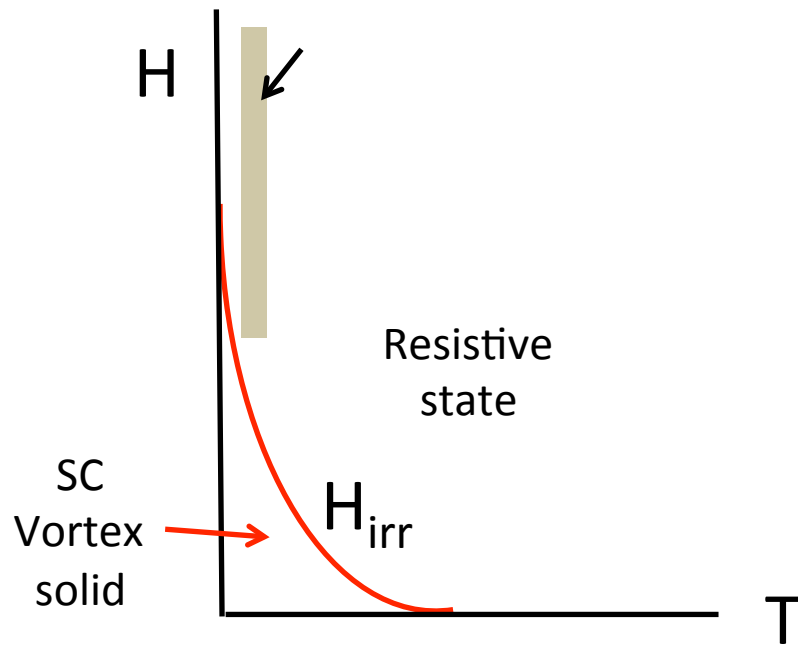
- * Quantum oscillations
- * Competing order parameters
& their relation to pseudogap

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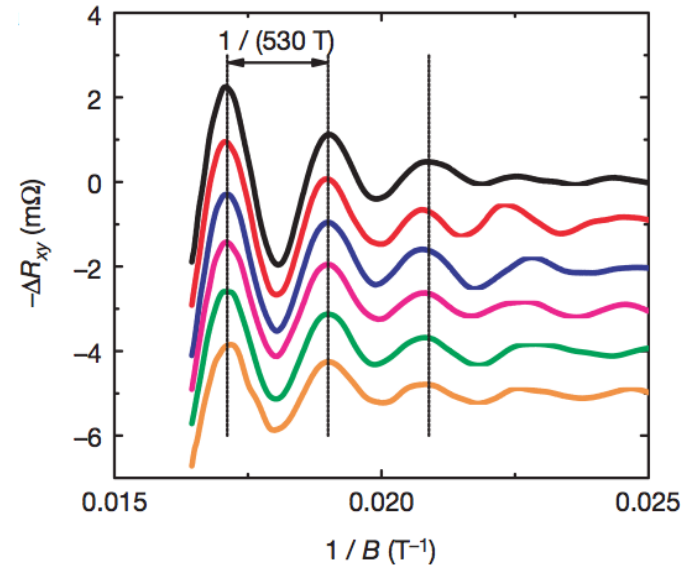
Destroy SC
with temp
vs.
Destroy SC
with B field

Quantum Oscillations in UD cuprates:



Field-induced
low temperature
resistive state

Quantum oscillations
Periodic in $1/H$

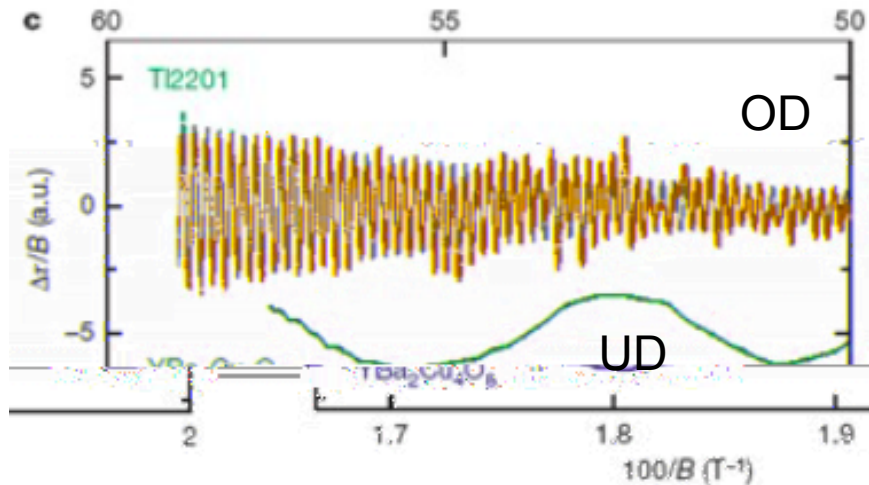


$T_c = 57.5$ K UD YBCO ($p=0.10$)

