Lecture 3

Mesoscopic effects in hopping conductivity

Topics

1. Anomalous transmission of a barrier

2. Hopping conductivity through an amorphous film

3. Distribution function of hopping conductivity of a finite-size sample

4. Hopping conductivity in 1D
system of filaments with random traps

\( D \) -probability to pass through a trap

\[
\langle \sigma \rangle = \exp[-\bar{N}(1 - D)] \\
\ln \langle \sigma \rangle = -\bar{N}(1 - D)
\]

average conductance exceeds exponentially the conductance of a typical filament

\[
\ln \langle \sigma \rangle > \langle \ln \sigma \rangle
\]

the product \( D^N p(N) \) is maximal for

\[
N = D \bar{N} = N_{\text{opt}} < \bar{N}
\]

sparse filaments with \( N \) close to \( N_{\text{opt}} \) determine the average conductance
electrons with broadly distributed energies are incident on a barrier with randomly positioned impurities

for a chain of \( N \) impurities

probability of formation of a chain increases with \( N \)

rigorous calculation yields

\[
v_0 = a^2 L
\]

\[
\ln \langle \sigma \rangle = -2 \left[ \frac{L}{a} \ln \left( \frac{1}{nv_0} \right) \right]^{1/2}
\]

unlike tunnel conductivity

\[
\Gamma_0 \propto \exp\left( -\frac{L}{a} \right)
\]

impurity level position

distance from the center

energy interval where transmission is high

\[
\sigma(\varepsilon) = \frac{4\Gamma_0^2}{(\varepsilon - \varepsilon_0)^2 + 4\Gamma_0^2 \cosh^2 \frac{2z}{a}'}
\]

miniband width

\[
\Delta_N \propto \exp\left( -\frac{L}{Na} \right)
\]

\[
\sigma_N \propto \Delta_N w_N \propto \exp\left[ -\frac{L}{Na} - N \ln \left( \frac{1}{nv_0} \right) \right].
\]
Hopping conductivity of an amorphous film

\[ R_{12} = \frac{1}{\sigma_0} \exp\left(\frac{2|\mathbf{r}_1 - \mathbf{r}_2| + \varepsilon_{12}}{a} - \frac{T_0}{T}\right) \]

\[ \varepsilon_{12} = \frac{1}{2}(|\varepsilon_1| + |\varepsilon_2| + |\varepsilon_1 - \varepsilon_2|) \]

Resistance of one hop

Resistance of one chain

\[ R_N = \frac{1}{\sigma_0} \exp\left(\frac{2L}{Na} + \frac{\delta z}{a} + \frac{(\delta \rho)^2 N}{La} + \frac{\delta \varepsilon}{T}\right) \]

Probability of formation

The product \( w_N R_N^{-1} \) is maximal for

\[ \delta \varepsilon = NT, \quad \delta z = Na, \quad \delta \rho = (La)^{1/2} \]

Applicability: chains dominate when

\[ \ln \langle \sigma \rangle < \left(\frac{T_0}{T}\right)^{1/4} \]

\[ L < \frac{1}{(gTa)^{1/2}} \]

\[ \lambda = \ln(\lambda/gTaL^2) \]

Solution of equation

We assumed that the chains are nearly straight:

\[ \delta z \approx \left(\frac{2La}{\lambda}\right)^{1/2} \]

and \( \delta \rho \approx (La)^{1/2} \) must be smaller than

\[ \frac{L}{N} \approx \left(\frac{La \lambda}{2}\right)^{1/2} \]

These conditions are also met when \( \lambda \gg 1 \)

---

Y. Park*  
Department of Materials Science and Metallurgy, University of Cambridge
Hopping transport in systems of finite thickness or length

A. S. Rodin and M. M. Fogler
University of California San Diego, 9500 Gilman Drive, La Jolla, California 92093, USA
(Received 18 July 2011; published 29 September 2011)

Evolution of a 2D network with increasing $T$

The network progresses from independent conducting strands to an interconnected grid.
Sample of a finite area

Since the distances between punctures are exponentially large, the situation may occur when no optimal puncture is present in a sample.

Then the conductivity of a typical sample will be determined by a few punctures of highest transmittance present in the sample.

Conductivity will strongly fluctuate from sample to sample.

Conductivity will strongly depend on the sample area.

