Optical Spectroscopy (II) [Armitage]

- Example: Simple Metals
  - In general, reflectivity looks like:
  - The screened plasma frequency is given by \( \omega_p = \frac{4\pi\varepsilon_0 n_e}{m^*} \)
  - Ag & Au has similar mass/density, but \( \omega_p \) differ by factors of 2, since Au has more interband processes, \( \Rightarrow \) higher \( \varepsilon_0 \).
  - Width of peak \( \sim \) scattering rate near \( \omega_p \).

- Semiconductors & band insulators
  - Dominated by interband transition at finite energy.
  - Phonons can also appear as peaks, but its energy will be different from that of excitons (excitons generally appear near the gap threshold).
  - While a semiconductor is doped, new "impurity" band appears.

- Hole donor & e⁻ donor will look similar, since optical spectroscopy creates electron-hole pairs, hence states related by particle-hole symmetry looks the same.

- Doping semiconductors, as doping (disorder) increases, there will be a conductor.

**Note:** At high freq, high disordered systems & low disordered ones look similar, since we're probing at length scale shorter than disorder scale.
For electronic glass (metal + disorder), in non-interacting

\[ \sigma \propto \omega \cdot U(r) + \omega^2 \quad \text{where} \quad U(r) = \frac{e^2}{r_0} \epsilon_{\infty} \]

But even with the functional form \( \sigma \propto \omega \cdot U(r) + \omega^2 \), the sharp transition from \( \omega \omega \) to \( \omega^2 \omega^2 \) are still not captured.

- These \( e^- \)-glasses may be understood as Fermi glass: the state is glassy because of the Fermi statistics, and interaction is non-essential (as oppose to Coulomb glass).
- At low freq. & low temperature, the electrons are localized by disorder. So conductivity is better understood as dissipation

- Mott Insulator

\begin{align*}
\text{Mott Insulator} & \\
\begin{array}{c}
\text{2N} \\
\text{N} \\
\text{2N-1}
\end{array} & \begin{array}{c}
\text{N} \\
\text{N-1} \\
\text{2N}
\end{array} & \begin{array}{c}
\text{2N} \\
\text{N-1} \\
\text{2N-1}
\end{array}
\end{align*}

\begin{align*}
\text{half-filled} & \\
\text{hole doped} & \text{e\textsuperscript{-} doped}
\end{align*}

\begin{align*}
\text{Band} & \\
\text{Mott}
\end{align*}
In Mott insulator, spectral weight transfer is faster as function of doping as compared to band insulator.

Example: Superconductor

- In perfect superconductor:
  \[ \sigma_1(\omega) = \frac{\pi}{2} \frac{N e^2}{m} \delta(\omega) \]
  \[ \sigma_2(\omega) = \frac{N e^2}{m \omega} \]

- At low energy, breaking of Cooper pairs.

- At finite temperature with disorder, one expects:
  \[ \sigma_{sc}(T) = \sigma_{1s}(T) + \frac{i N(T) e^2}{m \omega} + \sigma_{n}(T) \]
  \[ \delta\text{-fan piece} \]
  \[ \text{"normal" piece} \]

The more precise form given by Mattis–Bardeen formalism.

Case I & Case II differ by the coherence factors that enter into calculations. For case II, coherence factors cancel out the density-of-state singularity at 2\(T\). Case II occurs in s-swave sc.

For YBCO (d-wave)
Extended Drude Model

- Allow $m^*(\omega) \neq \tau(\omega)$ to be freq dependent (reasonable since interaction, e.g., phonon mediated attraction, can be time delayed)
  
  \[
  \frac{m^*(\omega)}{m} = -\frac{\omega_p^2}{4\pi^2} \Im \left\{ \frac{1}{\sigma(\omega)} \right\} 
  \]

  \[
  \frac{1}{\tau(\omega)} = \frac{\omega_p^2}{4\pi} \Re \left\{ \frac{1}{\sigma(\omega)} \right\} 
  \]

- Interaction increases optical mass over band mass at $\omega \to 0$ limit.

- Can use Matthiesen rule to estimate \( \frac{1}{\tau(\omega)} \), where the rates for different processes.

- To invert Drude formula to get $m^*(\omega) \neq \tau(\omega)$, we need the plasma freq. $\omega_p$ defined by \( \int_0^{\infty} \sigma_{\text{int}}(\omega) = \frac{\omega_p^2}{8} \). But it's hard to determine where $\sigma_{\text{int}}$ stop, so need to estimate.