N. Doiron-Leyraud ... Taillefer. *Nature* **447** 565 (2007)

S. Sebastian *et al.* *Nature* **454** 200 (2008)
New measurements up to 100 Tesla!

OD vs. UD Cuprates: FS reconstruction



Vignolle, ..., Hussey, Nature (2008)

OD TI2201

F = 18100 Tesla \sim 63% of BZ
Luttinger count $(1+x)$ holes

UD YBCO

* F = 530 Tesla \sim 3% of BZ
* Electron pocket (sign of Hall & Seebeck)

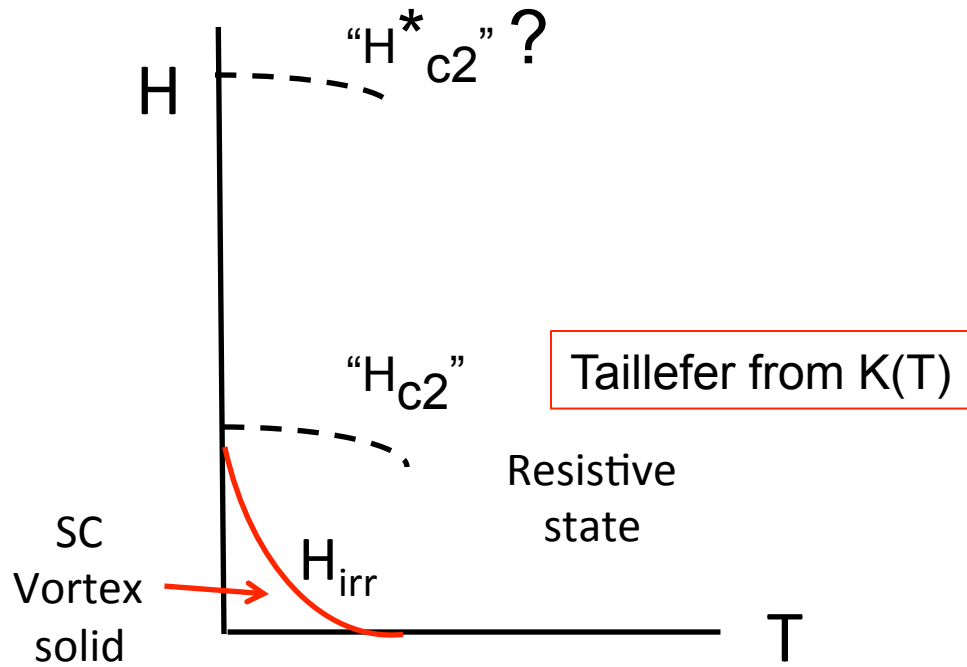
small electron pockets with area inconsistent with Luttinger count (in original Brillouin zone)

→ Translational symmetry breaking leads to Fermi surface reconstruction

[Millis & Norman. PRB (2007); Chakravarty & Kee PANS (2008); Yao, Lee & Kivelson. PRB (2011); Harrison & Sebastian. PRL (2012)]

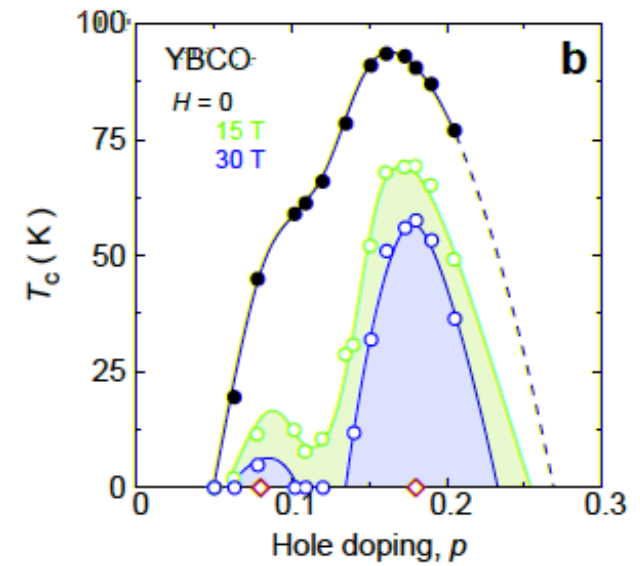
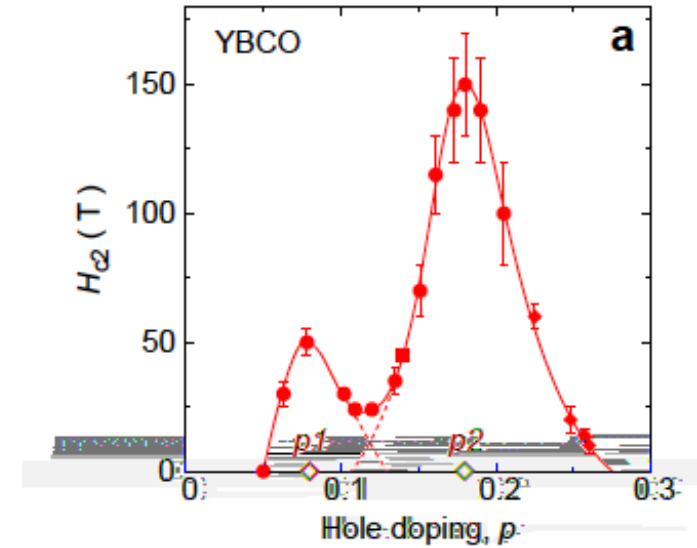
Field scales:

“upper critical field H_{c2} ”



Note:

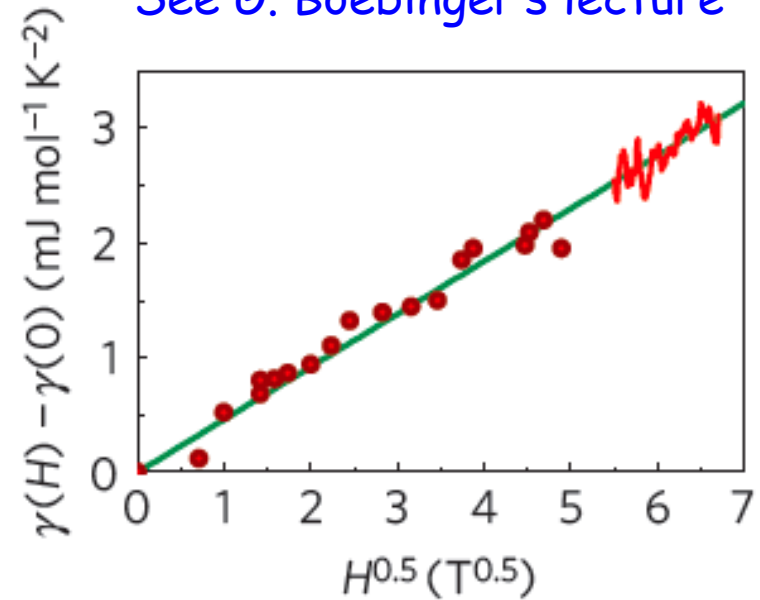
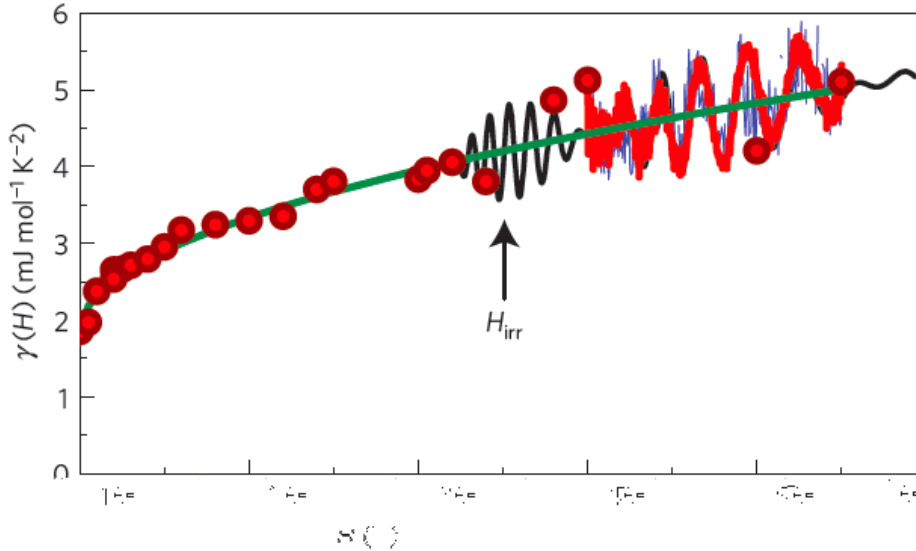
- “1/8 anomaly” (see below)
- Competing charge order
- Two QCPs



Grisonnanche ... Taillefer, Nature Comm. 5, 3280 (2014)

Heat Capacity in a Field

S. Riggs *et al.* Nature Phys. 7 332 (2011)
See G. Boebinger's lecture



* Quantum oscillations with a (constant + $H^{1/2}$) background

$\gamma \sim H^{1/2}$ Volovik effect \rightarrow SC nodes in resistive state?

