Lecture #1

"Microscopic" Theory of Superconductivity

* $CeCu_2Si_2$ (1979) magnetism mediated SC. Kondo Lattice.

* Mueller 1985 Cu−O

* (2008) Iron based SC Fe-As
everywhere metallic , Spin = 2

Superconductivity Order parameter

$$\Phi^{\sigma\sigma'}_{kk'} = \langle \psi_{k\sigma}(t)\,\psi_{k'\sigma'}(t)\rangle \Rightarrow \Phi^{\sigma\sigma'}_{k} = \langle \psi^{\sigma}_{k}\,\psi^{\sigma'}_{-k}\rangle$$

usual case $k=-k'$ (no net momentum) $= -\Phi^{\sigma'\sigma}_{-k}$ from fermi statistics

set SOC=0

1° Assume parity is good quantum #

$$\begin{cases}\Phi^{\sigma\sigma'}_{k} = \Phi^{\sigma\sigma'}_{-k} & \text{even parity} \Rightarrow \Phi^{\sigma\sigma'}_{k} = -\Phi^{\sigma'\sigma}_{k} \quad \text{Singlet pairing}\\ \Phi^{\sigma\sigma'}_{k} = -\Phi^{\sigma\sigma'}_{-k} & \text{odd parity} \Rightarrow \text{triplet pairing}\end{cases}$$

2° More Symmetry

a) free space SO(3) spatial rotational sym.

Φ_k transforms as $Y_{Lm}(\hat{k})$ (basis Function of rep. of O(3))

$\Rightarrow \Phi_k = f(|k|)\, Y_{lm}(\hat{k})$

↳ according to l ⇒ s, p, d, … wave pairing.

$$\begin{cases}\text{even } l\ (s,d\dots) \Rightarrow \text{spin singlet}\\ \text{odd } l\ (p,f\dots) \Rightarrow \text{spin triplet}\end{cases}$$

for $l>1$, there's $2l+1$ degeneracies. (family of order parameters)

b). Crystal: discrete rotational sym.

⇒ some representations l ⇒ some basis functions $\Gamma^{a}_{l}(\hat{k})$

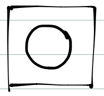
$$\Phi(k) \sim \Gamma^{a}_{l}(k)\, f(k)$$

↳ function with full sym of crystal.

$$\begin{cases}\text{"s-wave"} & \Gamma \text{ is trivial rep.}\\ \text{"non-s-wave"} & \text{some other rep.}\end{cases}$$

Non-s-wave must ⊥ to s-wave ⇒ generically has nodes (if it's real.)

e.g. 2D circle of F.S. Sym: O(2)



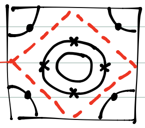
s 1

p p_x, p_y $(\cos\theta, \sin\theta)$

d $p_x^2-p_y^2,\ p_x p_y$ $(\cos 2\theta, \sin 2\theta)$

Sr_2RuO_4

nodal line



⇒ sym: C_4

s-wave Φ "same" everywhere $\sim f(k)$

extended s± wave $f(k)$ has sign structure.

$$\begin{cases}\bullet & +\\ \times & -\end{cases}$$

* Use the language of free space to describe pairing symmetry, but the degenerate properties are not the same as free space. In general, the degeneracy is lifted.

* $\Phi(R) = \int dk \, e^{ik \cdot R} \Phi(k)$, $\Phi(R=0) = \int dk \; \Phi(k)$

if we want $\Phi(R=0) = 0$ 1° $l > 1$ 2° $S\pm$ (change $f(k)$)

* Spin Orbit Couplings E. Blount PRB ~1984

1° TR $k\uparrow \simeq -k\downarrow$ each band is doable degenerate

$\quad\quad\quad\quad\quad\quad\quad\quad$ SI

2° I $k\uparrow \simeq k\downarrow$ ⇒ "pseudo-spin." $\hat{g}^{\alpha\beta}(k)$ spliting in H field

$\Phi^{\sigma\sigma'}(\hat{k}) \longrightarrow$ mapping $\longrightarrow$ topological

