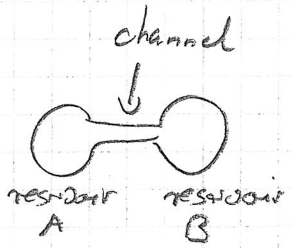
Dynamics in strongly correlated quantum gases

large variety exists

focus on two different setups

1) transport in 2 terminal setup



2) optical lattice systems

before I describe these in more detail

introduction : crash course on quantum gases

- quantum gas, [illegible]
- interaction between atoms
- light-atom interaction

Foot Atomic Physics
Pethick & Smith BEC
Stringari & Pitaevskii
Bloch, Dalibard Zwerger Rev Mod Phys 2008

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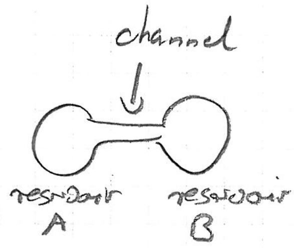

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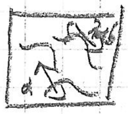
atomic quantum gases:

atom: - internal structure: ◎ nucleus + e^-s

- external degrees of motion

classical gas



quantum behaviour:

$$\omega := n\,\lambda_{dB}^3 \gtrsim 1$$

density $\sim 1/d^3$; de Broglie wavelength

in a thermal ensemble $\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$

define quantum degeneracy temp as $n \cdot \lambda_{dB}^3(T_c) :\overset{!}{=} 1$

$$\Rightarrow T_c = \frac{2\pi\hbar^2}{m k_B} n^{2/3}$$

examples:	m	$n\,[cm^{-3}]$	T_c [K]	[eV]	time
nuclear matter (neutron star)	m_n	10^{38}	10^{12}	30 MeV	
electron gas	m_e	10^{23}	$50 \cdot 10^3$	10 eV →	ps/fs
superfluid He	^4He	10^{22}	5	1 meV	
atomic gas	^{87}Rb	10^{13}	$100 \cdot 10^{-9}$ (nK) ↳ long time scales!	10 peV	ms

ms days

(NR: $\hbar = 1{,}05 \cdot 10^{-34}\,Js$, $k_B = 1{,}38 \cdot 10^{-23}\,JK^{-1}$, $Js^2 = 1\,kg\,m^2$

$\lambda_{dB,e} \approx \frac{0{,}7 \cdot 10^{-7} m}{\sqrt{T/K}}$, $m_p \approx 1{,}67 \cdot 10^{-27}\,kg$, $m_e = 9{,}1 \cdot 10^{-31}\,kg$

$m_p/m_e = 2000$)

How to get cool and avoid solidification?
(very low density)

atomic quantum gases:

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How to get cool and avoid solidification ?
(very low density)

How do get cool ?

