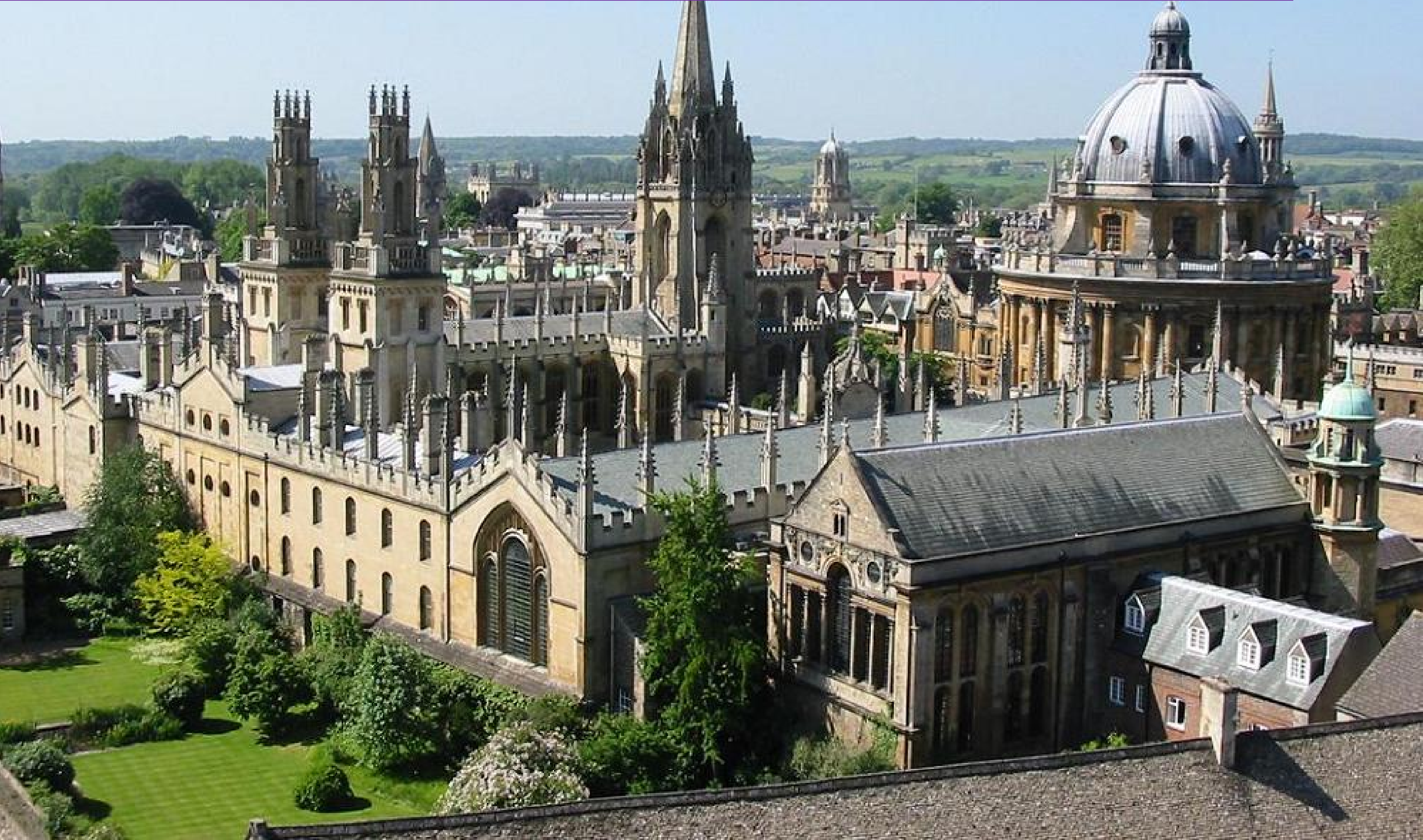
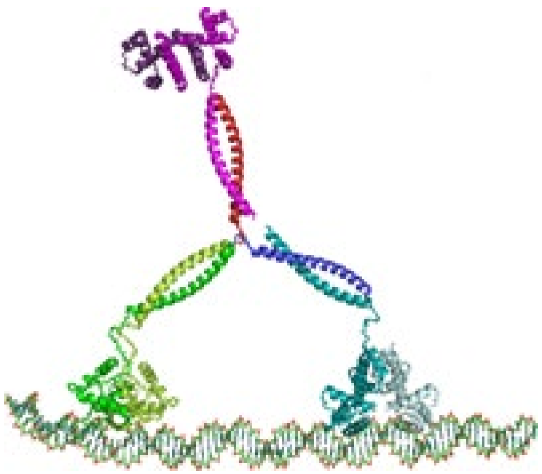
Active Nematics

