Nonequilibrium dynamics in Coulomb glasses near the metal-insulator transition

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Lecture I:  Metal-insulator transition and complexity in electronic systems

Lecture II: Studies of the electron dynamics near the 2D MIT: Relaxations of conductivity

Lecture III: Studies of the electron dynamics near the 2D MIT: Fluctuations of conductivity

- Phase diagram and aging in a 2DES in Si – brief summary
- Noise measurements as a probe of glassiness in low- and high-mobility samples:  - probability density functions
  - noise power spectrum
  - second spectrum
- Noise measurements in a magnetic field
- Theory, simulations, experiments in other systems
Phase diagram of a 2DES in Si

**Glassy Behavior** (for \( n_s < n_g \))

- **Insulating**
  \( \sigma(T \to 0) = 0 \)

- **Metallic** (Non-Fermi Liquid)
  \( \sigma(T \to 0) \neq 0 \)

- **Metallic** (FL? NFL?)

**Glassy regime:**
- slow, nonexponential relaxations
- diverging \( \tau_{eq} (T_g = 0) \)
- aging (change at the MIT)

Waiting time protocol for each density \( n_s \):
- fix \( n_s \) at 10 K
- cool down to meas. \( T \)
- change \( n_s \) to a different value during \( t_w \)
- back to the initial density and measure \( \sigma(t) \)
- warm up to 10 K; repeat for different \( t_w \)

What about aging at lower \( T \)?
Waiting time protocol at $T=0.24$ K

slowly relaxing
$\langle \sigma(t) \rangle$ subtracted
(only top 3 traces)

Much more noise after $t_w$ (during aging) than before

Before: signal **noisier**
at low $n_s$

Study fluctuations (noise) at low $T$
provides **complementary information**;
so far, only mean values of conductivity

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Noise in Laponite (colloidal suspension)

Probability density function (PDF)

\[ P(V) \]

Age \( t_{w1} < t_{w2} < t_{w3} \)

non-Gaussian \( \downarrow \)
Gaussian

as it ages and approaches equilibrium

(Buisson et al., Proc. SPIE 5469, 95 (2004); edited by Popović, Weissman, Rácz)

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Noise protocol:
- start from a fixed $n_s$ at high $T$ (*e.g.* 10 K)
- cool down to measurement $T$ (< 1 K)
- measure $\sigma(t)$
- warm up to ~1 K; change $V_g$ (density $n_s$) by a small amount – *no observable relaxations*
- cool down to measurement $T$ (< 1 K)
- measure $\sigma(t)$ for this new density
- repeat (warm up to ~1 K, change $n_s$ in a small step, cool down, measure $\sigma(t)$)

**Noise protocol:** *density changed in tiny steps at ~1 K (not at 10 K)*
Noise in low-mobility Si MOSFETs

[S. Bogdanovich and D. Popović, PRL 88, 236401 (2002)]

Conductivity

• relative fluctuations are of the order of 100% at low \( n_s \) and \( T \)

• noise decreases with increasing \( n_s \) and \( T \)

• non-Gaussian

• the character of the noise changes dramatically as \( n_s \) is varied: at high \( n_s \), the variance no longer varies with time

\[
\delta \sigma^2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle
\]

Large, slow, and non-Gaussian at low \( n_s \) and \( T \)!

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\[ \delta \sigma = \langle (\sigma - \langle \sigma \rangle)^2 \rangle^{1/2} \]

- a dramatic change of \( d(\delta \sigma)/dn_s \) at \( n_c \) (2D MIT)
- a dramatic increase of noise (\( \delta \sigma/\langle \sigma \rangle \)) at \( n_g \) (glass transition)
- glassy phenomena observed below \( n_g \)

\( T = 0.130 \) K
\( T = 0.196 \) K
\( T = 0.455 \) K
\( T = 0.805 \) K
Probability density functions (PDF) of fluctuations