* Specific heat strongly suppressed relative to
{high T, H=0} normal state \rightarrow "normal state" degrees of
freedom not recovered

$\gamma \ll \gamma_0 \approx 18 \text{ mJ/mol K}^2$ for $T > T_c$

J. Loram *et al.*, PRL 71 1740 (1993)

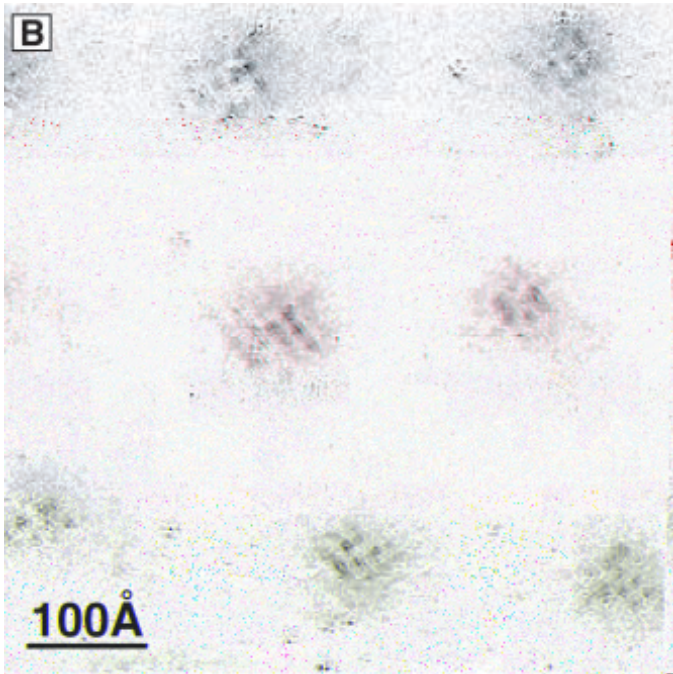
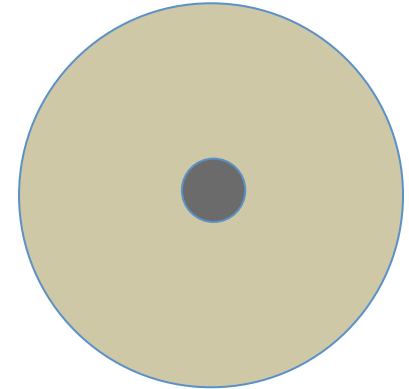
at 50 T

Implications of Phase Fluctuations for Vortex State

Two length scales:
core & halo $\xi_{\text{core}} = \frac{\hbar v_F}{\pi \Delta}$ $\xi_{\text{halo}} = \frac{\hbar v_F}{\pi T_c}$ (Superflow)

Lee & Wen, PRL (1997); Ioffe & Millis, PRB (2002)

Field scales: $H_{\text{irr}} \approx H_{c2}^{\text{halo}} \ll H_{c2}^{\text{core}}$