* Landau Theory:

$$\mathcal{F} = r|\Phi|^2 + \cdots = \sum_{l} \underset{\uparrow}{r_l} \sum_{a} |\Phi^a_m|^2 + \cdots$$

we can just consider the most unstable one

Example. Lonzarich / Monthoux ~1998

$$\mathcal{H} = \sum_{k,\sigma} \epsilon_k \psi^+_{k\sigma} \psi_{k\sigma} + \frac{1}{2}\int dr\,dr' \, V_\rho(r-r') \, n(r) n(r') + \frac{1}{2}\int dr\,dr' \, V_\sigma(r-r') \, \vec{\sigma}(r) \cdot \vec{\sigma}(r')$$

signs $\begin{cases} V_\rho(r-r') > 0 & \text{replusive} \\ V_\sigma(r-r') < 0 & \text{ferromagnetic} \end{cases}$

Hartree – Fock analysis

$$n(r) = \psi^+_\uparrow(r)\psi_\uparrow(r) + \psi^+_\downarrow(r)\psi_\downarrow(r)$$

F.T. ⇒ $n_Q = \sum_k \psi^+_{k+Q\uparrow}\psi_{k\uparrow} + (\uparrow \Leftrightarrow \downarrow)$

F.T. of V_ρ ⇒ $V_\rho(Q) \sum_{k,k'} \left[\left(\psi^+_{k+Q\uparrow}\psi_{k\uparrow} + (\uparrow\Leftrightarrow\downarrow)\right)\left(\psi^+_{k'-Q\uparrow}\psi_{k'\uparrow} + (\uparrow\Leftrightarrow\downarrow)\right)\right]$

$$= V_\rho(Q) \sum_{k,k'} \Big[\psi^+_{k+Q\uparrow}\psi^+_{k'-Q\uparrow}\psi_{k'\uparrow}\psi_{k\uparrow} + \psi^+_{k+Q\uparrow}\psi^+_{k'-Q\downarrow}\psi_{k'\downarrow}\psi_{k\uparrow} + (\uparrow\Leftrightarrow\downarrow) \Big]$$

we can also work out F.T. of V_σ.

in simple BCS

if no SC $E = \langle H \rangle = \langle KE \rangle$

Turn on SC , move ψ from wavefunc that optimizes KE

* ⇒ in SC state $|KE|$ goes down. (PE driven SC)

* May also have KE driven SC. ?

$\mathcal{H} = KE + V\langle\psi^+\psi^+\rangle\psi\psi + \cdots$ something else.

* it's very easy to put a variational parameter & make E go down

⇒ may or may not be an instability.

Define:

$$\Phi^{\sigma\sigma'}_p = \langle \psi_{p\sigma}\psi_{-p\sigma'} \rangle$$

$\Rightarrow$

$$(V_\rho(Q)+V_\sigma(Q))\left(\psi^+_{-p'\uparrow}\psi^+_{p'\uparrow}\,\Phi_p^{\uparrow\uparrow} + h.c. + (\uparrow\leftrightarrow\downarrow)\right) \rightarrow \text{triplet}$$

Heisenberg symmetry

$$\left\{\begin{array}{ll} \text{zz channel} & (V_\rho(Q) - 3V_\sigma(Q))\left(\psi^+_{-p'\uparrow}\psi^+_{p'\downarrow}\,\Phi_{\underline{\underline{p}}}^{\downarrow\uparrow} + h.c. + (\uparrow\leftrightarrow\downarrow)\right) \\ \pm\ \text{channel.} & 2V_\sigma(Q)\left(\psi^+_{-p'\uparrow}\psi^+_{p'\downarrow}\,\Phi_{\underline{\underline{p}}}^{\uparrow\downarrow} + h.c. + (\uparrow\leftrightarrow\downarrow)\right) \end{array}\right\} \text{singlet}$$

$$\Rightarrow \left\{\begin{array}{ll} \text{Singlet} & V_\rho - 3V_\sigma \\ \text{Triplet} & V_\rho + V_\sigma \end{array}\right.$$

Heisenberg sym helps with "3"

Lecture # 2

Remark: $KE = \sum_{k\sigma} (\epsilon_k - \mu) C^+_{k\sigma} C_{k\sigma}$

Non interacting G.S. $\langle KE \rangle < 0$

add S.C. $|\langle KE \rangle|$ decreases

Singlet BCS (Heisenberg) $(V_\rho - 3V_\sigma)\left[\psi^+_{-p'\uparrow} \psi^+_{p'\downarrow} \Phi^{\downarrow\uparrow}_p + h.c.\right]$

$$\mathcal{H} = \sum_{k,\sigma} \epsilon_k \psi^+_{k\sigma} \psi_{k\sigma} + \sum_p \Delta^{\downarrow\uparrow}_p \psi^+_{-p\uparrow} \psi^+_{p\downarrow} + h.c.$$

define: $\Delta^{\downarrow\uparrow}_{p'} = \sum_p (V_\rho - 3V_\sigma)_{p'-p} \quad \Phi^{\downarrow\uparrow}_p$

