Spins Dynamics in Nanomagnets

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Lecture 1
Outline

I. Magnetic Interactions
   – Exchange, Anisotropy and Dipolar Interactions
   – Zeeman Interaction

II. Micromagnetic Energy
   – Energy, Field and Length Scales
   – Examples of Magnetic Domain Structure

III. Magnetic Nanostructures
   – Single Domain Model
   – Experiments on Individual (classical) Nanomagnets
   – Thermal Activation and Quantum Tunneling of Magnetization

IV. Classical Magnetization Dynamics
   – Landau-Lifshitz-Gilbert Equation

References
Magnetic Interactions

- **Exchange**
  - Coulomb Interactions + Pauli Exclusion Principle

- **Magnetocrystalline Anisotropy**
  - Spin-orbit interactions

- **Magnetic Dipole Interactions**
  - Magnetic field associated with a magnetic moment

locks spins together

preferred direction in space

non-local
Magnetic Interactions

**Microscopic**

- **Exchange**
  \[ H_{\text{exc}} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]
  \[ J < 0 \quad \text{Ferromagnetic interactions} \]

- **Anisotropy**
  \[ H_{\text{anis}} = -d \sum_i S_{iz}^2 \]
  \[ d > 0 \quad \text{Easy axis anisotropy} \]
  \[ d < 0 \quad \text{Easy plane type anisotropy} \]

- **Dipolar Interactions**
  \[ H_{ij} = \frac{\mu_0}{4\pi r^3} (3(\vec{m}_i \cdot \hat{r}_{ij})(\vec{m}_j \cdot \hat{r}_{ij}) - \vec{m}_i \cdot \vec{m}_j) \]

**Continuum**

- **Energy**
  \[ E_{\text{exc}} = A \left( |\nabla m_x|^2 + |\nabla m_y|^2 + |\nabla m_z|^2 \right) \]
  \[ A = -JS^2c/a \]
  \[ A \approx kTc/a \quad \text{a(lattice constant)} \]
  \[ c(\text{lattice}): c=1(\text{sc}), 2(bcc), 4(\text{fcc}) \]

- **Magnetostatic Energy**
  \[ \vec{H}_d = -\tilde{D}\vec{m} \]
  \[ E_{\text{ms}} = -\mu_0 M_s \vec{m} \cdot \vec{H}_d \]
  \[ E_{\text{ms}} = \frac{\mu_0}{2} H_d^2 \]
Magnetic Interactions

• Zeeman Energy

\[ H_z = g \mu_B \sum_i \vec{S}_i \cdot \vec{H}_e \]

\[ E = -\mu_0 \vec{M} \cdot \vec{H}_e \]

Magnetostatic Energy

May sometimes be approximated by a local contribution to the energy

\[ E_z = \frac{\mu_0}{2} D_z M_s^2 \]

\[ E_y = \frac{\mu_0}{2} D_y M_s^2 \]

\[ E_{ms} = \frac{\mu_0}{2} (D_z - D_y) M_s^2 m_z^2 \]

Examples:
1) single domain particles
2) thin films with in-plane magnetization

Thin film
\[ D_z = 1, \ D_x = D_y = 0 \]

Infinite Cylinder
\[ D_z = 0, \ D_x = D_y = 1/2 \]

Sphere
\[ D_x = D_y = D_z = 1/3 \]

Analytic solutions for the micromagnetic energy, saddle states, rate of thermally activated reversal, etc. are then possible.

Martens, Stein, ADK, PRB 2006
Chaves, ADK, Stein, PRB 2009
Micromagnetic Energy

\[ E[m(r)] = A \int_{\Omega} |\nabla m|^2 \, d^3r - K \int_{\Omega} m_z^2 \, d^3r - \mu_0 M_s \int_{\Omega} H_e \cdot m \, d^3r + \frac{\mu_0}{2} \int_{\mathbb{R}^3} |\nabla U|^2 \, d^3r \]

Exchange  Anisotropy  Zeeman  Magnetostatic

\( \Omega \equiv \text{ferromagnet} \)

\[ E[m(r)] = \frac{\mu_0 l_{\text{ex}}^2}{2} \int_{\Omega} |\nabla m|^2 \, d^3r - K \int_{\Omega} m_z^2 \, d^3r - \mu_0 M_s \int_{\Omega} H_e \cdot m \, d^3r + \frac{\mu_0}{2} \int_{\mathbb{R}^3} |\nabla U|^2 \, d^3r \]

Exchange length  \( l_{\text{ex}} = \sqrt{2A/(\mu_0 M_s^2)} \)

Domain wall width  \( \lambda = \sqrt{2A/K} \)

Anisotropy/Magnetostatic  \( Q = \frac{2K}{\mu_0 M_s^2} \)

Magnetostatic Potential

\[ H = -\nabla U \]

\[ \nabla^2 U = \nabla \cdot M \]

+ boundary conditions

\[ U_{\text{in}} = U_{\text{out}} \]

\[ \frac{\partial U_{\text{in}}}{\partial n} - \frac{\partial U_{\text{out}}}{\partial n} = M \cdot \hat{n} \]
Energy, Field and Length Scales