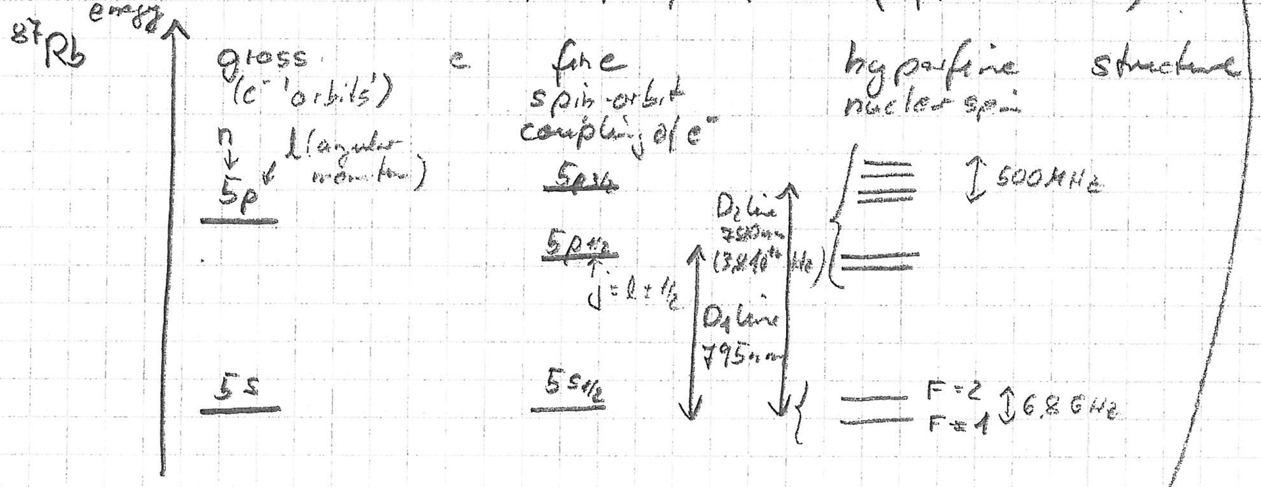
Laser cooling & evaporative cooling

use internal structure of atoms to manipulate external motion

brief reminder internal structure:

alkaline atoms : $1e^-$ in outer shell

Li, Na, K, Rb, Cs (often used)



further splitting in magnetic field

laser cooling: stop atoms by force ~~ex~~ of laser

evaporative cooling: like cup of coffee, let hot atoms escape
rethermalize to smaller temperature
reaches ~ 100 nK (v ~ 1 mm/s)

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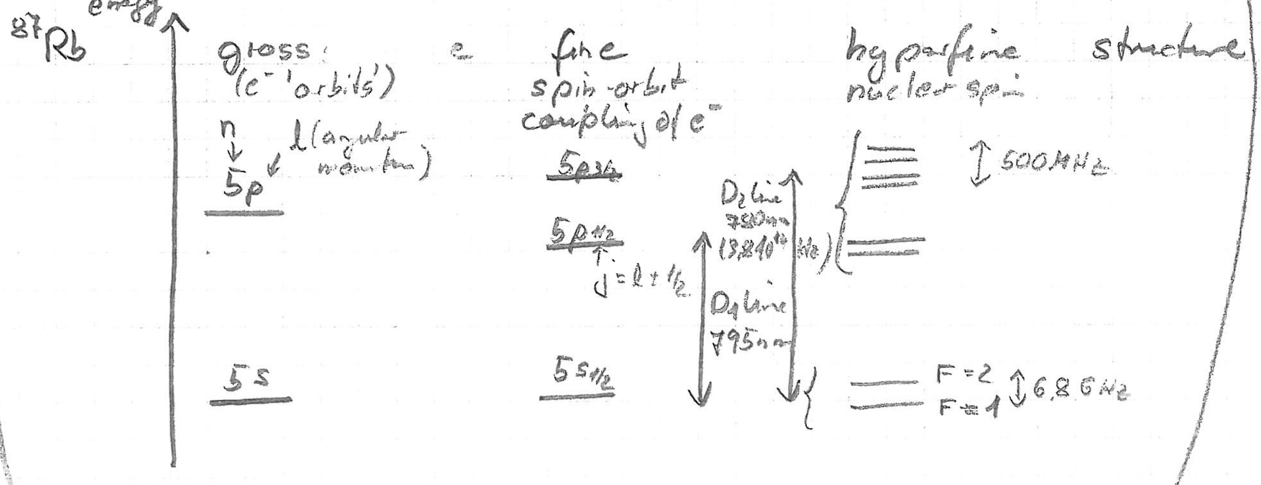

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How to get cool

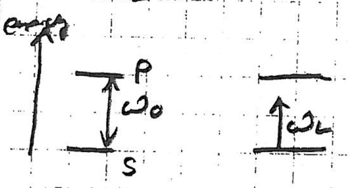
- problem: need to work at very low density to avoid solidification

new cooling methods needed:

Laser cooling:
proposed by Hänsch, Schawlow, Wineland, Dehmelt 1975)

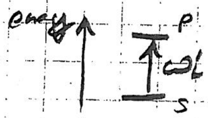
atom $\longrightarrow$ $p = mv$ laser $\longleftarrow$ $p = \hbar k$, ω_L

atom at rest:



restframe of moving atom: $\omega_L' = \omega_L + v \cdot k$

-> can be on resonance for a moving atom



atom absorbes photon

in 3D optimal detuning $\gamma/2$

$\bullet \longrightarrow$
$p = mv + \hbar k$

spontaneous emission (not directed)

