Landau Fermi Liquid and Heavy Fermion  [Coleman]

Overview

Increasing localization: 5d  4d  3d  5f  4f

- Interesting physics occur at the crossover between itinerant and localized system.
- f-spin are always localized
  \[ \begin{align*} 
  \text{high-T: local moment metal} \\
  \text{low-T: spins "quench" to form heavy fermions.} 
  \end{align*} \]
- Spin \( \equiv \) localized moment, when \( e^- \) lost all its charge degree of freedom
  e.g. Ce\(^{3+}\) (4f\(^1\)): \( L = 3, S = \frac{1}{2}, J = L - S = \frac{5}{2} \)
  At high-T, \( \chi = \frac{\eta M^2}{3T} \), \( M^2 = g_f^2 \mu_B^2 J(J+1) \).
  \[ S_0 = k_B \ln (2J+1) \]  [unquenched \( \equiv \) all states equally occupied]
- Under crystal field, 4 \( \rightarrow 2 \) Kramers degenerate

At low-T, these materials form Fermi liquid:

\[ E_F = \frac{p_F^2}{2m^*} \] \( N(0) = \frac{m^* p_F^2}{2 \pi^2 \hbar^3} \)

\[ \chi = \frac{\alpha N^*(0) / T}{1 - F_0^2} \] \( T_{\text{tr}} = \frac{2 \hbar^2}{3N^*(0)} \)

\[ W = \frac{\chi}{\chi} = 3 \left( \frac{4e^2 \hbar^2 k_B}{2 \pi^2 \hbar^3} \right) \frac{1}{1 + F_0^2} \]

Although 4f/5f metals vary 3 order of magnitudes in both \( \chi \) and \( N^*(0) \), the ratio \( W \) seems to fit on a line

most effects are accounted for by \( N^*(0) \)

NOTE: actual material below the ideal line

because of correction by \( g_f \) and \( F_0 \)

Local moment interact with itinerant \( e^- \) to form resonant states
- Possible Phases of heavy fermion system
  - metal, superconductor, Kondo insulator, quantum critical pts, "others"
  - UBe$_1$_$_3$: local moment $\rightarrow$ metal $\rightarrow$ superconductor
  - But traditionally (~BCS) local moment is thought to destroy SC.
  - Ce$_3$Pt$_4$Bi$_3$: Kondo insulator
  - Ce(Co, Rh, Ir) In [Pagliuso et al.]
  - YbRh$_2$Si$_2$: QCP
  - UPd$_2$Al$_3$: SC + AF coexistence

- Landau Fermi Liquid
  - Turn on interaction adiabatically: Pauli exclusion $\rightarrow$ Fermi surface robust. Excitation spectrum preserved.
  - Hence, $\left\langle \frac{e^{-}}{m} \mid \frac{1}{q} \right\rangle = \frac{n_{q}^{1}}{m} = \frac{N(q)}{N(0)} = 1 + \frac{F_{s}}{3}$

  - From 1950 to 1960, it is realized that one can drop the long-ranged part of Coulomb and capture the essential physics by remaining short-ranged interaction (e.g. Hubbard model, Anderson model, etc.)
  - This is facilitated by discovery/experiments on $^3$He.

- By Pauli exclusion, phase space restriction
  $\Rightarrow T^2(\varepsilon) = (\varepsilon^2 + T^2)$
\[ \Delta \text{ Thus, we have Landau energy functional:} \]
\[ \mathcal{E}_1 = \mathcal{E}_0 + \sum_{p, \sigma} (E_{p, \sigma}^{(0)} - \mu) \delta n_{p, \sigma} + \frac{1}{2} \sum_{p, p', \sigma, \sigma'} f_{p p' \sigma \sigma'} \delta n_{p, \sigma} \delta n_{p', \sigma'} \]

\[ \Delta \text{ Landau energy functional \sim "fixed point" Hamiltonian} \quad (\text{Shankar, RMP, 94'}) \]

\[ \Delta \text{ Warning: } \epsilon_{p, \sigma}^{(0)} = E_{p, \sigma}^{(0)} + \sum_{p', \sigma'} f_{p p' \sigma \sigma'} \delta n_{p, \sigma} (E_{p', \sigma'}^{(0)} - \mu) \neq \epsilon_{p, \sigma}^{(0)} \]

This is a feedback (aka self-consistent) condition & lead to renormalization \& excitation energy.

\[ \Delta \text{ Entropy: } S = -k_B \sum_{p, \sigma} \left( n_{p, \sigma} \ln n_{p, \sigma} + (1 - n_{p, \sigma}) \ln (1 - n_{p, \sigma}) \right) \]

\[ \Rightarrow \quad n_{p, \sigma} = \frac{\epsilon_{p, \sigma}^{(0)}}{e^{\epsilon_{p, \sigma}^{(0)} / k_B} + 1} = f(\epsilon_{p, \sigma}^{(0)}) \quad \text{renormalized energy} \]

As \( T \to 0 \), \( \delta n_p \to 0 \), \( n_{p, \sigma} = f(\epsilon_{p, \sigma}^{(0)}) \)

\[ v_p = \frac{\partial \mathcal{E}_1}{\partial p} = -\frac{\mu}{m^*} \]

\[ \Delta \text{ This give rise to linear heat capacity.} \]

\[ \Delta \text{ By rotation invariance,} \quad \{ f_{p p' \sigma \sigma'} = f_{p' p \sigma' \sigma} \}

\[ f_{s a} = f_{s a} (\cos \theta) = \hat{p} \cdot \hat{p}' \]

\[ \Rightarrow \quad f_{s a} (\cos \theta) = \frac{1}{N_{s a}^{(0)}} \sum_{l=0}^{\infty} (2l+1) F_{s a}^{s a} P_{l} (\cos \theta) \quad \text{landau parameters} \]

\[ \Delta \text{ Let } \delta E_{p, \sigma}^{(0)} = \beta E_{p, \sigma} \]

\[ \Delta \text{ From this we find} \]

\[ \xi_s = \frac{\mu \beta}{1 + F_{s}^{s a}} = \mu \beta N_{s a}^{(0)} (1 - A_{s}^{a}) \]

\[ \xi_c = \frac{N_{c}^{(0)}}{1 + F_{c}^{s a}} = N_{s a}^{(0)} (1 - A_{c}^{s}) \]

\[ \text{large in heavy fermion} \]

where \( A_{s}^{a} = \frac{F_{c}^{s a}}{1 + F_{s}^{s a}} \)

(The \( A_s \)'s can be interpret as T-matrix amplitude for s-wave scatter)

\[ \Delta \text{ Since } \xi_s \text{ large while } \xi_c \text{ unrenormalized, } 1 - A_{s}^{a} \approx 0. \]
Heavy Fermion & Local Landau Fermi Liquid

$$\Delta A_{\text{pop}} = \frac{1}{N^{(0)}} (A_0^2 + A_0 \sigma \sigma')$$

$$A_0^{(0)} = A_0^2 + A_0^a \approx 0 \quad \text{and} \quad \kappa \phi \approx 0$$

$$\Rightarrow A_0^a = - A_0^a \approx 1$$

$$\Delta \langle p(T) \rangle = p_0 + A T^2 \quad ; \quad \frac{A}{\phi} = \text{approx. const.} \quad \& \quad \text{is universal}$$

$$\langle C(T) \rangle = T^2$$