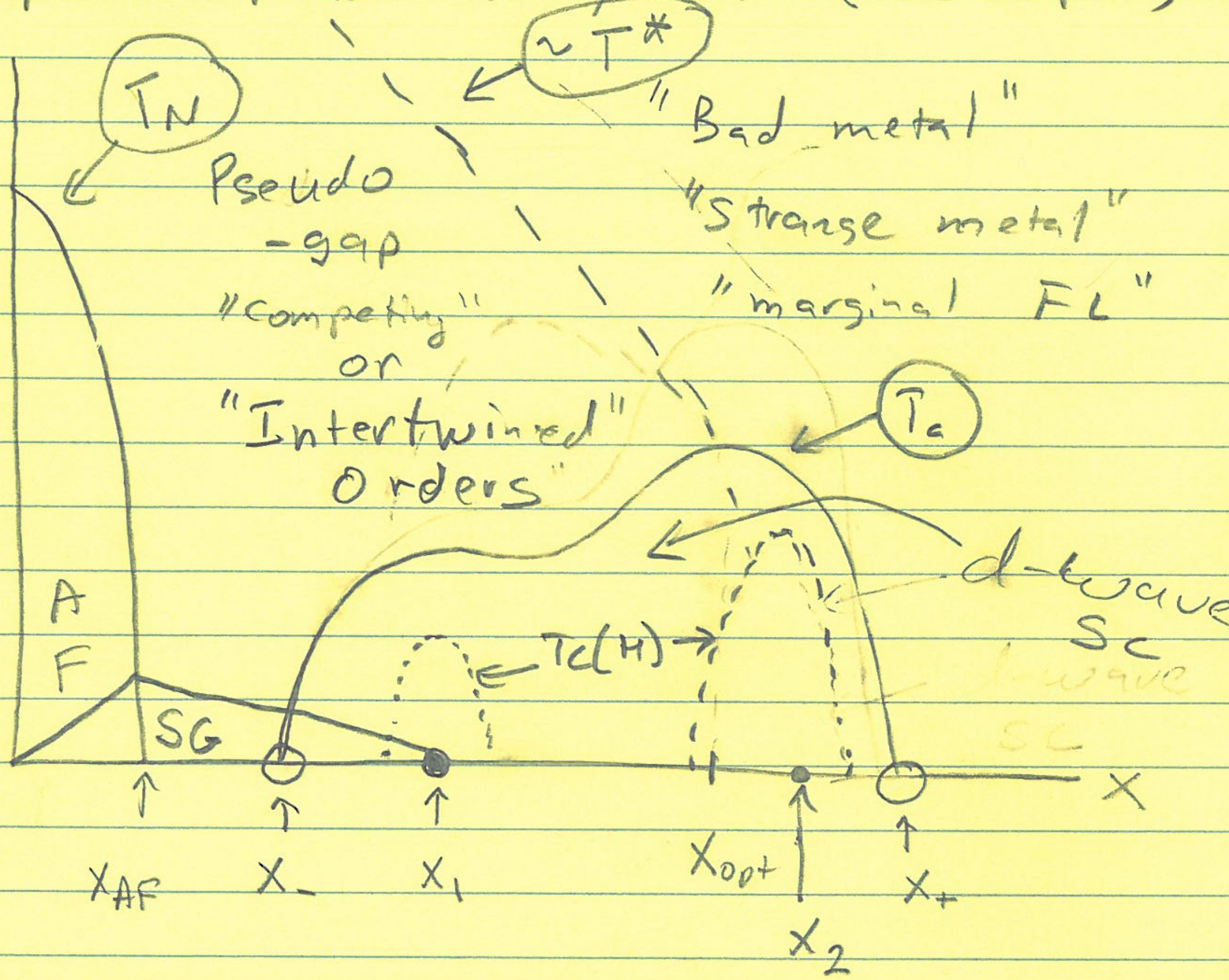
Lecture #1 : Questions

Cuprate Phase diagram (hole doped)



① Why is T_c so large?

② Why (and how) is it associated with antiferromagnetism?

③ Why is it d-wave SC?

④ What is a pseudo-gap? What, if any, is its relation to fluctuation SC?

X ⑤ What other orders are intertwined with SC.? Why.

{ Will not cover this, but will post draft review / colloquium }

a) CDW SRO.

b) SDW SRO.

c) "Nematic" (point group symmetry breaking)

electronic vs structural.

LRO vs SRO

"vestigial" vs "primary"

d) Spin glass

e) Pair-density-wave (PDW) ?

f) Orbital current order ?

g) Some form of topological order ?

7) What are the "universal" aspects?

a) Compare different hole doped cuprates

b) Compare electron & hole doped cuprates

{News! CDW order found in NCCO

$Q_{NCCO}(x=-0.15) \approx Q_{Bi2212}(x=+.13)$}

c) Cuprates & Fe based SC's

d) Other unconventional SC's

e) Other SC's with high T_c's

6) What is the nature of the various prominent QCP's

x_{AF}, x_-, x_1, x_2, x_+, ...

