Introduction

These four lectures describe quantum dots in various regimes from large dots containing perhaps several thousand electrons, to small dots containing 0,1,2... electrons. The interest in these systems is the ability to control electrons, often one at a time, in a regime in which quantum interference, electron-electron interaction and chaotic classical dynamics combined. Because electron numbers can range from 0 to tens of thousands, these systems are well-suited to explore problems at the heart of mesoscopics, namely, the crossover from microscopic to macroscopic, from, from discrete to continuous, from coherent to classical.

The focus of these talks will be principally toward experimental results, with occasional pointers to the theoretical literature, as I know it. No doubt, I will leave out crucial references both to relevant theory and to complementary experiments from other groups. I apologize up front for that. The primary probes of these systems is transport, i.e. measuring the current-voltage characteristics of these devices in response to various external conditions. In part of Lecture 3 and Lecture 4, I will mention charge sensing as an alternative way of measuring what is going on inside the dot.

Lecture 1. Mesoscopic Quantum Dots

The structures described are fabricated on a two dimensional electron gas (2DEG) wafer (material grown by our collaborators) consisting of epitaxial layers of AlGaAs and GaAs, as shown in Figs. 1-3. Typical device parameters for the 2DEG are given in table 1. Notice that even for moderate mobilities, the electron transport mean free path exceeds the typical device size by at least an order of magnitude. Devices are fabricated using electron beam lithography, following several steps of photolithograph during which ohmic contacts are applied, larger gate structures deposited, and alignment marks placed. Following these lithographic steps, the device is bonded into a commercial chip carrier (custom made by Kyocera to contain no nickel), which inserts into a chip carrier socket (also made with no nickel, see Fig. 3) and cooled in a dilution refrigerator. The various fridges around the lab use a common strategy for keeping the electrons cool. The dc lines pass through ~1-10kΩ resistors that divide isolated segments of a metallic enclosure—usually the hollow core of the cold-finger is used for this purpose—which provides both thermal sinking through the resistors and electromagnetic isolation. Isolating coaxial lines for high-frequency operation involves many more tricks and will be discussed briefly below, in the section on ac transport.

Depending on the impedance of the device, we use either a four-wire lock-in measurement, best suited for the moderate impedance (~10kΩ) of open quantum dots and quantum point contacts (QPC’s) or a two-wire current measurement (usually using an Ithaco 1211 preamplifier) for higher-impedance (~1MΩ)
### Table 1

A possible table format might look like this:

<table>
<thead>
<tr>
<th>2DEG Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective mass</td>
<td>m*</td>
<td>0.067</td>
</tr>
<tr>
<td>Spin degeneracy</td>
<td>g_s</td>
<td>2</td>
</tr>
<tr>
<td>Valley degeneracy</td>
<td>g_v</td>
<td>1</td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>ε</td>
<td>13.1</td>
</tr>
<tr>
<td>Effective Lande g-factor</td>
<td>g'</td>
<td>-0.44</td>
</tr>
<tr>
<td>Density of states</td>
<td>ρ(E)</td>
<td>2.8 x 10^{-6} cm^3 meV^{-1}</td>
</tr>
<tr>
<td>Level spacing</td>
<td>1/ρ(E)</td>
<td>3.57 μm m^3</td>
</tr>
<tr>
<td>Fermi wave vector</td>
<td>k_F</td>
<td>1.1 x 10^{-6} cm</td>
</tr>
<tr>
<td>Fermi energy</td>
<td>E_F</td>
<td>7.0 μeV</td>
</tr>
<tr>
<td>Thermal diffusion length</td>
<td>l_D</td>
<td>81 nm</td>
</tr>
<tr>
<td>Diffusion constant</td>
<td>D</td>
<td>7 x 10^{-6} cm^2/s</td>
</tr>
<tr>
<td>Cyclotron energy</td>
<td>h_o</td>
<td>1.73 meV/B</td>
</tr>
<tr>
<td>Cyclotron radius</td>
<td>l_C</td>
<td>20 nm/B</td>
</tr>
<tr>
<td>Magnetic length</td>
<td>l_M</td>
<td>26 nm/B</td>
</tr>
<tr>
<td>Zeeman energy</td>
<td>g'μ_B</td>
<td>25.5 μeV/B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>296 mK/B</td>
</tr>
</tbody>
</table>

...continued...
The same origin as universal conductance fluctuations in disordered mesoscopic metallic samples, namely, interference of multiple paths passing through the device.