Elemental transition metal ferromagnets

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<thead>
<tr>
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<th>8</th>
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<th>10</th>
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<td>26</td>
<td>II</td>
<td>VIIIB</td>
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<tr>
<td>Co</td>
<td>27</td>
<td>II</td>
<td>VIIIB</td>
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<tr>
<td>Ni</td>
<td>28</td>
<td>II</td>
<td>VIIIB</td>
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[Ar].3d^6.4s^2 .3d^7.4s^2 .3d^8.4s^2

Partially filled 3d shell

• Exchange  \( \sim 1000 \text{ K, } 100 \text{ T} \)
  Curie Temperature

• Anisotropy  \( \sim 10 \text{ K, } 0.01 \text{ to } 10 \text{ T} \)

• Dipolar Interactions  \( \sim 1 \text{ K, mT} \)

Exchange length  \[ l_{\text{ex}} = \sqrt{\frac{2A}{\mu_0 M_s^2}} \sim 4 \text{ nm} \]

Domain wall width  \[ \lambda = \sqrt{\frac{2A}{K}} \sim 5 \text{ to } 100 \text{ nm} \]
Examples

- Competing interactions lead to the formation of magnetic domains: exchange, anisotropy and magnetostatic

(110) Fe Thin Films

\[ Q = \frac{2K}{\mu_0 M_s^2} \]

- \( Q << 1 \) flux closure domains
- \( Q >> 1 \) stripe domains

Examples
Examples

- Domain configurations as function of:
  - Linewidth
  - Magnetic history

After transverse saturation

H=0

- Neel-cross-tie walls
- (Asymmetric) Bloch walls
- Canted Bloch walls

Examples

Photoelectron emission microscopy--PEEM

Fe(110) Wire on Mo (110)/[Al₂O₃]

[110]

[001]

100 nm Al

100 nm Fe

100 nm Mo

sapphire

Closure Domain
Length 1500 nm

Observed: 800 nm

Circular polarized light (right)

Circular polarized light (left)

magnetization direction

Difference Image
(enhanced contrast)

60 µm x 60 µm

60 µm x 60 µm

60 µm x 60 µm

PEEM-II

PEEM-II

MFM

PEEM-II

Circular polarized light (right)

Circular polarized light (left)

Difference Image
(enhanced contrast)

60 µm x 60 µm

60 µm x 60 µm

60 µm x 60 µm
Ni/Co (111) Single Crystalline Superlattices

Magneto-optical images

sapphire substrate/bcc V (110)/fcc Au (111)-10/fcc (111)×(Ni₃/Coₓ)₁₀/Au

S. Girod et al. APL 2009

FIG. 2. (top) Kerr images obtained during magnetization reversal close to the coercive field for a (a) SL with Co coverage equal to 0.75 ML and (b) for a SL with Co coverage equal to 2 ML. (bottom) Corresponding hysteresis loops measured by SQUID on these samples.

FIG. 3. (Color online) Data for a series of SLs with 3 ML of Ni, Co coverage varying from 0 to 4 ML, repeated ten times. (a) Saturation magnetization per unit surface area versus Co layer thickness from magnetometry (open circles) compared to the calculated value assuming magnetization value of the bulk Co and Ni—(solid line) (b) Frequency dependence of the resonance field for the multilayer film with 2.5 ML Co. The x-intercept gives the perpendicular anisotropy constant. (c) Energy density (Perpendicular magnetocrystalline anisotropy constant and demagnetization energy density) vs Co layer thickness. The anisotropy constant is deduced from magnetometry (open circles) and FMR measurements (full circles). The dash lines are guide to the eyes. The demagnetization energy density (full line) is calculated assuming magnetization value of the bulk Co and Ni.
INTRODUCTION

Fig. I.1 Scale of size which goes from macroscopic down to nanoscopic sizes. The unit of this scale is the number of magnetic moments in a magnetic system (roughly corresponding to the number of atoms). The hysteresis loops are typical examples of magnetization reversal via nucleation, propagation and annihilation of domain walls (left), via uniform rotation (middle), and quantum tunneling (right).

Image from, W. Wernsdorfer, Advances in Chemical Physics 2001 and ArXiv:0101104
Single Domain Model

Uniform M--no exchange energy

\[ E = -K m_z^2 - \mu_0 M_s \vec{m} \cdot \vec{H} \]

When does the metastable state become unstable?

\[ h_{sw}^0 = \frac{H_{sw}^0}{H_a} = \frac{1}{(\sin^{2/3} \theta + \cos^{2/3} \theta)^{3/2}} \]

\[ H_a = 2K/(\mu_0 M_S) \]

\[ h_x^{2/3} + h_y^{2/3} = 1 \]

also known as the Macrospin or Stoner-Wohlfarth Model
Single Domain Model

Experimental study of an individual Co nanoparticle

Fig. 2.4 High Resolution Transmission Electron Microscopy observation along a [110] direction of a 3 nm cobalt cluster exhibiting a f.c.c.-structure.