8) How can we change T_c?

{ including rather remarkable insensitivity to most forms of disorder }

9) Can theory provide any guidance in the search for new HTC superconductors?

*** 10) What is the bad metal?

a) Ubiquitous in (many) highly correlated electrons systems

b) It is not a FL.

c) It is largely incoherent.

5

The pros & cons of controlled solutions of simple models.

1) The Holstein model $\Longleftrightarrow$ the electron-phonon problem.

a) Weak coupling by RG.

BCS instability

Effect of retardation μ^*

b) Strong coupling

Bipolarons & CDW.

Breakdown of Eliashberg theory.

c) Optimal T_c at pairing to phase coherence crossover.

d) Why is T_c low for conventional SC's ?

2) What do we know about

2d Hubbard model?

** a) Weak coupling limit

$\Rightarrow$ origin of dwave SC.

$\Rightarrow$ band structure considerations

some inferences about pseudo-gap.

b) Absence of SC in the strong coupling limit.

c) Intrinsic frustration at intermediate coupling

d) A little about numerics.

3) Quasi-1D models.

a) A solvable problem with a MFL "normal" state

b) A solvable problem with a pseudo-gap regime

c) Again — Max T_c as a dimensional crossover.

phase-pairing crossover.

If time permits

4) Some thoughts about some of the QCP's

Holstein model

$$H = -\sum_{\substack{\langle ij \rangle \\ \sigma}} \left[c^{\dagger}_{i\sigma} c_{j\sigma} + h.c \right]$$

$$+ \sum_{j} \left\{ \frac{p_i^2}{2M} + \frac{K}{2} x_j^2 \right\}$$

$$- \alpha \sum_{j} x_j \hat{\rho}_j$$

$$+ U \sum_{j} \hat{\rho}_j \left[\hat{\rho}_j - 1 \right] \leftarrow \text{(Holstein-Hubbard)}$$

$$\hat{\rho}_j = \sum_{\sigma} c^{\dagger}_{j\sigma} c_{j\sigma}$$

Weak-coupling: step #1: integrate out phonons.

$$S_0 = S_{el} + S_{ph} + S_{el\text{-}ph.}$$

$$S^{eff}[\bar{c}, c] : \quad e^{-S^{eff}} = e^{-S_{el}} \langle e^{-S_{el\text{-}ph}} \rangle_{ph}$$

$\hbar = 1$

$$S^{eff} = S_{el} + \cancel{\langle S_{el-ph} \rangle_{ph}}$$

$$- \frac{1}{2} \left\langle \left[S_{el-ph} - \langle S_{el-ph} \rangle \right]^2 \right\rangle_{ph}$$

$$+ \ldots$$

$$= S_{el} - \frac{\alpha^2}{2} \int d\tau \, d\tau' \sum_j' \rho_j(\tau) \rho_j(\tau') \, G(\tau - \tau')$$

$$+ \ldots$$

$$G(\tau - \tau') = \langle T_\tau \{ X(\tau) \; X(\tau') \} \rangle$$

$$\nu = \frac{2\pi n}{\beta}$$

$$S_{ph} = \sum_\nu' \left\{ \frac{M}{2} \nu^2 + \frac{K}{2} \right\} |X_\nu|^2$$

$$\boxed{0 \le \tau \le \beta}$$

$$G(\tau) = \frac{1}{M\beta} \sum_\nu' \frac{e^{i\nu\tau}}{\nu^2 + \omega_0^2}$$

$\omega_0^2 = K/M$

$$= \frac{-1}{M} \int \frac{d\nu}{2\pi i} \left[\frac{1}{e^{\beta\nu} - 1} \right] \frac{e^{\nu\tau}}{\nu^2 - \omega_0^2}$$

$$= \frac{1}{2M\omega_0} \left\{ \bar{n}(\omega_0) e^{\omega_0 \tau} + [1 + n(\omega_0)] e^{-\omega_0 \tau} \right\}$$

For $\beta\omega_0 >> 1$

$$G(\tau) \approx \frac{1}{2M\omega_0} e^{-\omega_0|\tau|}$$

$$S^{eff} = \ldots + \int d\tau d\tau' \sum_j \rho_j(\tau) \rho_j(\tau') V^{eff}(\tau-\tau')$$

$$V^{eff}(\tau) = V\delta(\tau) - \frac{\alpha^2}{2} G(\tau)$$

Case 1: $\omega_0 >> E_F$ (anti-adiabatic)

$$\int d\tau \int d\tau' \rho_j(\tau) \rho_j(\tau') V^{eff}(\tau-\tau')$$

$$\approx \int d\tau \rho_j(\tau) \rho_j(\tau) \int d\tau' V^{eff}(\tau')$$

$$\int d\tau' V^{eff} = V - \frac{\alpha^2}{2M\omega_0^2} = V - \frac{\alpha^2}{2K}$$

"Negative V^{eff}" if $\alpha^2/2K > V$

$$\boxed{V^{eff} = V - \frac{\alpha^2}{2K}}$$ indep of M

Case #2 $\omega_0 \ll E_F$

$\Rightarrow$ retardation is significant.