$\Gamma/2$

$\bullet \longrightarrow$

$p = mv + \hbar k + \hbar k'$ with $\langle \hbar k' \rangle = 0$

energy difference: $\langle \Delta E \rangle = -\hbar v |k|$

works very well within a few seconds cooled to ~ 0,1 mK (determined by spontaneous emission rate γ)
many different atoms (Alkaline)
(limits since spontaneous emission random walk -> $\langle k'^2 \rangle \neq 0$)
Doppler limit $\gamma/2k_B$
Nobel prize 1997: Steven Chu, Claude Cohen-Tannoudji, William Philips
'for development of methods to cool & trap atoms with lasers'

not yet quantum degenerate

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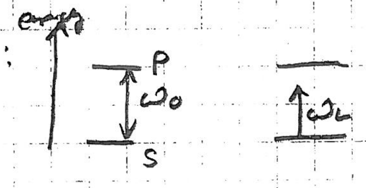

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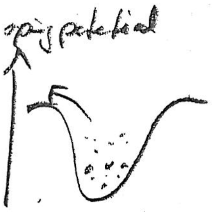
Evaporative cooling

leave out

- principle: like cup of coffee



fast atoms 'spill' over

rethermalization

(magnetic traps: use rf knife to switch atoms to $m_F = 0$; dipole traps:

continuous process, limited by atom losses

reaches approx ~100 nK ($v \sim 1\,\mathrm{mm/s}$)

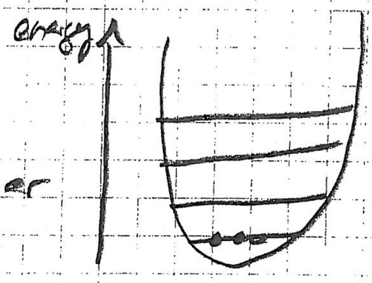
-> bosons & fermions become quantum degenerate

comments:
- works very well with bosons, since good scattering properties
- less well with fermions due to Pauli principle (s-wave scattering suppressed)

-> need different hyperfine states or mixtures from bosons and fermions

How do detect quantum degeneracy for bosons?

Bose-Einstein condensation:



ideal Bose gas zero temperature

general criterium of Penrose & Onsager

one-particle density matrix

$\rho(r,r') = \langle \psi^\dagger(r)\,\psi(r') \rangle$, ψ boson operator

shows 'off-diagonal long range order'

$\lim_{|r-r'|\to\infty} \rho(r,r') \Rightarrow \langle n_0 \rangle$ with $\langle n_0 \rangle$ of order N

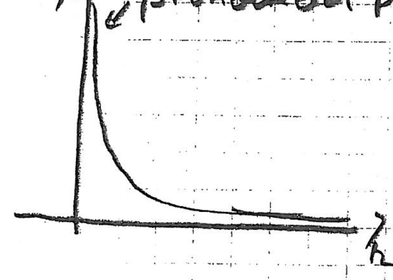
macroscopic occupation of $k=0$ state

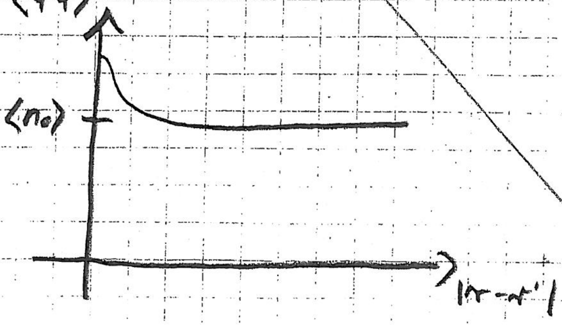
(all other contributions oscillate out

$\rho(r,r') \propto \sum_{k,k'} e^{ikr - ik'r'} \langle \psi_k^\dagger \psi_{k'} \rangle$)

-> momentum distribution

$n(k) = \langle n_0 \rangle\, \delta_{k,0}$ + reg. fluctuations





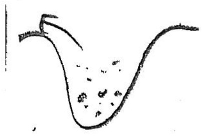
Evaporative cooling

leave out

(p5)