Julia Yeomans
University of Oxford

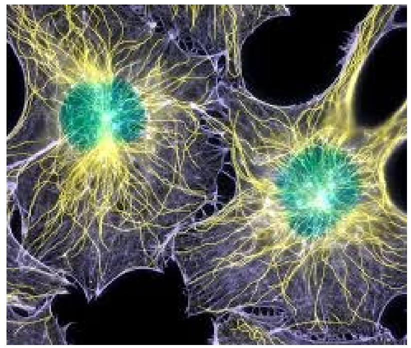


Active matter:
takes energy from the surroundings on a single particle level

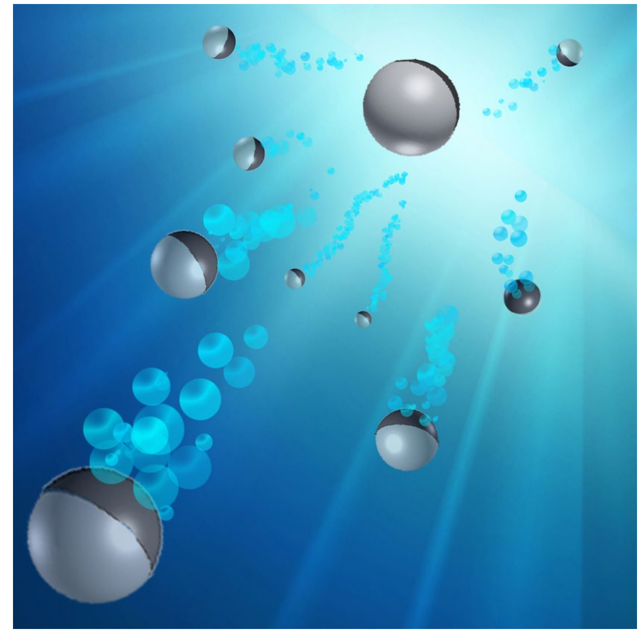
molecular motors



cells



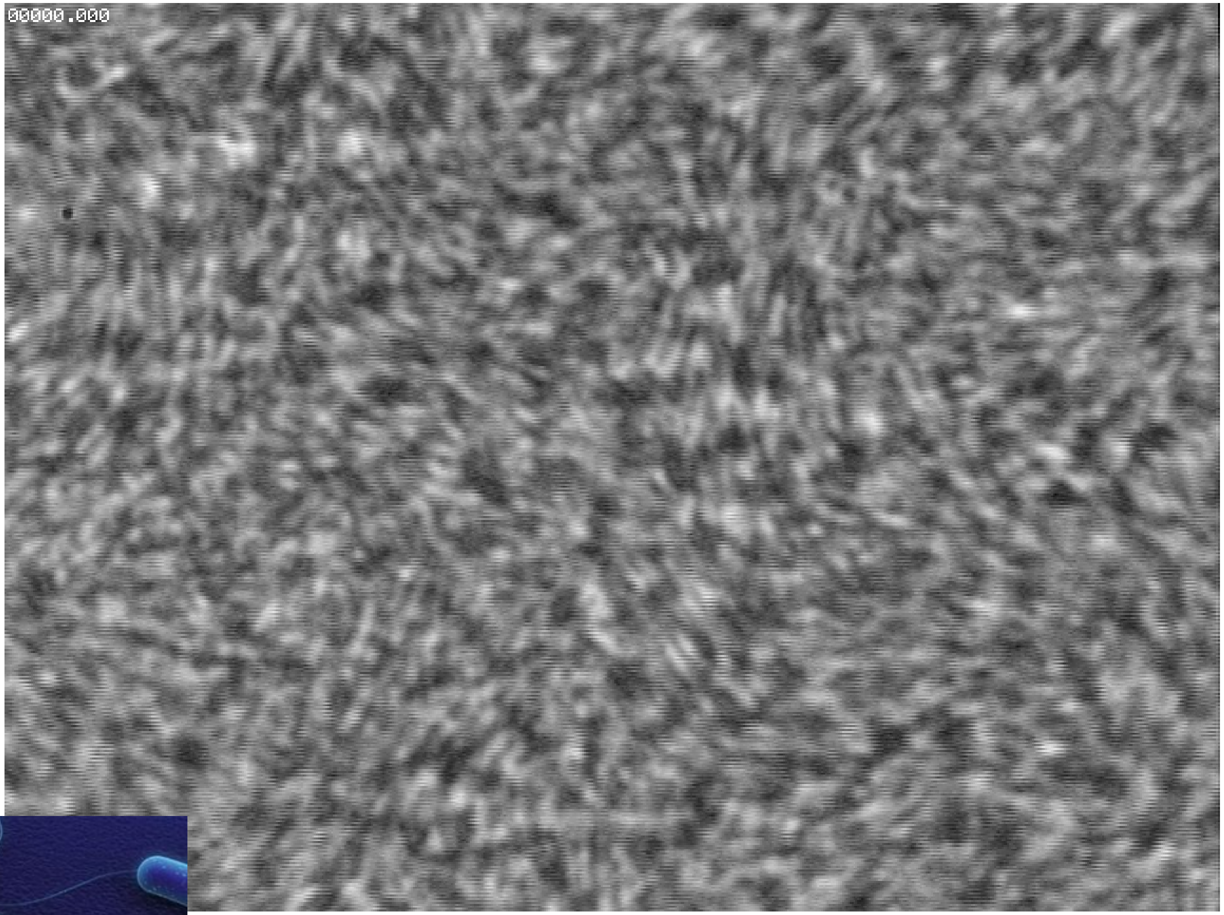
active colloids



microswimmers

animals

Active turbulence: bacteria

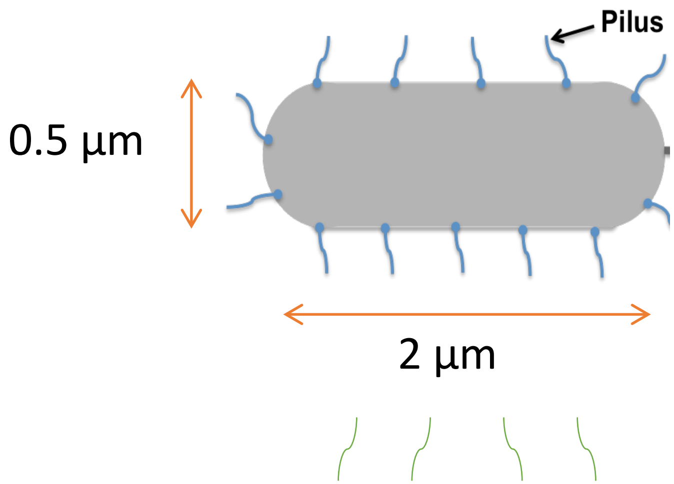


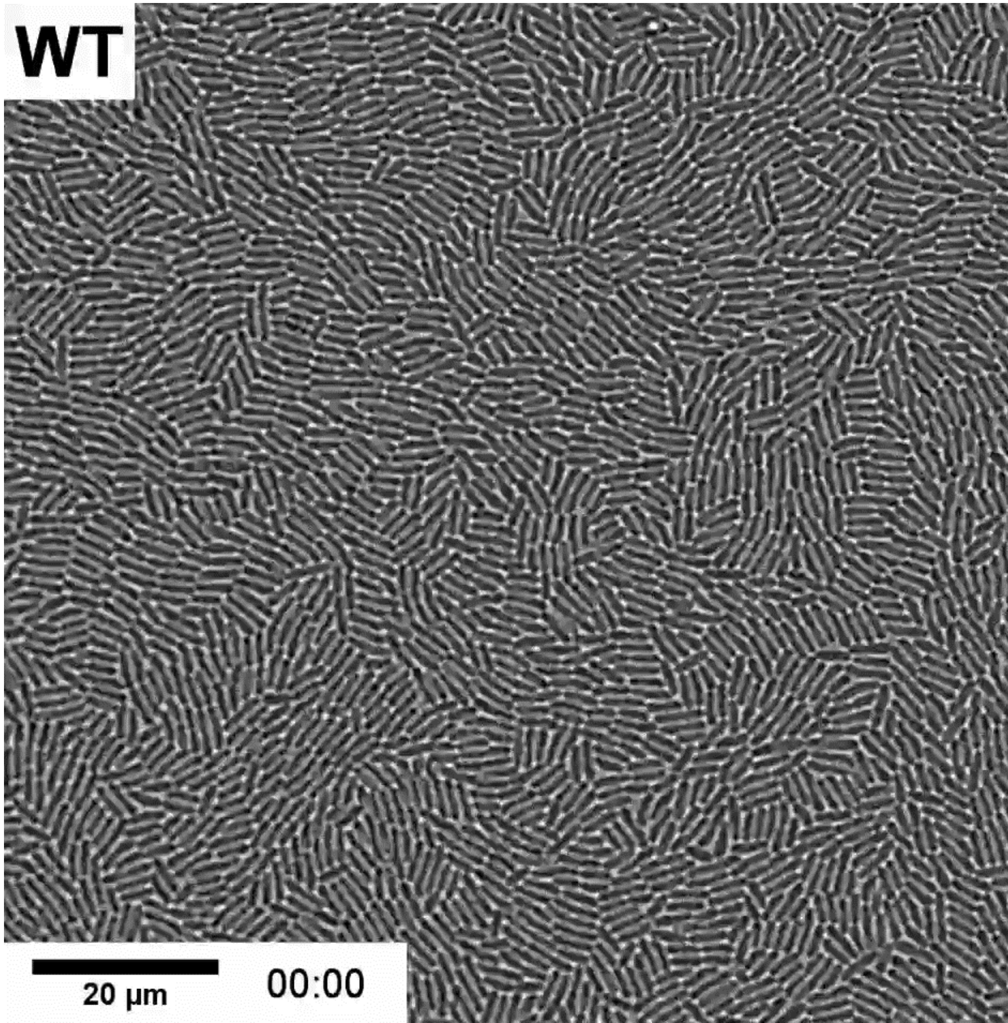
Dense suspension of microswimmers

Pseudomonas aeuriginosa

twitching motility using Type IV Pili

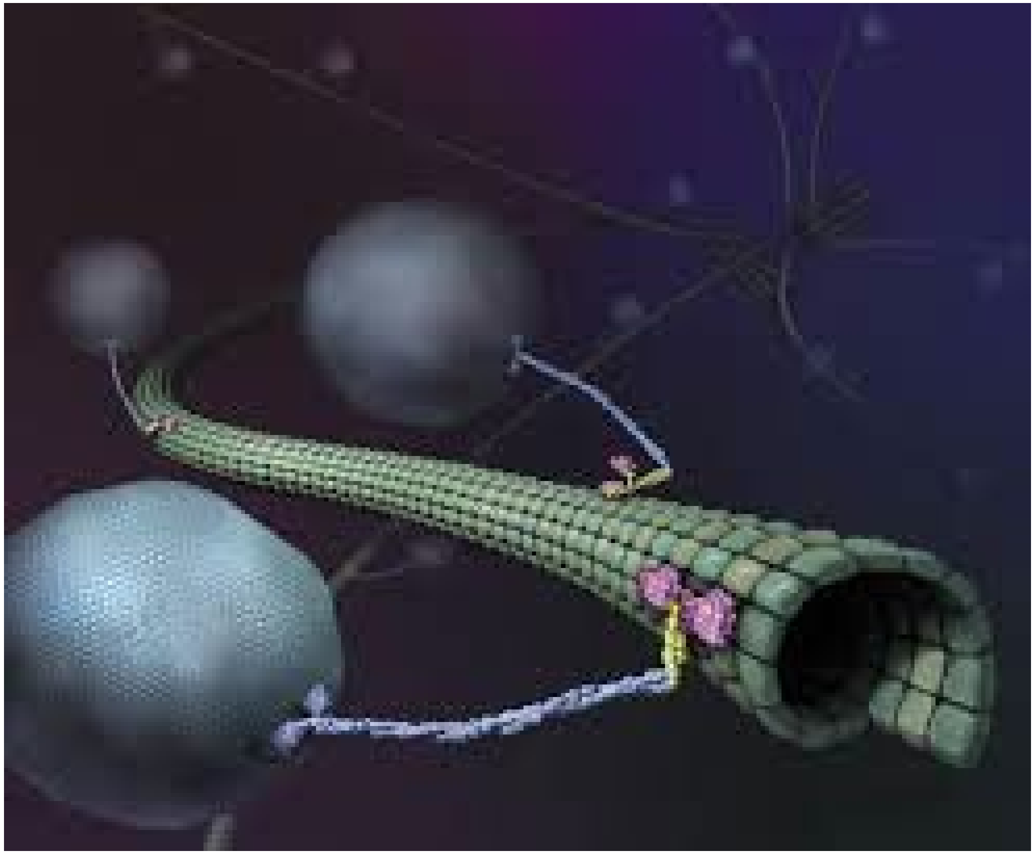
reversals

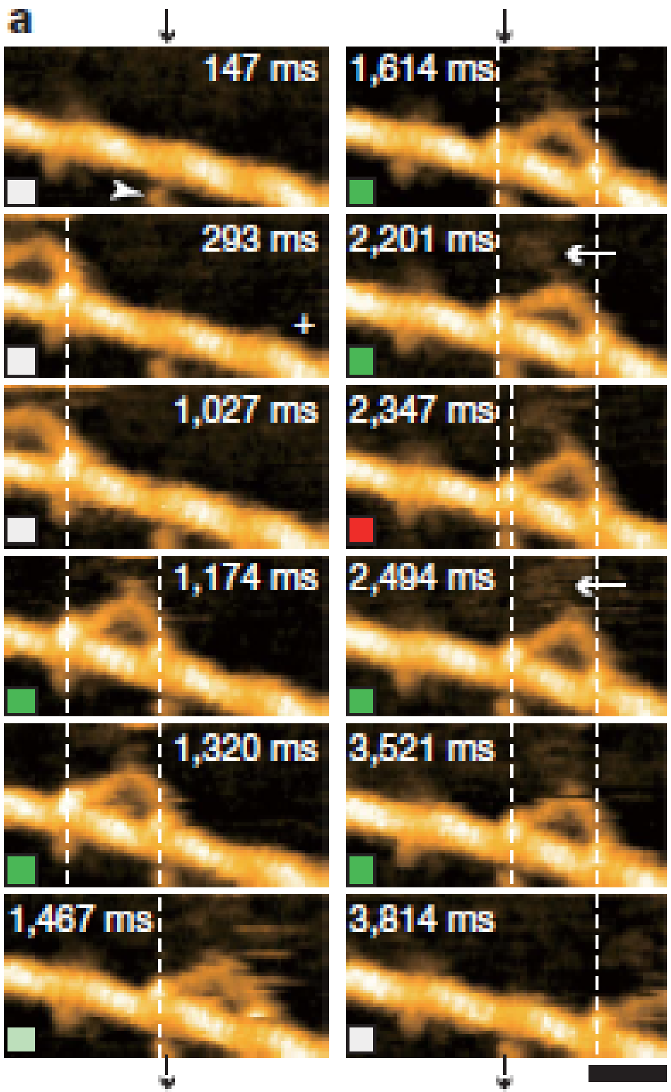




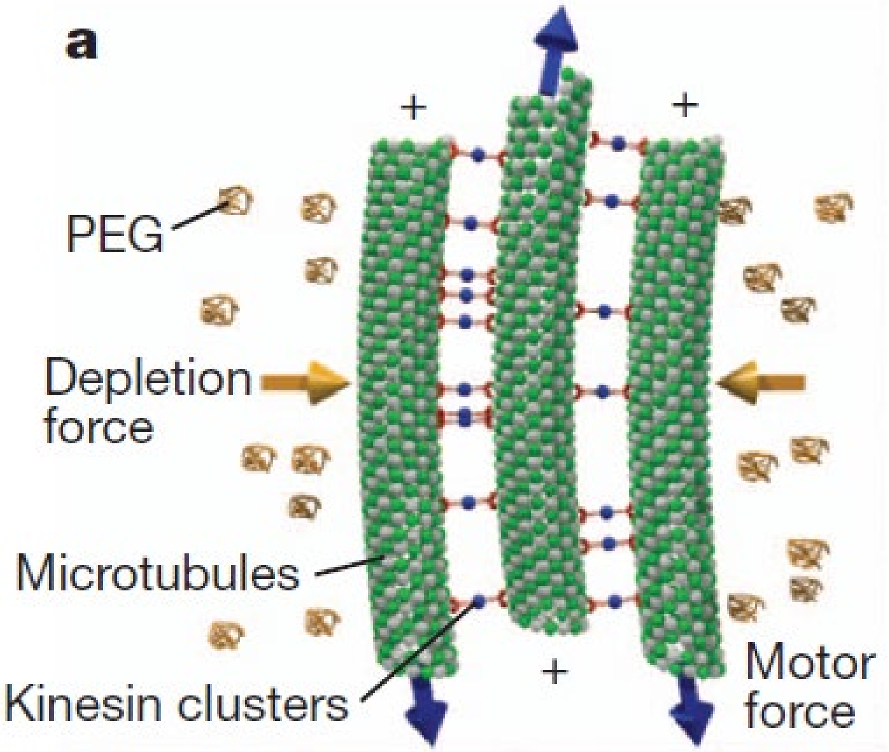
Motor proteins

Kinesin walking, from Inner Life of a Cell

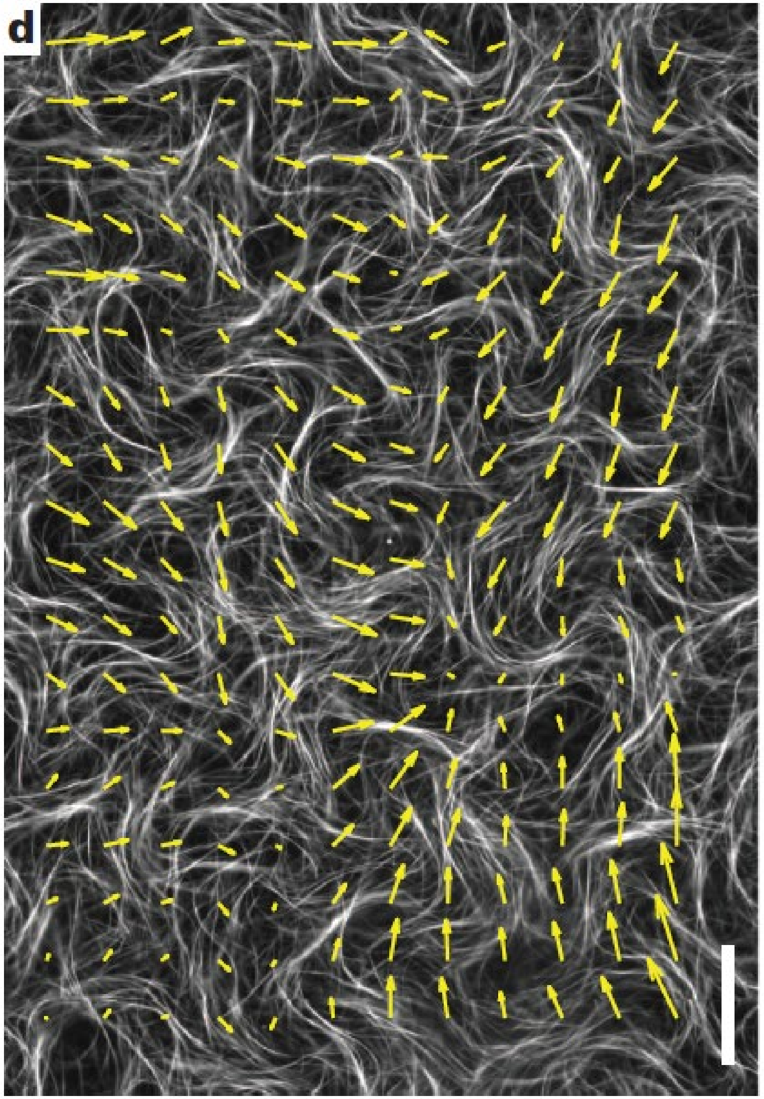



a
147 ms
293 ms
1,027 ms
1,174 ms
1,320 ms
1,467 ms
1,614 ms
2,201 ms
2,347 ms
2,494 ms
3,521 ms
3,814 ms