- PDF: - Gaussian for \( n_s > n_g \)
  - non-Gaussian for \( n_s < n_g \)

Low-mobility sample

\( T = 0.13 \, \text{K} \)

\( n_c < n_s (10^{11} \, \text{cm}^{-2}) = 5.58 < n_g \)

(metallic glassy phase)

Each time interval \( \approx 35 \, \text{minutes} \)

- structure changes with time

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PDF for sixteen different 35 minute intervals:
- broadens with the increasing sampling time but never becomes Gaussian

• nonergodic on experimental time scales

Low-mobility sample

$T=0.13\ \text{K}$

$n_c < n_s (10^{11}\ \text{cm}^{-2}) = 5.58 < n_g$

Each time interval $\approx 35\ \text{min}$
Noise power spectrum (low-mobility samples):

Power spectrum of \[ \frac{\sigma(t) - \langle \sigma \rangle}{\langle \sigma \rangle} \]:

\[ S_I \propto \frac{1}{f^\alpha} \]

- a sudden and dramatic slowing down of the electron dynamics for \( n_s < n_g \approx 7.5 \times 10^{11} \text{ cm}^{-2} \).
Noise exponent $\alpha$ has a **sudden jump** at $n_s = n_g$!

\[ S_I \propto 1/f^\alpha \]

- A sudden shift of the spectral weight towards lower $f$ indicates a sudden (sharp) and dramatic slowing down of the electron dynamics at $n_g$.
- Observable only at low enough temperatures.

**Use noise to identify the glass transition!**

[Bogdanovich and Popović, PRL 88, 236401 (2002)]
Resistance noise and power spectra in high-mobility (low disorder) Si MOSFETs

\[ S_R \propto \frac{1}{f^\alpha}, \quad 1 \leq \alpha \leq 2 \]

\[ T = 0.24 \, \text{K} \]

[Jaroszyński, Popović, Klapwijk, PRL 89, 276401 (2002)]
Noise in samples with low disorder:

- a dramatic increase of the low-frequency noise at low $n_s$ and $T$
- a rapid increase of the exponent $\alpha$ with decreasing $n_s$

$\Rightarrow$ Glass transition at $n_g \approx 10 \times 10^{10} \text{ cm}^{-2} \geq n_c$

Glass transition and the MIT almost coincide – no intermediate phase

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Noise in high-mobility Si MOSFETs:

- non-Gaussian at low $n_s$ and $T$

- variance changes with time

\[ \langle \rho - \langle \rho \rangle \rangle \delta \rho \]

\[ \delta \rho^2 = \text{variance} \]

$T = 0.24$ K

$n_s (10^{10} \text{ cm}^{-2}) =$

- 8.7
- 9.27
- 9.56
- 9.63
- 9.86
- 10.43
- 11.15
- 11.87

check higher order correlations

[Weissman: Rev. Mod. Phys. 60, 537 (1988); Rev. Mod. Phys. 65, 829 (1993)]
Second spectrum – “noise of the noise”
(Voss & Clarke, Restle & Weissman, Seidler & Solin)

- $S_2(f_2, f)$: power spectrum of the fluctuations of $S_R(f)$ with time
  - i.e. Fourier transform of the autocorrelation function of the time series of $S_R(f)$; fourth-order noise statistic

- probes correlations between fluctuators:
  - uncorrelated fluctuators
    - $S_2(f_2, f)$ white;
  - correlated fluctuators
    - $S_2 \propto 1/f_2^{1-\beta}$ (non-Gaussian)
Second spectrum in a 2DES in Si:

- a dramatic change in $S_2$ near $n_g$: $S_2$ white – uncorrelated at high $n_s$, $S_2$ nonwhite – correlated below $n_g$
- high-mobility samples: $n_c \leq n_g \sim n_s^*$ (no intermediate phase)
- low-mobility samples: $n_c < n_g \approx 1.5n_c < n_s^*$ (wide intermediate phase)

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Scaling of the second spectra

- no systematic dependence on $f$ ⇒ scale invariance
- consistent with the hierarchical picture of glassy dynamics
- rules out interacting two-state systems (droplets, clusters, defects, …) as possible sources of noise

(measured at $f = (f_L, 2f_L)$)

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Noise measurements in parallel magnetic fields:
spin or charge?