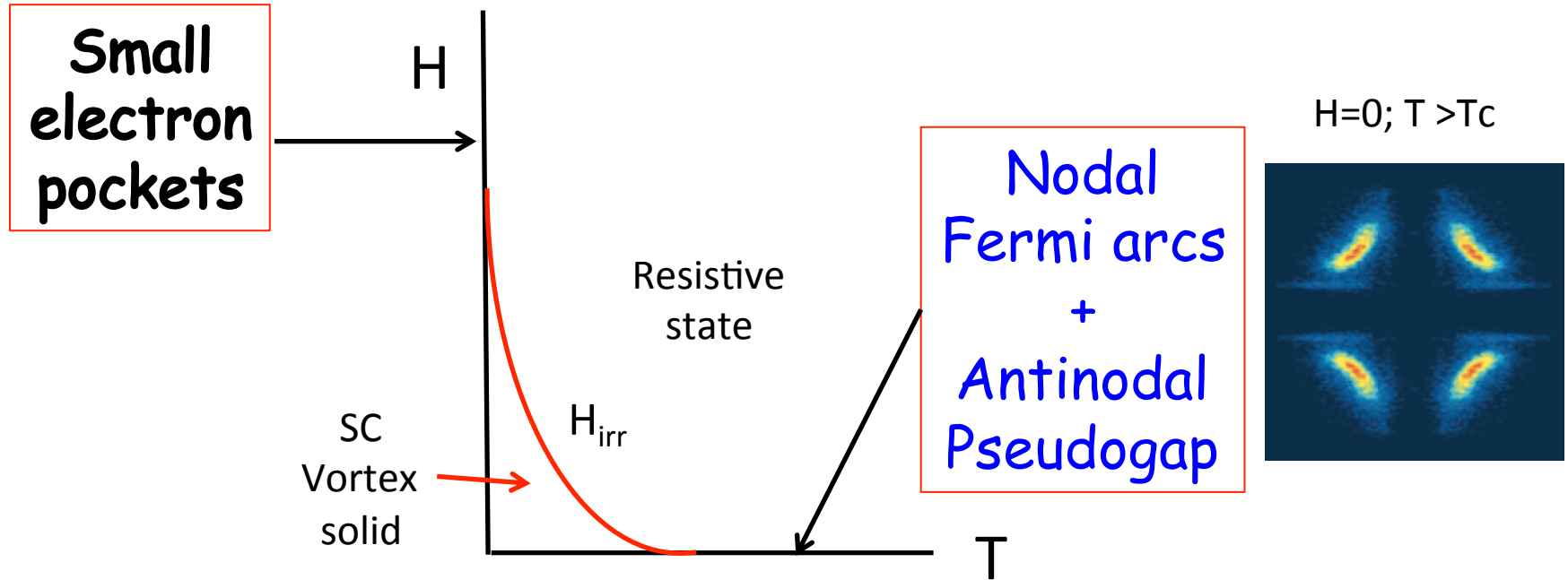


Halo region:

- suppressed superconductivity
- enhanced competing order

Neutrons (LSCO): Lake et al, Science (2001)
STM (Bi2212): Hoffman et al, Science (2002)

How to reconcile the electronic excitations seen by **quantum oscillations** and **ARPES**?



Pseudogap ~ 50 meV ~ 500 K T_c
Fields ~ 50 T ~ 50 K

Senthil and Lee PRB (2009)
Banerjee, Zhang & Randeria, Nature Comm. (2013)
Allais, Chowdhury, Sachdev, arXiv (2014)

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- Quantum Oscillations

- Competing order parameters

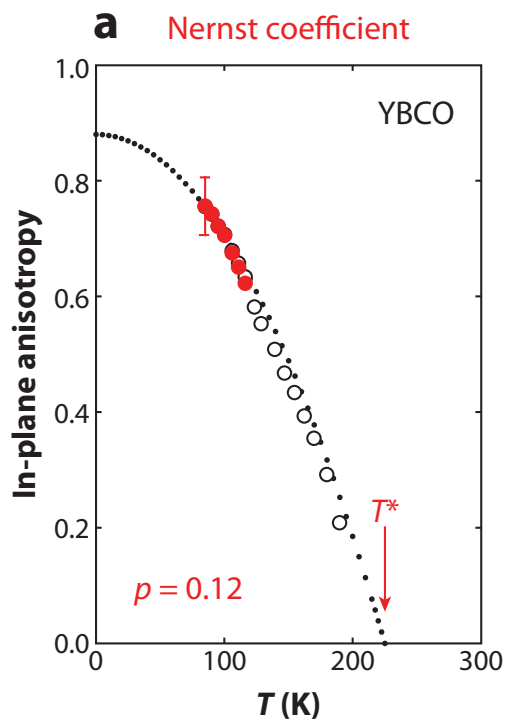
Plethora of broken symmetries in pseudogap state

- * Translational -- stripes, CDW
- * Rotational -- nematicity
- * Time-reversal -- will not discuss
 - * neutrons (Bourges, Greven...) → Orbital Currents (Varma)
 - * polar Kerr (Kapitulnik)
- Is $T^*(x)$ a Crossover or Phase Transition?
- Which of these broken symmetries - if any - are responsible for large antinodal "pseudogap" in $H=0$?
- Broken translational required for large H by quantum oscillation frequency

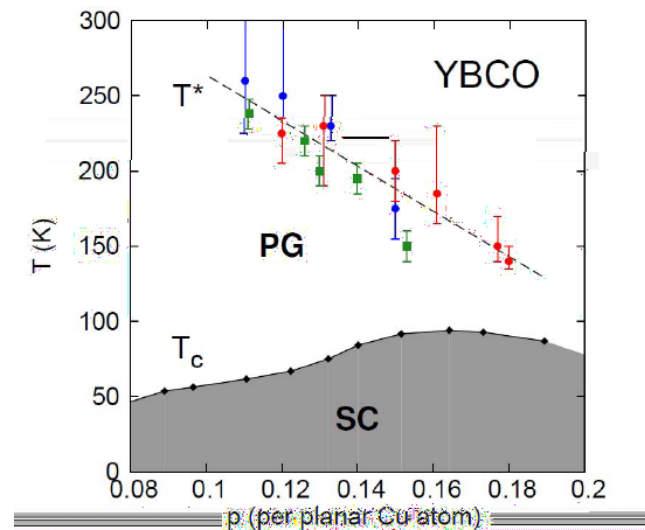
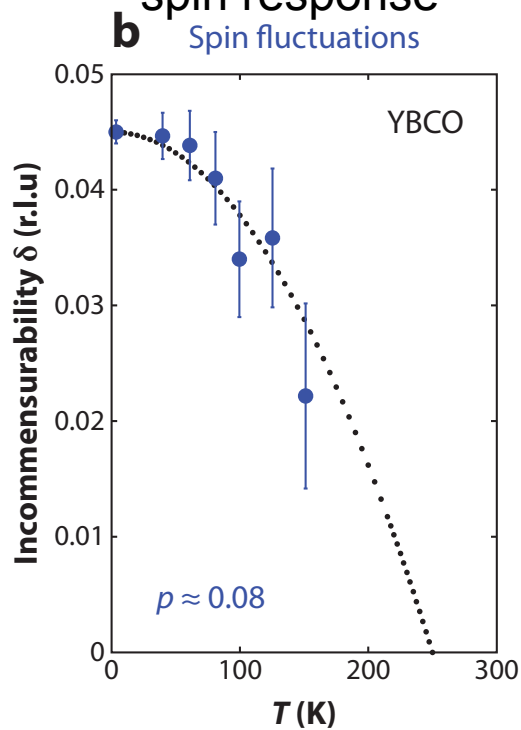
Broken Rotation Symmetry: Electronic Nematic Order