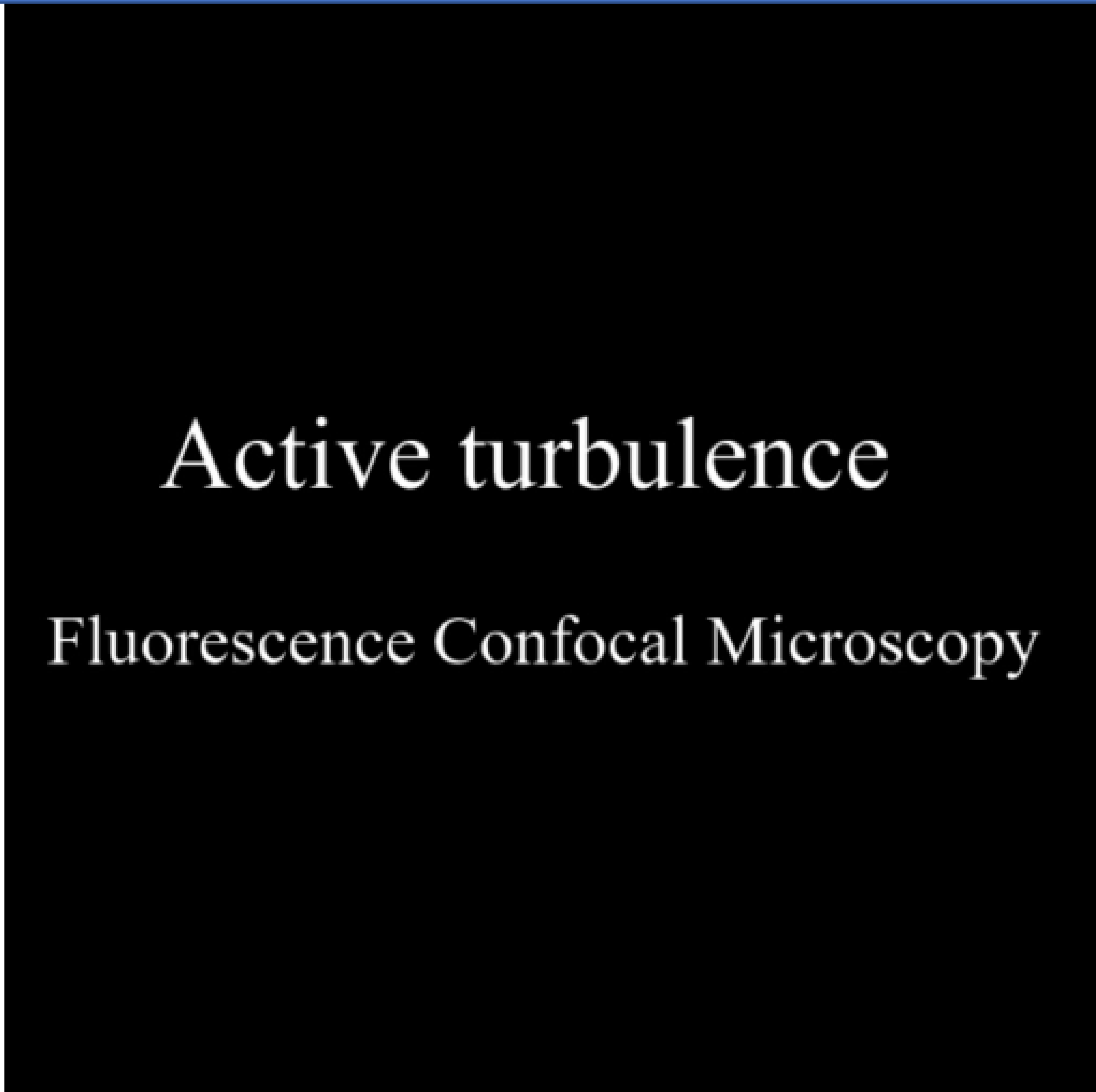
Motor proteins



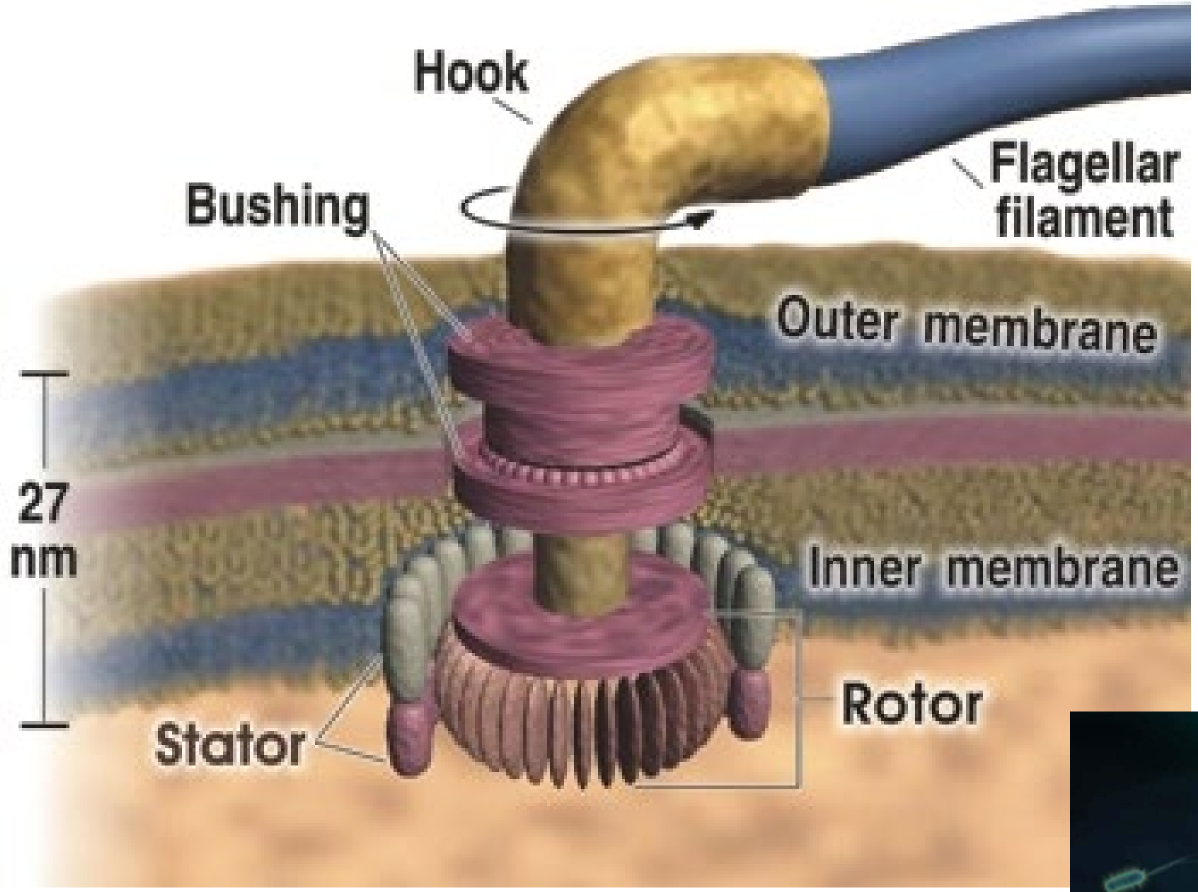
Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012

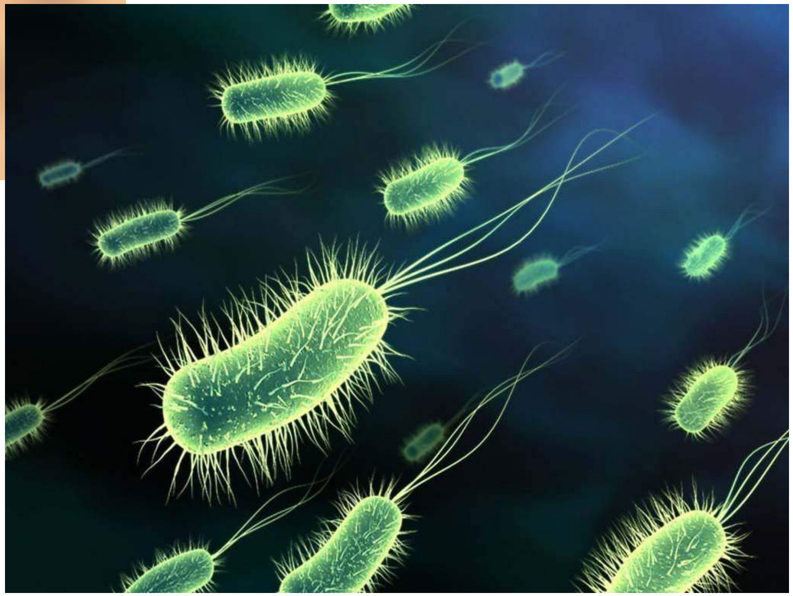


Active turbulence: motor proteins

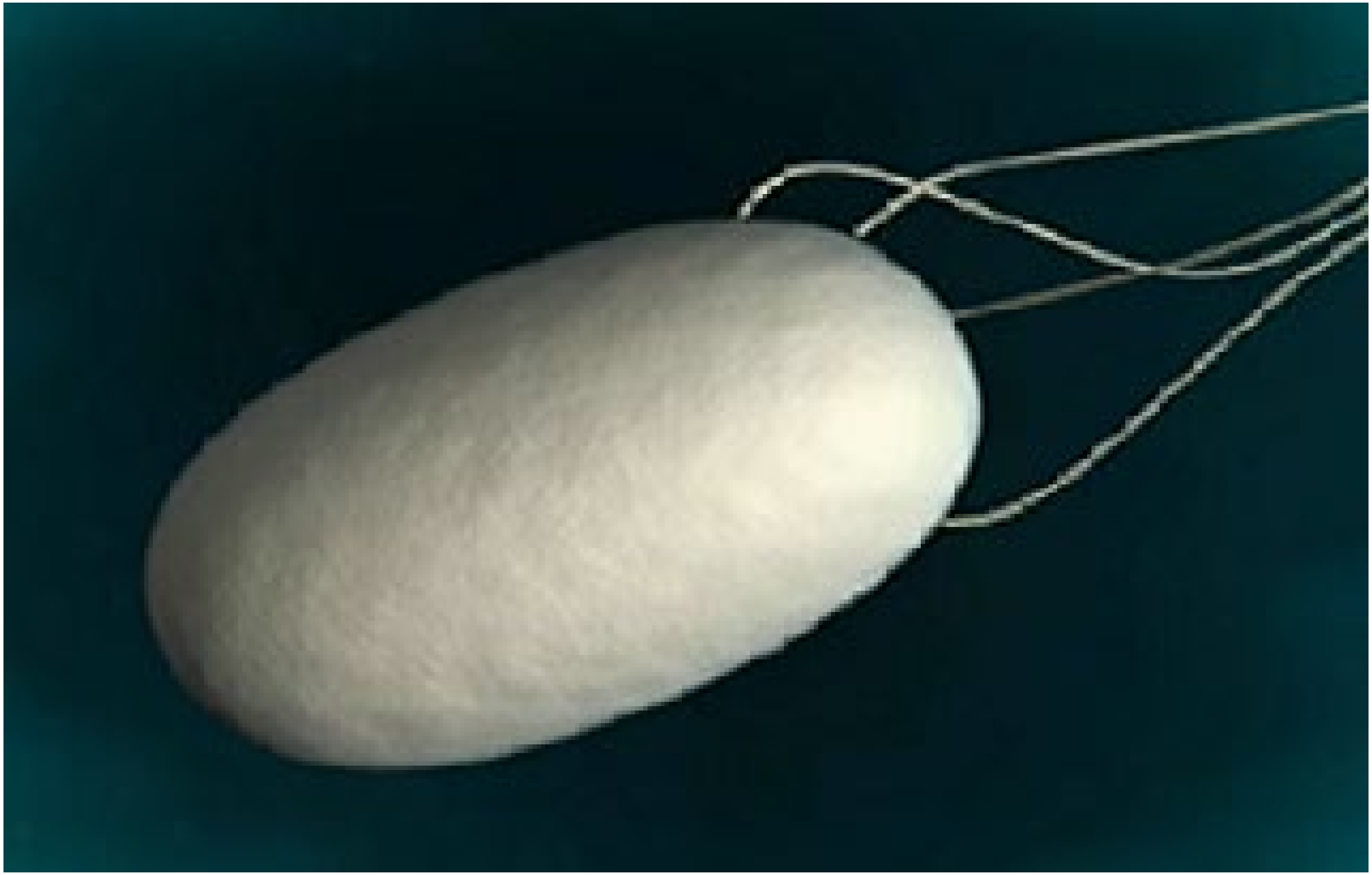


Francesc Sagues
Pau Guillamat
Jordi Ignes-Mullol

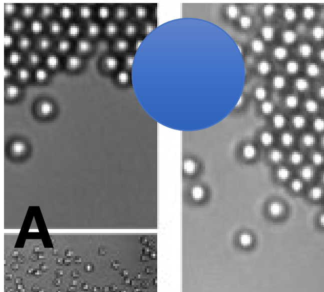




Bacterial flagellar motor
(100 revs per sec unloaded)

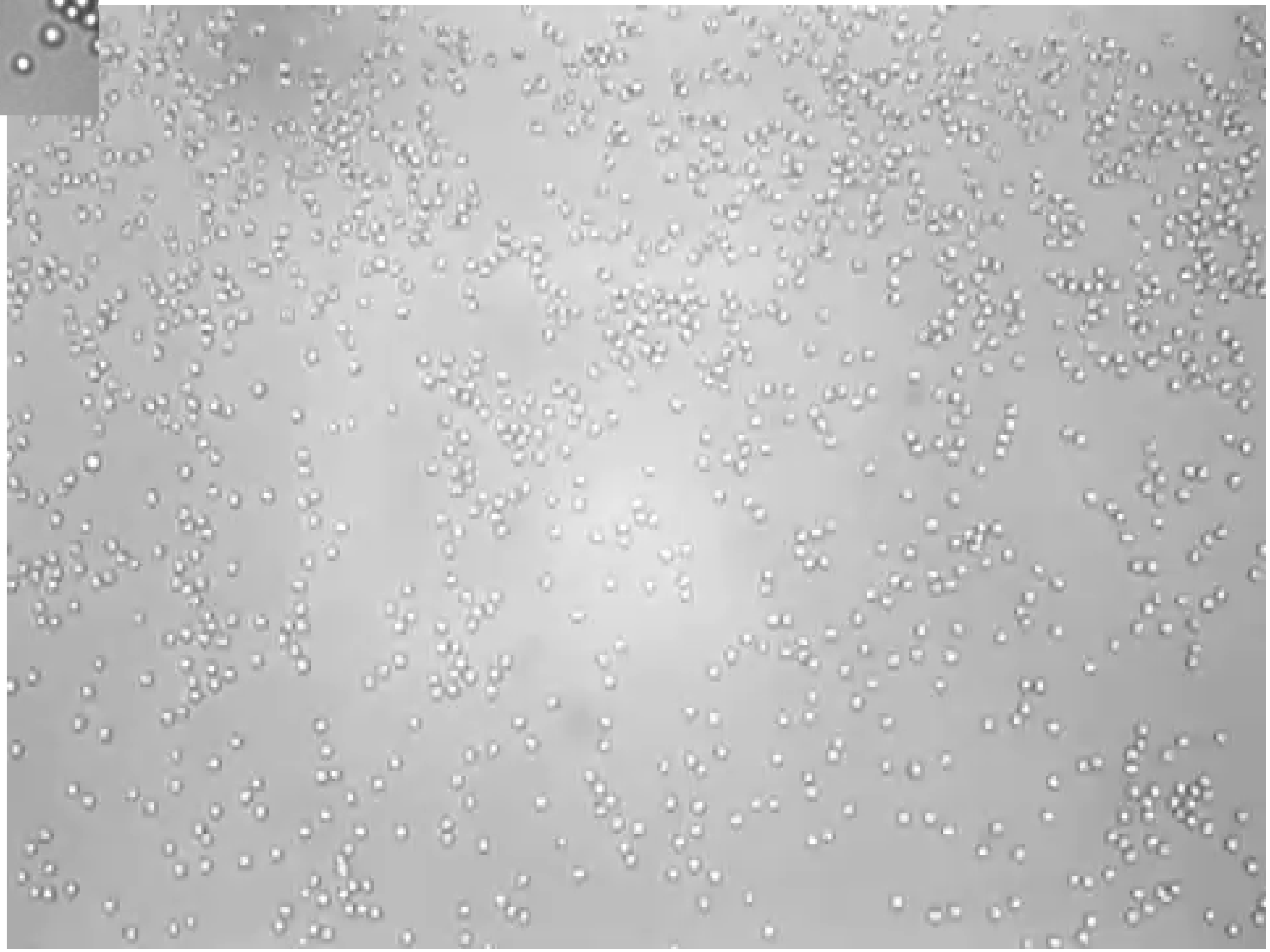


Collective motion: macroscopic animnals



heamatite

hydrogen peroxide



Pine group, New York

Active matter

Meant to be out of thermodynamic equilibrium

Why is it interesting?