Can get S.C. even if

$\alpha^2/2K < U$!

Step 2 : integrate out "fast"

fermion modes.

$$\psi(\vec{R},\tau) = \psi_{f}(\vec{R},\tau) + \psi_{s}(\vec{R},\tau)$$

$$\psi_{f} = \sum_{\substack{|\epsilon_{\vec{k}}| > \Lambda \\ |\nu| > \Lambda}}' \frac{e^{i\vec{k}\cdot\vec{R} - i\nu\tau}}{\sqrt{N\beta a^d}} C_{\vec{k}\nu\sigma}$$

$$\psi_{s} = \psi - \psi_{f}$$

$$\tilde{S}[\bar{\Psi}_s, \Psi_s] = \langle S^{eff} \rangle_f$$

$$\pm \frac{1}{2} \langle [S^{eff} - \langle S^{eff} \rangle_f]^2 \rangle_f$$

$$+ \cdots$$

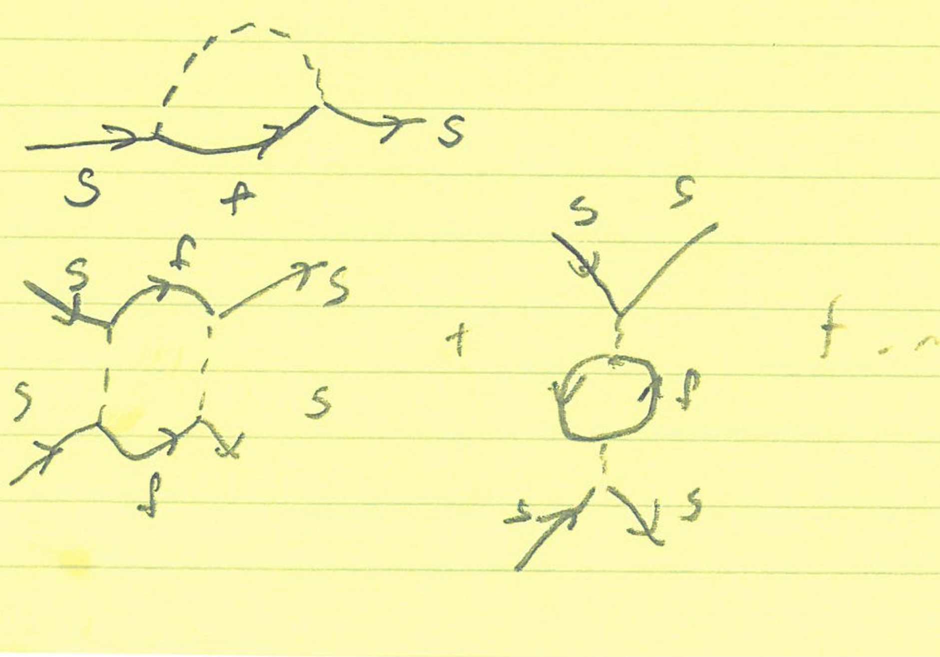
$$= S^{eff}[\bar{\Psi}_s, \Psi_s] + \text{const.}$$

$$\int d\tau \, d\tau' \int d\vec{R} \; \bar{\Psi}_s(\vec{R}\tau) \Psi_s(\vec{R}\tau')$$

$$V^{eff}(\tau - \tau') \langle \bar{\Psi}_f(\vec{R}\tau) \Psi_f(\vec{R}, \tau') \rangle_f$$

$$+ \cdots$$

Diagrams



This gives us a QFT

The result is $\tilde{S}$

Narrow band of "slow" fermions

with cutoff $\Lambda \ll E_F$

& <u>weak</u> interactions & weakly

renormalized parameters

$$\tilde{V} = V + V\,K(\Lambda)\,V + \ldots$$

$\chi =$  $\sim \rho(E_F)$

negligible

depend on band structure

$\underline{P}(\Lambda) =$  $= \rho(E_F) \ln\left[\frac{E_F}{\Lambda}\right] + P_0$

Require $\left|V \rho(E_F)\right| \ln\left[\frac{E_F}{\Lambda}\right] \ll 1$

$\tilde{S}$ is a problem that has been heavily analyzed.

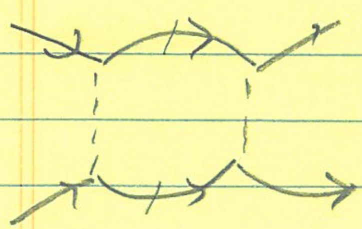
Forward scattering terms are marginal

$\Rightarrow$ F.L. parameters

(but since they are small, we can ignore them.)

Step #3 Peturbetive RG

Scattering in Cooper channel is revelant even though $\tilde{V}$ is small.