Difference between a & b axis responses strongly enhanced in pseudogap state

Anisotropy of Nernst



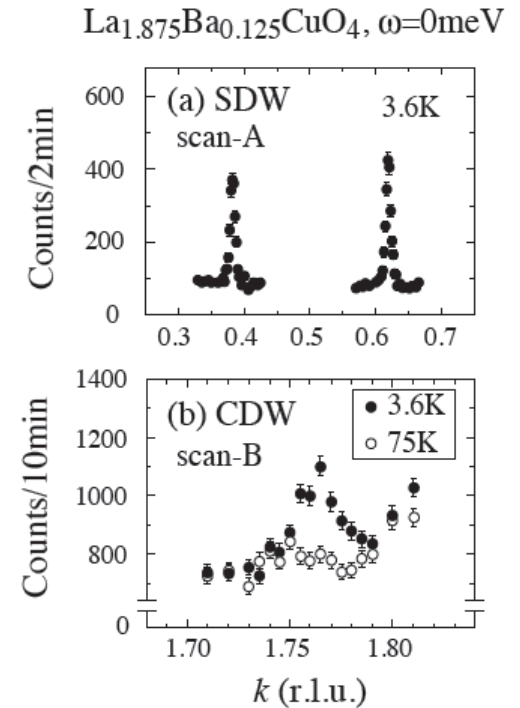
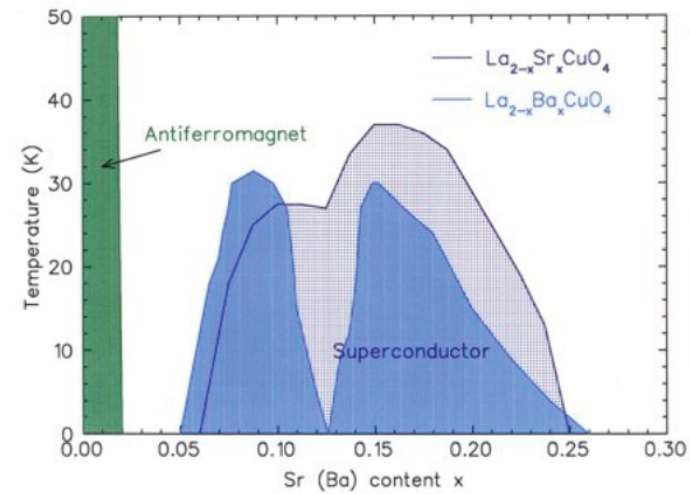
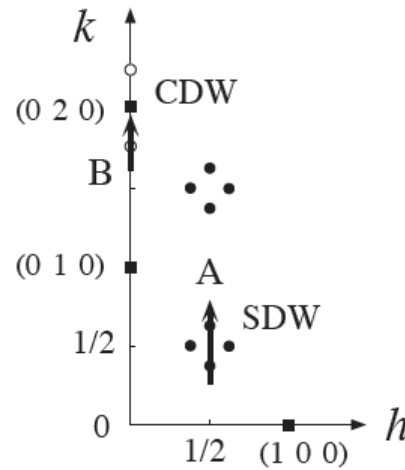
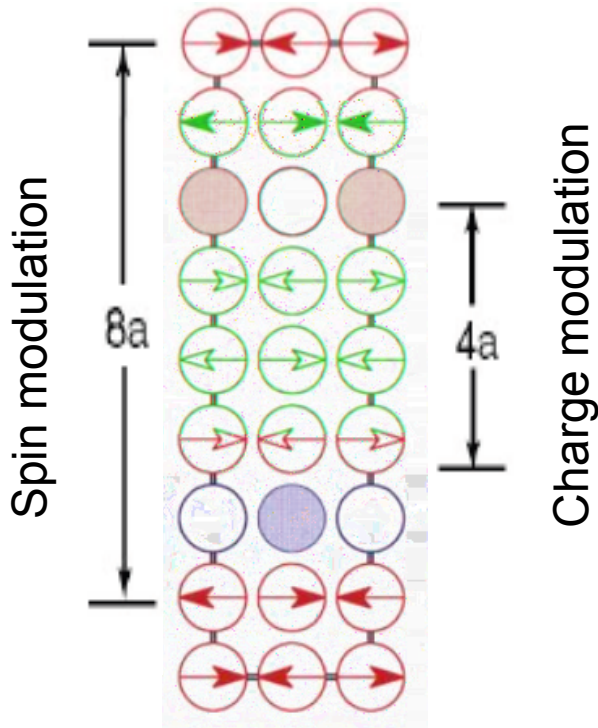
Unidirectional
Incommensurate
spin response