Active matter

Meant to be out of thermodynamic equilibrium

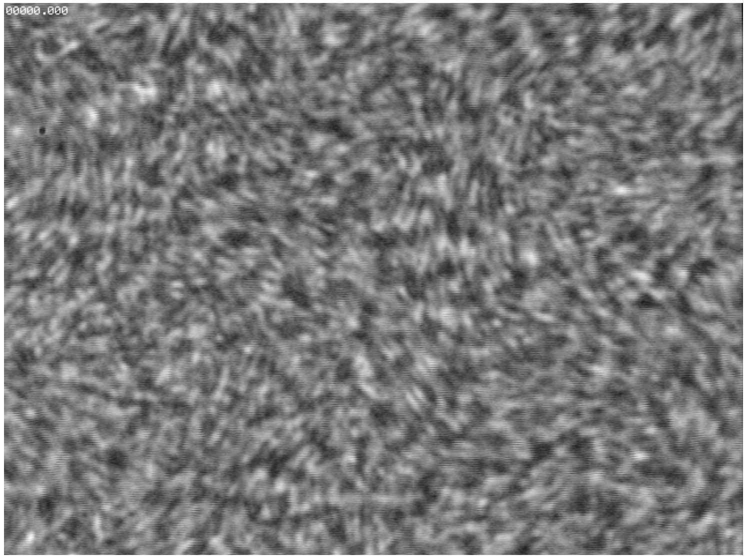
Why is it interesting?

1. to understand biological systems: biomechanics and self-assembly

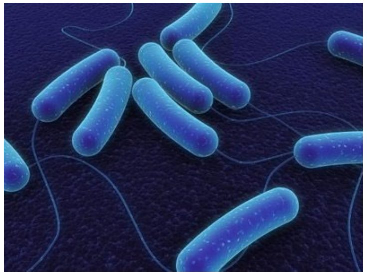
2. To create new types of micro-engines

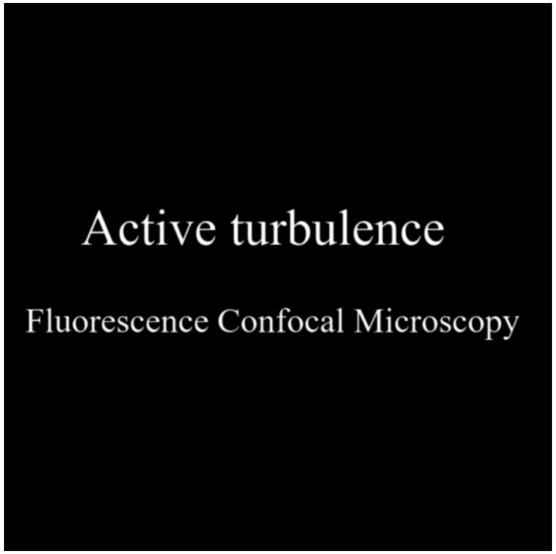
3. As examples of non-equilibrium statistical physics

Active turbulence



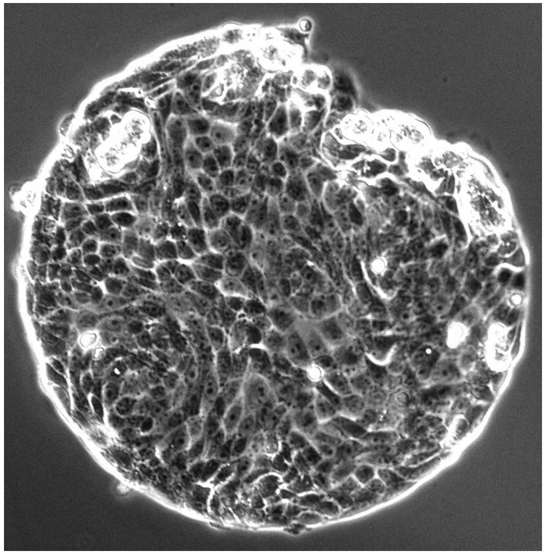
Dense suspension of microswimmers





Microtubules driven by motor proteins

Confluent cell layer



1. Introduction

2. Active turbulence and active topological defects

- Background 1: Swimming at low Re
- Background 2: nematic liquid crystals
- Active stress
- Active topological defects
- The hare and the tortoise, and other examples

3. 3D 4. confined systems 5. mechanobiology

Low Reynolds number hydrodynamics

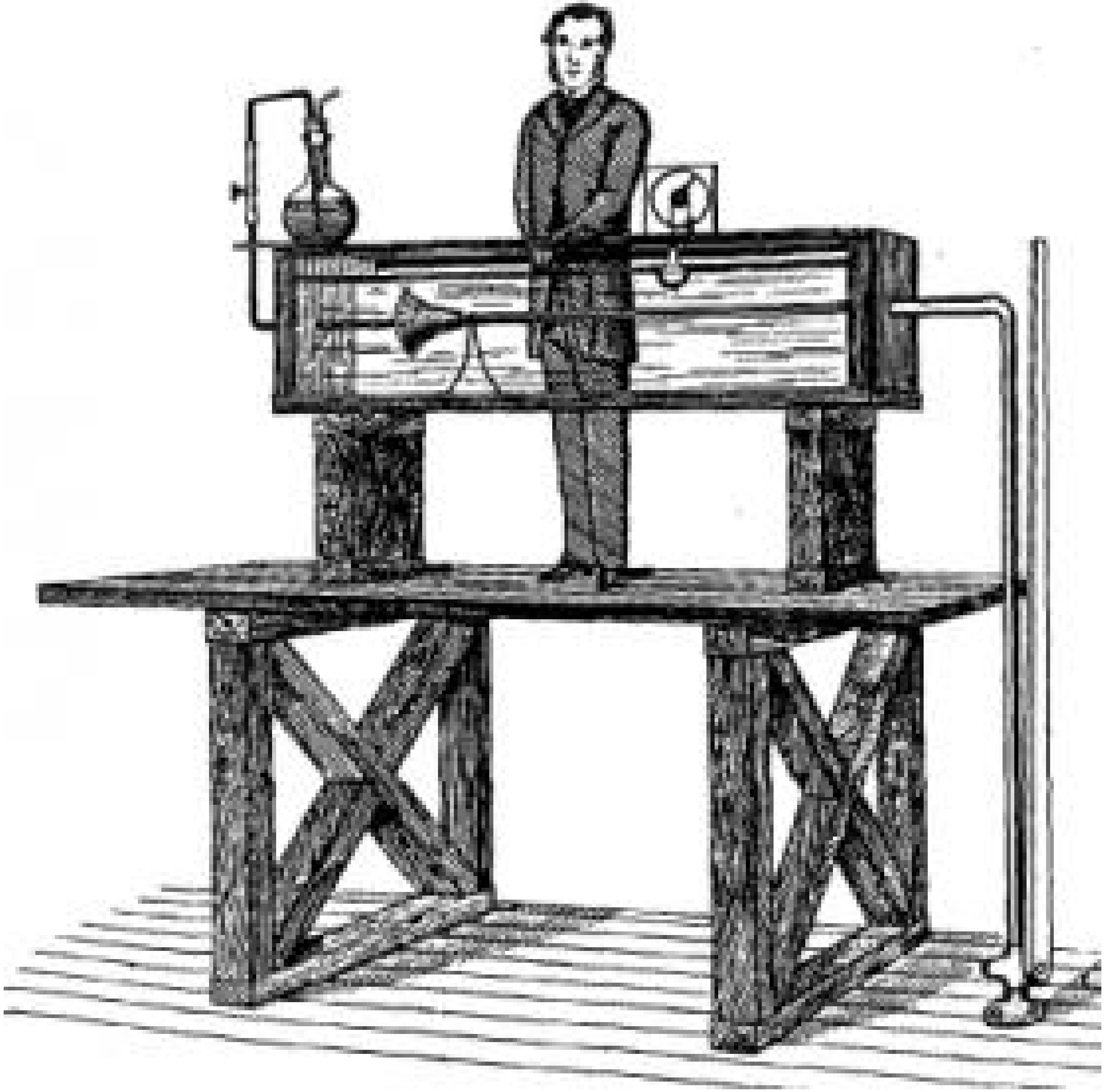
$$\rho\left\{\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right\} = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f}$$