Ensemble of samples should be characterized by the distribution function.
Peak position of the distribution function

Cross section of transmission of a puncture

$$A = A_0 \exp(-u)$$

Areal density of punctures

$$\rho(u) = \lim_{s \to \infty} \frac{1}{S} \sum_i \delta(u - u_i)$$

For punctures with anomalously high transmission

$$\rho(u) = \frac{1}{S_0} \exp[-\Omega(u)]$$

characteristic puncture area

$$\Omega(u)$$ decreases with increasing $u$

Sample-averaged conductivity

$$\langle \sigma \rangle = A_0 \int_0^\infty du \, e^{-u} \rho(u) = \frac{A_0}{S_0} \int_0^\infty du \, \exp[-u - \Omega(u)]$$

The integrand has a sharp maximum at $u = u_{opt}$ such that

$$\Omega'(u_{opt}) + 1 = 0$$
Applies when the number of optimal punctures within the sample area is large.

Since \[ |\ln(S_0 \rho(u_{opt}))| = \Omega(u_{opt}) \gg 1 \]
the condition of applicability reduces to \[ v > 1 \]

For \[ v < 1 \]
the conductivity of a typical sample will be dominated by a few punctures with highest transmission present in the sample.

This condition defines \[ u = u_f > u_{opt} \]
such that \[ S \rho(u_f) \sim 1 \]
\[ \Omega(u_f) = v \Omega(u_{opt}) \]

\( u_f \) defines the lower limit in \[ \int_{u_f}^{\infty} du \rho(u) \exp(-u) \]

\[ \ln \left( \frac{S_0}{A_0} \sigma(v) \right) = -v \Omega(u_{opt}) - u_f(v) \]

\[ S \rho(u_{opt}) \gg 1 \]
Width of the distribution function

\[ \langle (\delta \sigma)^2 \rangle = \langle \sigma^2 \rangle - \langle \sigma \rangle^2 = \frac{A_0^2}{S} \int_0^\infty du \, \rho(u) e^{-2u} \]

from Poisson’s statistics

\[ \langle n_i^2 \rangle - \langle n_i \rangle^2 = \langle n_i \rangle \]

The integrand has a sharp maximum at \( u = u_d \) such that

\[ \Omega'(u_d) + 2 = 0 \]

\[ u_d < u_{opt} \]

determines the width of the distribution function if

\[ S \rho(u_d) \geq 1 \]

\[ v > v_d \]

where \( v_d \) is the solution of

\[ \varphi(v_d) = 2 \]

\[ \Omega(u_f) = v \Omega(u_{opt}) \]

punctures responsible for variance are exponentially more sparse than punctures responsible for average

\[ \varphi(v) = -\Omega'(u_f(v)) \]

\[ v = \frac{\ln(S/S_0)}{\Omega(u_{opt})} \]
The width of the distribution function of $\ln \sigma$ for $\nu > \nu_d$

$$\Delta_0 \sim \frac{1}{\sigma(\nu)} \left( \frac{A_0^2}{SS_0} \int_{u_f}^{\infty} du \exp[-2u - \Omega(u)] \right)^{1/2} = \frac{A_0}{S_0 \sigma(\nu)} \exp[-u_f(\nu) - \nu \Omega(u_{\text{opt}})]$$

for $\nu < \nu_d$

The width is inversely proportional to $S^{1/2}$

Integration only over punctures present in a typical sample

$1 < \nu < \nu_d \implies \sigma(\nu) = \langle \sigma \rangle \implies \Delta_0 \sim \exp[-u_f(\nu) + u_{\text{opt}} + (1 - \nu)\Omega(u_{\text{opt}})]$

$\nu < 1 \implies \ln \left( \frac{S_0}{A_0} \sigma(\nu) \right) = -\nu \Omega(u_{\text{opt}}) - u_f(\nu) \implies \Delta_0 \sim 1$

For $\nu \leq 1$, uncertainty in $u_f$ determines the width of the distribution function

