# BOULDER 2017 I

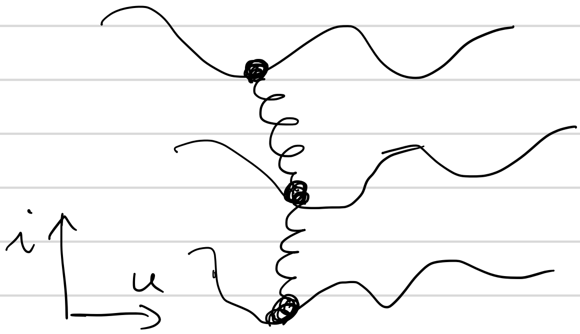
particle 1d $\quad u \quad H = V(u) \quad \cancel{m\ddot{u}} + \eta\dot{u} = -V'(u) + f$

TInv

$$\overline{V(u)V(u')} = R_0(u-u')$$

gaussian $\qquad$ SR or LR

elastic line in RP (discrete) $u_i$




$$\overline{V_i(u)V_j(u')} = \delta_{ij} R_0(u-u')$$

SR internal space

elast. energy

$$H = \sum_i \frac{c}{2}(u_{i+1}-u_i)^2 + V_i(u_i)$$

$$\eta\partial_t u_i = -\frac{\delta H}{\delta u_i} = c(u_{i+1}+u_{i-1}-2u_i) - V_i'(u_i) + f$$

$u_i \to u(x)$

$$H_V[u] = \frac{1}{2}\int d^dx\, c(\nabla_x u)^2 + V(x,u(x))$$

substrate, break TI, couples $u$ $\neq \nabla u$ struck. disorder

general continuum model of DES

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x')R_0(u-u')$$

pinning

deform. displacement $\to \in \mathbb{R}^N$

1) $u(x) \to \vec{u}(\vec{x})$ $\quad \hookrightarrow \mathbb{R}^d$ internal $\qquad N=1$ interfaces, $d=1$ line

2) $\frac{1}{2}\int_q c q^2 u_q u_{-q} \longrightarrow |q|^\alpha u_q u_{-q}$ LR elasticity

$\int_q = \int \frac{d^dq}{(2\pi)^d}$

quadratic form $N>1$

$u=0$ GS

contact line

magnets dipolar

cracks

3) $R_0(u)$ smooth $r_f$ (important role pinning)

$R_0(u)$ SR random bond (RB) LR random field (RF)

$R_0(u) = R_0(u+a)$ lattice + disorder (RP)

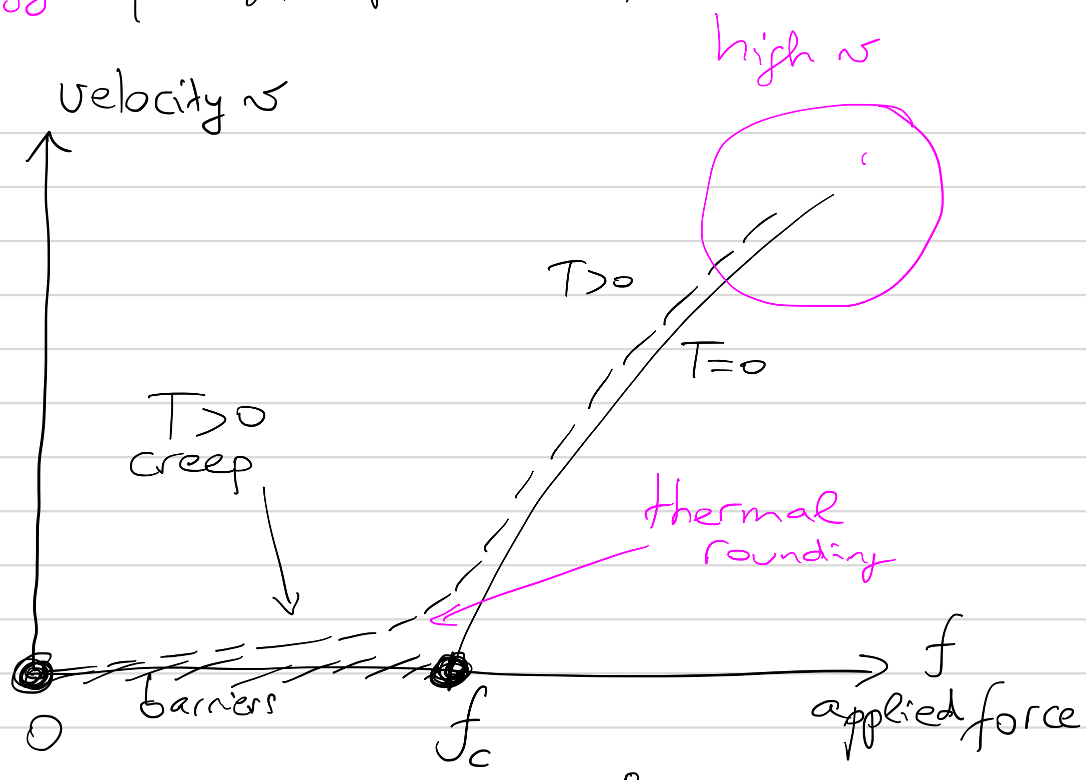
$T=0$ statics $\min_u H = E_{min}$ $\quad u_{min}(x)$

dyn

$$\eta\partial_t u(x,t) = c\nabla_x^2 u(x,t) + F(x,u(x,t)) + f$$

$$F(x,u) = -\partial_u V(x,u)$$

phenomenology    pinning, depinning, creep




$v \sim (f-f_c)^\beta$ order parameter

two T=0 critical points    statics $f=0$
depinning $f=f_c$

scale invariance
RG fixed points    $\overline{(u(x)-u(0))^2} \sim x^{2\zeta_{eq}}$    $T=0$ $u_{min}$, $T>0$
FRG    $\sim x^{2\zeta_{dep}}$