R.G. transformation

Case #1 : $g_0 = V^{eff} \rho(E_F)$

$$+ () \left[V^{eff} \rho(E_F) \right]^2$$

$$+ \left[V^{eff} \rho(E_F) \right]^2 \ln \left[E_F / \Lambda \right]$$

$$+ \ldots$$

Reduce $\Lambda \to \Lambda' = \Lambda e^{-\ell}$ & rescale.

$$\frac{dg}{d\ell} = -g^2 + \ldots$$

if $g_0 > 0$ $\quad g \to 0$ as $\ell \to \infty$.

$$g(\ell) = \frac{g_0}{1 + g_0 \ell} = \frac{g_0}{1 + g_0 \ln[\Lambda/\Lambda']}$$

$$\frac{dg}{d\ell} = -\frac{g_0^2}{(1+g_0\ell)^2} = g^2 \quad ✓$$

if $g_0 < 0$ $\quad g \to -\infty$ as $\ell \to \infty$

$$g(\ell^*) = -1 = \frac{-|g_0|}{1 + |g_0|\ell^*}$$

$$1 + |g_0|\ell^* = |g_0|$$

$$\ell^* = \frac{1 - |g_0|}{|g_0|} = \frac{1}{|g_0|} - 1$$

$$\Lambda^* = \Lambda e^{-\ell^*} = \Lambda \exp\left\{-\frac{1}{|g_0|} + 1\right\}$$

This seems to depend on Λ.
let $\lambda = -V^{eff}\rho(E_F)$

$$g_0 = -\lambda - (\,)\lambda^2 - \lambda^2 \ln\left[\frac{E_F}{\Lambda}\right]$$

$$\frac{1}{|g_0|} = \frac{1}{\lambda[1 + (\,)\lambda^2 + \lambda^2 \ln(E_F/\Lambda)]}$$

$$= \frac{1}{\lambda} - (\,) - \ln\left[\frac{E_F}{\Lambda}\right]$$

$$\Lambda^* = \Lambda \exp\left\{-\frac{1}{\lambda} + 1 - (\,) - \ln\left[\frac{E_F}{\Lambda}\right]\right\}$$

$$= E_F \exp\left\{-\frac{1}{\lambda} + 1 - (\,)\right\} \checkmark$$

Attractive U Hubbard.

$$T_c \sim E_F\, e^{-1/\rho(E_F)|U^{eff}|} \checkmark$$

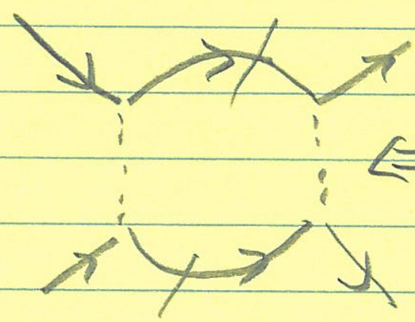
Case # 2 Retarded $E_F \gg \Lambda \gg \omega_0$

$$g_0 = V\rho(E_F) - (\,)[V\rho(E_F)]^2$$

$$- [V\rho(E_F)]^2 \ln[E_F/\Lambda]$$

$$+ \dots$$

$$\tilde{g}_0 = -\frac{\alpha^2}{2K}\rho(E_F)$$



⇐ only g

$$\frac{dg}{dl} = -g^2 + \dots$$

$$\frac{d\tilde{g}}{dl} = 0.$$

Integrate until $\Lambda' \sim \omega_0$

$$g(\omega_0) = \frac{g_0}{1 + g_0 \ln[\Lambda/\omega_0]} \equiv \mu^*$$

$$\mu^* = \frac{V \rho(E_F)}{1 + V \rho(E_F) \ln[E_F/\omega_0]}$$

$$+ \cdots$$

$$\simeq \frac{1}{\ln[E_F/\omega_0]} \quad !$$

Now we have reached a case with non-retarded int.

$$\lambda = \frac{\alpha^2}{2K} - \mu^*$$

Step # 4 R.G. again.

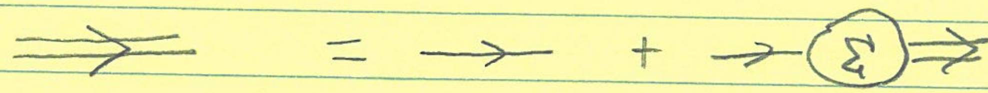
if $\lambda > 0$.

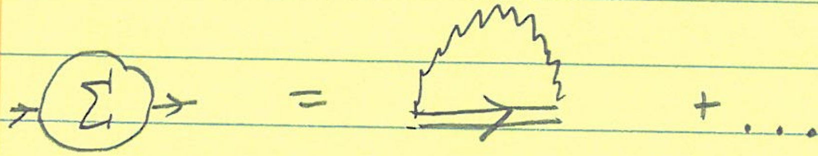
$$T_c \sim \omega_0 \, e^{-1/\lambda}$$

if $\lambda < 0$

$$V \to 0.$$

False Statements about Eliashberg theory: $\{ \text{ignore } U \}$





Claim can ignore ... if

$\omega_0 \ll E_F$ regardless of λ

$\lambda = \frac{\alpha^2}{K} \rho(E_F)$.