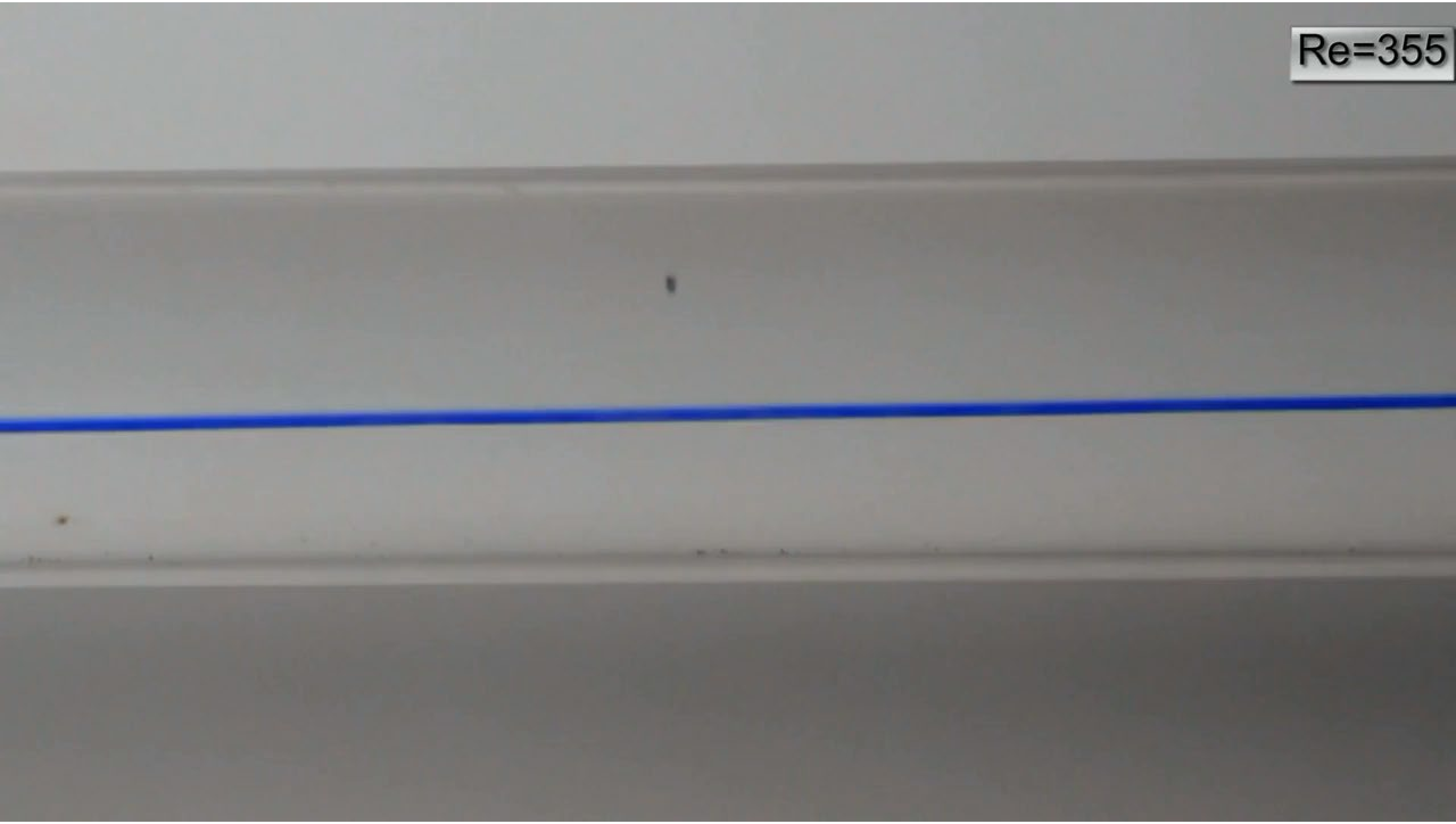
inertial term

viscous term

Reynolds number = inertial term / viscous term

= velocity x length / viscosity



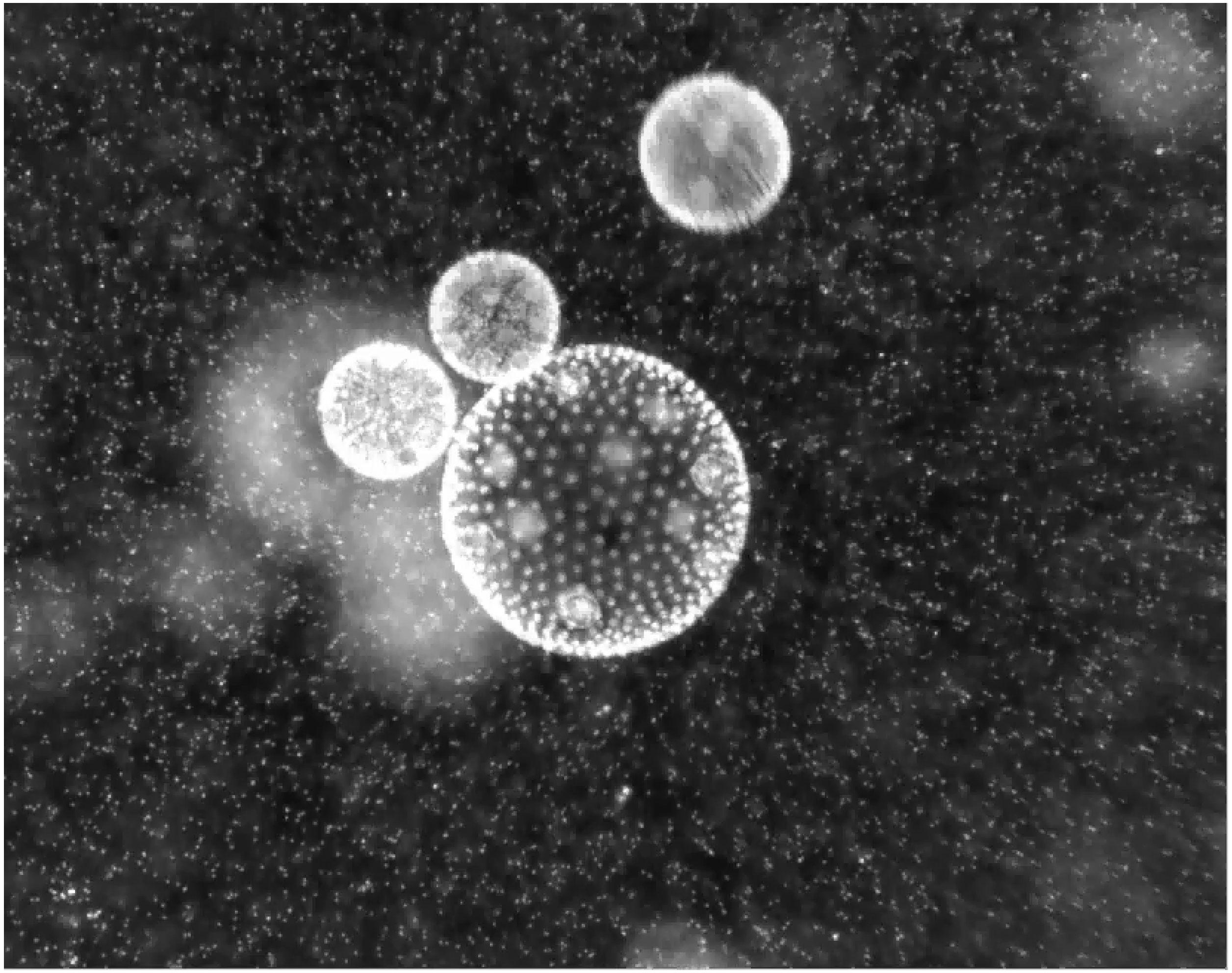

Re=355

Large length scales:
High Re

$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

Small length scales
Low Reynolds numbers

$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

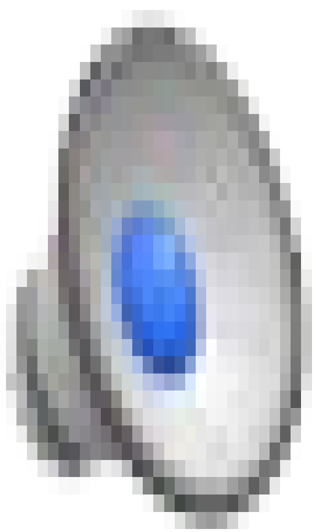


Goldstein group, Cambridge

Low Reynolds number hydrodynamics

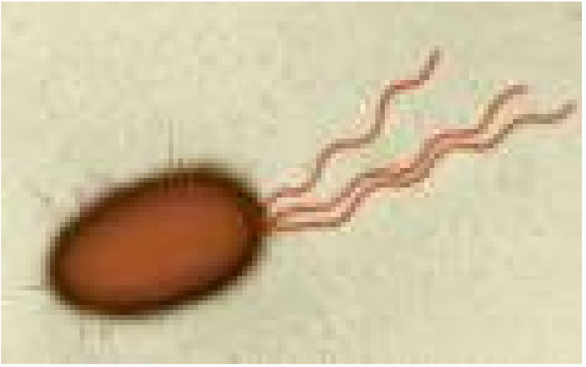
small length scales, high viscosities

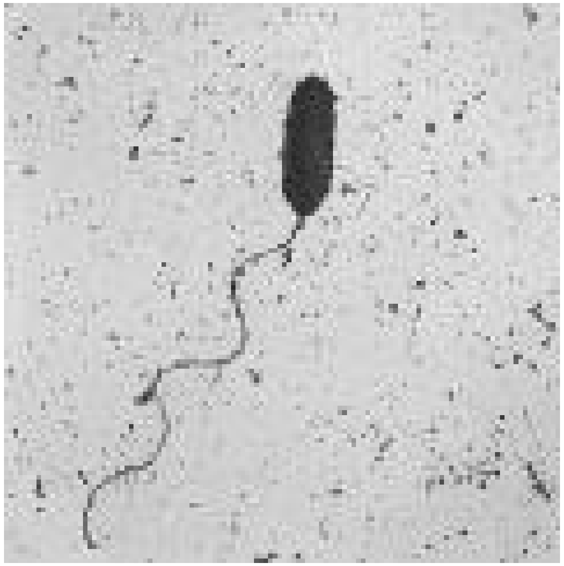
Stokes equations: $\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f}$ $\quad \nabla \cdot \mathbf{v} = 0$

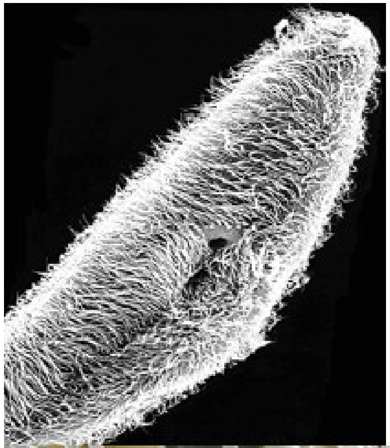


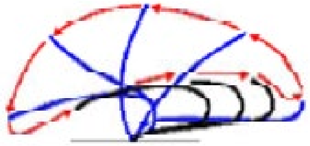
Purcell’s Scallop Theorem

Swimmer strokes must be non-invariant under time reversal





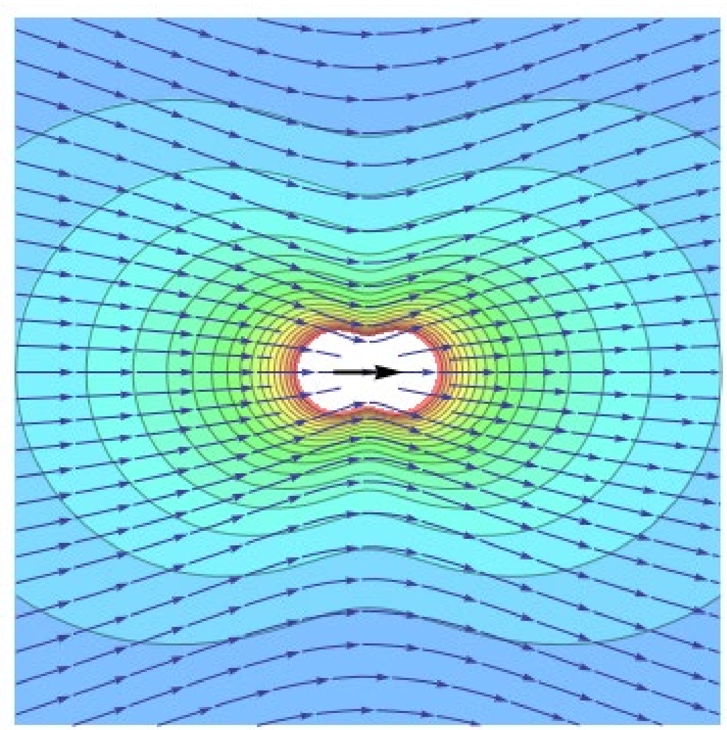




Hydrodynamics of active systems

Stokes equations

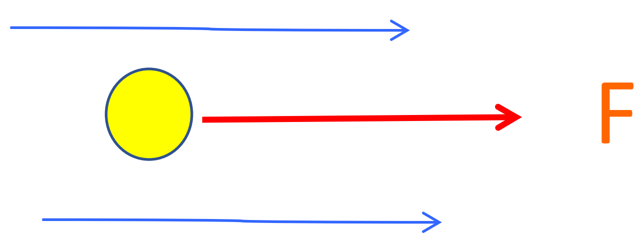
$$\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f}$$



Stokeslet

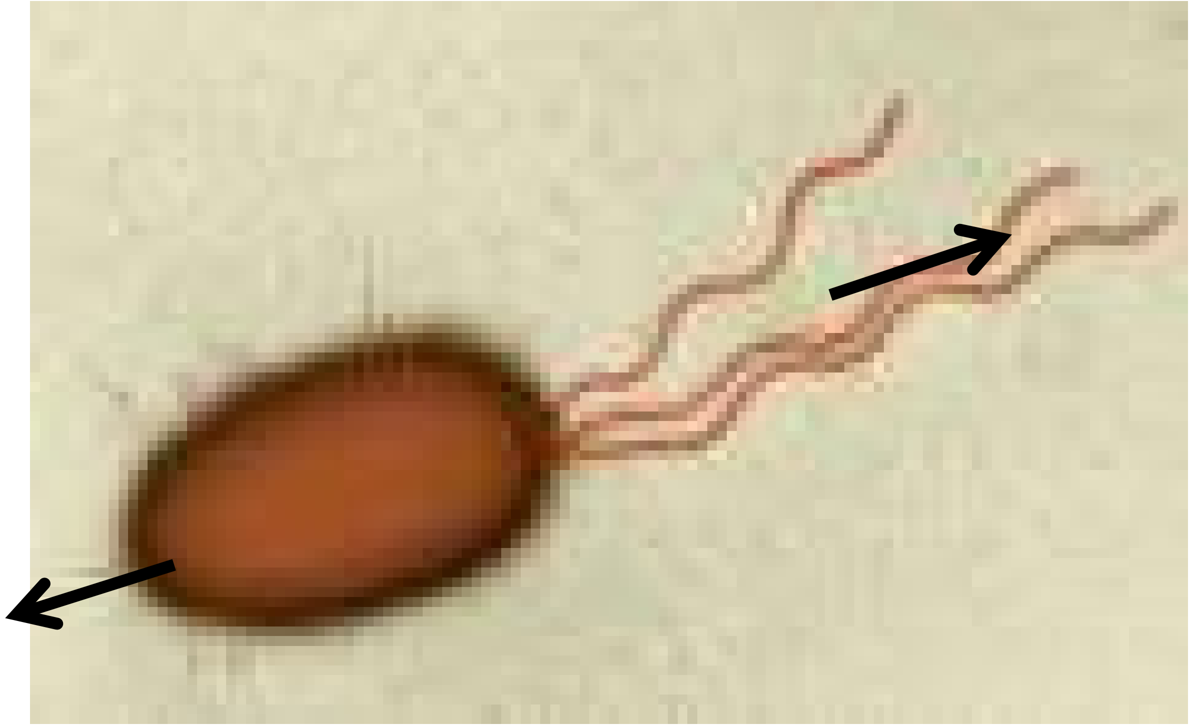
$$\mathbf{v} = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r} + \frac{\mathbf{rr}}{r^3} \right)$$

$$v_i(\mathbf{r}) = \frac{f_j}{8\pi\mu} \left(\frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3} \right)$$



