As far as I know solving 2D Hubbard for U not small can only be done numerically.

① Turns out to be simple at $U \to \infty$ because no intertwined orders.

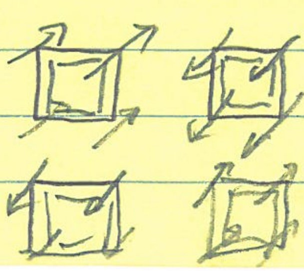
Li et al, PRL 108, 126406 (2012)

DMRG, on 2–6 leg ladders.

for $x \leq 0.25$ results indep of # of legs.

Ferromagnetism for $0 < x < x_c \sim 0.2$

Checkerboard insulating CDW+SDW at $x \sim 0.25$



(bond density wave)

Phase separation for $x_c < x < 1/4$

(2) For $U \sim 8t$

Reason for optimism:

No reason for long ξ

Why it has proven hard:

intertwined orders.

"Best" variational treatment

of $t-J$ mode with $J \sim t$

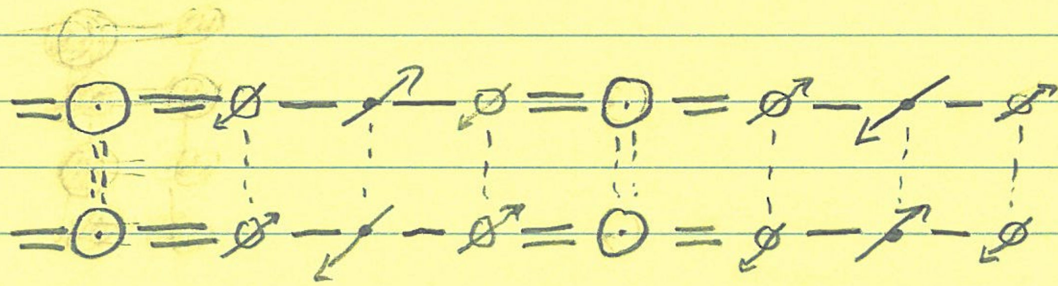
Corboz et al arXiv 1402.2859

Uses tensor network states with

large bond dimension.

① 3 states with energy differences too close to call

ⓐ Striped S.C. : SC + CDW + SDW



Vertical site centered stripes.

$$\lambda \cong \left[\frac{x_0(J/t)}{x} \right]$$

lowest energy

$x_0 \cong 1$ for $J/t = .8$
insulating

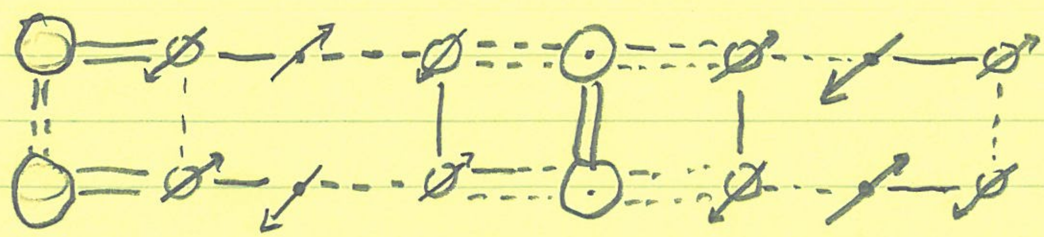
$x_0 \approx 0.5$ for $J/t = 0.2$

No diagonal Stripes or checkerboards.

d-wave-like S.C.

SDW has anti-phase domain walls at peaks of CDW

b) PDW



$$\Delta = 0 \qquad |\Delta_Q| = |\Delta_{-Q}| \neq 0.$$

$$\frac{E_{PDW}}{N} \approx \frac{E_{stripe}}{N} + x\,t\,\alpha_1$$

$$\alpha_1 \approx 0.001 \quad !$$

c) Uniform SC $\nearrow$ + AF $x < x_c \approx 0.1$; $\searrow$ plain $x > x_c$

$$\frac{E_{SC}}{N} = \frac{E_{stripe}}{N} + x\,t\,\alpha_2$$

$$\alpha_2 \approx 0.01$$

To access "strong coupling"

physics in a controlled manner

$\Rightarrow$ Quasi 1D models.

Quasi 1D "Hubbard" models.

(Attractive chains)

Arrays of 2-legs repulsive ladders

Step 1 Solve the 1D problem.

1DEG : (1) Never has Q.P. description at $T \to 0$.

Breaks down even in weak coupling.

(2) Assymptotic separation of spin & charge.

(3) Bosonization gives assymptotically exact QFT description.

$$(\varphi_s, \theta_s) \qquad (\varphi_c, \theta_c)$$

In both the examples I am considering

$\Delta_s > 0$.

For $T > \Delta_s \Rightarrow$ Luttinger liquid

$T < \Delta_s \Rightarrow$ Luther-Emery liquid.

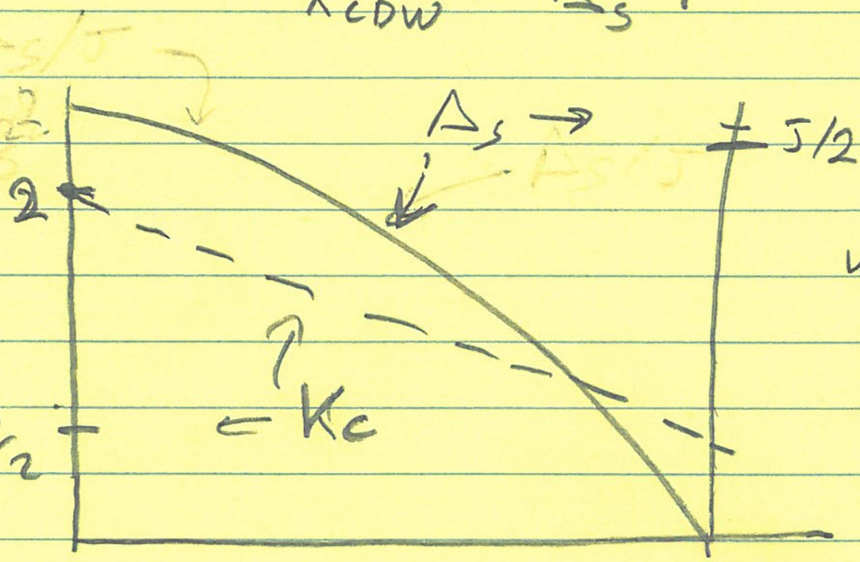
at $T > \Delta_s$ $\quad \chi_{ss} \sim \rho(E_F)$

