For Boulder Summer School  July, 2010

Strongly-correlated quantum many-body physics
with ultracold neutral atoms.

(people also cool molecules, ions)

Realized so far:

• Paired-fermion superfluids (I'll focus here, mostly)
• Mott insulators
• Luttinger liquids (1D)

with some aspects of control not available in condensed matter systems.

Atoms: Bosons or Fermions (fermionic atoms can "imitate" electrons)

mass range $^6\text{Li} \leftrightarrow ^{133}\text{Cs}$ (or more, but alkalis are easiest to work with)

internal "spin" states are hyperfine states of nuclear spins + electrons in incomplete shells,

densities $10^{11} - 10^{14}$ atoms $\text{cm}^{-3}$ dilute gas by usual standards

but cooled to quantum degeneracy $10^{-6} - 10^{-9}$ K,

thus not dilute on their own terms.

Thermal de Broglie wavelength $\geq$ interatomic spacing $\approx 1$ mm
Gas is metastable against forming molecular bound states:

2 atom collision:

\[ \overset{-P}{A} \overset{P}{B} \rightarrow E > 0 \]

Molecule formation is forbidden by conservation of \( E + P \)

But 3-atom collision can form molecule:

and 3rd atom takes up released energy.

This limits the density of the gas.

Experiments last up to 10s of seconds, but often less than 1 second.

2-atom scattering time \( \tau \geq 10^{-6} \) seconds is smallest "microscopic" time.

Makes far-from-equilibrium behavior quite accessible, and sometimes makes equilibrium difficult to access.
Good control of optical potential: (single-atom "external" potential)

\[ V_x(\vec{r}) = - \sum_{\omega} P_x(\omega) I(\vec{r}, \omega) \]

- atomic species \( x \)
- lasers \( \omega \)
- real part of \( x \)'s polarizability
- intensity of "light" at \( \vec{r} \)

Use:
- focussed beams to make smooth potentials
- standing waves,
- interference, speckles, holograms to make optical lattices or other potentials that vary rapidly with \( \vec{r} \) (\( \propto \) on scale \( \geq \lambda \) of photons)

Keep \( \omega \) away from absorption lines to avoid scattering, heating.

Can \( \# \) change \( I(\omega) \)'s quickly.

Optical potential can "imitate" crystalline lattice of a material.
Interactions between neutral atoms are short-ranged (van der Waals or Bonding). \( r_{\text{inter}} \ll \lambda_B \equiv \text{\mu m} \)

Usually negligible between identical fermions, due to anti-symmetry of wavefunction.

Can be greatly enhanced near a Feshbach resonance (FR) where they are on verge of forming a weakly-bound state. This (FR) makes the interactions controllable (via B-field) in certain cases.

For bosons, this enhancement of interactions also enhances 3-atom loss processes, but for case of two species of fermions "Pauli blocking" (2 of the 3 must be identical) prevents this, allowing tuning to strong interaction (divergent scattering length = "unitarity")
Evaporative cooling + depolarization

Final stage of cooling to lowest temperature/entropy is "simple" evaporative cooling:

\[ V(r) \]

optical + magnetic potential

\[ R^2 \]

need atom-atom interaction to continuously produce high-energy atoms via scattering. Doesn't work for one-component Fermi gas. Or generally

Two component gas (e.g. Fermions):

\[ M^\uparrow, M^\downarrow \]

evaporation here cools + depolarizes towards \[ M^\uparrow \rightarrow M^\downarrow \]

Cooling + Thermometry here are not accurately controlled/compared to doing low-T physics with materials.

This is an area where good new ideas/techniques are needed.
One-body + 3-body losses make holes.

For fermions this produces a lot of heating and entropy.

For weakly-interacting Bose condensate losing atoms from the condensate does not produce much heating or entropy (no shift below μ).

So cooling fermions to very low T has been less successful than bosons.

Mott insulator was first produced with bosons in an optical lattice, only later (~6 years) with fermions.

Magnetically-ordered phases of Mott insulators in optical lattice have not yet been produced. Will it again be easier with bosons?

People are pushing on both.
Paired-fermion superfluids.

The simplest BCS model for paired-electron superconductivity has a momentum-independent attraction between $\uparrow + \downarrow$ electrons at the Fermi surface.

Momentum-independent = short-range in real space (8-function).

This not accurate for electrons, but is realized accurately for two-species Fermi gases $^6\text{Li}$, $^{40}\text{K}$, e.g.

The two species are usually called $\uparrow$ and $\downarrow$, although they are really 2 different hyperfine states + usually differ more in the nuclear spin state than the electrons.

Control: interaction (via Feshbach resonance)

densities $n_\uparrow, n_\downarrow$.