Nambu: $\Psi_k = \begin{pmatrix} \psi_{k\uparrow} \\ \psi^+_{-k\downarrow} \end{pmatrix} \qquad \mathcal{H} = \sum_k \Psi^+_k \begin{pmatrix} \epsilon_k & \Delta_k \\ \Delta^+_k & -\epsilon_k \end{pmatrix} \Psi_k$

$\Phi^{\downarrow\uparrow}_p = \langle \psi_{p\downarrow} \psi_{-p\uparrow} \rangle = \langle \Psi^+_p \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Psi_p \rangle$ (even parity)

Assume $\Delta^+ = \Delta$

$$\mathcal{H} = \epsilon_k \tau_3 + \Delta \tau_1 \Rightarrow E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$$

in the basis that diagonalize $\mathcal{H}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ eigenvalue $-E_k$, occupancy $f_F(-E_k)$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ E_k $f_k(E_k)$

let R be the diagonalization matrix

$$\Psi_- = R\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \Psi_+ = R\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Phi^{\downarrow\uparrow}_p = f(-E_k)\left[\langle 0 , 1| \underset{\downarrow \text{ depends on } \Delta}{R^+} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} R \left| \begin{matrix} 0 \\ 1 \end{matrix} \right\rangle\right] + f(E_k)\left[\langle 1 0 | R^+ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} R \left| \begin{matrix} 1 \\ 0 \end{matrix} \right\rangle\right]$$

self consistency $\Delta^{\downarrow\uparrow}_p = \sum_{p'} (V_\rho - 3V_\sigma)_{p-p'} \Phi^{\downarrow\uparrow}_p$

take $R = \cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\tau_2$

$$\Rightarrow \Delta_p = \sum_{p'} \underset{\underset{\text{must be negetive}}{\downarrow}}{\underbrace{(V_\rho - 3V_\sigma)}}_{p-p'} \underset{\text{negetive}}{\underline{\left[-\frac{1}{2} f(-E_{p'}) + \frac{1}{2} f(E_{p'})\right]}} \frac{\Delta_{p'}}{\sqrt{\epsilon_{p'}^2 + \Delta_{p'}^2}}$$

linearize gap function

$$\Delta_p = -\sum_{p'} V_{p-p'} \tanh\left(\frac{|\varepsilon_{p'}|}{2T}\right) \frac{\Delta_{p'}}{|\varepsilon_{p'}|}$$

$$\Delta_p = f(|p|) Y_{lm}(\hat{p}) \Rightarrow 1 = -\sum_{p'p'}\left[Y^*_{lm}(p) V Y_{lm}(p')\right] \ln\frac{\omega}{T}$$

must be negetive

$V(p-p')$ if $|p|=p_f$

$$= V_0 + \underbrace{\sum_m V_1 Y_{1m}^* Y_{1m}}_{\text{not relevant (parity)}} + \cdots V_2 Y_{2m}^* Y_{2m}$$

$$|p-p'|^2 = 2p_F^2\left(1-\cos(\theta_1-\theta_2)\right)$$

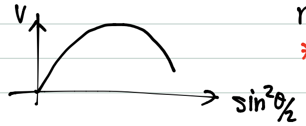
when V_2 is negetive

$$\Rightarrow \quad V = V\left(\sin^2\frac{\theta_1-\theta_2}{2}\right) = V_0 + V_1 \sin^2\frac{\theta_1-\theta_2}{2} + \boxed{V_2} \sin^4\frac{\theta_1-\theta_2}{2}$$

$$\frac{\partial V}{\partial \theta} = V_1 \frac{1}{2}\sin\theta + V_2 \sin\theta \sin^2\frac{\theta}{2}$$

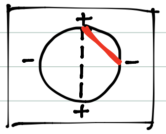
if $V_1 V_2 < 0 \quad \Rightarrow \sin^2\frac{\theta}{2} = -\frac{V_1}{V_2}$

$V_1 > 0,\ V_2 < 0$



repulsive everywhere, but $V_2 < 0$

* repulsive but with structure.



using the most repulsive interaction, make Δ change sign to gain energy.