The symmetry in magnetic field, $B$, applied perpendicular to the plane of the 2DEG is a consequence of the Landauer-Büttiker symmetry for a two-lead device, $R_{ij,ij}(B) = R_{ij,ij}(-B)$. Even though the device is measured in a four-wire configuration, the majority of the voltage is dropped across the device, which is a two-lead device.

The trajectories depicted in the micrographs in the insets of Figs. 5 and 6, are of course schematic, but reflect a semiclassical view of ballistic transport. From this kind of analysis much can be learned, including, for instance, the functional form of the correlation function (or equivalently, the power spectrum) of these fluctuations. Another theoretical approach is random matrix theory (RMT) which applies in the case of chaotic classical dynamics within the dot. The beautiful, interconnected story of RMT, level repulsion, and conductance fluctuations has been reviewed in the literature and will not be summarized here. Instead, we take only the results we need to allow conductance fluctuations to be used to learn about the dephasing time $\tau_\phi(T)$.

Whereas the rms amplitude of conductance fluctuations depends on $\tau_\phi(T)$ as well as $T$ explicitly, the weak localization correction, appearing as a dip at zero magnetic field in the average conductance, is only temperature dependent through $\tau_\phi(T)$. In fact, RMT models that include dephasing through the inclusion of voltage-probe leads allow $\tau_\phi(T)$ to be

---

2 For review of RMT as applied to quantum dots, see C. W. J. Beenakker, Reviews of Modern Physics 69, 731 (1997); Y. Alhassid, Reviews of Modern Physics 72, 896 (2000). For a review of RMT in disordered systems, see the text by P. Mello, Oxford University Press.
weak localization correction
\[ \delta g \simeq \frac{e^2}{\hbar} \frac{N}{2N+1+\gamma_\varphi} \]

conductance fluctuations
\[ \text{var}(g) = \frac{\Delta}{6kT} f(\gamma_\varphi), \]
\[ f(\gamma_\varphi) = \frac{2}{2+\gamma_\varphi} \left( \frac{1}{\sqrt{3}+\gamma_\varphi} \right)^2 \]

dehasing channels
\[ \gamma_\varphi = \frac{2\pi\hbar}{\tau_\varphi \Delta} \]

mean level spacing
\[ \Delta = \frac{2\pi\hbar^2}{m^* A_{\text{dot}}} \]

**Table 2** Random matrix theory formulas for weak localization correction and variance conductance fluctuations (in units of $e^2/h$). Weak localization formula is valid for any number of modes in each lead, $N > 1$, whereas the approximate formula for conductance fluctuations is valid for $N=1$ only. Various forms of these expressions, valid in various limits are known.

extracted directly from $\delta g = \langle g(B\neq0) \rangle - \langle g(B=0) \rangle$, with formulas that depend only on the number of quantum channels connecting the dot to the reservoirs, and the area of the dot, $A_{\text{dot}}$, which determines its mean level spacing. Formulas for the case of single-mode leads are given in Table 2.\(^3\)

The measured dephasing time $\tau_\varphi(T)$ measured from these formulas (or ones like them, depending on experimental parameters) is show in Fig. 8. Two features are worth noting: the first is that over much of the measured range, $\tau_\varphi(T) \propto 1/T$. This result disagrees with relevant theory (which predicts $\tau_\varphi(T) \propto 1/T^2$).\(^4\) However, the theory addressed closed devices, explicitly disallowing scattering with momentum transfer smaller than the inverse dimension of the dot.\(^4\) The second thing to notice is that the data appears to saturate at low temperature. As was demonstrated in Ref


4, this saturation is not the result of a saturating electron temperature. Here, the explicit temperature dependence of var(g) is useful as a thermometer: we see a continued $\sim 1/T$ rise in var(g) between 100 mK and 40mK.

It is interesting to note that RMT predicts not just the mean and variance of conductance fluctuations, but the full distribution function, $P(g)$. This is perhaps the best illustration of the universality of RMT. The distribution of conductances is not dependent on material (as long as spin-orbit effects are weak, though that too can be accounted for, as discussed below), device shape (as long as it generates classical chaos), or size (except insofar as area affects $\gamma_{\Phi}$ through $\Delta$). Only the temperature, dephasing rate, and number of channels in the leads are parameters of the theory. As seen in Fig. 8, the experimental distributions are in good agreement with theory, once finite temperature and dephasing are included in the model.\footnote{A. G. Huibers, et al., Phys. Rev. Lett. 81, 1917 (1998).}
Deviations from RMT have been investigated in transport measurement, and have been associated theoretically with self-similar structure in conductance fluctuations. The experimental evidence for self-similar, or fractal, conductance fluctuations is not firmly established in my opinion.