This is wrong.

... is non singular as

$\omega_0 / E_F \to 0$ [Migdal's theorem] but not small

if λ not small.

Eliashberg theory implies

$$T_c \to \infty \quad \text{as} \quad \lambda \to \infty .$$

$\{$ i.e. for large enough λ

$T_c > \omega_0$ $\}$

This would be great. But it is false.

Strong Coupling limit. $\lambda \gg 1$.

$$H_0 = \alpha \sum_i{}' \hat{\rho}(i) x_i + \frac{1}{2} \sum_i{}' \left[\frac{p_i^2}{M} + K y_i^2 \right]$$

$$H' = -t \sum_{\substack{\langle ij \rangle \\ \sigma}}{}' \left[c_{i\sigma}^{+} c_{j\sigma} + h.c. \right]$$

"Small"

$$H_0 = \underline{\underline{K}} + \sum_i{}' \frac{K}{2} \left[x_i + \frac{\alpha \hat{\rho}(i)}{K} \right]^2$$

$$\left[U - \frac{\alpha^2}{2K} \right] \sum_i{}' \hat{\rho}(i) \hat{\rho}(i)$$

Diaganize H_0

$$U = \prod_i e^{-i p_i \alpha \hat{\rho}(i)/\kappa}$$

$$U^+ p_j U = p_j \qquad U^+ \hat{\rho}(i) U = \hat{\rho}(i)$$

$$U^+ x_j U = x_j - \frac{\alpha \hat{\rho}(j)}{\kappa}$$

$$U^+ H_0 U = \frac{1}{2} \sum_j{}' \left[p_j^2 + \kappa x_j^2 \right]$$

$$+ U^{eff} \sum_j{}' \hat{\rho}(j) \hat{\rho}(j)$$

$$H' = U^+ H' U = -t \sum_{\substack{\langle ij \rangle \\ \sigma}}{}' \left[c_{i\sigma}^+ c_{j\sigma} e^{+i\frac{\alpha}{\kappa}(p_i - p_j)} + h.c. \right]$$

Unperturbed problem

$$U^{eff} > 0 \Rightarrow \text{polarons}$$

$$U^{eff} < 0 \Rightarrow \text{bipolarons.}$$

Degenerate 1st order P.T.

plolarons:

$$t^{\text{eff}} = t \langle e^{i(P_1 - P_2)\alpha/K} \rangle_0$$

$$= t \exp\left[-\frac{\alpha^2}{2K^2}(\langle P_1^2\rangle + \langle P_2^2\rangle)\right]$$

$\frac{\langle p^2\rangle}{2M} = \frac{\omega_0}{4}$

$$= t \exp\left[-\frac{\alpha^2}{2K^2} M\omega_0\right]$$

$$= t \exp\left[-\frac{\alpha^2}{2K\omega_0}\right]$$

$$\Downarrow$$

$$E_p/\hbar\omega_0$$

"Self-localized" in limit $\frac{\alpha^2}{K} \gg \hbar\omega_0$!

bipolarons $t^{\text{eff}} = 0.$

must go to second order.

degenerate P.T. (like ~~Hei~~ superexchange.)

Result of 2nd order degen. P.T.