$\zeta(d, N; \alpha, \begin{smallmatrix} RB \\ RF \\ RP \end{smallmatrix} \ldots)$    $\zeta_{eq}(d=1, N=1, SR) = 2/3$

$T>0$    thermal activation    $\tau \sim e^{E_b/T}$

Conjecture    1) $v \sim v_0 e^{-E_b/T}$ finite barrier
$\sim f$

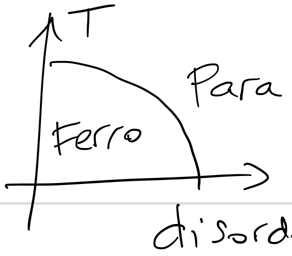
$\mu = \dfrac{d-2+2\zeta_{eq}}{2-\zeta_{eq}}$    2) $E_b(f) \sim U_c (f_c/f)^\mu$ diverging barrier glass
$f \to 0$

$\mu_{d=1} = 1/4$    $v \sim v_0 e^{-\frac{U_c}{T}(f_c/f)^\mu}$

Ising

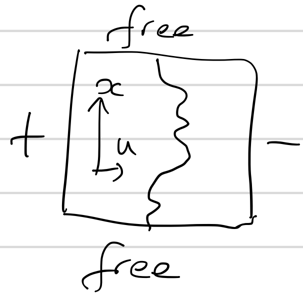
RB $D=2,3$

RF $D=3$




$$H = \sum_{\langle ij \rangle} (J+\delta J_i) S_i S_j - h_i S_i$$

RB RF




$E_{DW} \sim 2J$ length

$$\sim 2J \int \frac{dx}{a} \sqrt{1+\left(\frac{du}{dx}\right)^2}$$

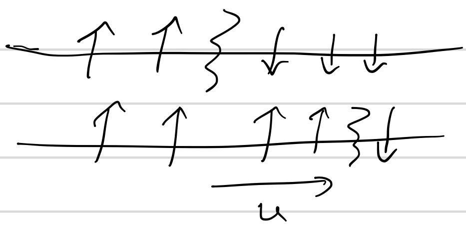
$$\sim cst + \frac{J}{a}\int dx\, (\nabla_x u)^2$$

RB $V(x,u) \simeq \frac{2}{a}\delta J(x,u)$ $R_0$ SR

RF

$D=0+1$

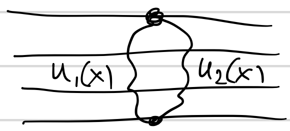
2 phases
≠ energy




$$V(u)-V(0) = 2\sum_{i=1}^{u} h_i$$

$$\overline{(V(u)-V(0))^2} \sim |u|$$

$D=1$




area swept

$$\Delta V = \sum_x V(x,u_1(x)) - V(x,u_2(x))$$

$$V(u,x) = 2\sum_{i=1}^{u} h_{x,i}$$

$$\overline{V(u,x)V(u',x')} = \delta_{xx'} R_0(u-u')$$

$$Z = \int Du(x)\, e^{-H_V[u]}$$

$$\overline{Z^n} = \int \prod_{a=1}^{n} Du_a(x)\, e^{-S[u^a]}$$

$$S[u^a] = \frac{1}{2T} \sum_a \int_x (\nabla u_a(x))^2 - \frac{1}{2T^2} \sum_{ab} \int_x R_0(u_a(x) - u_b(x))$$

$$R_0(u_a - u_b) \sim \underbrace{\frac{1}{2} R''(0)(u_a - u_b)^2}_{\text{Larkin}} + \frac{1}{4!} R^{(4)}(0)(u_a - u_b)^4$$

$$+ u_{ab}^6 + u_{ab}^8 + \dots$$

all relevant $d < d_{uc} = 4$

$u(x) \longrightarrow x(t)$

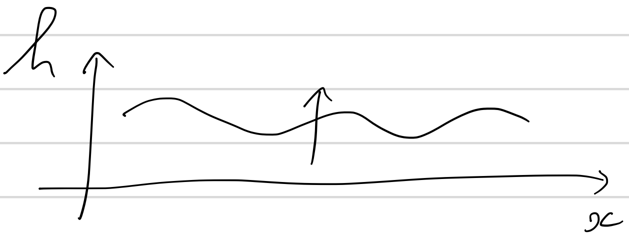
$$Z(x,t\,|\,0,0) = \int_{x(0)=0}^{x(\tau)=x} \mathcal{D}x(\tau)\, e^{-\int_0^t d\tau\, \frac{1}{4}\left(\frac{dx(\tau)}{d\tau}\right)^2 - V(x(\tau),\tau)}$$

$c=1$
$T=1$

$$\partial_t Z = \partial_x^2 Z - V(x,t) Z(x,t)$$

$$Z(x,t) = e^{h(x,t)}$$

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 - V(x,t)$$




growth interface
+ noise

$$h(0,t) = vt + t^{1/3}\chi$$

$$P_{stat}(h) = e^{-\frac{1}{2}\int dx (\nabla h)^2} \qquad h \sim x^{1/2}$$

$u = -\partial_x h$ Burgers $\partial_t u + \partial_x u^2 = \partial_x^2 u - V(x,t)$

$h \sim x^{\theta/\zeta}$

$h \sim t^{\theta}$

STS $\theta = d-2+2\zeta = 2\zeta - 1$

FDT $\theta/\zeta = 1/2$

$\Longrightarrow \theta = 1/3 \quad \zeta = 2/3$