- A sufficiently strong magnetic field is expected to destroy the spin glass order; 2DES fully spin polarized for $B \approx 2-4$ T

![Graph showing noise measurements](image)

$S_R \propto 1/f^\alpha$

$\alpha = 1.58$
$\alpha = 1.24$
$\alpha = 0.96$
$\alpha = 0.58$

$n_s = 11.9 \times 10^{11} \text{ cm}^{-2} > n_c(B=0)$

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The same criteria for glass transition:

- a sudden and dramatic increase of $S_R$, a rapid rise of $\alpha$, and a change of $(1-\beta)$ at $B_g(n_s)$ at low $B$ and $n_g(B)$ at high $B$
Phase diagram in B

- glass transition persists in high parallel B, where the 2DES is spin polarized

$n_g$ – glass transition

- charge, as opposed to spin, responsible for glassy ordering

Coulomb glass
Global phase diagram (theory)

- (2D) MIT as a Mott transition with disorder (DMFT picture)
- glass as a precursor of MIT
- melting of glass even at T=0 (by quantum fluctuations)
- Metallic glass phase: $\sigma(T) - \sigma(0) \sim T^{3/2}$
- hierarchical, correlated dynamics

Physical trajectory: $E_F \sim n_s; U \sim n_s^{1/2}; W \sim \text{const.}$

1. High-mobility samples
2. Low-mobility samples

[V. Dobrosavljevic et al.: PRL 83, 4642 (1999); PRB 66, 081107 (2002); PRL 90, 016402 (2003); PRL 91, 066603 (2003); EPL 67, 226 (2003); PRL 94, 046402 (2005)]
Simulations

- **Molecular Dynamics** [C. Reichhardt and C. J. Olson Reichhardt, PRL 93, 176405 (2004): a classical model of interacting electrons in 2D with disorder]

- increase of noise power and $\alpha$ with decreasing density and $T$

- non-Gaussianity at low $n_s$ and $T$

Similar to experiments in 2DES in Si

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Noise power and $\alpha$ maximum

Trajectories change with time: dynamical inhomogeneities

$\bullet$ Monte Carlo [Kolton, Grempel, Dominguez, PRB 71, 024206 (2005)]:
3D Coulomb glass - heterogeneous dynamics

FIG. 5. Electron trajectories for a fixed period of time for fixed $T = 0.09$ at (a)$N_s/N_p = 1.67$, (b) 1.37, (c) 0.5, and (d) 0.3.
Experiments in other systems

- $1/f^\alpha$ noise in 2D holes in GaAs (much less disorder than in Si) – some similarities, but no changes in $\alpha (\approx 1)$ and $(1-\beta) (\approx 0)$
- not low enough $p_s$, $T$, $f$, or disorder?

[Leturcq, L’Hôte, Tourbot, Mellor, Henini, PRL 90, 076402 (2003)]

\[ p_c = 1.44 \times 10^{10} \text{cm}^{-2} \]
\[ p_s^* = 1.57 \times 10^{10} \text{cm}^{-2} \]
• $1/f^\alpha$ noise in bulk Si:P(B) - a huge increase of noise and the onset of non-Gaussianity at the MIT!!! Similar to 2DES in Si!

Emergence of nanoscale inhomogeneities!

High temperature superconductors

[C. Panagopoulos and V. Dobrosavljević, PRB 72, 014536 (2005)]

Glassy insulator?

Metal-insulator transition?

Inhomogeneous, conducting glassy state?