$$H_{b_i - p.} = - t^{eff} \sum_{\langle ij \rangle}' [b_i^+ b_j + h.c.]$$

$$+ V^{eff} \sum_{\langle ij \rangle}' b_i^+ b_i b_j^+ b_j$$

$$+ [\infty] \sum_j' b_j^+ b_j [b_j^+ b_j - 1]$$

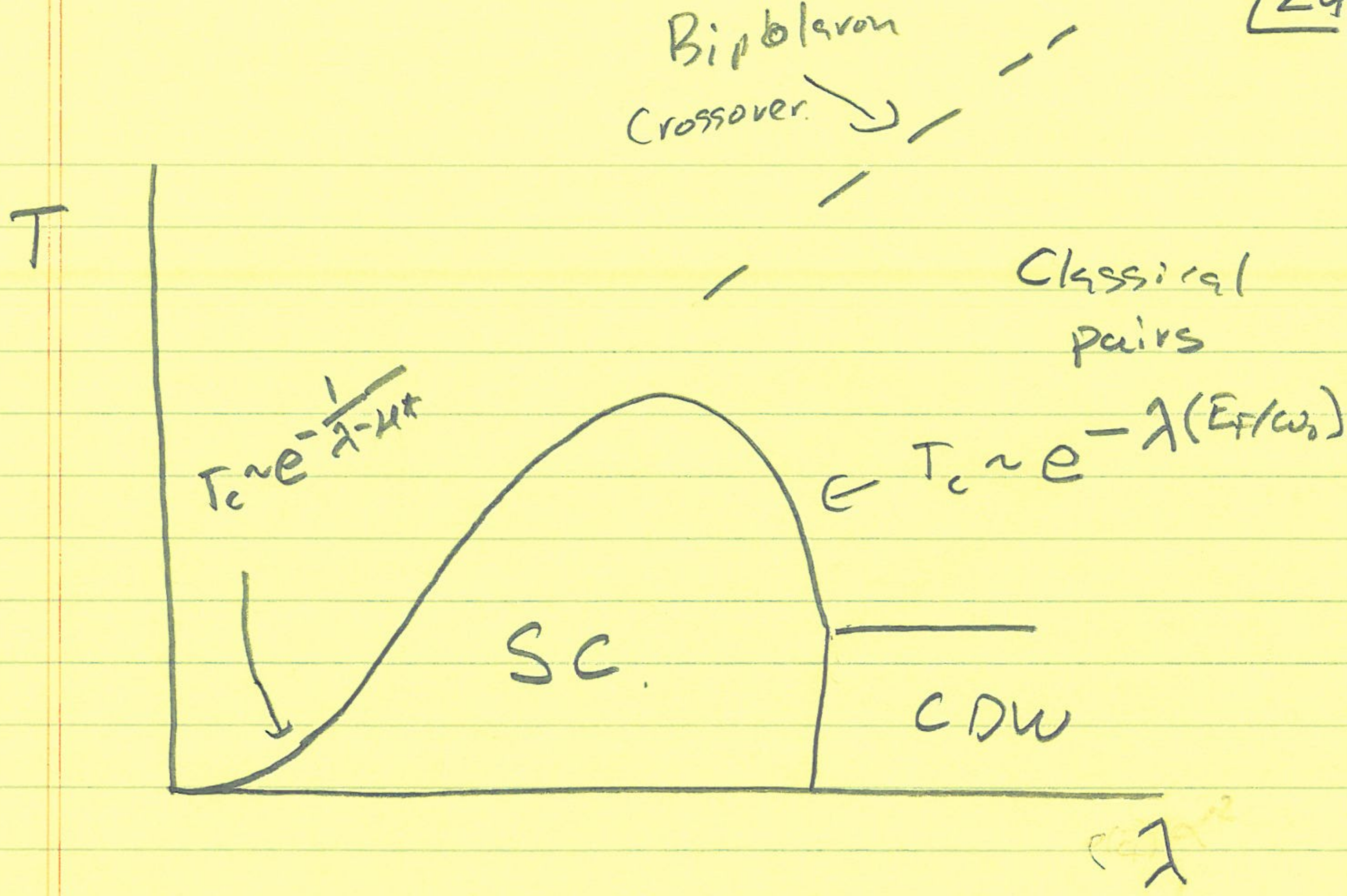
$$t^{eff} = \frac{2 t^2}{|V^{eff}|} F_+ (|V^{eff}| / \omega_0)$$

$$V^{eff} = \frac{4 t^2}{|V^{eff}|} F_- (|V^{eff}| / \omega_0)$$

$$F_\pm (X) = \int_0^\infty dt \, \exp \left\{ - t - X [1 \pm e^{-t/X}] \right\}$$

$F_+(X) \rightarrow e^{-2X}$ as $X \rightarrow \infty$.

$F_-(X) \rightarrow 1$ as $X \rightarrow \infty$



Why is T_c small?

Table of S.C's from Allen & Dynes.