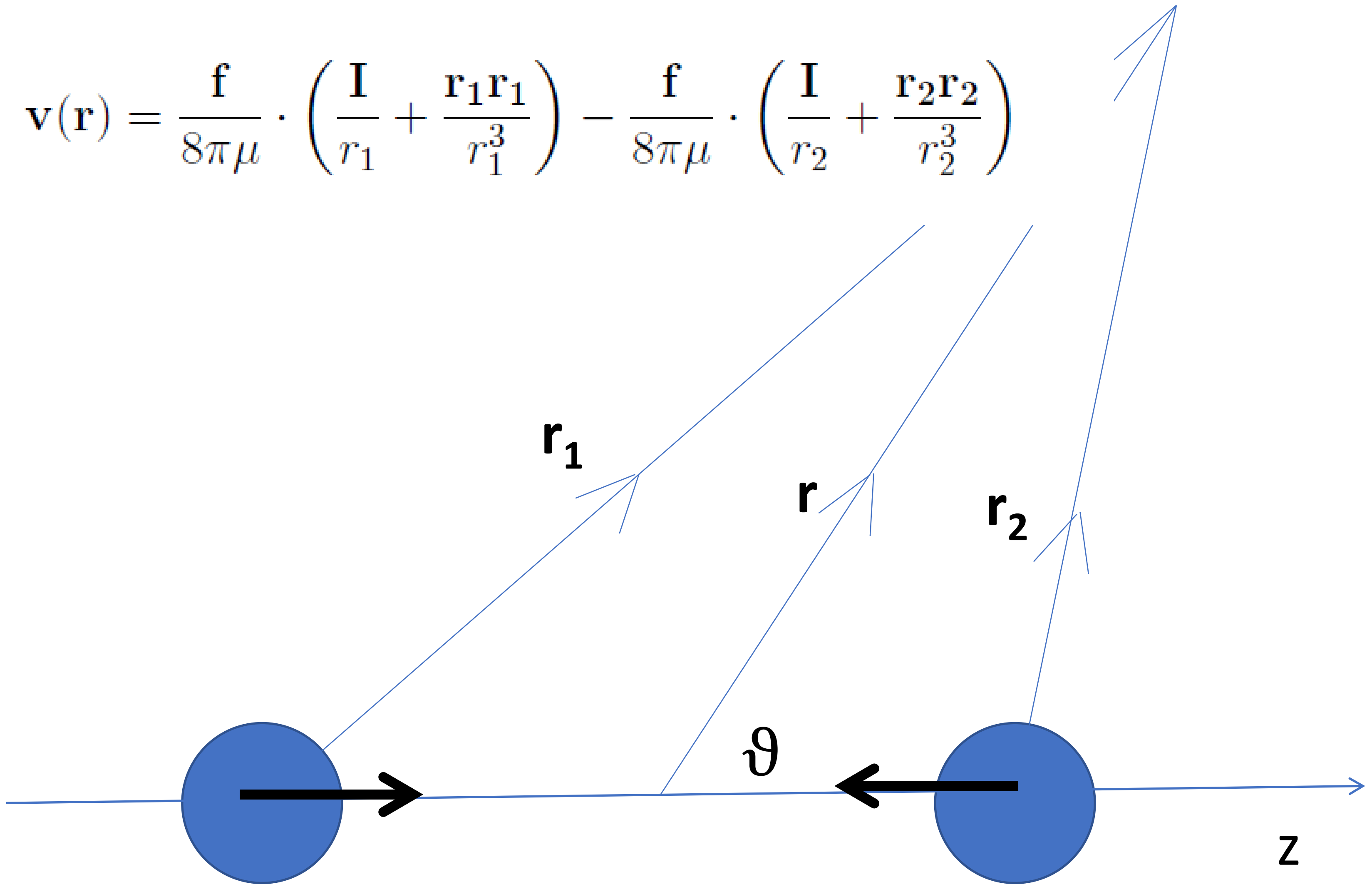
Swimmers have no external forces or torques acting on them.

So all forces they exert on the fluid must act in equal and opposite pairs.



Far flow field of a swimmer

$$\mathbf{v}(\mathbf{r}) = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_1} + \frac{\mathbf{r}_1\mathbf{r}_1}{r_1^3} \right) - \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_2} + \frac{\mathbf{r}_2\mathbf{r}_2}{r_2^3} \right)$$

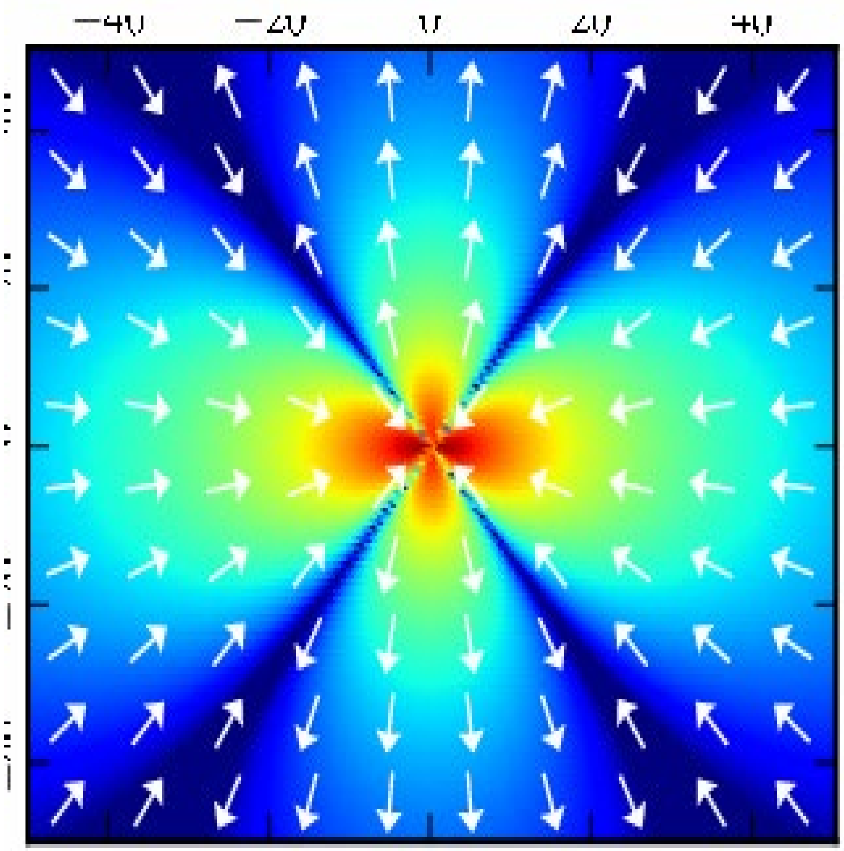


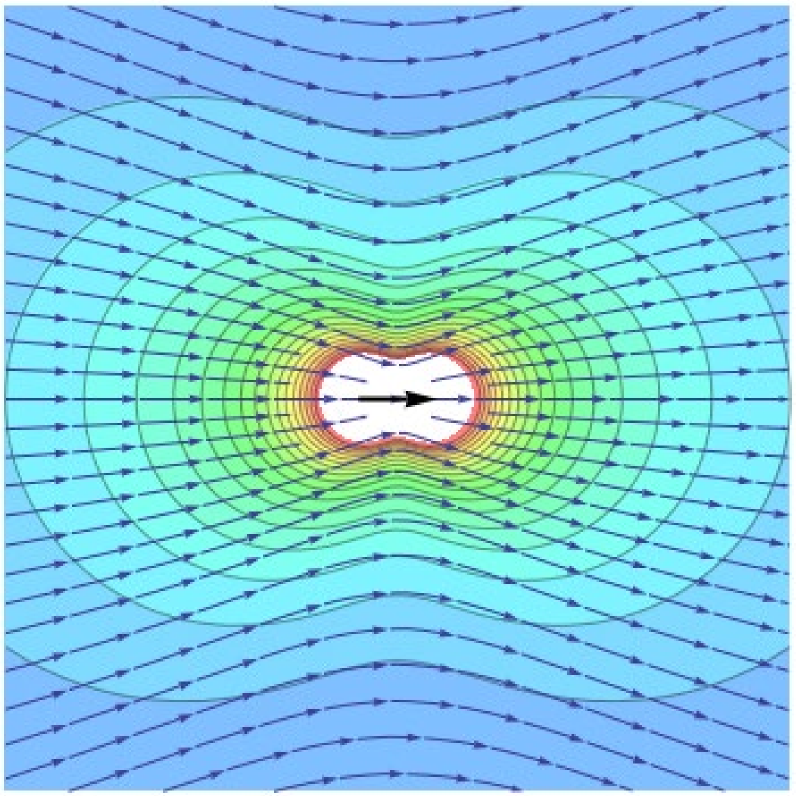
Far flow field of a swimmer

$$v_r = \frac{f}{4\pi\mu}\ \frac{L}{r^2}\ (3\cos^2\theta - 1)$$

Swimmers have dipolar far flow fields because they have no net force acting on them