$\chi_{CDW} \sim \rho(E_F)$

at $T < \Delta_s$

* $\chi_{SC} \sim \Delta_s T^{-(2-K_c^{-1})}$

$\chi_{CDW} \sim \Delta_s T^{-(2-K_c)}$



$J = \frac{1}{3} t$
White & Affleck et al
PRB (2002)
PRL (1997)

weak inter-ladder coupling.

J = Josephson

V = $2k_F$ CDW coupling

$t_\perp$ = single particle hopping.

if $t_\perp < \Delta_s$ = "irrelevant"

J & V both relevant

(for some range of K_c)

J relevant for $K_c > 1/2$

V " " $K_c < 2$

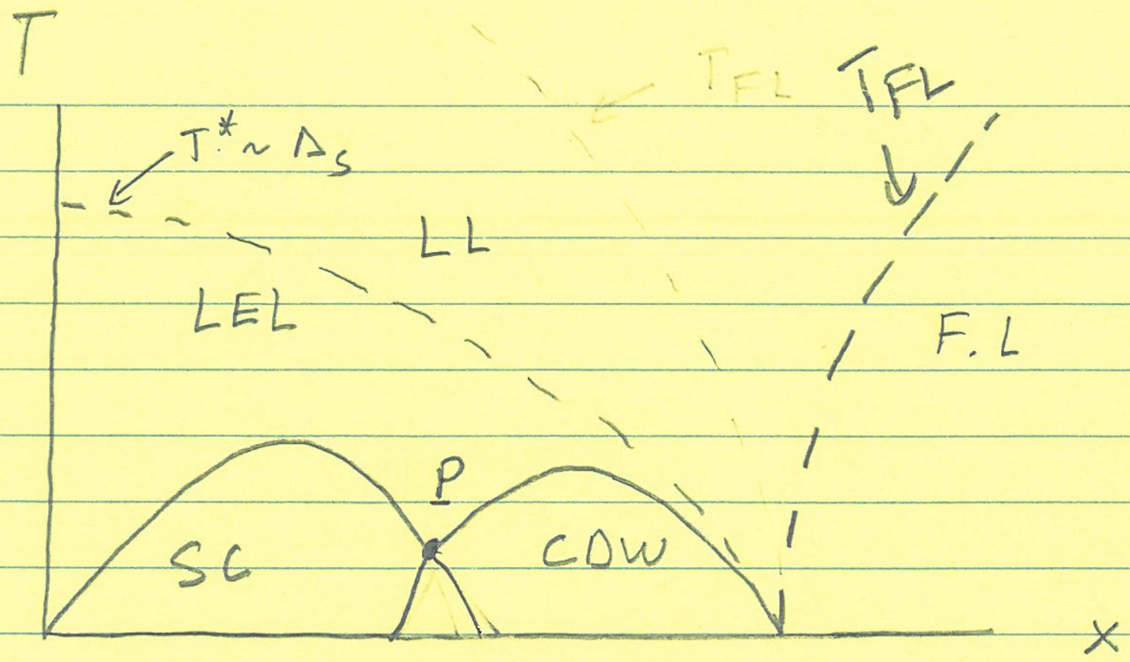
Interladder MFT

$$T_c \sim x \, \Delta_s \left(J / t \right)^{\alpha_{sc}} \qquad \alpha_{sc} = \frac{K_c}{2K_c - 1}$$

Arrigoni et al

$$T_{CDW} \sim x \, \Delta_s \left(V / t \right)^{\alpha_{CDW}} \qquad \alpha_{CDW} = \frac{1}{2 - K_c}$$

This factor involves more work. It is related to Mott insulator at $x = 0$.



Arrigoni et al (2004)

Description of multi-critical pt. P.

Jaefari et al (2010) in 2d.

n = SO(4) vector. $n \cdot n = 1$

$n_1 / n_2 = \tan(\theta_{sc})$

$n_3 / n_4 = \tan(\theta_{cdw}$

$|n_1|^2 + |n_2|^2 \propto |\Delta|^2$

$$S[n] = \frac{1}{2g}(\partial_\mu n)^2$$

$$+ h_1 [n_1^2 + n_2^2 - n_3^2 - n_4^2]$$

$$+ h_2 [(\partial_\mu n_1)^2 + (\partial_\mu n_2)^2 - (\partial_\mu n_3)^2 - (\partial_\mu n_4)^2]$$

$$+ \cdots$$

$h_2 \propto (K_c - 1)$ $\qquad$ $h_1 \propto \Delta_s [J - V]$

Both are "relevant"

① Multi-critical pt. $\underline{P}$ has approximate $SO(4)$ sym. only under very fine tuned circumstance!

② The pseudo-gap is a "spin-gap"

It enhances both CDW & SC susceptibilities.

③ T_{FL} is where Q.P. looses integrity. Here this occurs by dimensional crossover. This crossover is not a solved problem even in this context!