$$T_c \ll \omega_0 \ll E_F$$

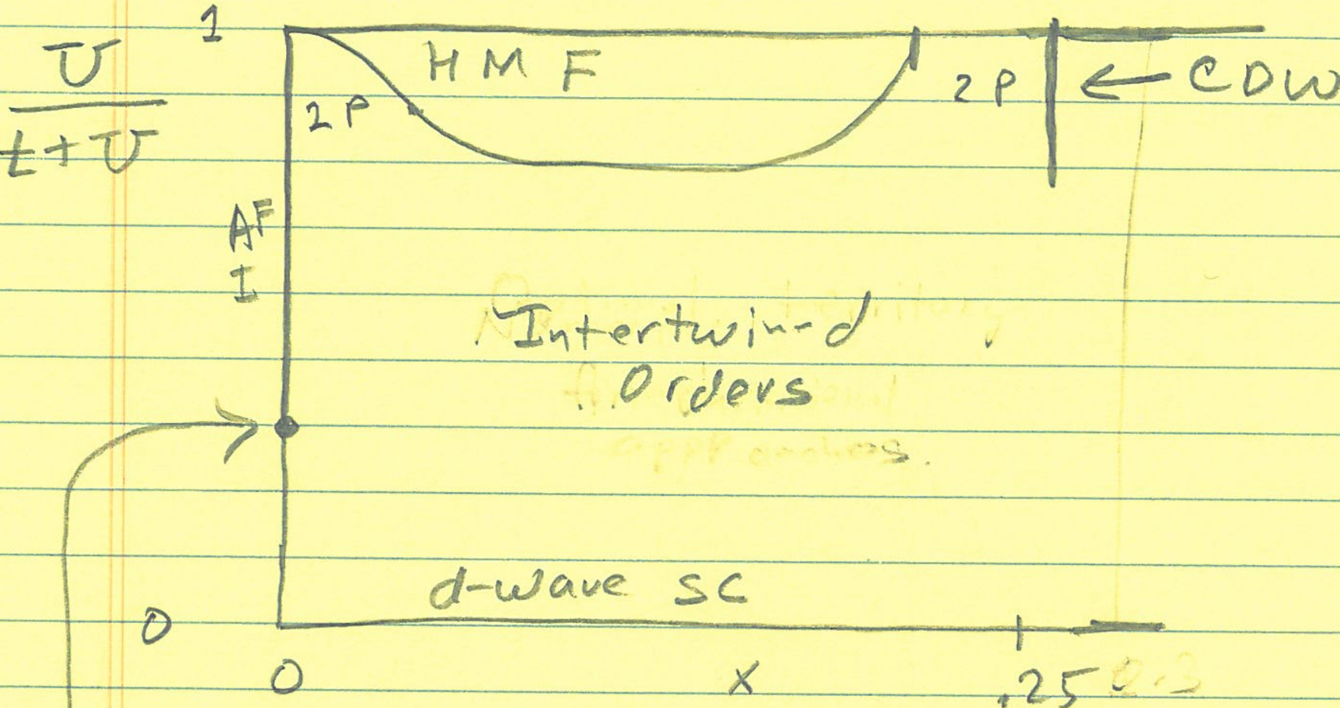
Because weak induced int.

to renormalize μ^*

actually $\lambda \approx 1$ $\mu^* \approx 0.12$

$\boxed{T_c \sim \omega_0 \, f}$ $f \sim \frac{1}{5} - \frac{1}{20}$

2'd Hubbard model. $T=0$



(we will imagine $0<|t'|\ll t$

for amusement

Senthil land?

1st order land (me & Vic)?

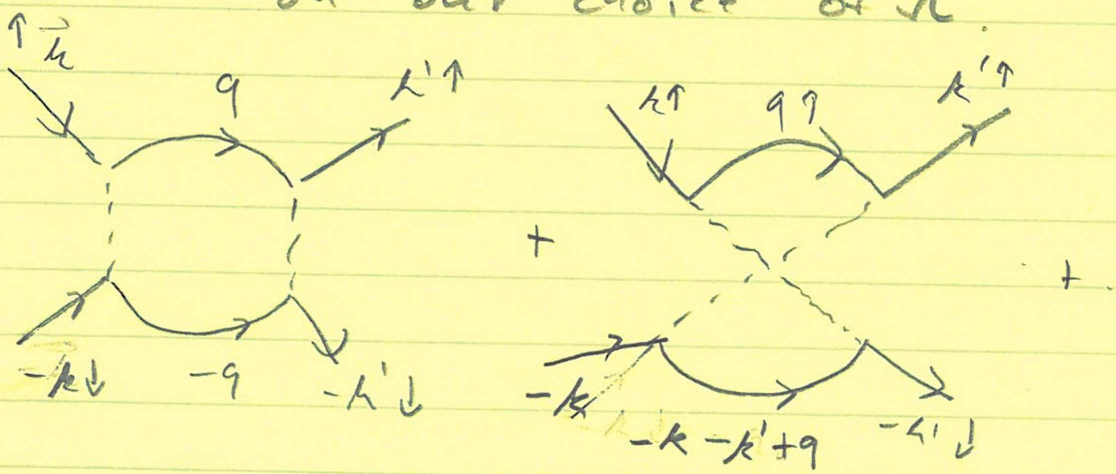
Weak coupling. $U/t \ll 1$

Step I: integrate out down to cutoff Λ

Λ is again unphysical.

it is where we define our QFT.

Must make sure nothing depends on our choice of Λ.



$$V^{eff}(k\,k') = U + U^2 P(\Lambda) + U^2 X(k+k'; \Lambda) + \ldots$$

$$\rho(\Lambda) = \rho(E_F)\left\{ \ln\left[\frac{E_F}{\Lambda}\right] + \alpha \right\}$$

$$\chi(k+k', \Lambda) = \underbrace{\chi(k+k', 0)}_{\chi(k+k')} + \mathcal{O}(\Lambda)$$

Choose Λ s.t.

$$\rho(E_F)U^2 \gg \Lambda \gg E_F e^{-\frac{1}{\rho(E_F)U}}$$

We thus acheive a QFT with

weak int. and linearized

dispersion.

The interactions are still repulsive

(dominated by 1^{st} order U)

but k dependenq.