For triplet pairing:

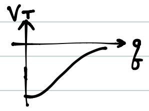
$$V_T(p-p')\left(\psi_{-p\uparrow}^+ \psi_{p\uparrow}^+ \Phi_{p'}^{\uparrow\uparrow} + h.c.\right) \qquad \Phi = \langle \psi_{p\uparrow} \psi_{-p\uparrow} \rangle$$

$$\Psi = \begin{pmatrix} \psi_{p\uparrow} \\ \psi_{-p\uparrow}^+ \end{pmatrix} \Rightarrow \mathcal{H} = \frac{1}{2}\Psi_K^+ (\epsilon_K \tau_3 - \Delta \tau_1) \Psi_K + \text{spin down.}$$

we can have fully polarized state with SC. (in Principle, but not found)

$\Rightarrow$ self consistent eq. $\quad \Delta_p = -\frac{1}{2}\sum_{p'} V_T(p-p') \tanh\left(\frac{|\epsilon_{p'}|}{2T}\right) \frac{\Delta_{p'}}{|\epsilon_{p'}|}$

ferromagnetic interaction $\rightarrow$ p-wave



Typical Solid, typical spacing $a \sim 2Å \Rightarrow \begin{cases} KE \sim \frac{\hbar^2}{2m_e a^2} \sim 3\text{-}5eV \\ PE \sim \frac{e^2}{\epsilon a} \end{cases} \qquad \frac{PE}{KE} = \frac{a}{a_B} \equiv r_s$

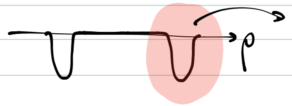
perturbation good for $r_s \lesssim 1$

typical solids $\quad r_s \sim 2\text{-}6$

actually $r_s \sim 30$ instabilities happen. Wagner Crystal State (electron crystal)

Transition Metal Oxide. (or heavy fermion)

d-electron $\sim 0.5Å$ lattice space $4Å$



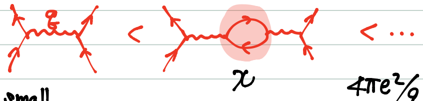
$\frac{e^2}{a}$ estimate is wrong here.

RG fixed point → free fermion fixed point

Coulomb Interaction:

$$\mathcal{H} = \sum_{k\sigma} \epsilon_k \psi^{\dagger}_{k\sigma} \psi_{k\sigma} + \sum_{k,k',q} \frac{4\pi e^2}{\underline{\underline{q^2}}} \psi^{\dagger}_{k+q\sigma} \psi^{\dagger}_{k'-q\sigma'} \psi_{k'\sigma'} \psi_{k\sigma}$$

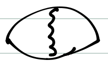
important to screen Coulomb Interaction



geometric series.

1° when q small diverges

$$V_{eff} = \frac{4\pi e^2 / q^2}{1 - \frac{4\pi e^2}{q^2}\chi}$$

2°  $\int d^3q \, \frac{1}{q^2} \rightarrow$ finite

so we don't need to care.

Let's calculate χ

$$\chi(q,\omega) = 2\sum_p \frac{f(\epsilon_p) - f(\epsilon_{p+q})}{\omega - (\epsilon_{p+q} - \epsilon_q) - i\varepsilon}$$

$$\cong 2\sum_p \frac{-\frac{\partial f}{\partial \epsilon}\, \vec{v}_p \cdot \vec{q}}{\omega - \vec{v}_p \cdot \vec{q}} \quad \text{for } \varepsilon_{q+p} = \varepsilon_p + \vec{v}_p \cdot \vec{q}$$

1° $\omega \ll v_F q$, $\chi \cong -N_0 \Rightarrow V_{eff} = \frac{4\pi e^2}{q^2 + 4\pi e^2 N_0} = \frac{q_{TF}^2}{q^2 + q_{TF}^2} \frac{1}{N_0} \sim O(1)$

$\lambda_{TF} \sim 2\mathring{A}$

2° $\omega \gg v_F \cdot q$ $\quad \chi \sim \frac{N_0 v_F^2 q^2}{\omega^2} \Rightarrow V_{eff} = \frac{1}{1 - \frac{4\pi e^2}{q^2}\frac{N_0 v_F^2 q^2}{\omega^2}} = \frac{\omega^2}{\omega^2 - \Omega_p^2}$

Ω_p → plasma frequency ~ typically $\gg \varepsilon_f$
10 ~ 15eV

Where does this go wrong?