- principle: like cup of coffee

trap potential



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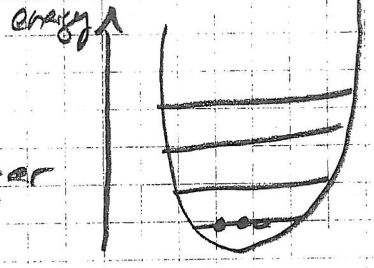

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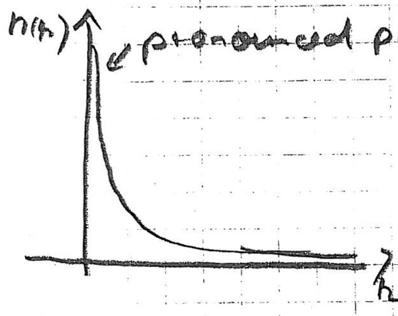
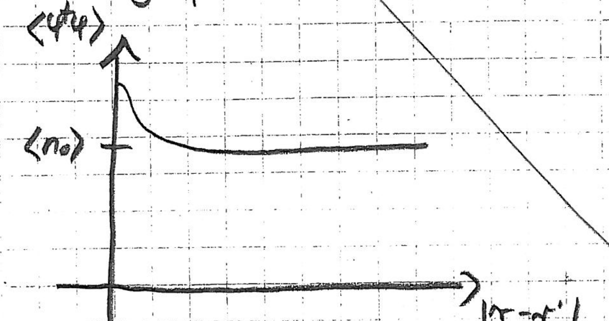
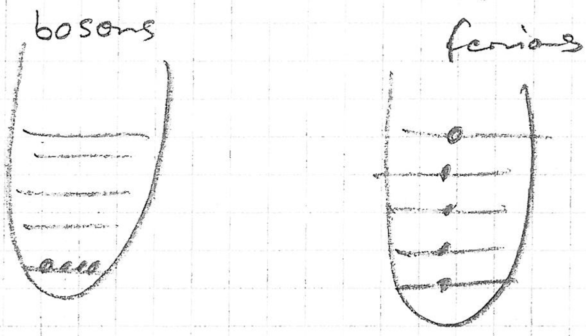




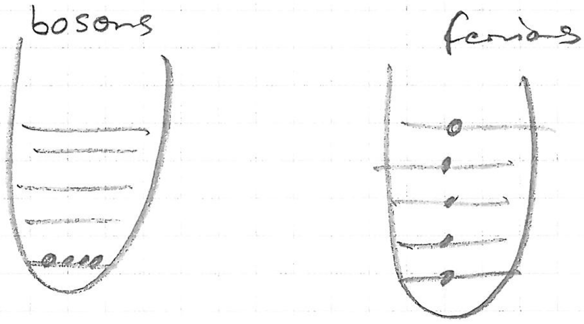
Quantum degenerate gases



Bose-Einstein
condensate

slide

Quantum degenerate gases

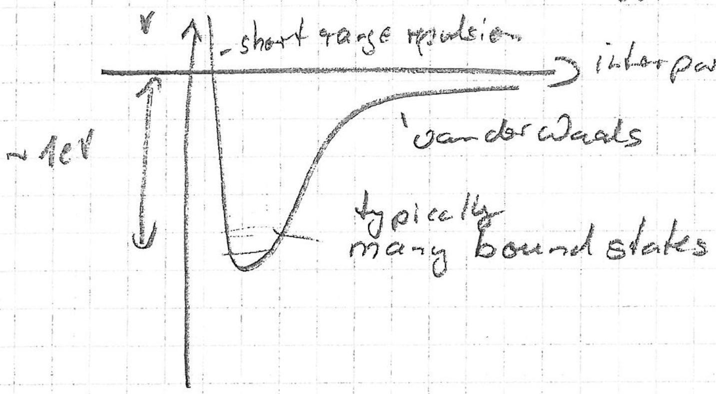


Bose-Einstein condensate

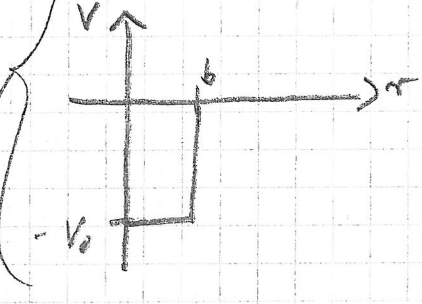
slide

scattering between atoms

neutral atoms : van der Waal potential $V(r) \sim \frac{C_6}{|r|^6}$ interparticle