Step II Perturbative RG.

$$g_{\hat{k}\hat{k}'} = \sqrt{\frac{V}{V_{\hat{k}}}}\; V^{eff}(\vec{k}, \vec{k}')\sqrt{\frac{V}{V_{\hat{k}'}}}$$

pts. of Fermi surface

$$\Lambda' = \Lambda e^{-l}$$

$$\frac{d g_{\hat{k}\hat{k}'}}{dl} = -\sum_{\hat{p}}' g_{\hat{k}\hat{p}}\, g_{\hat{p}\hat{k}'} + \ldots$$

$$\frac{d \underline{g}}{dl} = -\underline{g}\cdot\underline{g} + \ldots$$

Not sure why there is no matrix

$$\frac{d g_{\hat{k}\hat{k}'}}{dl} = -g_{\hat{k}\hat{p}}\, \beta_{\hat{p}\hat{p}'}\, g_{\hat{p}'\hat{k}'} + \ldots$$

contrast quadratic band touching problem ⇒ Vafek

why is $\beta_{\hat{p}\hat{p}'} = \delta_{\hat{p}\hat{p}'}$?

$$\underline{g}^{(0)} = \underline{g}(\Lambda)$$

$$\underline{g}^{(0)}_{\hat{k}\hat{k}'} f^{\alpha}_{\hat{k}'} = \lambda^{(0)}_{\alpha} f^{\alpha}_{\hat{k}}$$

$$g^{(0)}_{\hat{k}\hat{k}'} = \sum_{\alpha}' \lambda_{\alpha} f^{\alpha}_{\hat{k}} f^{\alpha}_{\hat{k}'}$$

$$\frac{d\lambda_\alpha}{d\ell} = -\lambda_\alpha^2 + \ldots$$

$$\lambda_\alpha = \frac{\lambda_\alpha^{(0)}}{1 + \lambda_\alpha^{(0)} \ln[\Omega/\Omega']}$$

each chanel is independent

Find most negative $\lambda_\alpha^{(0)}$

$$T_c \sim \Lambda e^{-1/|\lambda_\alpha^{(0)}|}$$

{ Home-work prove that for Λ in the stated range, this result is independent of Λ }

Only $\underline{1}$ channel gets large

$$\Rightarrow \quad \Delta_{\hat{k}}(T) = \Delta_0(T)\, f_{\hat{k}} \quad !$$

$$g_{\hat{k}\hat{k}'} \sim V(\hat{k}, \hat{k}') = [U + U^2 P(\Omega)] + U^2 X(\hat{k} + \hat{k}') + \ldots$$

1^{st} term is large & positive

$\Rightarrow$ need $\sum_{\hat{k}} \frac{1}{\sqrt{V_{\hat{k}}}} f_{k} = 0.$

$\Rightarrow$ d-wave , p-wave, g-wave

...

or very complicated extended

s-wave.

Subject to this.

$$\underbrace{U^2 \frac{\chi(\hat{k}+\hat{k}')}{\sqrt{V_k V_{k'}}}}_{g_{kk'}} f_{k'} = \lambda f_{k'}$$

For circular F.S. in 2d.

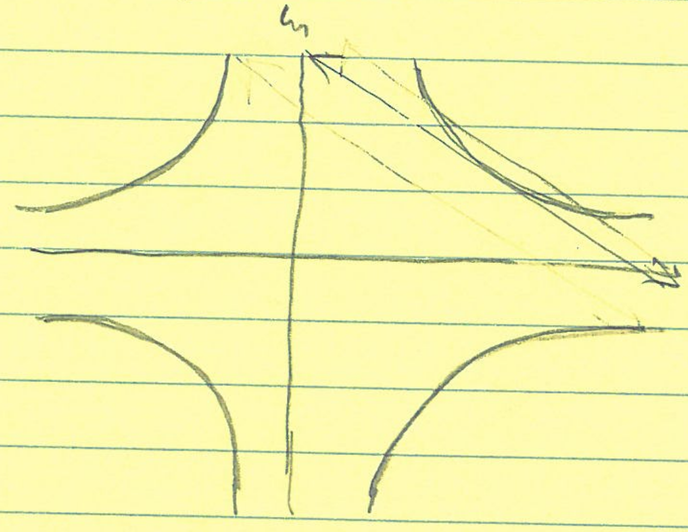
$\chi(k) = \text{consta for } |k| \leq 2k_F$

$V_k = \text{const.}$

$\Rightarrow$ No solution with neg. eigenvalue!

Band structure effects.

$\max[\chi(\vec{k})] \Rightarrow \vec{k} \sim (\pi,\pi)$
$\min[\chi(\vec{k})] \Rightarrow \vec{k} \sim 0.$



Cuprate
$\leftarrow$ F.S.

perfect for d-wave.

(Short-range A.F. fluctuations,

$$\chi(\vec{k}) \Rightarrow \frac{\chi(\vec{k})}{1 - J\chi(\vec{k})}$$ or some such thing.

This enhances pairing.

Other bad things happen, however, if ω_0 small or ξ_0 large.

What is missing?

$$T_c \sim E_F \exp\left[-\frac{\alpha}{|\rho(E_F)\,U|^2}\right]$$

very small!

Normal state is good F.L.

No pseudo-gap

No intertwined orders.