$S\rho(u_f) \sim 1-2 \implies \delta u_f \sim [S\rho'(u_f)]^{-1} \implies \Delta_0 \sim \delta u_f \sim 1/\varphi(\nu) \geq 1$
Example:

\[ \sigma_N = \exp\left[-\frac{2L}{Na}\right] \]

\[ \nu_N = C^{-N} \]

\[ \rho(u) = \frac{1}{S_0} \exp[-\Omega(u)] \]

\[ \Omega(u) = \frac{Q_0^2}{4u} \]

\[ u_{opt} = \frac{Q_0}{2} \]

\[ \Omega(u_f) = \nu \Omega(u_{opt}) \]

\[ u_f(v) = \frac{Q_0}{2\nu} \]

\[ \varphi(v) = -\Omega'(u_f(v)) \]

\[ \varphi(v_d) = 2 \]

\[ v_d = 2^{1/2} \]

\[ \ln \sigma(v) = -\frac{Q_0}{2} \left(v + \frac{1}{v}\right) \]

\[ 1 < v < 2^{1/2} \]

\[ \Delta_0 \sim \exp\left[-\frac{1}{2}Q_0(v + v^{-1} - 2)\right] \]

\[ \Delta_0 \sim \exp\left[-\frac{1}{4}Q_0(v - 4 + 2^{3/2})\right] \]

\[ v > 2^{1/2} \]

peak position

evolution of the width of the distribution function with area
Analytical expression for the distribution function

\[ Q = -\ln(S_0 \sigma / A_0) \]

\[ f(Q) = e^{-Q} \sum_{n_i=0}^{\infty} \delta \left( e^{-Q} - \frac{S_0}{S} \sum_i n_i \exp(-u_i) \right) \prod_k p(n_k) \]

Fourier transform of the \( \delta \) function

\[ f(Q) = \frac{e^{-Q}}{2\pi} \int_{-\infty}^{\infty} dt \exp(it e^{-Q}) \times \sum_{n_i=0}^{\infty} \prod_i \frac{\exp(-\tilde{n}_i)}{n_i!} \left[ \tilde{n}_i \exp\left(-\frac{itS_0}{S} \exp(-u_i)\right) \right]^{n_i} \]

Summation over \( n_i \)

In the continuous limit

\[ f(Q) = \frac{e^{-Q}}{2\pi} \int_{-\infty}^{\infty} dt \exp\left\{ it e^{-Q} + S \int_{0}^{\infty} du \rho(u) \left[ \exp\left(-\frac{itS_0}{S} e^{-u}\right) - 1 \right] \right\} \]

Each type of punctures is Poisson-distributed

\[ p(n_i) = \frac{\exp(-\tilde{n}_i)}{n_i!} \tilde{n}_i^{n_i} \]
For $\nu > \nu_d$ the distribution function is gaussian

$$\Delta = \ln \sigma - \ln \langle \sigma \rangle$$

$\Delta_0$ distribution function of $\ln \sigma$

$$f(\ln \sigma) = \frac{\exp\left(-\Delta^2/2\Delta_0^2\right)}{(2\pi)^{1/2} \Delta_0}$$

$1 < \nu < \nu_d$ non-gaussian but the peak position is still determined by optimal punctures

$$f(\ln \sigma) = \frac{1}{\pi \Delta_0} \int_0^\infty dt \exp\left(t^\phi \cos \frac{\pi \phi}{2}\right) \cos\left(t \frac{\Delta}{\Delta_0} + t^\phi \sin \frac{\pi \phi}{2}\right)$$

$\nu < 1$ the peak position is determined by punctures with $u = u_f > u_{opt}$

$$f(\ln \sigma) = \frac{e^\Delta}{\pi} \int_0^\infty dt \exp\left(-t^\phi \cos \frac{\pi \phi}{2}\right) \cos\left(te^\Delta - t^\phi \sin \frac{\pi \phi}{2}\right)$$
For $\nu < 1$ the variance is given by

$$D = \langle (\ln \sigma)^2 \rangle - \langle \ln \sigma \rangle^2$$

Asymptotic form for small area, $\nu \leq 1$

$$D(\nu) = \frac{\pi^2}{6} \left( \frac{1}{\varphi^2(\nu)} - 1 \right)$$

The probability density function is

$$f(\ln \sigma) = \varphi \exp(-\varphi \Delta - e^{-\varphi \Delta})$$

The cumulative distribution function is

$$F(\sigma) = \frac{\varphi}{\sigma} \left( \frac{\sigma_v}{\sigma} \right)^{\varphi} \exp\left[ -\left( \frac{\sigma_v}{\sigma} \right)^{\varphi} \right]$$

longer tail towards large conductivities

$\varphi(\nu) = 0.5$

$\varphi(\nu) = 0.25$

$\varphi(\nu) = 1.44$

$\varphi(\nu) = 1.69$

$\varphi(\nu) = 1.96$
Is it possible to measure the distribution function experimentally?