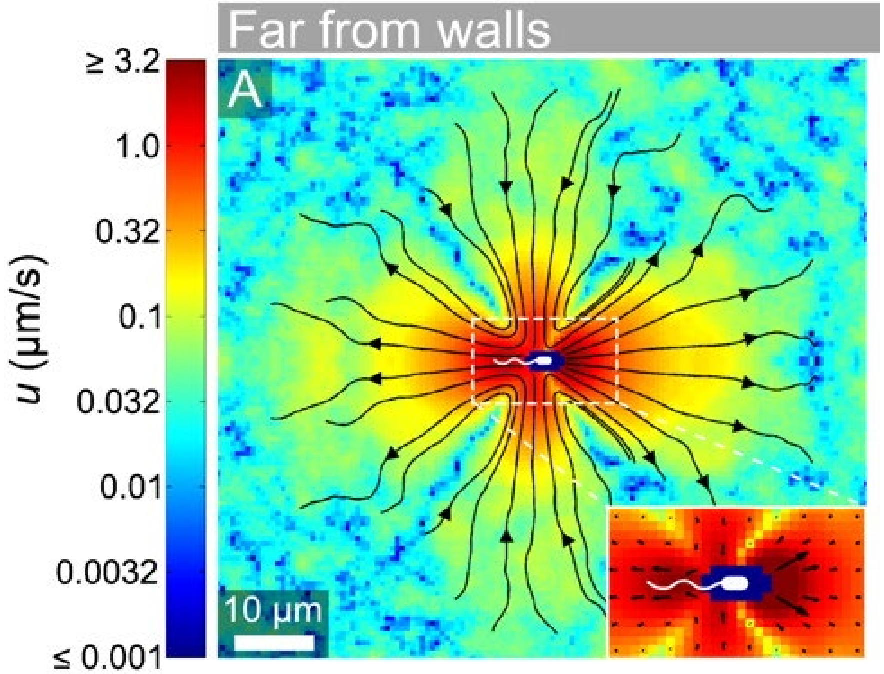
Swimmer and colloidal flow fields

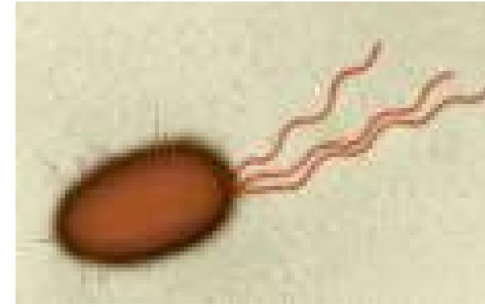




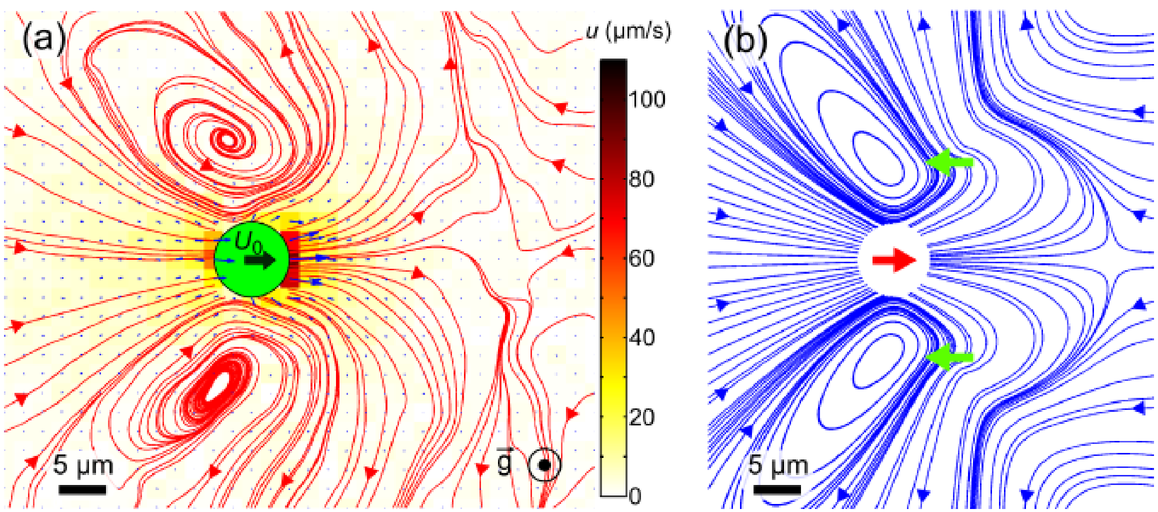
$$v \sim \frac{1}{r^2}$$

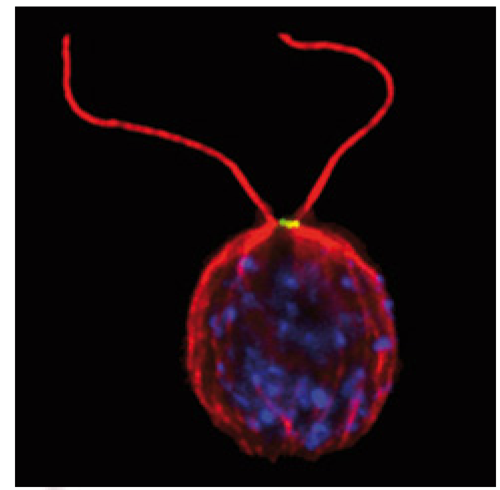
$$v \sim \frac{1}{r}$$





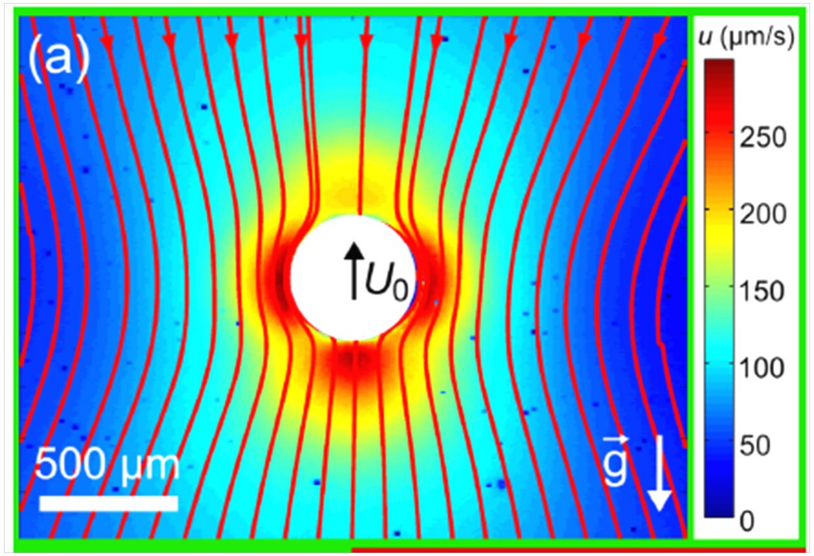
E-coli



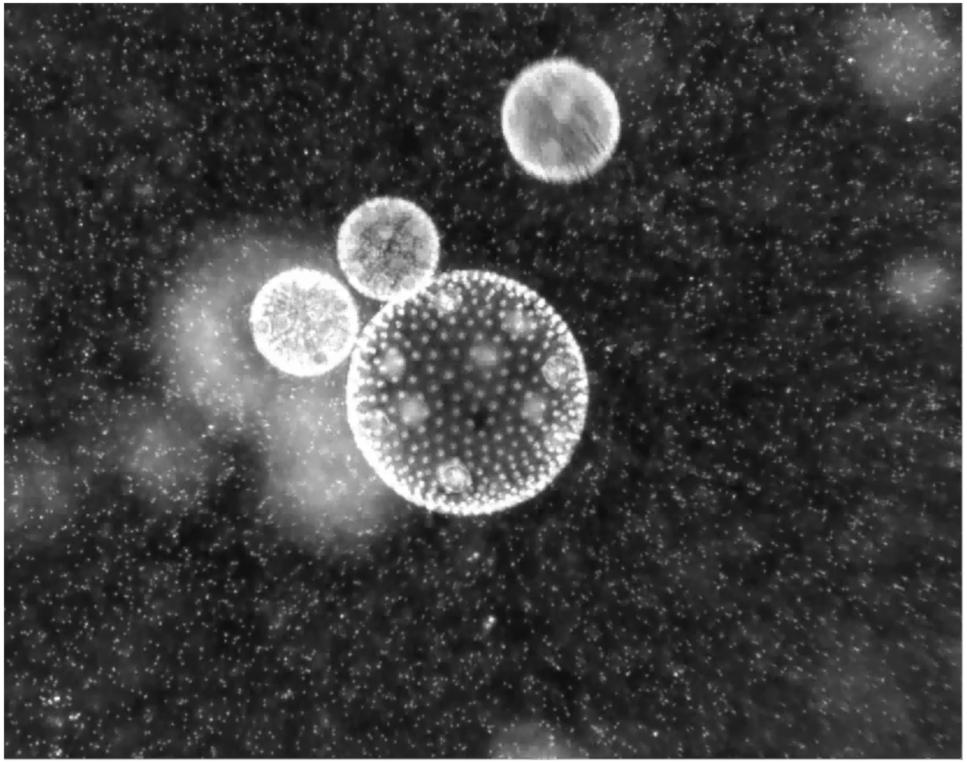


Chlamydomonas

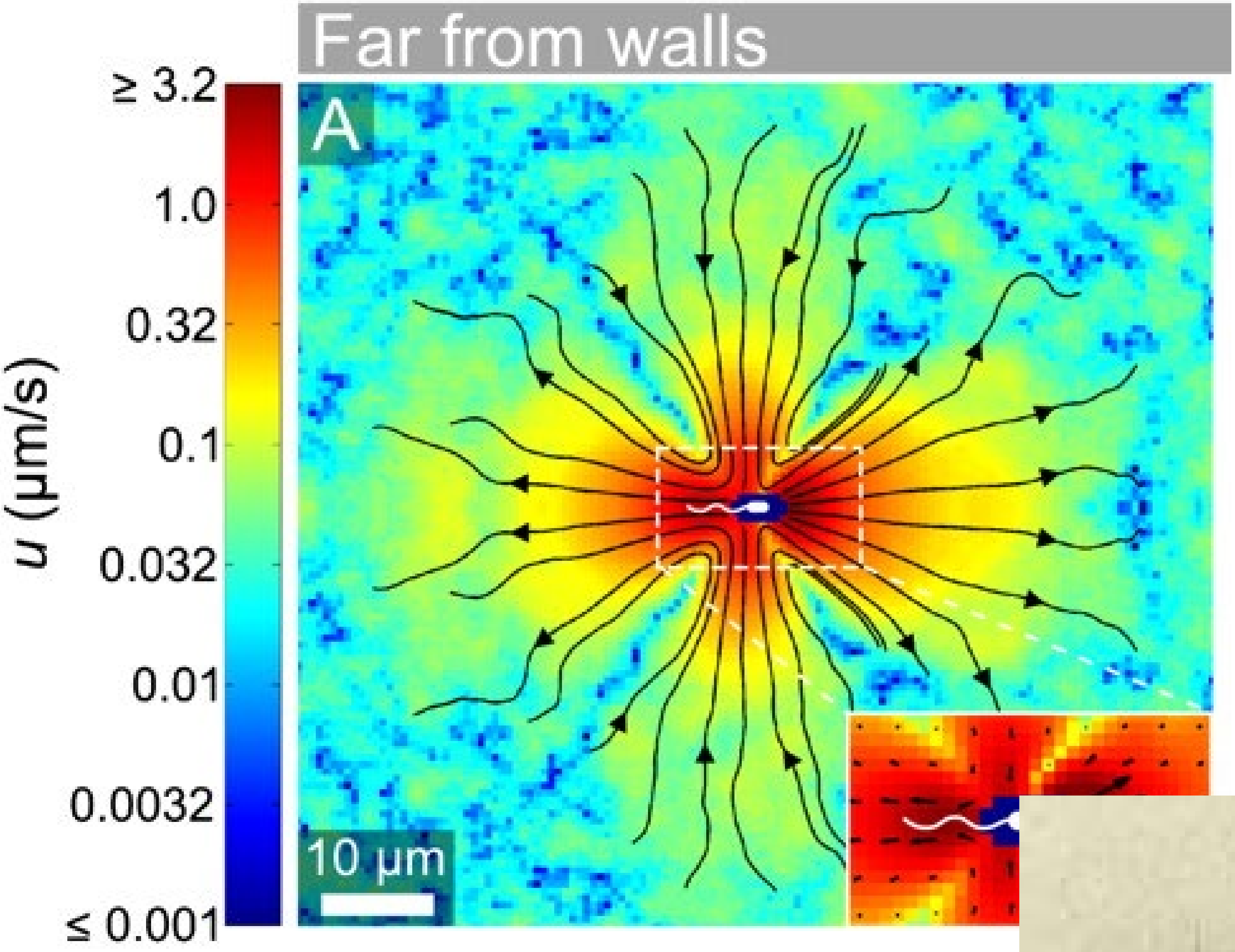
Goldstein group



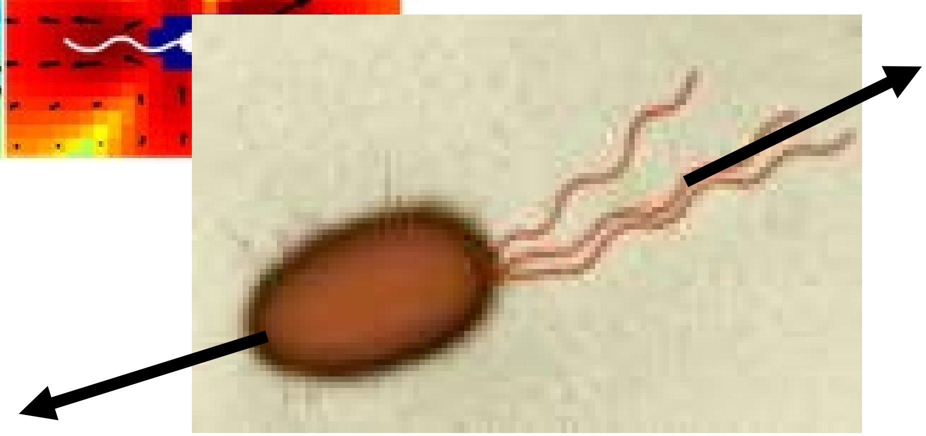
Volvox



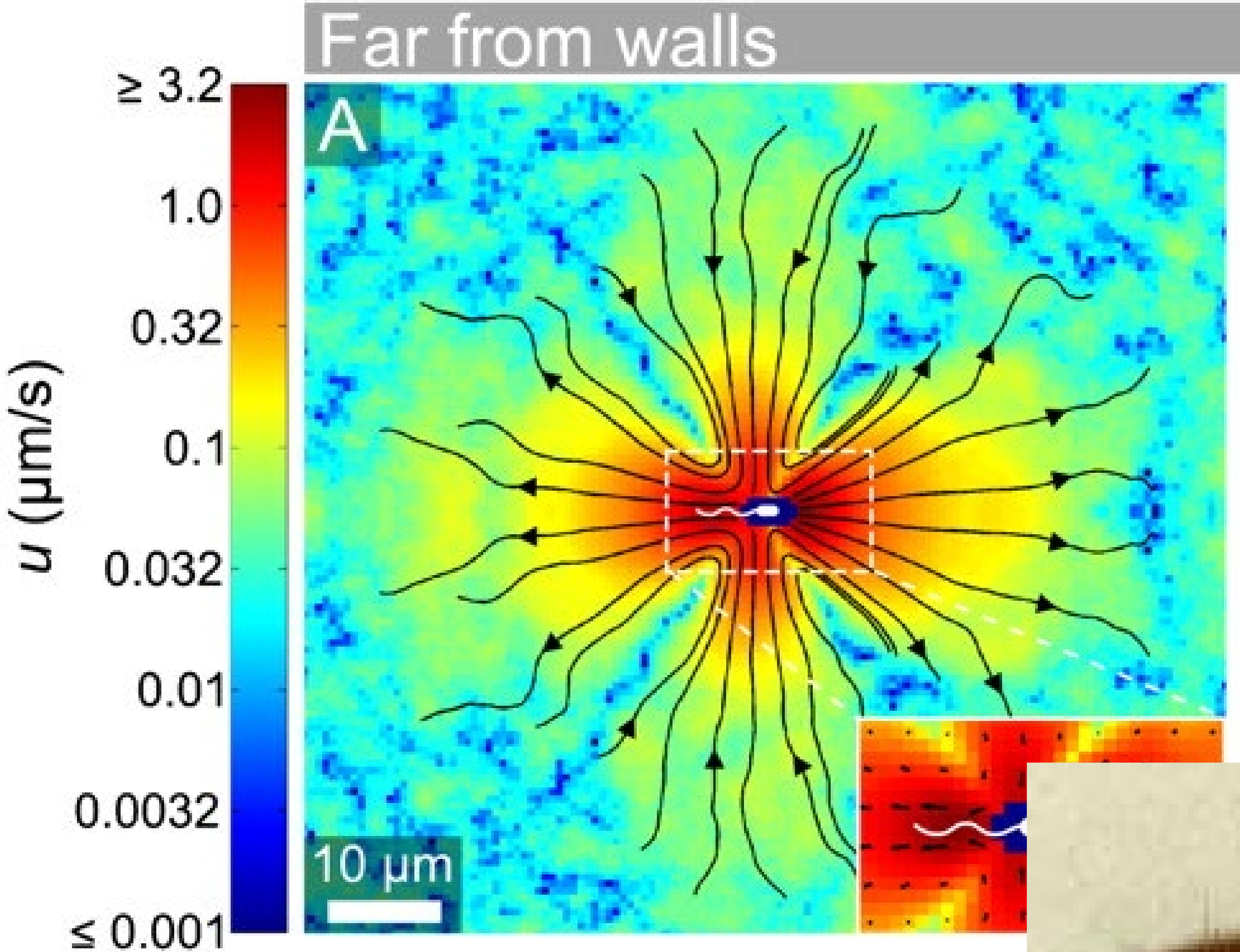
Dresher et al, PRL 105 (2010)
PNAS 108 (2011)