reminder : scattering at low energies



scattering state

$$\psi(r) \sim \underset{\substack{\uparrow \\ \text{incident} \\ \text{plane wave}}}{e^{ikr}} + f(k) \underbrace{\frac{e^{ikr}}{r}}_{\text{spherical wave}}$$

at low energy dominated by

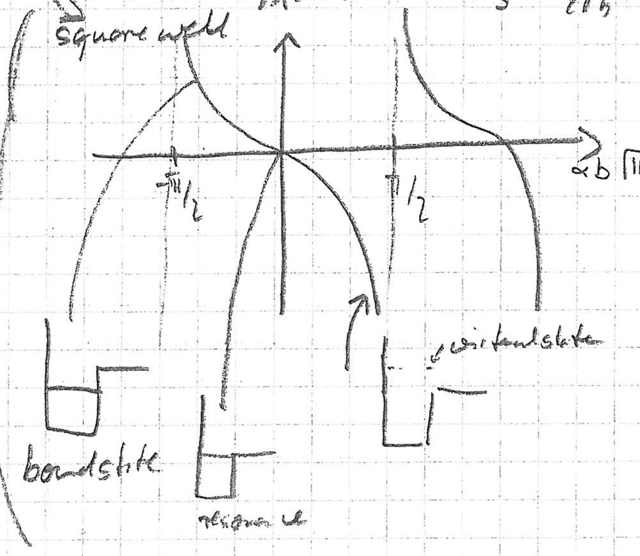
scattering length : $a_s := -\lim_{k \to 0} f(k)$

Born approx : $a_s = \frac{m}{2\pi\hbar^2} \int V(r')\, d^3r'$

home work check : (Dalibard & Basdevant, QM)

$$a_s = b - \frac{\tan Kb}{K} \quad , K = \frac{\sqrt{2m|V_0|}}{\hbar}$$

square well



comments :
- scattering resonance
- sign of scattering lengths can change

↳ describe scattering of atoms by local pseudo potential with same scattering length $\quad g\,\delta(r) \left(\frac{\partial}{\partial r} r\, \psi(r)\right)$ 'sloppy'

with $g = \frac{4\pi\hbar^2 a_s}{m}$

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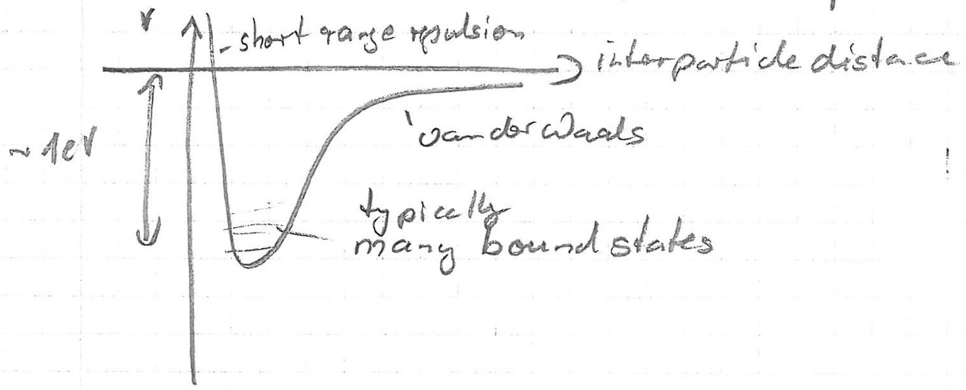