A single sample of a small area in external field $v < 1$ with random component

transmissions of two most transparent punctures within a given sample have different rate of change with the difference in log-transmissions exceeds the width of the distribution function at

```
\delta F \sim F_0 \left[ \beta \Omega(u_f) \right]^{1/2}/u_f \varphi(v) = F_c
```

\[ \delta u = -u_f \delta F/F_0 \]
\[ \delta u' \sim u_f \frac{\delta F}{F_0 \left[ \beta \Omega(u_f) \right]^{1/2}} \]

\[ \delta F \sim F_0 \left[ \beta \Omega(u_f) \right]^{1/2}/u_f \varphi(v) = F_c \]

$\Rightarrow$ “individuality” of the sample is forgotten

distribution function of mesoscopic fluctuations of conductivity of a given sample yields the distribution functions over samples
Hopping Conductivity in One Dimension

Juhani Kurkijärvi

Laboratory of Solid State Physics and School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14850

(Received 31 August 1972)

Mott’s law in 1D

\[
\ln \sigma = - (T_0 / T)^{1/2}
\]

\[T_0 = 1 / ga.\]

resistance of 1D sample

a break in a one-dimensional chain
For a given resistance of a break its area is the smallest if it is rhombus-shaped with the diagonals $\frac{au}{2}$ and $2uT$ the resistances between two sites arranged symmetrically on the opposite sides of the rhombus are the same.
Any decrease of the rhombus area results in the decrease of its resistance.

The area of the rhombus $A = \frac{1}{2} a Tu^2$

Contribution of breaks to the resistance $R(u) \propto \exp(u) \exp(-gaTu^2/2)$

The product is maximal for $u = (gTa)^{-1} = T_0/T$ $\ln \langle R \rangle = T_0/2T$ much bigger than $\langle \ln R \rangle = \left(\frac{T_0}{T}\right)^{1/2}$.
Conductance in Restricted-Dimensionality Accumulation Layers

A. B. Fowler, A. Hartstein, and R. A. Webb

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 20 August 1981)

FIG. 1. The upper part shows an idealized plan of a sample. The two $n^+$ regions are the source and drain. The $p^+$ regions are the control electrodes. In this case the $n$-type substrate was $10$-$\Omega$-cm Si. The width between the controls was $1-2$ $\mu$m. The length of the controls is $14$ $\mu$m. The lower part shows a section through the device along the dotted lines. The diffusions were about $1$ $\mu$m deep and the oxide was $300$ A thick. Potential lines are sketched for a positive gate voltage.

FIG. 3. The log of conductance plotted as a function of $T^{-1/2}$ for various gate voltages ($V_g$) with the substrate and controls grounded to the source. The solid lines are the best least-squares fits to the data.

FIG. 4. The power laws that best fit the data in Fig. 3 as a function of gate voltage. The error bars are estimated by deleting points from the ends of the fits. The error from the least-squares fit parameters is better. The open triangles are fits to a restricted low-temperature range only.

R.A. Webb
Upon decreasing temperature:

**FIG. 2.** Conductance as a function of gate voltage for three temperatures.

**FIG. 1.** Conductance as a function of gate voltage on an expanded gate voltage scale for selected temperatures. Only the large peak is displayed for the 65- and 36-mK curves.

No flat-topped peaks characteristic of one-hop-controlled processes were observed.
Nonmonotonic Variations of the Conductance with Electron Density in ~70-nm-Wide Inversion Layers

R. F. Kwasnick, (a) M. A. Kastner, (a) J. Melngailis, and P. A. Lee (a)

Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 18 August 1983)

The conductance of metal-oxide-silicon field-effect transistors with ~70-nm-wide inversion layers exhibits nonmonotonic variations with electron density below 15 K. The variations are largest at low electron concentrations and are the result of variations of the activation energy $E_A$. When $E_A$ is largest the current is found to be limited by spatial barriers which contain tunneling channels at discrete energies, as in the model of Azbel.