Following work by Sivan and coworkers, we have constructing a spectrometer using a small dot adjacent to a big dot, as seen in Fig. 9. Differential conductance $dI/dV$ measured through the single electron dot serves as a spectrometer of the large dot, giving a measure of the wave function of the large dot at the location of the spectrometer at an energy set by the energy level of the single-electron dot relative to the Fermi sea of the large dot.

In the magnetic field range ~50-120 mT a strong series of ridges appears in the data (Fig. 10). Classical phase space analysis of a simplified version of the problem shows that a strong periodic orbit with a concave diamond develops within the dot in this range of magnetic field. The spacing of the stripes in the data correspond well to the expected period Aharonov-Bohm oscillations with an area given by the diamond, while the slope of the stripes agrees with the expected dependence in the field-energy plane to keep the number of wavelengths around the orbit constant on each stripe.

It is evident by inspection that the oscillations, and overall conductance fluctuations, are stronger when the spectrometer is near the Fermi surface of the large dot (toward the bottom of the bottom panel of Fig. 10). Assuming the interpretation of the stripe features to be valid, we can use energy-dependent amplitude of the periodic oscillations to extract a energy dependent dephasing rate, $\tau_\phi(\epsilon)$, where $\epsilon$ is the distance of the spectrometer window (the single particle level) below the Fermi surface of the large dot.

Following work by Sivan and coworkers, we have constructing a spectrometer using a small dot adjacent to a big dot, as seen in Fig. 9.

| Table 3 | Model for extracting energy dependent dephasing from the amplitude of periodic oscillations. |

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_N(\epsilon) = A_0 \exp\left(-N\epsilon \tau^{-1}(\epsilon)\right)$</td>
<td>Amplitude of Nth harmonic.</td>
</tr>
<tr>
<td>$t = L / v_F \sim 18$ ps</td>
<td>Period of one cycle.</td>
</tr>
<tr>
<td>$\frac{A_1(0)}{A_2(0)} = \exp(t \tau^{-1})$</td>
<td>Ratio of first harmonic to second harmonic.</td>
</tr>
<tr>
<td>$\tau_{\text{esc}} \sim 17$ ps</td>
<td>From fit to power spectrum of $g$ in striped region.</td>
</tr>
<tr>
<td>$\tau^{-1}(\epsilon) = \tau_{\text{esc}}^{-1} + \tau_\phi^{-1}(\epsilon)$</td>
<td>Model all energy dependence as reflecting dephasing.</td>
</tr>
<tr>
<td>$\tau_\phi(0) \sim 2$ ns $\gg \tau_{\text{esc}}$</td>
<td>Find zero-energy value from weak localization.</td>
</tr>
</tbody>
</table>

By defining an effective temperature $T = \epsilon / k_B$ we can compare $\tau_\phi(T)$ from the spectroscopic measurement to $\tau_\phi(T)$ from weak localization. The two compare surprisingly well, with the notable lack of a $T^{-2}$ part in the $\tau_\phi(T)$, perhaps because the substrate remains at base temperature.
surface of the dot. The procedure is outlined in Table 3, and the results shown in Fig. 11. If we make the connection to previous measurements of \( \tau_0(T) \) through the simple association \( \varepsilon \leftrightarrow T \) (without the 3.5 from the derivative of the Fermi function), then, remarkably, the data from this experiment sits essentially on top of the weak localization data. The data also has a slope \( \tau_0(\varepsilon) \propto 1/T \), in this case extending across a broader temperature range. Theory also exists to extract \( \tau_0(\varepsilon) \) from the chaotic parts of phase space, but that analysis has not yet been carried out.

Table 4 Device parameters that determine the symmetry class within RMT. See Table 5 below.

Table 5 Chart of parameter ranges for RMT including Zeeman and spin-orbit effects. The large numbers in each square characterize the relative size of var \((g) \sim \delta(\Sigma)\).

So far, there has been no mention of the spin of the electron. In material with weak spin-orbit coupling, we can consider either RMT or semiclassical formulations that ignore spin. As the strength of spin-orbit coupling increases, spin effects on quantum interference become dramatic. This is, of course, well known from the mesoscopics of bulk disor-
dered metals and semiconductors, where, for instance, weak localization effects change sign in the presence of strong. The origin of this change of sign in quantum corrections to average conductivity is the fact that as an electron spins evolves through $2\pi$ (for instance, by following the motion of the electron around a loop due to spin-orbit coupling), its overall wave function of the electron undergoes a sign change, and it takes a rotation by $4\pi$ to bring the sign back to its original. This change in sign reverses the sign of coherent backscatter to destructive interference producing a peak, rather than a dip, in conductance, at zero magnetic field (where all loops interfere).