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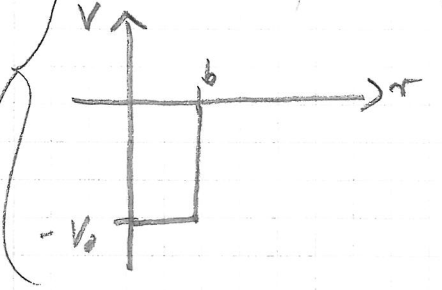

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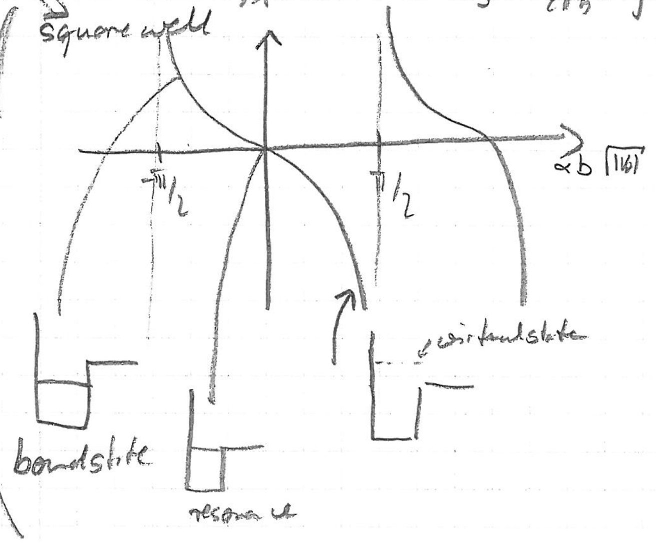

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with $g = \frac{4\pi\hbar^2 a_s}{m}$

scattering length a_s typically hard to calculate, but can be measured

Feshbach resonance: a_s can be tuned by magnetic field, coupling differently to shifted spin configurations

Rev Mod Phys 78, 1311 Köhler et al

<u>slide example</u>

$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

a_{bg} : background scattering length

ΔB width of resonance

B_0 : position of resonance

Li K

description of <u>external motion</u>

single particle operator

$$H = \int d^3r \left(-\frac{\hbar^2}{2m} \Psi^\dagger(\underline{r}) \nabla^2 \Psi(\underline{r}) + g \Psi^\dagger(\underline{r}) \Psi^\dagger(\underline{r}) \Psi(\underline{r}) \Psi(\underline{r}) + V(\underline{r}) \Psi^\dagger(\underline{r}) \Psi(\underline{r}) \right)$$

confinement

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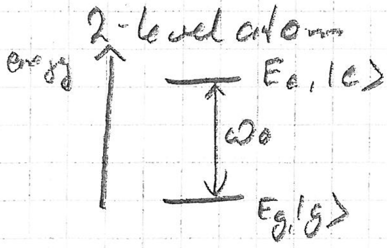
single particle operator

consider next

Effect of atom light interaction

(Cohen-Tannoudji, QM) Foot Atomic Phys

2-level atom



atom

$H_a = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$

shine in laser light $\underline{E}(r,t) = E_0 \cdot \hat{\varepsilon} \cos(\omega_L t + \cancel{kz})$

treat laser classically (not single photon)

Dipole approx: varies slowly c/t the size of atom, $\lambda \approx 500\,nm$, $a_0 \approx 50\,pm$

$H_I = -\underline{d}\,\underline{E}(t)$ ← induced dipole moment of atom, $\hat{d} = e\hat{r}$

$\hookrightarrow \quad H_{tot} = \hbar \begin{pmatrix} -\omega_0/2 & \Omega\cos(\omega_L t) \\ \Omega^* \cos(\omega_L t) & \omega_0/2 \end{pmatrix} \qquad \Omega = \dfrac{\langle g | e\hat{r} E_0 | e\rangle}{\hbar}$

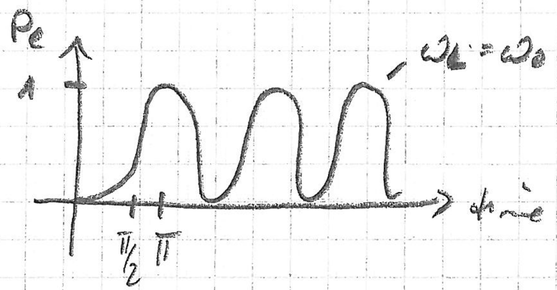
2 situations: i) close to resonance $\omega_0 \approx \omega_L$, $|\omega_0 - \omega_L| \ll \omega_0, \omega_L$