**FIG. 1.** Left: Schematic top view of the narrow-gate MOSFET. The resistivity of the $p$-type Si substrate is 3 Ω-cm. Right: Cross section through the device along the dotted line in the left figure. Shown are the narrow inversion layer situated under the narrow gate and the boundary of the depletion region.

The narrow gate is created by first reactive-ion etching a 50-nm step down into the 100-nm-thick gate oxide using photoresist as the mask, and then evaporating Al into the step at a glancing angle to the surface.

As sketched in Fig. 1, wide gates overlap the $n^+$ regions so that electrical contact to the narrow inversion layer is made through ~1-mm-wide inversion layers.

**FIG. 2.** Current vs gate voltage. The arrow indicates the gate voltage (2.585 V) of the deep minimum explored in this Letter.

In Fig. 2 we show an expanded version of the structure near threshold. When $V_G$ is increased beyond the first few maxima the conductance decreases by as much as three orders of magnitude at 2 K with a gate-voltage change of 0.05 V. Note that the inversion layer contains only $10^3$–$10^4$ electrons because it is so narrow, and the large decrease in the conductance is achieved with an increase of $V_G$ corresponding to the addition of only ~200 electrons to the inversion layer.
Variable-Range Hopping in Finite One-Dimensional Wires

Patrick A. Lee

Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 15 June 1984)

\[ \alpha = 0.02 \]

\[ -0.5 < E_i < 0.5 \]

\[ \ln(R_{ij}/\gamma) = 2\alpha|x_i - x_j| + (1/2kT)(|E_i - \mu| + |E_j - \mu| + |E_i - E_j|) \]

dependence of the resistance between two sites on the gate voltage

percolation simulation in 1D

at low temperatures plateaus are absent

current flows when resistors with \( R_{ij} < R_c \) are switched on

in experiment
the sample length \(~\sim 10 \mu m\)
exceeds \(~\sim 20-70\) times the localization length

Average log-resistance obeys a 1D Mott’s law

\[ \langle \ln R \rangle = \left( \frac{T_0}{T} \right)^{1/2} \]

over Fermi level positions
Qualitative picture of mesoscopic fluctuations: switching of most resistive breaks

![Schematic diagram](image)

The rhombus resistance will fall off as
\[ \exp(-|\mu - \mu_0|/T) \]

**absence of plateaus**: sites on opposite sides of the Fermi level never determine the resistance

**Quantitative description**

\[ \rho(u) = g Tu e^{-\Omega(u)} \]

\[ \Omega(u) = \frac{1}{2} g T a u^2 = Tu^2/2T_0 \]

\[ v < 1 \]

\[ L \rho(u_f) \sim 1 \]

**Mott’s law**

\[ \ln \left( \frac{R(v)}{R_0} \right) = u_f(v) = \frac{v^{1/2} T_0}{T} = \left\{ 2 \frac{T_0}{T} \ln \left[ \frac{L}{a} \left( \frac{T}{T_0} \ln \frac{L}{a} \right)^{1/2} \right] \right\}^{1/2} \]

**Fig. 15.** (a) Schematic of internal and mutual break switching in a one-dimensional chain with variation of the Fermi level; (b) Fluctuations of the chain ln \( \sigma \) with variation of the Fermi level. The dashed lines specify the variation of the resistances of individual breaks.
Width of the distribution function:

$$\varphi(v) = v^{1/2} = \Omega'(u_f(v))$$

$$\Delta \sim \frac{1}{\nu^{1/2}} = \left(\frac{T_0}{2T}\right)^{1/2} \ln^{-1/2} \left[ \frac{L}{a} \left(\frac{T}{T_0} \ln \frac{L}{a}\right)^{1/2} \right]$$

$$\frac{\Delta}{\langle\ln R\rangle} = \frac{T}{T_0 \nu} = \frac{1}{2 \ln \left[ \frac{L}{a} \left(\frac{T}{T_0}\right)^{1/2} \right]}$$

On the other hand, this difference should be $\sim \Delta$

“Period “of mesoscopic fluctuations

For a general $\mu$ the resistance is dominated by two breaks shifted in energy by $2\delta\mu_c$ their log-resistances differ by $\sim \delta\mu_c / T$

On the other hand, this difference should be $\sim \Delta$
New aspects of variable-range hopping in finite one-dimensional wires