In heterostructure-based quantum dots, the dominant spin-orbit effects show an important cancelation effect due to the winding and unwinding of spin rotations associated with the confined motion of electrons in zero dimensions. The cancellation, which applies only to Rashba-type and linear Dresselhaus spin-orbit coupling eliminates the first-order contribution, but leaves second order effects, which take the form of a spin dependent Aharonov-Bohm effect. Higher-order term (in the quantity $(L/\lambda_{so})$ where $L$ is a typical linear dimension of the device, and $\lambda_{so}$ is roughly the length over which the spin rotates by $2\pi$ due to spin-orbit coupling). An extended RMT for spin-orbit coupling in quantum dots, including Zeeman fields, was given by Aleiner and Falko.\(^9\)

The extended RMT gives the variance of conductance fluctuations (at $T = 0$) in terms of symmetry parameters: $\text{var}(g) \sim s/(\beta \Sigma)$, where $\beta$ is the conventional (Dyson) parameter describing time-reversal symmetry, $s$ is the Kramers degeneracy parameter and $\Sigma$ characterizes mixing of different spins when Kramers degeneracy is already broken. Spin rotation symmetry is classified as either not

broken ($s = 2$, $\Sigma = 1$), partially broken ($s = 1$, $\Sigma = 1$) or completely broken ($s = 1$, $\Sigma = 2$).

The variance of conductance is reduced by a factor of two when a crossover into the class with next-lower symmetry occurs. The Kramers degeneracy can be lifted by a Zeeman field as well as spin-orbit coupling if $B_\perp = 0$. Once Kramers degeneracy is broken ($s = 1$), mixing of spins ($\Sigma = 2$) due to spin-orbit coupling is possible at $B_\parallel = 0$ for strong spin-orbit coupling or can be induced by $B_\parallel$ when spin-orbit coupling is suppressed by confinement near $B_\parallel = 0$. Finite temperatures and decoherence strongly reduce $\text{var}(g)$, but the relative reduction factor $\text{var}(B_\perp = 0, B_\parallel = 0)/\text{var}(B_\perp = 0, B_\parallel \neq 0)$ is only affected weakly.

Figure 12 demonstrates the suppression of weak localization effects by confinement in high-density GaAs quantum dots. These wafers happened to have a large spin-orbit coupling by virtue of the high density, which makes the Fermi velocity larger, and the details of the heterostructure interface. For large dots made from this wafer, spin-orbit effects

---

reverse the sign of coherent backscattering, giving antilocalization (a conductance peak at B = 0). For the small dot in Fig. 12(c), made from the same material, spin-orbit effects are suppressed by confinement and no antilocalization is observed. The device in 12(b) has an interior gate that allows it to be controllably switched from a large dot with antilocalization to a small dot with localization. One could imagine a class of devices that take advantage of rapid turning on and off of spin orbit coupling. The rapid change in area provided by the internal gating allows such a process. The solid curves in Fig. 12 are fits to RMT, showing essentially perfect agreement between theory and experiment.

The extended RMT also can account for combine spin-orbit and Zeeman coupling, the latter produced experimentally by applying a purely in-plane field. At low in-plane fields, B < 200 mT, experiment and RMT agree essentially perfectly. At higher field, the fact that B also couples to orbital electronic states (even for a perfectly 2DEG) needs to be accounted for. With this modification, good agreement over the full range of experimental parameters, for both average and variance of conductance, is obtained, is seen in Fig. 13. For average conductance there is essentially perfect agreement between the modified RMT (denoted RMT+FJ in Fig. 12) and experiment; for var(g) there is some minor discrepancy that is not understood.

So far we have restricted our attention linear-response transport (except for the spectrometer experiment) which, among other things, obey Landauer-Büttiker symmetry in applied magnetic field. We now consider two departures. If we apply a relatively large dc bias to a two-lead quantum dot, we are no longer guaranteed that the Landauer-Büttiker sym-

---

**Fig. 16** Standard deviation of interaction parameter (left axis) and dimensionless interaction parameter (right axis) as a function of the number of modes in the leads. The dash-dot line indicates the theoretical estimate. Whereas theory gives a $\delta \alpha$ decreasing with the number of modes in each lead as $N^{-2}$, the experiment shows $\delta \alpha$ increasing. The likely explanation for this result is that the theory does not include thermalization, which occurs for small N. For larger N, the distribution of electron energies in the dot are out-of-equilibrium due to the relatively rapid escape.

