Outline of the series

**American Football, Barber Poles and Clouds**

- Systems with more than one driven species
  - Variety of models with multiple species of particles
  - Surprises in “bare bones” NESM models with just two species
    - "Charged" particles driven in opposite directions
    - Phase transitions in the "ABC" model
- Summary and Outlook: "Come and join in the fun!"

Outline of ABC

- Motivations
  - Fundamental issues in Non-equilibrium Statistical Mechanics
  - Potential applications to physical/biological systems
- Surprises from a 2-D “Bare bones” model
- More surprises from 1-D: one vs. two lanes
  - Long range order … or not??
  - Subtleties in coarsening
  - Effects of lane preference
- What else can we look forward to?

Motivations

Driven Diffusive Systems

- Diffusion of one or more particle species on a lattice
  - Relevant for many applications (biology, chemistry, …)
  - Simple local order parameter, satisfies conservation law
- …under non-equilibrium conditions
  - Driven by external forces
  - Many varieties … but only one (easy example) example here
  - Many surprises, even for one species (driven Ising lattice gas)
- Two species of particles (Potts lattice gas)
- Driven by external "vicious" field $E$ - i.e., uniform bias
- Periodic boundary conditions (so that NESS can be achieved)

Motivations

examples of Potential Applications

- Physical Systems
  - Pedestrian or vehicular traffic
  - Driven colloidal systems
- Biological Systems
  - Molecular motors on microtubules
  - Gel-electrophoresis (reptation model)

(C) Godrèche and S. Sandow, 1998 unpublished

2-D bare bones model

Two "charged" particle species (+, -, 0) with excluded volume, diffusing under an external, “electric” field $E$

<table>
<thead>
<tr>
<th>$E$</th>
<th>all particle-hole exchanges with rate: 1 except jumps against $E$: $\exp(-E)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>all “charge” exchanges with rate: $\gamma e^{-E}$ except jumps against $E$: $\gamma e^{-E}$</td>
</tr>
</tbody>
</table>

…on an $H \times L$ lattice with PBC

control parameters: $H, L, \gamma, \gamma$, and

$\left(N^+ - N^+\right)/HL \quad q = (N^- - N^-)/HL$

here, mostly $E=\infty, \quad m \approx 0.5$ and $q = 0$

One dimension (1×L) … “one lane road”

exact solution ($E=\infty, \gamma > 0$) available


- density homogeneous (i.e., no transition to jam)
- exponential distribution of (particle) cluster sizes:

$\bar{\rho}(s) \sim s^{-\frac{3}{2}} \exp(-s/\bar{q})$

black (○) → ← gray (○)
Two dimensions ($H \times L$) ... “multilane road”
only MC and MFT (BS-KPZ: 1991-4910)

* anomalous, anisotropic, long-range correlations
* transitions to jammed states!
* variety of ordered states
  - "Barber poles" for $H \gg L$
  - "American football" for $H \approx L$
  - drifting structures ($q \neq 0$)

* both continuous; discontinuous transitions
* interesting *coarsening* phenomena ("clouds")
* Continuum Theory qualitatively adequate

"Old" surprises...

- Disordered state: weird $S(k)$’s
- Barber poles: winding number distributions?
- Clouds: dynamic scaling or not??


Structure Factors $S(k)$
in the homogeneous phase of an ordinary system ($\varrho=\text{const.}$)

Ornstein-Zernike $S(k, k_f)$

Isotropic Lorentzian:
$$1/(1+k^2/\xi^2)$$

FT $$\Rightarrow$$
$$\exp(-r/\xi)$$

"Old" surprises

Structure Factors $S(k)$
in the homogeneous phase of this driven system ("clouds")

$S_\omega(k)$ FT of $(\varrho(0)\varrho(x))$

Positive (necessary)
Anisotropic (no-surprise)
Discontinuity at origin!
generic for DDS
due to violation of FDT

Imaginary parts are always positive!
WHY ???

Anisotropic (no-surprise)
Discontinuity (like before)
Some change signs!
comes out of Langevin description + FDT violation
Distributions of Structure Factors

- How do the averages come about? (especially for the negative SF’s)
- SF for each snapshot is a mess (speckle patterns from, e.g., laser scattering)
- What is the whole distribution of SF’s?

Asymmetries in Structure Factor Histograms
G. Korniss, B. Schmittmann and R.K.P. Zia

Histogram of Barber Poles
So far, NO reasonable explanation!

Coarsening (dynamic scaling)

1000 x 1000
ω = 1/2, q = 0
L = 50

NO explanation, except for large k
Rescale both S and k
No scaling for any other series!