R. A. Serota, R. K. Kalia,* and P. A. Lee
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 16 August 1985)

\[ \Delta^2 = \langle (\ln \rho - \langle \ln \rho \rangle)^2 \rangle \]

\[ \Delta^2 = 4.9 \]

It is evident that the distributions are asymmetric with longer high-resistance tails, which is a characteristic of all the systems. These distributions also become narrower with the increase in length of the systems.

\[ D(\nu) = \frac{\pi^2}{6} \left( \frac{1}{\phi^2(\nu)} - 1 \right) \]
\[ \phi = 0.5 \]

\[ f(\Delta) = \frac{1}{2\pi^{1/2}} \exp \left( -\frac{\Delta}{2} - \frac{1}{4} e^{-\Delta} \right) \]

**FIG. 1.** Distributions of \( \ln \rho \) for chains of \( N = 2000 \) sites (3! ensembles) and \( N = 9000 \) sites (36 ensembles), respectively. Note the high-resistance tails, and that the distribution of the longer chain is narrower.

*Igor Ruzin*
Wide samples

Fig. 21. Mesoscopic fluctuations in the conductance of MOSFET with variation of gate voltage (Laiko et al. 1987) for different temperatures.

Fig. 20. (a) Cross-section of MOSFET (Orlov et al. 1986, Laiko et al. 1987): 1, conducting channel in GaAs; 2, depletion region; 3, semi-insulating GaAlAs layer; 4, conducting substrate; (b) Band diagram of the structure.

\( w = 200 \, \mu m \quad l \approx 1.5 - 2 \, \mu m \)
Statistical properties of mesoscopic conductivity fluctuations in a short-channel GaAs field-effect transistor


Institute of Radio Engineering and Electronics, USSR Academy of Sci.
Submitted 15 June 1989

FIG. 1. Conductivity fluctuations in the channel of a GaAs FET at various temperatures $T$: 1.5; 2.0; 2.5; 3.0; 3.5; 4.2; 6.0; 8.0 K (curves 1–8).

Alexander Savchenko
1951 - 2010

FIG. 3. Histogram of the distribution of the log conductivity of the experimental dependences for the three temperatures 1.5 (a), 4.2 (b) and 8 K (c) (curves 1, 6 and 8 of Fig. 1) after subtracting the monotonic part from them. The smooth curves show the corresponding approximating functions.
The experimental histograms for all eight temperatures are approximated by the same function (11), in which the parameter \( w(T) \) is determined each time by minimizing the mean-square deviation of the theoretical curve from the experimental histogram.

\[ f(Q) = \frac{1}{\pi w} \int_{0}^{\infty} \exp \left( -\frac{\pi x}{2} \right) \cos \left( \frac{x \Delta_1}{w} - x \ln x \right) dx \]

for \( |1 - \varphi| \ll \left( \Omega''(U_{opt}) \right)^{1/2} \)

FIG. 4. Histogram resulting from averaging over the data reduced to \( T = 1.5 \) K for eight temperatures, and the corresponding theoretical distribution function of the log conductivity.

FIG. 5. Temperature dependence of the channel conductivity for various values of \( |V_g| \): 1—1.105, 2—1.117; 3—1.148; 4—1.157; 5—1.195; 6—1.200; and 7—1.241 V (curves 1–6 are arbitrarily shifted along the ordinate axis).
as the temperature is increased a transition should occur from the regime of chainlike conduction to a regime where the sample conductivity is determined by a rather small number of regions having the form of branched clusters.

FIG. 6. Temperature dependence of the experimental value of the width $w$ of the distribution function and the calculated parameters $w$ and $Q_0$ for two values of the frequency $\tau_{ph}^{-1}$: $10^{11}$ sec$^{-1}$ (curve 1) and $10^{12}$ sec$^{-1}$ (curve 2).
Distribution-function analysis of mesoscopic hopping conductance fluctuations

R. J. F. Hughes
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom
and Minerva Center, Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics,
Bar-Ilan University, Ramat-Gan 52900, Israel