**Fig. 17** (a) Modest effect of microwaves on the symmetry of conductance, shown for +B (solid blue) and -B (dashed). (b) dc current induced by the application of rf or microwave gate voltage. At 10MHz, the symmetry of the current reveals its origin to be rectification. At 5.56 GHz, there is very little symmetry in B. Here, photovoltaic effects dominate transport.
metry $g(B) = g(-B)$ will be obeyed. It was recently pointed out while at large bias this symmetry is not guaranteed, that destroying it requires electron-electron interactions. The reason, simply put, is that without interaction, each energy level, $E$, within transport window will separately obey $g(E,B) = g(E,-B)$, so that integrating over $E$ to account for finite bias will not affect this symmetry.

If we consider the first departure from the even symmetry $g(B) = g(-B)$. Expanding the current $I = g(B) V + \zeta(B) V^2 + \ldots$, we can expand $\zeta(B)$ as $\zeta_0 + \alpha B + \ldots$. The term proportional to $\alpha$ is thus linear in $B$ and even in source-drain voltage, and corresponds to a nonlinear conductance term $g_\alpha(B) = \alpha BV$. As shown in Refs. XX, $\alpha$ is proportional to electron-electron interaction strength.

In the mesoscopic regime, one expects (and indeed we find) that an ensemble average over shapes for field of $\alpha$ vanishes. We therefore characterize this quantity by its the standard deviation, $\delta\alpha$. We investigate the component of conductance that is antisymmetric in $B$, denoted $g_\alpha = (g(B) - g(-B))/2$, and extract the component linear in $B$, as shown in Fig. 15. After gathering statistics on these nonlinear components, we are able to find $\delta\alpha$ and its dimensionless counterpart $\delta\alpha'$, defined

$$
\delta\alpha = \delta\alpha' \frac{1}{\sqrt{N}} \frac{e}{A} \frac{e^2}{\Delta \phi_0} \frac{1}{\hbar},
$$

where $N$ is the number of modes in each lead, $A$ is the device area, $\phi_0$ is the flux quantum. The quantity $\delta\alpha'$ has been calculated in the limit of perfect screening, with a predicted value $\pi/2$. Experimental values are in reasonable agreement for $N=4$, but for smaller $N$, where equilibration occurs within the dot before the escape time, asymmetry is reduced (as expected).

Another instance of nonlinearity that violates magnetic field symmetry is the appearance of both rectification and photovoltaic effects in quantum dots. Both effects produce dc current in response to an applied ac gate voltage. Depending on the frequency and strength of ac voltages, components that are both symmetric and antisymmetric in magnetic field appear.\textsuperscript{11}

Rectification produces a dc current in response to a single ac applied gate voltage. It arises when a voltage applied to a gate,

$$
V_g(t) = V_g^{dc} + V_g^{ac} \sin(\omega t)
$$

parasitically couples to the source drain voltage, with a possible phase shift,

$$
V_{ds}(t) = \alpha V_g^{ac} \sin(\omega t + \phi)
$$

leading to an induced dc current

$$
I_{\text{rect}} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \alpha V_g^{ac} \sin(\omega t + \phi) g(V_g(t)) dt
$$

are characterized by a lack of ±B symmetry, examples of which are seen in Figs. 17 and 18.

There is some subtlety regarding using symmetry do disentangle rectification and photovoltaic effects: At intermediate microwave frequencies, conductance remains symmetric while the induced dc current becomes asymmetric. At higher frequencies, or larger amplitudes of microwave gate excitation, conductance becomes asymmetric as well, resulting in asymmetric rectification. In this regime separating photovoltaic and rectification effects cannot rely on magnetic field symmetry.

through the dot. When the applied gate voltage is small compared to the gate voltage correlation length in conductivity (~10 mV), this current can be approximated as

$$I_{\text{rect}} = \alpha \cos(\phi) \left( \frac{V_{ac}}{V_g} \right)^2 \frac{dg}{dV_g}.$$  

Rectification does not require particularly high frequencies, and is the dominant source of dc current resulting from ac gate voltage at MHz frequencies. Examples of rectification are seen in the top panels of Fig. 17 and 18.

In contrast, beyond the adiabatic regime, when the ac frequency becomes comparable to various relaxation times in the problem, a photovoltaic mechanism leading to dc currents becomes dominant. Photovoltaic effects