Broken Translation & Rotation: Stripes

Nd:LSCO; LBCO ($x=1/8$)

Static stripes kill superconductivity



Review: J. Tranquada, AIP Conf. Proc. 1550, 114 (2013)
see S. Kivelon's lectures

Broken translation: Charge Density Wave

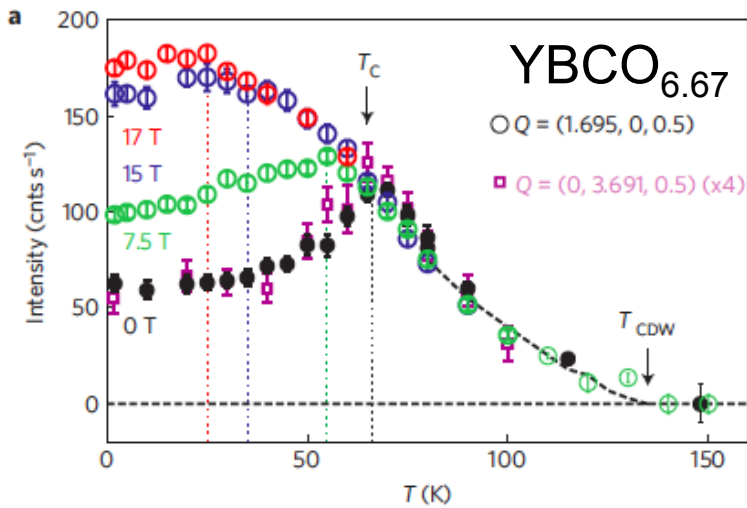
STM: checkerboard CDW -- Bi2212, Bi2201, Oxchlorides
vortex core - Bi2212

High field NMR - YBCO

Xray diffraction - YBCO

REXS - YBCO, Bi2212

- * Finite correlation length in $H=0$ (disorder or intrinsic?)
- * Long range order in high fields:
Fermi surface reconstruction in Qtm Oscillations



UD YBCO (ortho-II, -III & -VIII)

- * **Hard x-ray diffraction** (100keV)
J.Chang et al., Nature Phys (2012)

Bi-directional CDW

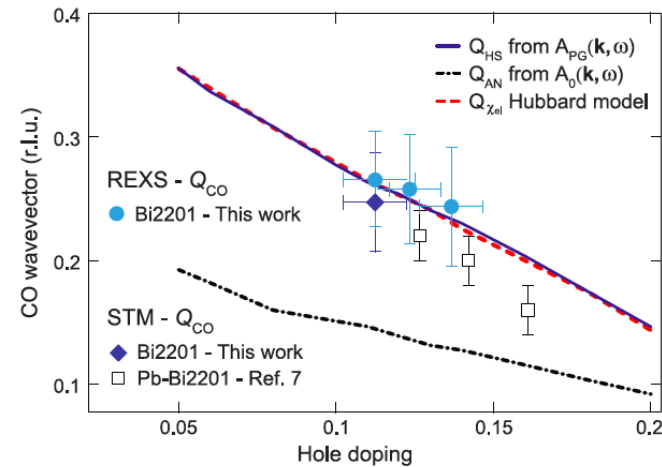
$$\mathbf{q}_1 = (0.31, 0, 0.5)$$

$$\mathbf{q}_2 = (0, 0.31, 0.5)$$

- * **Resonant elastic x-ray scattering**
(soft xrays 930eV) YBCO
G.Ghiringhelli et al, Science (2012)