A. K. Savchenko
Department of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

J. E. F. Frost, E. H. Linfield, J. T. Nicholls, and M. Pepper
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

E. Kogan and M. Kaveh
Minerva Center, Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics, Bar-Ilan University,
Ramat-Gan 52900, Israel

\[
\langle Q^2 \rangle - \langle Q \rangle^2 = \frac{\pi^2}{6} \left( \frac{1}{\nu} - 1 \right)
\]

\[
s = \langle (\ln R - \langle \ln R \rangle)^2 \rangle^{1/2} \sim \left( \frac{T_0}{T} \right)^{1/2} \left[ \ln \left( \frac{2L}{\xi} \right) \right]^{-1/2}
\]

FIG. 2. (a) Part of the experimental characteristic on a logarithmic conductance scale from a 19.4 \times 0.6 \mu m 1D Si MOSFET. Dotted lines demarcate gate-voltage intervals used for averaging.
FIG. 3. Conductance fluctuations from a $1.8 \times 0.2 \mu m$ 1D GaAs device and experimental DFs obtained from five adjacent gate-voltage intervals spanning the characteristic. At low gate voltages, the distribution is no longer exponentially wide while at high gate voltages the conductance becomes too small to measure. In between is a region where good fits to the theoretical 1D DF (solid curves) can be obtained.

FIG. 8. (a) DFs obtained from a $2 \times 100 \mu m$ Si MOSFET as a function of magnetic field and their fits to the 2D theory. (b) The average magnetoconductance obtained by the direct and fitting methods is negative. (c) The standard deviation of the distribution obtained by the two methods increases slightly.
FIG. 5. Temperature dependence of the fluctuations from a 19.4 × 0.6 μm Si MOSFET. (a) Experimental DF's fit by the 1D theoretical form. (b) Fit to Eq. (1) of the average of lnG obtained both directly (hollow circles) and from the position $\Delta_0$ of the fitted DF’s (filled circles). (c) Temperature dependence of the fluctuation amplitude $s$ both measured by the standard deviation of the data points (hollow circles) and calculated from the fits to the 1D DF’s (filled circles). The gradient of the latter yields a $T^{0.65}$ power law. (d) Fitting the fluctuation amplitude to a $T^{1/2}$ power law yields the prefactor 0.35.

Fits of these data to Eq. (1) averaged over the marked gate-voltage intervals together with the values of $T_0$ extracted below 0.3 K.
FIG. 4. Distribution functions obtained from three Si MOSFET's fabricated on the same chip showing the characteristic 1D and 2D asymmetries. Lithographic channel dimensions are (length \times width): (a) 2 \times 100 \mu \text{m} fit by 2D DF, (b) 5 \times 2 \mu \text{m} fit by 1D DF, and (c) 1.5 \times 100 \mu \text{m} fit by a Gaussian. Inset are the regions of the characteristic from which the histograms were obtained.
Conductance fluctuations in large metal-oxide-semiconductor structures in the variable-range hopping regime

Dragana Popović*
Department of Physics, Brown University, Providence, Rhode Island 02912

A. B. Fowler and S. Washburn
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

P. J. Stiles
Department of Physics, Brown University, Providence, Rhode Island 02912

Conductance fluctuations due to variable-range hopping have been studied in 8-mm-wide silicon inversion layers of large area (3.2 mm²). The temperature dependence of the average logarithm of conductance \( \langle \ln G \rangle \) varies with the carrier density \( N_s \) from nearly activated to very weak. Fluctuations of \( \ln G \) with the chemical potential \( \mu \) occur on two different scales. The distribution function of the fluctuations in \( \ln G \) is also analyzed, and the results are consistent with the model of conduction via exponentially rare, highly conducting, quasi-one-dimensional chains of hops.

on much larger samples with length \( L = 0.4 \text{ mm} \) and width \( W = 8 \text{ mm} \). Several features of our data are consistent with the model of conduction via quasi-1D chains (punctures).

FIG. 1. Conductance vs gate voltage at (a) \( T = 0.555 \text{ K} \), (b) \( T = 0.420 \text{ K} \), (c) \( T = 0.330 \text{ K} \), (d) \( T = 0.090 \text{ K} \). Inset: The “best” exponent \( n \) for different gate voltages. The dashed line is a guide to the eye.
FIG. 2. (a) Fluctuations in the conductance logarithm with the gate voltage at $T = 330$ mK. (b) Histograms of the distribution of the conductance logarithm for $0.40 \, \text{V} \leq V_g \leq 0.42 \, \text{V}$ at two temperatures.