1° Strong Correlated d, f electrons.
2° low density of electron

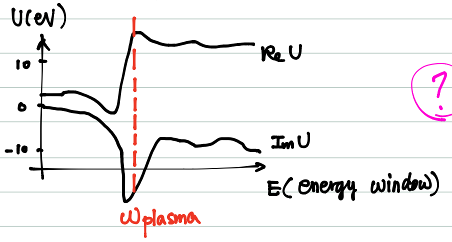
Lecture #3

Remarks:

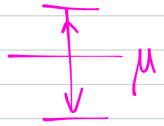
* $V_{SCR} = \frac{1}{N_0} \frac{q_{TF}^2}{q^2 + q_{TF}^2}$, most $q \sim k_F \gg q_{TF}$

* $(a\, q_{TF})^2 = \underset{60\,eV}{\boxed{\frac{4\pi e^2}{a}}} a^3 N_0 \sim 20$ relatively big number

* example: Screened U $SrVO_3$



?



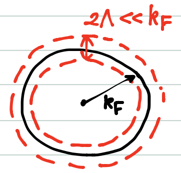
* $\chi_{physical} =$  $+ \cdots$

$$= \frac{\chi_0}{1 - \frac{4\pi e^2}{q^2}\chi_0} = \frac{q^2}{q^2 + q_{TF}^2}$$

density fluctuation is suppressed @ Length range

⇒ as uniform as possible.

Landau Fermi Liquid Theory:



RG:

1° Reduce shell width $\Lambda \to \Lambda/b$
Integrate out modes in eliminated region

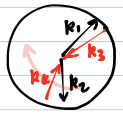
2° Rescale momentum and fields to preserve the quadratic part of action

3° see how interaction scales.

$$S = \int d\tau \int dk \left(\psi^+ \partial_\tau \psi - \epsilon_k \psi_k^+ \psi_k\right)$$
$$+ \int dt \int dk_1 \cdots dk_4 \, \delta(k_1 + k_2 - k_3 - k_4) \, U_{k_1 k_2 k_3 k_4} \psi_{k_1}^+ \psi_{k_2}^+ \psi_{k_3} \psi_{k_4}$$

* phase space for the interaction is limited. (most interaction goes away)

Special cases: #1 $k_1 \simeq k_3$, $k_2 \simeq k_4$
#2 $k_1 \simeq -k_2$, $k_3 \simeq -k_4$



as RG flows , #1 & #2 are more & more important
$U_{k_1 k_2 k_3 k_4}$ flows to #1 & #2

* this can go bad if F.S. has special structure.

$d^2k = dK\, d\Omega$ $K = K'/b$, $t = t' \cdot b$ (assume $\epsilon_k \sim V_F(\Omega) \cdot K \Rightarrow z = 1$)

$\Rightarrow \psi = b^{1/2} \psi'$ (we also rescale μ to keep density of electron)

Consider #1 & #2
$U(\Omega_1, \Omega_2)\, \psi_{\Omega_1}^+ \psi_{\Omega_1} \psi_{\Omega_2}^+ \psi_{\Omega_2}$ ⇒ Marginal , don't scale.

what's left:

1° charge channel amplitude $U(\Omega_1, \Omega_2)\, n_q(\Omega_1)\, n_{-q}(\Omega_2)$ $q < \Lambda$

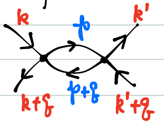
2° Spin channel

3° Pairing channel

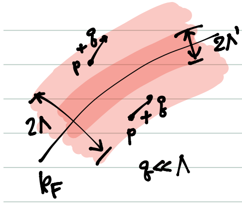
* $\frac{\chi}{1 - U\chi}$ if $1 - U\chi \sim 0$ ⇒ unstable

$\mu \psi^\dagger \psi$ ⇒ μ will diverge ⇒ we need a counter term to control μ.