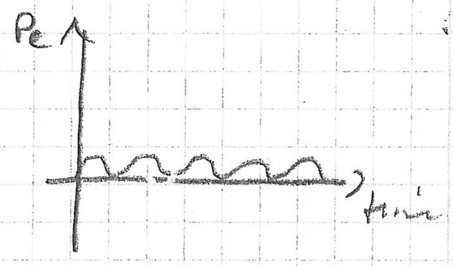
ii) far off resonance $|\omega_0 - \omega_L| \gg \omega_0, \omega_L$

i) solution to find atom in state $|e\rangle$ (rotating wave approx) ← go into rotating frame of atom

(initially prepared in $|g\rangle$)

$$P_e(t) = \frac{\Omega^2}{\Omega^2 + (\omega_L - \omega_0)^2} \sin^2\left(\frac{\sqrt{\Omega^2 + (\omega_L - \omega_0)^2}}{2} t \right)$$





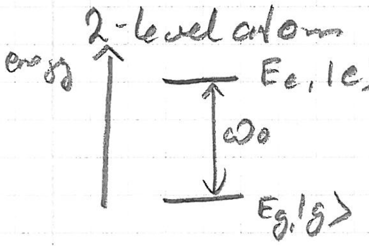
Rabi oscillations : π pulse : preparation in $|e\rangle$

$\pi/2$ pulse : superposition of $|e\rangle, |g\rangle$

important for preparation of different spin states (qbits)

Effect of atom light interaction

(Cohen-Tannoudji, QM) Foot Atomic physics



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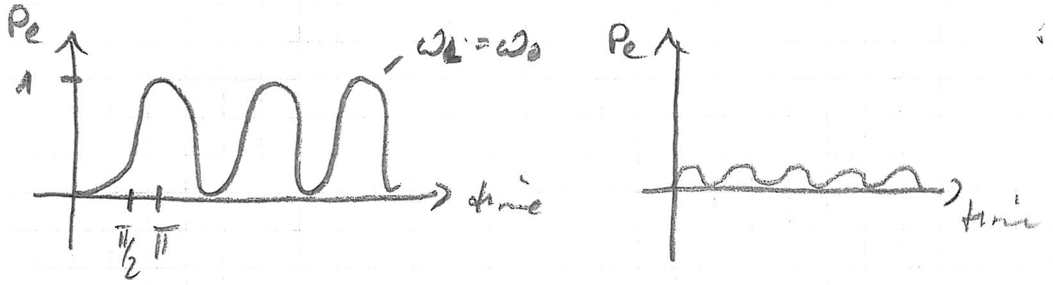

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$\pi/2$ pulse: superposition of $|e\rangle, |g\rangle$

important for preparation of different spin states (cf. ...)

ii) for off resonant: light-shift (ac-Stark shift)

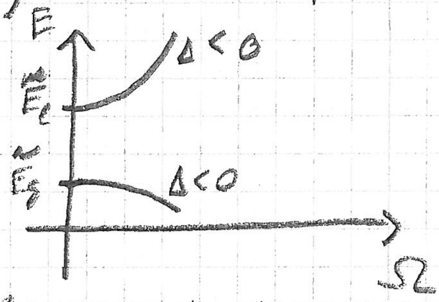
(go into rotating frame ω_L

$$\tilde{H} = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega^* & -\Delta \end{pmatrix} \qquad \Delta = \omega_L - \omega_0$$
)

new eigenstates (stroboscopic) $|\Delta| \gg |\Omega|$ for detuned

$$\tilde{E}_g \approx -\frac{\hbar\omega_0}{2} + \frac{\hbar\Omega^2}{4(\omega_L-\omega_0)}$$

$$\tilde{E}_e \approx \frac{\hbar\omega_0}{2} - \frac{\hbar\Omega^2}{4(\omega_L-\omega_0)}$$