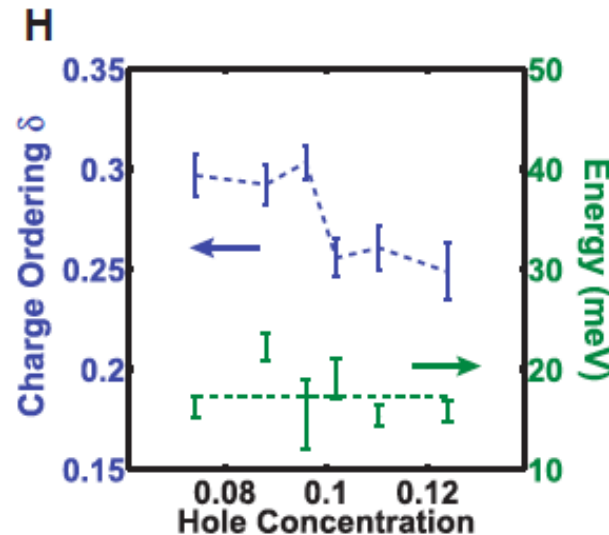
in-plane ξ (\AA) ~ 60 (at T_c)

Evidence for CDWs



UD Bi2201
REXS
STM
ARPES
Comin...
Damascelli
Science
(2014)

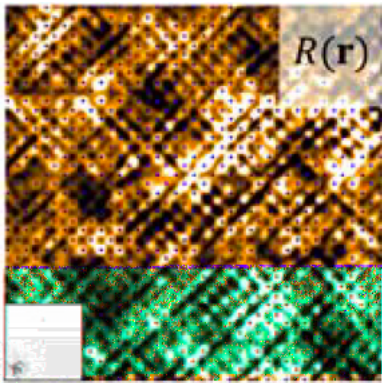
$$\xi \sim 20 - 30 \text{\AA}$$



UD Bi2202
REXS
STM
Silva Neto
...Yazdani
Science
(2014)

STM → Nematic + Bond-centered CDW

Fujita ... Sachdev, Davis, arXiv1404.0362;



"d-wave" CDW
[Sachdev]

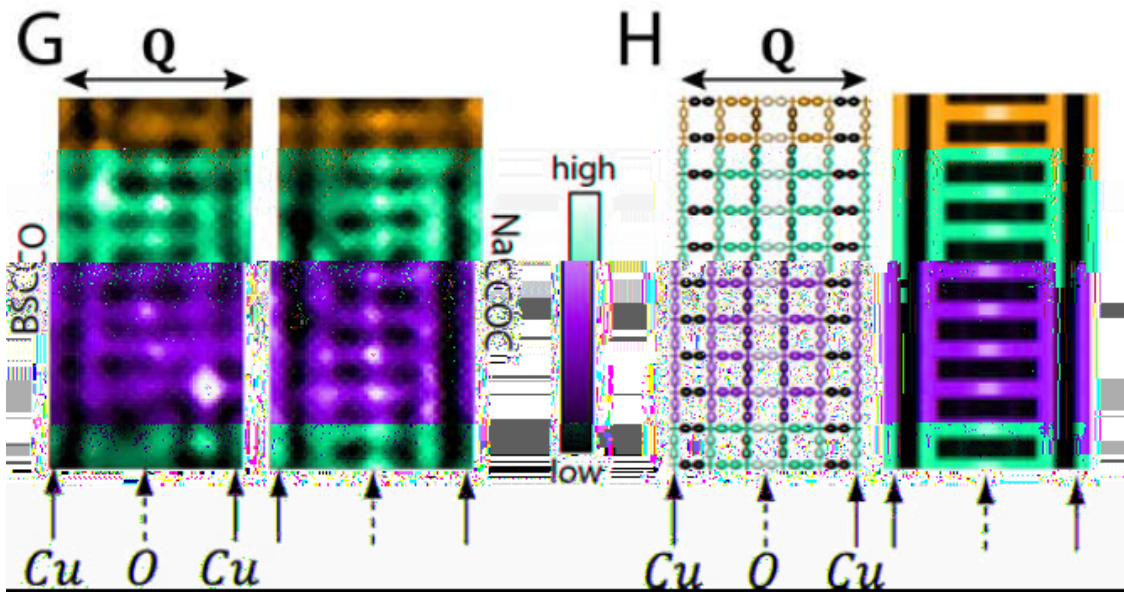
$$c_{r\alpha}^\dagger c_{r'\alpha}$$

$$(r + r')/2 \quad \mathbf{Q}$$

CDW wavevector

$$(r - r')$$

→ Form factor for
internal structure
of CDW



Pseudogap: More questions; few definitive answers

Here are my thoughts:

- What is responsible for the antinodal pseudogap $\sim 60\text{meV}$?

- proximity to Mott & spin pairing (RVB, DMFT)

- competing orders are not the cause of the pseudogap but rather competing instabilities in the PG state

- Broken translational symmetry required by quantum oscillations

- bidirectional CDW order stabilized in a field

- \leftrightarrow FS reconstruction seems the best proposal