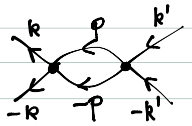
* one loop result:



$q < \Lambda$



$f(\epsilon_p) - f(\epsilon_{p+q}) \simeq 0$ ⇒ charge & Spin are strictly marginal



$\int dp \frac{1}{\omega^2 + \epsilon_p^2}$

$$\Rightarrow \frac{dU(\Omega,\Omega')}{d\ln b} = - \int d\Omega'' \, U(\Omega,\Omega'')\, U(\Omega'', U')$$

$$U \sim \sum_L U_L Y^*_{Lm}(\Omega)\, Y_{Lm}(\Omega')$$

decouples into different representation channels.

Lattice version:

$$U(\Omega,\Omega') = \sum_a U_a(\hat{\Omega},\hat{\Omega}')\, \hat{\Gamma}^\dagger_a(\hat{\Omega})\, \hat{\Gamma}_a(\hat{\Omega}')$$

for the most strong channel:

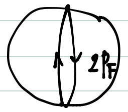
$$\frac{dU_L}{d\ln b} = -cU_L^2 \Rightarrow U(b) = \frac{U(b=1)}{1 + cU(b=1)\ln b}$$

$\begin{cases} \text{repulsive} \sim \frac{1}{c \ln b} & \text{weaker} \\ \text{attractive} \to \text{stronger} & \end{cases}$

pairing instability

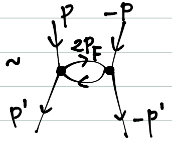
el-ph interaction

$$\ln b \sim \ln E_f / \omega_{pl}$$



Kohn-Luttinger Theorem

$\delta U \sim$



$\underline{U_{BCS}(\Omega,\Omega')} - \delta U$ < 0 for some angular momentum channel l

decay faster in l

* depend on 1° circular F.S. 2° infinite l channels.

El-ph interaction:

Atoms i, X_i

$$\delta E = \frac{1}{2} \sum_{ij} K(i-j) X_i X_j + \frac{1}{2} \sum_i \underset{\substack{\downarrow \\ M \gg m_e}}{M_i} \dot{X}_i^2$$

phonon Hamiltonian.

Coupling: $\int dt \int dp\,dp'\, C_i^{pp'} e^{i(p-p')\cdot R_i} X_i \cdot \sum_\sigma \psi^\dagger_{p\sigma} \psi_{p'\sigma}$

*1. δE doesn't scale with Λ, independent degree of freedom.

*2. Coupling is marginal.

Lecture # 4

$$\int dt \int dp\,dp'\, \underline{C^{i}_{pp'}}\, X_i \cdot \sum_\sigma \psi^\dagger_{p\sigma}\, \psi_{p'\sigma}\, e^{i(p-p')\cdot R_i}$$

* marginal

* assume no $\dot{X}\psi^\dagger\psi$, reason: $\dot{X}$ is small for the ions.

Second quantize: $X_a = (b_a^\dagger + b_a)\sqrt{\dfrac{\hbar \omega_a}{2K_a}}$

$$\Rightarrow \mathcal{H}_I = \sum_{a,p,p'} \underset{\substack{\downarrow \\ g = c^{pp'}_{a}\sqrt{\frac{\hbar\omega_a}{2K_a}}}}{g_a^{pp'}} (b_a^\dagger + b_a)\, \psi^\dagger_{p-a,\sigma}\, \psi_{p\sigma}$$

In the S.C states:

$$\langle \Psi(t)\Psi^\dagger(0)\rangle = \underline{\underline{G}}(\omega) = \begin{bmatrix} \overset{\langle\psi\psi^\dagger\rangle}{\overset{\mathbin{/\!/}}{G^N(\omega)}} & \overset{\langle\psi\psi\rangle}{\overset{\mathbin{/\!/}}{G^A(\omega)}} \\ G^{A\dagger}(\omega) & -G^{N*}(-\omega) \end{bmatrix}$$

$$\Rightarrow G^{-1} = G_0^{-1} - \Sigma$$

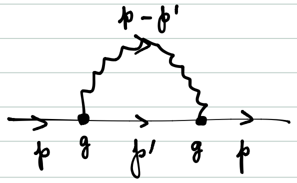
$$= (\omega - \Sigma_0^N(\omega)) - (\varepsilon_K + \Sigma_e^N(\omega))\,\tau_3 - \vec{\Sigma}_A \cdot \vec{\tau}_1$$

$$\Sigma(\omega) = \begin{bmatrix} \Sigma^N(\omega) & \Sigma^A \\ \Sigma^A & -\Sigma^{N*}(-\omega) \end{bmatrix} \qquad \begin{aligned} \Sigma_0 &= \tfrac{1}{2}\left[\Sigma^N(\omega) - \Sigma^{N*}(-\omega)\right] \\ \Sigma_e &= \tfrac{1}{2}\left[\Sigma^N(\omega) + \Sigma^{N*}(-\omega)\right] \end{aligned}$$