Homogeneous metal

Quantum glass transition?
High temperature superconductors

Glassy insulator | Metallic glassy phase ($|k_F| < 1$; “bad” metal) | Metal

$T_N$ | Marginal Fermi Liquid | $T_m$

$T^*$ | Pseudogap | $T_c$

Antiferromagnet

Spin Glass

Superconductor

SC-Glass

MIT in a 2DES in Si at $T=0$

[Coulomb glass; $T_g=0$]

$n_c$ | metal-insulator transition | $n_g$ | glass transition

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Lightly doped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \)

\( x \geq 0.02 \): short-range magnetic order for \( T > T_{sg} \)
(\( T_{sg} \) - spin glass transition)

- AF domains; holes along domain walls

3D long-range antiferromagnetic (AF) order:

\( x = 0.03 \) carrier concentration

\( T_{sg} \sim 7-8 \) K

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Resistance fluctuations (noise); LSCO

- noise Gaussian at “high” T (e.g. T > 0.18 K for $R_{ab}$ noise)
- at low T, non-Gaussian noise metastable states (out-of-equilibrium)
Noise in $R_{ab}$

- Structure depends on the observation time – different states contribute $\Rightarrow$ nonergodic

$T = 0.082 \text{ K}$
$B = 0 \text{ T}$

3 h intervals

$\Delta R / R(10^{-4})$
- increase sampling time to 12 hours, but never becomes Gaussian at low $T$

- nonergodic, does not reach equilibrium on experimental time scales at low $T$

Onset of glassiness in transport at $T \ll T_{SG}$: suggests spin and charge glass not directly related.
Noise statistics: T and B dependence

- Power spectrum: $S_R \sim 1/f^\alpha$

\[ \alpha \text{ increases as } T \text{ is reduced; no effect of } B! \]

fewer metastable states that dominate at low $T$ in the exp. time window

\[ \text{Slowing down of the dynamics as } T \to 0 \]
Second spectrum $S_2(f_2, f)$

- the power spectrum of the fluctuations of $S_R(f)$ with time

1) white ($1-\beta = 0$) for uncorrelated fluctuators (Gaussian)

2) $S_2(f_2, f) \propto 1/f_2^{1-\beta}$ for interacting fluctuators (non-Gaussian)

Increase of correlations as $T \to 0$

Noise statistics independent of both $B$ and magnetic history

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(Partial) summary of noise results in LSCO

- Slowing down of the dynamics as $T \to 0$
- Increase of correlations as $T \to 0$
- Slowing down of the dynamics as $T \to 0$
- Increase of correlations as $T \to 0$

Glass transition at $T=0$

- Noise statistics independent of both $B$ and magnetic history (unlike conventional spin glasses) $\Rightarrow$ charge, not spin!
- Onset of hysteretic and memory effects in magnetoresistance:
  $$T_{\text{onset}} \ll T_{\text{sg}}$$

$\Rightarrow$ Charge glass transition $T_{\text{cg}} = 0$

[I. Raičević et al., PRL101, 177004 (2008)]

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Scaling of the second spectra

- can distinguish between different models:
  - droplet approach
  - hierarchical, tree-like model

$S_2$ decreases with $f$ for a fixed $f_2/f$, consistent with droplet picture (short-range interactions)

Spatial segregation of holes as a result of competing interactions on different length scales

Cluster charge glass
Summary of Lecture III

- Noise study in a 2DES in Si:
  - sharp transition to a glassy phase as a function of density
  - glass transition at $T=0$
  - glassy phase: slow, correlated, hierarchical dynamics
  - glassy ordering due to charge, not spin $\Rightarrow$ Coulomb glass

- Noise study in lightly doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$:
  - transition to a glassy phase at $T=0$
  - glassy phase: slow, correlated dynamics; clusters
  - glassy ordering due to charge, not spin $\Rightarrow$ Coulomb glass

- Noise as a probe of glassy dynamics in other systems (e.g. doped Si)

_Glasses everywhere…_