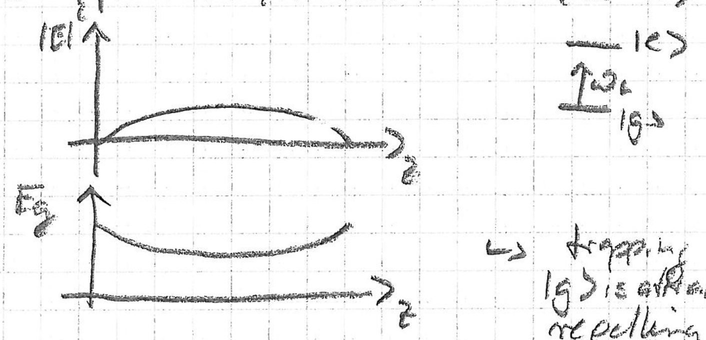
light field causes shift of energy levels
↳ induced dipole potential

$$V = -\tfrac{1}{2}\,\alpha\,|E|^2$$

polarizability $\propto \frac{1}{\omega_L-\omega_0}$

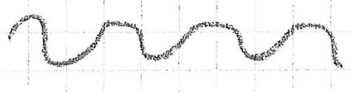
→ homework: recover both solutions
Foot atomic physics

assume focused, red detuned ($\Delta<0$) laser beam



↳ trapping potential for $|g\rangle$, $|g\rangle$ is attracted to intensity max., repelling for $|e\rangle$

- by shaping laser field many different geometries possible
- rapidly tunable
- since dependence on Δ can be diff't for diff't states/species (but not numbers)

optical lattice →  ⇐

counterpropagating beams ↳ standing wave $V \sim V_0 \sin^2(kx)$

$\lambda \sim 1000\,nm$

↳ pancakes

many diff't geometries possible

ii) for off resonant: light-shift (ac-Stark shift)

$$\left(\begin{array}{l} \text{go into rotating frame } \omega_L \\ \tilde{H} = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega^* & -\Delta \end{pmatrix} \qquad\qquad \Delta = \omega_L - \omega_0 \end{array} \right)$$

new eigenstates (stroboscopic) $|\Delta| \gg |\Omega|$ far detuned

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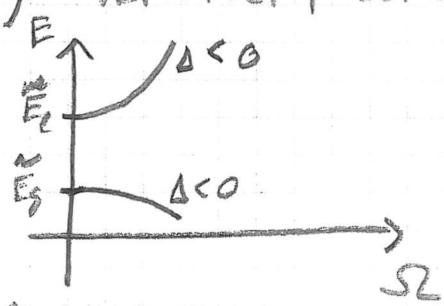

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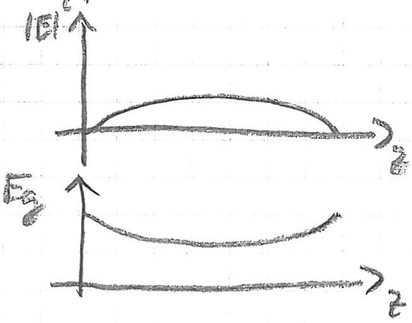

↳ trapping potential for $|g\rangle$
$|g\rangle$ is attracted to intensity max.
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- by shaping laser field many different geometries possible
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optical lattice

counterpropagating beams ↳ standing wave $V \sim V_0 \sin^2(k_L z)$
$\lambda \sim 1000\,nm$
↳ pancakes

many diff. geometries possible

Cold atoms

$$H = \sum_\sigma \int d^3r \left(-\frac{\hbar^2}{2m} \psi_\sigma^\dagger(r) \nabla^2 \psi_\sigma(r) + g\, \psi^\dagger \psi^\dagger \psi \psi + V(r)\, \psi^\dagger \psi \right)$$

knobs : - interaction strength $g \sim a_s$

- tunable geometries : - lattices
  - disorder
  - 1D, 2D, 3D
  - linear potential
  - ⋮

'internal' : state conversions, spin flip (local/global)

momentum kick, ...

choose your atoms : bosons/fermions, spins, masses...

comments :

- well decoupled from environment
  ↳ no thermal bath ⇒ think of entropy and not so much temperature