$\underline{\underline{G}}(k,z) = \displaystyle\int \frac{dx}{\pi}\, \frac{\underline{\underline{G}}''(k,x)}{z-x}$ spectrum function , $\underline{\underline{G}}''$ is imaginary part.

k dependence $\Rightarrow$ replace $k \to k_F + \delta k$

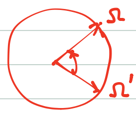
$$\int_{-\Lambda}^{\Lambda} dk_\perp\, \underline{\underline{G}}(k,\omega) = \frac{(\omega - \Sigma_0) + \Sigma^A \cdot \tau_1}{\sqrt{(\omega-\Sigma_0)^2 - \Sigma^{A2}}} = \frac{\omega - \overset{\to\ \text{gap func. } \Delta(\hat{\Omega})}{\frac{\Sigma^A}{Z}}}{\sqrt{\omega^2 - (\frac{\Sigma^A}{Z})^2}}\,\frac{1}{v_F} \, , \; Z = 1 - \frac{\Sigma_0}{\omega}$$



$$\underline{\underline{\Sigma}} = T \sum_{\omega'} \int d\Omega'\, g^2\, D(\underbrace{p\Omega - p'\Omega'}_{\text{only depends on angle difference}}, \varepsilon - \omega')\; \underbrace{\frac{\omega' - \Delta}{\sqrt{\omega'^2 - \Delta^2}}\, N_0(\Omega')}_{\int dk_\perp G(k,\omega)}$$

Linearize version

$$\Sigma^N(\omega) = T \sum_{\omega'} \int d\Omega'\, g^2\, D(\Omega, \Omega', \omega - \omega')\, \text{sgn}(\omega')\, N_0(\Omega')$$



$|p| = |p'| = k_F$

$$\Downarrow$$

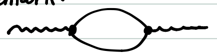
$$Z\Delta = T \sum_{\omega'} \int d\Omega' \left[g^2 D(\Omega, \Omega'; \omega - \omega') + \underset{\substack{\Downarrow \\ \text{el-el interaction.}}}{V(\Omega, \Omega')} \right] \frac{\Delta^A(\Omega', \omega')}{|\omega'|}$$

$$\Sigma^N(\omega) \sim g^2 N_0 f(\omega/\omega_D)$$

$\sim -g^2 N_0\, \omega/\omega_D$ for ω small

$$\Rightarrow Z = 1 - \frac{\Sigma_0}{\omega} \sim 1 + \underset{\lambda \sim O(1)}{\frac{g^2 N_0}{\omega_D}} \; , \quad G = \frac{1}{\omega - \varepsilon_p - \Sigma} \sim \frac{1}{Z}\, \frac{1}{\omega - \frac{\varepsilon_p}{Z}}$$

Remark:



$g^2 N_0 \sim \lambda \omega_D$

$\Rightarrow$ an $O(1)$ renormalization to phonon?

* usually we don't consider this

$\Rightarrow$ $K(i-j)$ contains the effect of electrons already!

* in spin fluctuation mediating Sc. we need to include this.

$$\boxed{Z\Delta - \int (D-V)\times\frac{\Delta}{\omega} = 0}$$

$$\Big[\underbrace{\omega \overset{1+\lambda_W}{Z}}_{\text{Normal Part}} - \underbrace{(\overset{\lambda_s}{D} - \overset{\mu^*}{V})}_{\text{Pairing Part}}\Big]\underbrace{\frac{\Delta}{\omega}}_{\text{eigenstate}} = 0$$

phonon

$$\Rightarrow \frac{1+\lambda_W}{\lambda_s-\mu^*} = -\ln\frac{\omega_c}{T_c} \Rightarrow T_c \sim e^{-\left(\frac{1+\lambda_W}{\lambda_s-\mu^*}\right)}$$

* increase el-ph doesn't always increase T_c.

* ferro $\sim \frac{1+2\lambda}{\lambda}$ hard to SC.

$\Rightarrow$ ising $\sim \frac{1+\lambda}{\lambda}$