As far as I know solving 2D Hubbard for $U$ not small can only be done numerically.\(\uparrow\)

- Turns out to be simple at $U \to \infty$ because no intertwined orders.

Li et al., PRL 108, 126406 (2012)

DMRG on $2^{-6}$ log ladders for $x \leq 0.25$ results indep. of # of logs.

Ferromagnetism for $0 < x < x_c \sim 0.2$

Checkerboard insulating CDW, SDW at $x = 0.25$ (bond density wave)
Phase separation for $x_c < x < \sqrt[3]{4}$

2 For $U \sim 8t$

Reason for optimism:

No reason for low $p$

Why it has proven hard:

interwined orders.

"Best" variational treatment

of t-J mode with $J = t$

Corboz et al. arXiv 1402.2859

Uses tensor network states with

large bond dimension.
1. States with energy differences too close to call

2. Striped S.C.: SC + CDW + SDW

Vertical site centered stripes.

\[ \lambda \approx \left[ \frac{x_0 (J/t)}{x} \right] \]

\( x_0 \approx 1 \) for \( J/t = 0.8 \) insulating

\( x_0 \approx 0.5 \) for \( J/t = 0.2 \) No diagonal Stripes or checkerboards.

d-wave-line S.C.

SDW has anti-phase domain walls at peaks of CDW
b) PDW

\[ \Delta = 0 \quad (|\Delta_\alpha| = |\Delta_{-\alpha}| \neq 0) \]

\[ \frac{E_{PDW}}{M} \approx \frac{E_{stripe}}{N} + x \cdot \alpha_1 \approx 0.001 \]

\[ \alpha_1 \approx 0.001 \]

c) Uniform SC + AF \quad x < x_c \approx 0.1

\[ \frac{E_{SC}}{N} = \frac{E_{stripe}}{N} + x \cdot \alpha_2 \]

\[ \alpha_2 \approx 0.01 \]
To access "strong coupling"

physics in a controlled manner

⇒ Quasi 1D models.
Quasi 1D "Hubbard" models.

(Attractive chains)

Arrays of 2-leg repulsive ladders

Step 1: Solve the 1D problem.

1DEG: ① Never has Q.P. description at $T \to 0$.

Breaks down even in weak coupling.

② Asymptotic separation of spin and charge.

③ Bosonization gives asymptotically exact QFT description.

$(\Phi_s, \Theta_s) \quad (\Phi_c, \Theta_c)$

In both the examples I am considering $\Delta_s > 0$.

For $T > \Delta_s \Rightarrow$ Luttinger liquid

$T < \Delta_s \Rightarrow$ Luttinger-Energy liquid.
at $T > \Delta_s$, $\chi_{ss} \sim P(E_F)$

$\chi_{CDW} \sim P(E_F)$

$\Delta_s \rightarrow \Delta_s T^{-1}$

$\chi_{sC} \sim \Delta_s T^{-2-\kappa_c}$

$\chi_{CDW} \sim \Delta_s T^{-2-\kappa_c}$

\[ J = \frac{1}{3} t \]

White & Affleck

et al.

PRB (2002)

PRL (1997)
weak inter-ladder coupling.

\[ J = \text{Josephson} \]

\[ \mathcal{V} = 2 k F \text{ COW coupling} \]

\[ t_1 = \text{single particle hopping} \]

\[ \hat{U} t_1 < \Delta_s = \text{"irrelevant"} \]

\[ J \& \mathcal{V} \text{ both relevant} \]

(for some range of \( K_c \))

\[ J \text{ relevant } \mathcal{V} \quad K_c > \frac{1}{2} \]

\[ \mathcal{V} \text{ irrelevant } K_c < \frac{1}{2} \]

Inter-ladder MFT

\[ T_c \sim x \Delta_s \left( \frac{J}{t} \right) \quad \Delta_{sc} = \frac{K_c}{2K_c - 1} \]

\[ T_{\text{CW}} \sim x \Delta_s \left( \frac{\mathcal{V}}{t} \right) \quad \Delta_{\text{CW}} = \frac{1}{2 - K_c} \]

This factor involves more work. It is related to Mott insulator at \( x = 0 \).

Description of multi-critical point.


$\mathbf{N} = SO(4)$ vector, $\mathbf{n} \cdot \mathbf{n} = 1$

$\frac{n_1}{n_2} = \tan(\Theta_{sc})$

$\frac{n_3}{n_4} = \tan(\Theta_{cow})$

$|n_1|^2 + |n_2|^2 \propto \Delta^2$
\[ \sum n_j = \frac{1}{2g} (\delta_n n_1^2) + h_1 \left[ n_1^2 + n_2^2 - n_3^2 - n_4^2 \right] + h_2 \left[ (\delta_n n_1)^2 + (\delta_n n_2)^2 - (\delta_n n_3)^2 - (\delta_n n_4)^2 \right] + \cdots \]

\[ h_2 \propto (K_c-1) \quad h_1 \propto \Delta_s [\bar{J} - \bar{V}] \]

Both are "relevant"

1. Multi-critical pt. \( P \) has approximate \( SO(4) \) sym.

   only under very fine tuned circumstance!

2. The pseudo-gap is a "spin-gap"

   It enhances both couplings.
TFL is where Q.P. loses integrity.

Here this occurs by dimensional crossover.

This crossover is not a solved problem even in this context!