E-coli

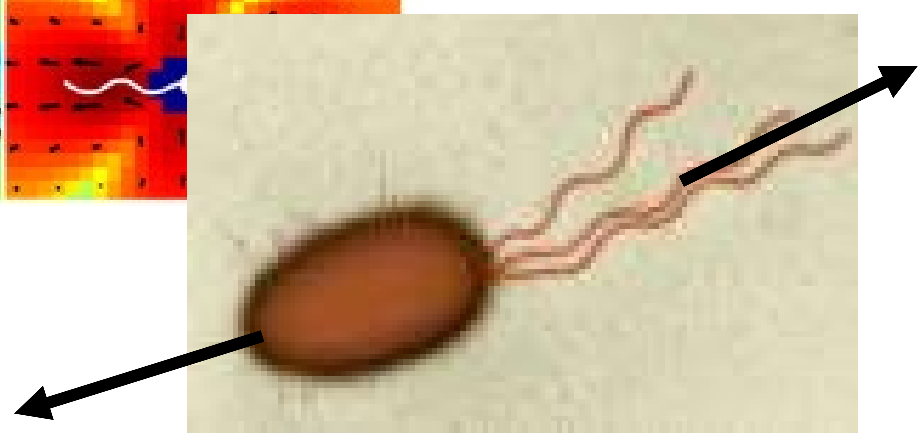


Goldstein group, Cambridge

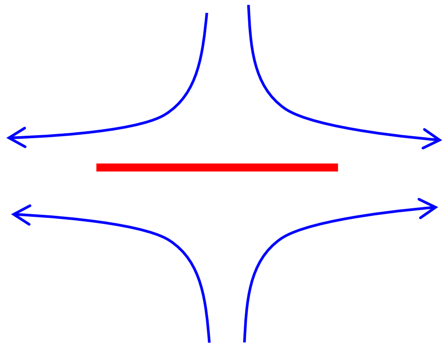


E-coli

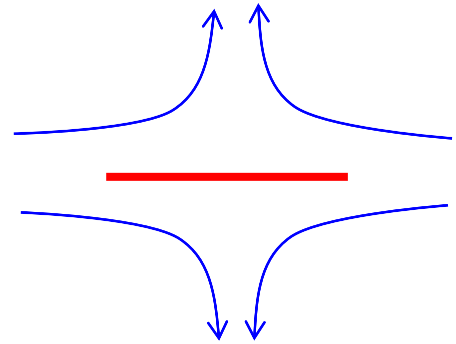
NB nematic symmetry



Goldstein group, Cambridge



Extensile



Contractile

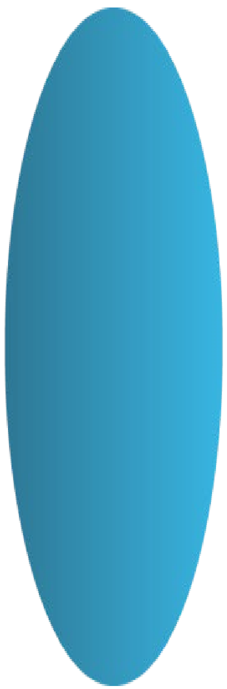
1. Introduction

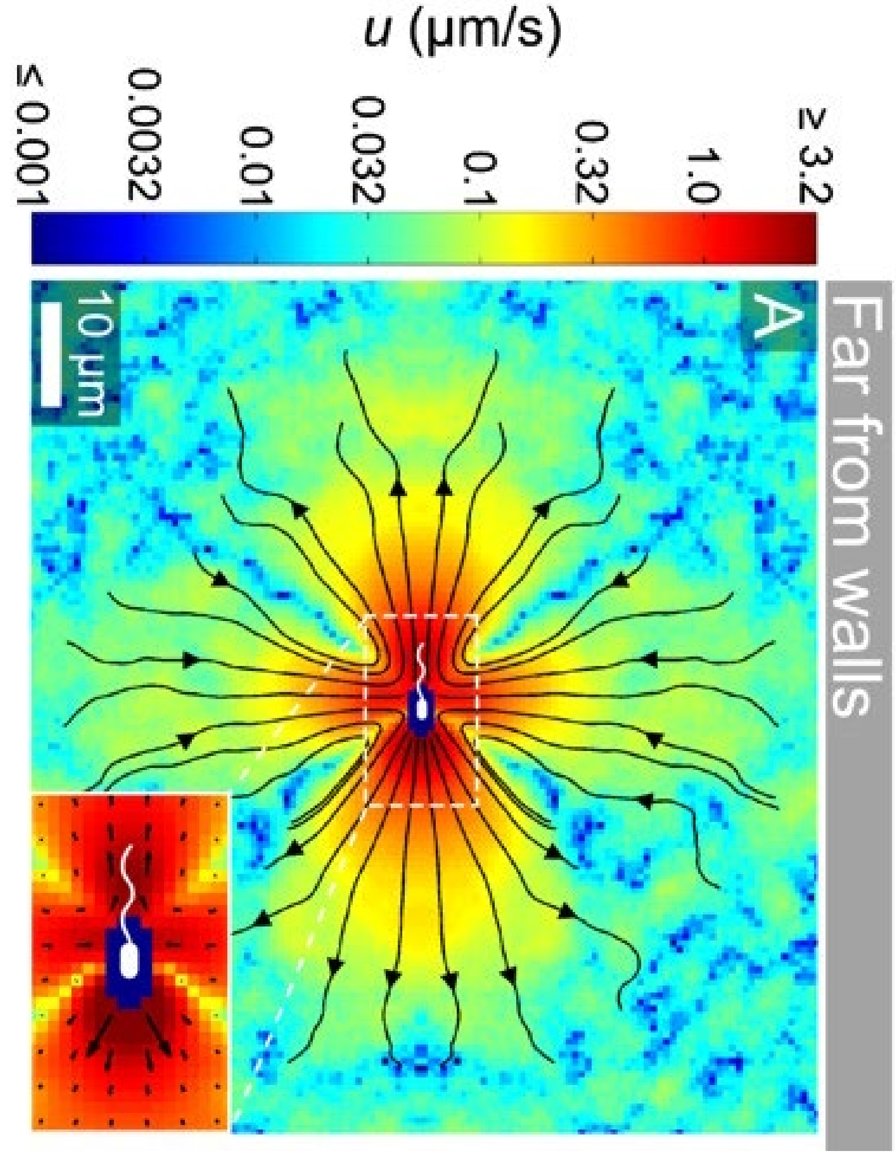
2. Active turbulence and active topological defects

- Background 1: Swimming at low Re
- Background 2: nematic liquid crystals
- Active stress
- Active topological defects
- The hare and the tortoise, and other examples

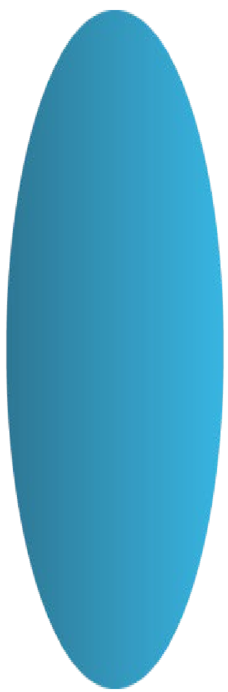
3. 3D 4. confined systems 5. mechanobiology

nematic symmetry

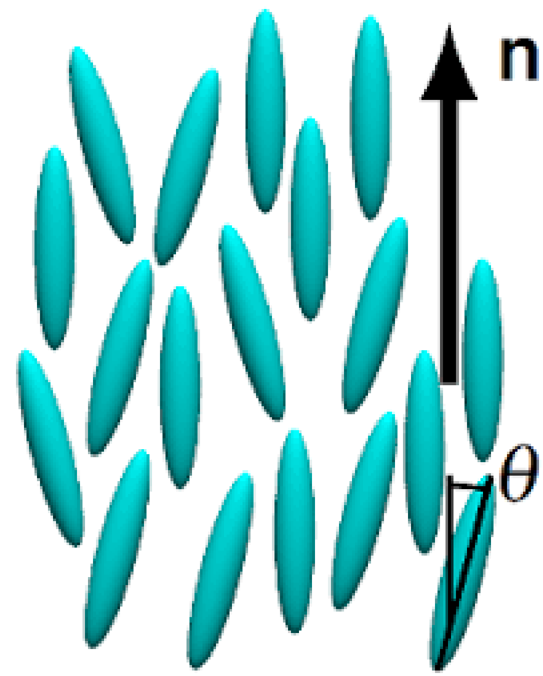




nematic symmetry



nematic phase



$$Q_{ij} = \frac{3}{2}\langle n_i n_j - \frac{\delta_{ij}}{3} \rangle$$


alamy stock photo

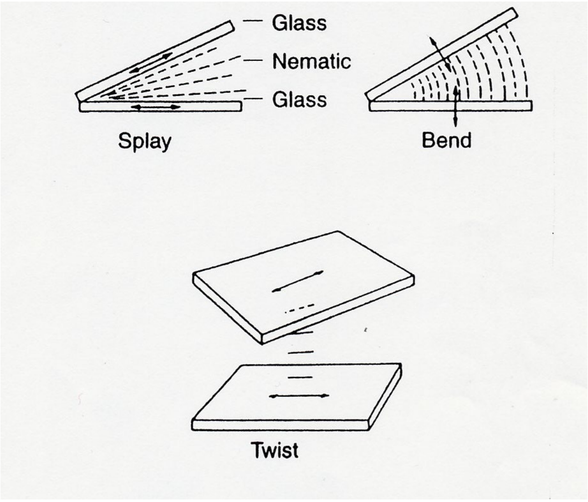
Tensor order parameter, Q

$$Q_{ij} = \frac{3}{2}\langle n_i n_j - \frac{\delta_{ij}}{3}\rangle$$

Landau-de Gennes free energy

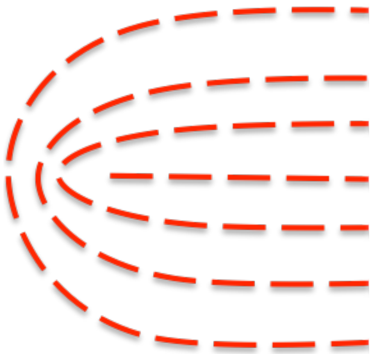
:

$$F = \frac{K}{2}(\partial_k Q_{ij})^2 + \frac{A}{2} Q_{ij} Q_{ji} + \frac{B}{3} Q_{ij} Q_{jk} Q_{ki} + \frac{C}{4}(Q_{ij} Q_{ji})^2$$

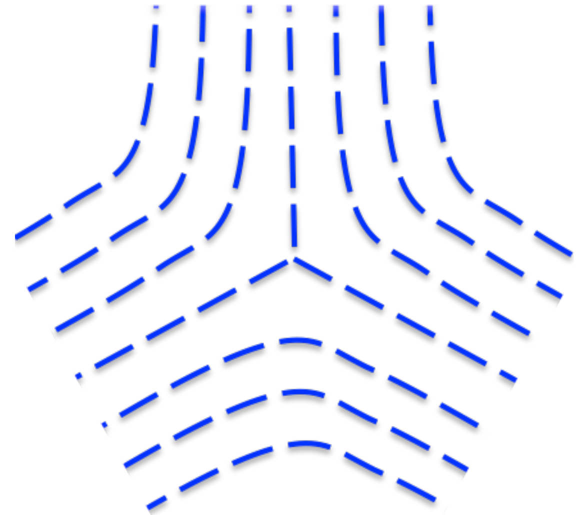


An ‘elastic liquid’

Topological defects in nematic liquid crystals



$$m = +\frac{1}{2}$$



$$m = -\frac{1}{2}$$

topological charge

$$m = \frac{1}{2\pi}\int_{dS} d\theta$$

defects annihilate in pairs to give perfect nematic order

Continuum equations of liquid crystal hydrodynamics

$$\rho(\partial_t + u_k\partial_k)u_i = \partial_j\Pi_{ij}$$

$$\Pi_{ij}^{viscous} = 2\mu E_{ij}$$

$$\Pi_{ij}^{passive} = -P\delta_{ij} + 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl}H_{lk}) - \lambda H_{ik}(Q_{kj} + \delta_{kj}/3)$$
$$-\lambda(Q_{ik} + \delta_{ik}/3)H_{kj} - \partial_i Q_{kl}\frac{\delta\mathcal{F}}{\delta\partial_j Q_{lk}} + Q_{ik}H_{kj} - H_{ik}Q_{kj}$$

Tumbling parameter

Continuum equations of liquid crystal hydrodynamics

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}$$

$$S_{ij} = (\lambda E_{ik} + \Omega_{ik})(Q_{kj} + \delta_{kj}/3) +$$

$$(Q_{ik} + \delta_{ik}/3)(\lambda E_{kj} - \Omega_{kj}) - 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl}\partial_k u_l)$$

$$E_{ij} = (\partial_i u_j + \partial_j u_i)/2$$

$$\Omega_{ij} = (\partial_j u_i - \partial_i u_j)/2$$

$$H_{ij} = -\delta \mathcal{F}/\delta Q_{ij} + (\delta_{ij}/3)\mathrm{Tr}(\delta \mathcal{F}/\delta Q_{kl})$$

$$\mathcal{F} = K(\partial_k Q_{ij})^2/2 + A Q_{ij} Q_{ji}/2 + B Q_{ij} Q_{jk} Q_{ki}/3 + C(Q_{ij} Q_{ji})^2/4$$

Continuum equations of liquid crystal hydrodynamics

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}$$

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\rho(\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij}$$

viscous + elastic

Continuum equations of active liquid crystal hydrodynamics

$$(\partial_t + u_k\partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}$$

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\rho(\partial_t + u_k\partial_k)u_i = \partial_j \Pi_{ij}$$

viscous + passive + active stress

$$\Pi_{ij}^{active} = -\zeta Q_{ij}$$

Active stress => active turbulence

$$\Pi_{ij}^{active} = -\zeta Q_{ij}$$

Gradients in the magnitude or direction of the order parameter induce flow.

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

1. Introduction

2. Active turbulence and active topological defects

- Background 1: Swimming at low Re
- Background 2: nematic liquid crystals
- Active stress
- Active topological defects
- The hare and the tortoise, and other examples

3. 3D 4. confined systems 5. mechanobiology