Horrible comparison with continuum theory!
unlike the case for models A,B

More surprises from 1-D: one vs. two lanes
Snapshots of $H=1,2$ systems
(both in steady state)

$E = \infty, \gamma = 0.1, L = 1000$

Cluster size scales with $L$.

Natural Questions:

No transitions in 1-d vs. ordered states in 2-d reminiscent of Ising model …

- How does cross-over occur here?
- What happens if we “gradually” progress from one to the other?
- Study “two lane” system, $2 \times L$;
- …then to “multi-lane” cases.
...a few details of this two-lane system

- \( m = 0.5 \), \( E = \infty \), \( \gamma = 0.1 \), \( L \leq 10,000 \)
- length of "jam" ~ 0.47L
  \[ \Rightarrow 94\% \text{ of particles are in the "jam"} \]
  \[ \Rightarrow 6\% \text{ of particles are "travelers"} \]

...describe clusters via the residence distribution, \( p(l) \):
probability for particle to be in cluster of length \( l \)...
Snap shots of a small 2×L system

- Essentially no holes in the “jam” “travelers”
- So, it’s easy to identify a cluster and its size (s) and/or length (l)

Residence for 1×500 and 1×1000

- New surprises: Fast coarsening

Comparison with 1×L

- From residence distribution $p(\ell, t)$
- Find growth rate of average cluster size:
  $$\bar{\ell}(t) = \sum \ell p(\ell, t)$$
- See if dynamic scaling exists:
  $$\bar{\ell}(t)p(\ell, t) = f(x) \quad x = \ell/\bar{\ell}(t)$$
- Check L independence (using good collapse)

...brief summary
of published material on fast coarsening in $2 \times L$

- $m = 1/2, q = 0, E = \infty, \gamma = 0.1, L \leq 10,000$
- coarse grained clusters grow: $\ell \sim \rho^\gamma$
- dynamic scaling ok for $p(\ell, t)$
  ...with scaling function $\sim$ theory for 1 species
  but wrong exponent
- improved “theory” gives reasonably good fit to both exponent and $p(\ell, t)$
  but no analytic understanding

Lane preference
- “cars/trucks” (sometimes) tend to stay in “fast/slow” lane
- $p$ probability for choosing “preferred” lane
- $p = 0, 1$ cases are clear:
  - jam as before: free flow
  - equal mix (on the average) $\rightarrow$ pure cars/trucks
    i.e., $Q = “\text{excess”} = 0 \rightarrow 1$
- expect $Q(p)$ to be monotonically increasing, e.g.,

Profiles at ‘small’ $p$
(CM of entire ‘jam’ centered at 500, then averaged)

Profiles at ‘large’ $p$

brief remarks
- Simple model seems to show effects we might see (should expect) on two-lane roads
- Numerical integration of MFT display qualitatively same behavior
- Need a better understanding of ‘negative response’
- Need better theories for quantitative predictions
What else can we look forward to?

- Despite its simplicity, this model continues to present many interesting & challenging issues:
  - Many already raised and …
  - What about 3 lanes? and … 13 lanes?
  - Exclusion at larger distances (big trucks), interactions…
  - Inhomogeneous jump rates (gravel patches, road works…)
- Other “ABC” models: driven in parallel, overtaking cyclically
- Multispecies models: widely differing driver preferences

Outline of the series

- Overview/Review “Equilibrium SM vs. Nonequilibrium SM”
- An Ising-like model in DDS “Shattered expectations”
- DDS in one-dimension “Bare bones NESM”
- Systems with more than one driven species “American football, Barber poles, and Clouds”

Summary and Outlook

- Adding a “simple” drive to the equilibrium Ising model adds dimensions far beyond expectations.
- Even a “bare bones” system (i.e., “non-interacting”) provided many amazing phenomena, while adding other species leads to further surprises.
- Potential applications to wide range of systems in nature exist.
- Some new insights have been garnered, but the goal of an overarching framework for NESM is far from being just “around the corner”.

Summary and Outlook

- Equilibrium SM is not an easy subject, but “full” Nonequilibrium SM is really far out !!
- Nonequilibrium SM topics here form a tiny corner, in which
- Driven Diffusive Systems occupy a minute part, in which
- Models presented in these lecture are a small fraction, with…

Lots of open questions ⇒

- Lots of work to be done
- Lots of ideas to pursue
- Lots of interesting phenomena, waiting to be discovered!
- Lots of ways/levels to participate:
  - Computer simulations
  - Numerical/analytical approaches to ODEs,PDEs,SDEs
  - Field theory (QFT,SFT)
  - Rigorous mathematical methods

Come and join in the fun!!