Prob, of not switching in a time t:

\[ P(t) = e^{-t/\tau} \]

\[ \tau = \tau_0 \exp\left(\frac{U}{k_B T}\right) \]

Insect: examples of the probability of not-switching of magnetization as a function of time for different applied fields and at 0.5 K. Full lines are data fits with an exponential function: \( P(t) = e^{-t/\tau} \).

Fig. 3.3 Temperature dependence of the switching field of a 3 nm Co cluster, measured in the plane defined by the easy and medium hard axes \((H_y - H_x)\) plane in Fig. 2.6. The data were recorded using the blind mode method (Sect. 1.2.6) with a waiting time of the applied field of \( \Delta t = 0.1 \) s. The scattering of the data is due to stochastic and in good agreement with Eq. 3.10.

Fig. 3.2 Scaling plot of the mean switching time \( \tau(H_w, T) \) for several waiting fields \( H_w \) and temperatures \( 0.1 \text{s} < \tau(H_w, T) < 60 \text{s} \) for a Co nanoparticle. The scaling yields \( \tau_0 \approx 3 \times 10^{-9} \text{s}. \) Inset: examples of the probability of not-switching of magnetization as a function of time for different applied fields and at 0.5 K. Full lines are data fits with an exponential function: \( P(t) = e^{-t/\tau} \).
Thermal Activation and Quantum Tunneling

Quantum Tunneling of Magnetization in Small Ferromagnetic Particles

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Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 25 October 1987)

The probability of tunneling of the magnetization in a single-domain particle through an energy barrier between easy directions is calculated for several forms of magnetic anisotropy. Estimated tunneling rates prove to be large enough for observation of the effect with the use of existing experimental techniques.

\[ \Gamma \sim e^{-U/k_B T} \]

\[ \Gamma \sim e^{-B(0)} = e^{-U/k_B T_c} \]

\[ T_c = U/k_B B(0) \]

Thermal Quantum

Thermal relaxation (over the barrier)

see lecture 3

also, Enz and Schilling, van Hemmen and Suto (1986)
Classical Magnetization Dynamics

Effective Field \( \mu_0 H_{\text{eff}} = -\delta E/\delta M \)

Single Domain Model

\[ E = -Km_z^2 - \mu_0 M_s \vec{m} \cdot \vec{H} \]

\[ \vec{H}_{\text{eff}} = \vec{H} + \frac{2K}{M_s} m_z \]

\[ \frac{d\vec{L}}{dt} = \text{Torque} = \mu_0 \vec{M} \times \vec{H}_{\text{eff}} \]

\[ \vec{M} = -\gamma \vec{L} \]

\[ \gamma = \left| \frac{g\mu_B}{\hbar} \right| = \frac{g|e|}{2m} \]

\[ \frac{d\vec{M}}{dt} = -\gamma \mu_0 \vec{M} \times \vec{H}_{\text{eff}} \]
Magnetization Dynamics

Landau Lifshitz Gilbert Equation

\[ \frac{\partial \mathbf{M}}{\partial t} = -\gamma \mu_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \]

**Gyroscopic term**

- \( \mathbf{M} \) is aligned with the effective field.

**Damping term**

- \( \mathbf{M} \) precesses about \( \mathbf{H}_{\text{eff}} \) at the Larmor frequency:
  \[ \omega_0 = \gamma H_{\text{eff}} \]
- Decrease of the angle of precession due to damping.

**Resonance**

- Maintain the precessional motion with rf field (\( \omega \)):
  - When \( \omega = \omega_0 \rightarrow \theta \) is max.

FERROMAGNETIC RESONANCE
FMR: Landau-Lifshitz-Gilbert Dynamics

\[ \frac{\partial \mathbf{M}}{\partial t} = -\gamma \mu_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \]

\vspace{1cm}

**FMR:** fixed frequency rf field \( \perp \mathbf{H} \)

**Precession:** \( \mathbf{M} \times \mathbf{H}_{\text{eff}} \)

\[ \gamma / 2\pi = 28 \text{ GHz/T} \]

**Linewidth:** Homogeneous or intrinsic linewidth

\[ \Delta H = 4\pi \alpha f / (\mu_0 \gamma) \]

with inhomogeneous broadening

\[ \Delta H = \Delta H_0 + \frac{4\pi \alpha}{\mu_0 \gamma} f \]
Spin Waves

Example: $\theta_H = 45^\circ$ situation

Uniform Precession
 Finite k spin waves

Spin wave band (at FMR)
Spin wave wave frequency (GHz)

$\theta_H = 45^\circ$

$\theta_k = 90^\circ$

$\theta_k = 0^\circ$

Band of modes in between

Spin wave wave number $k$ (rad/cm)

$H \perp B_s$

$H \parallel B_s$
Summary of Topics Covered

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References

Introduction to the Theory of Ferromagnetism, A. Aharoni, Oxford 1996

Lectures on Magnetism, M. Chudnovsky and J. Tejada, Rinton 2006