New:
Can go to much stronger attraction than electrons, a liquid $^3\text{He}$

Can explore highly-polarized Fermi gas $n_\uparrow \gg n_\downarrow$ (spin-flip rate is very slow)

$$H = \sum_{k, \sigma} \frac{k^2 r^2}{2m} \hat{c}_k^\dagger \hat{c}_k + g \sum \int d\vec{r} \hat{c}_{\uparrow}(\vec{r}) \hat{c}_{\downarrow}(\vec{r})$$

"universal" limit:

$g \Lambda \rightarrow 0$ as $\Lambda \rightarrow \infty$ with s-wave scattering length $(2 \text{ atoms at E=0})$
BCS - BEC crossovers: \( n_\uparrow = n_\downarrow \) \( k_{F\uparrow} = k_{F\downarrow} = k_F \)

**Diagram:**
- \( q_5 < 0 \) BCS limit
- \( q_5 = 0 \) unitarity
  - (where 2-atom bound state forms at \( k_F = 0 \))
- \( q_5 > 0 \) BEC limit

\( T_c \sim \Delta \sim T_F \) for \( k_{F\uparrow} = k_{F\downarrow} \)

- Size of Cooper pairs >> \( \frac{1}{k_F} \) interatomic spacing smaller pairing

For \( k_{F\uparrow} = k_{F\downarrow} \) Fermi surfaces are perfectly nested.

Superfluid at \( T = 0 \) for any \( -g_{\Delta} < 0 \)

- Strong attraction between atoms \( \Rightarrow \) molecules
- Weak pairing

\( \frac{1}{k_F q_5} \) Bose condensate of weakly-interacting \( \text{\textbullet} \) molecules

- Spin gap \( \Delta \sim \frac{\hbar^2}{m a_s^2} > \frac{1}{T_F} \)
  - Binding energy of molecules<< size of Cooper pairs \( \text{\textbullet} \) molecules.

\( \frac{1}{k_F} \sim k_F \) pairing of \( \frac{1}{k_F} \) with \( k_F \) bosonic Cooper pairs that condense.
Polarized superfluid: $\eta_\uparrow > \eta_\downarrow$  $k_{F\uparrow} > k_{F\downarrow}$

well over

On "BEC side" makes Bose-Fermi mixture of

\[ \uparrow \downarrow \]

and excess $\uparrow$ fermions.

all minority atoms are in molecules at $T=0$

when bosons are strongly bound: $0 \leq k_{F\tilde{g}} < 1$

strongest "repulsion" is $\uparrow - \uparrow$ Pauli exclusion.

"Sarma phase":

Fermi sea of excess $\uparrow$'s fully mixed into superfluid (BEC) of $\uparrow \downarrow$'s

(very low $T$: $\uparrow \downarrow$'s mediate $P$-wave pairing of $\uparrow$'s,

Bulgar et al.)

\[ k_x \rightarrow k_{FF} \]

non-interacting

strong pairing

\[ k_{F} = \sqrt[3]{k_{FF}^3 - k_{F\tilde{g}}^3} \text{ of excess } \uparrow \text{'s} \]

(in 3D)

This regime

produced by Ketterle's group @ MIT
Strongly confine these molecules + atoms in 2 directions, with optical lattice but let them move freely in the 3rd direction $\Rightarrow$ 1D system

(Hulet's group, Rice U.) has produced this in the lab)

Still a Bose-Fermi mixture, but an important difference:

- **Bo*so**n  
- contains one of these fermions

antisymmetry of fermions changes sign when boson passes fermion in 1D

Ground state of bosons in presence of the fermions in 1D signed $\Psi_{boson}$ for a given set of fermion positions

average wave number of Bose-coordinates of Cooper pairs $\Psi^*$'s

This is the 1D version of Larkin-Ovchinnikov-Fulde-Ferrell pairing (LOFF, or FFLO)

1D version is a type of Luttinger liquid (divergent quantum fluctuations due to 1D)

Weak coupling view:

Two condensates $+Q$, $-Q$ make a standing wave, and excess fermion pair-hole.

Pair $+k_F$ with $-k_F$ $\Rightarrow$ Cooper pairs with momentum $Q = k_F - k_F$
Back to 3D.

On "BEC side" at $T=0$, have molecules $\uparrow\downarrow$ bosons and Fermi sea of excess $\uparrow\uparrow$ fermions. (at low energy)

All interactions are repulsive due to "Pauli blocking"

- boson-boson
- boson-fermion
- fermion-fermion

Far from unitarity molecule is tightly bound, so fermions it is at high momentum + orthogonal to other fermions $\Rightarrow$ weak repulsion.

Fermion-fermion repulsion dominated, facing mixing of boson+fermions.

Closer to unitarity boson-fermion repulsion dominated, covering phase separation

[Diagram]

- Zero-temperature phase diagram:
  - Fermi gap ($P$-wave superfluid $\Rightarrow$ at very low $T$.)
  - FFLO$^?$
  - Phase separation (first order transition)
  - Bose-fermi mixture (BEC + Fermi sea)
  - Fermions $\Rightarrow$ p-wave SF at low $T$

BCS side

BEC side
With a large gas cloud in a large harmonic trap; see the
phase diagram/equation of state: e.g. at unitarity \( q_s \rightarrow 0.0 \)
\[
\text{at global equilibrium.}
\]

MIT, Paris, Rice

On BEC side, jump in density at interface SF/N can be
more than factor of 2 in a dilute gas!

In 3D \( P=0 \) superfluid is dense (heavier) sits at
bottom of trap

\( P>0 \) normal is lighter, sits on top

In 1D it is the other way around:

P>0 FFLO is dense than \( P=0 \) superfluid.

In 1D fermions fill in tobose fluid CE make it dense.

In 3D they don't: fermi-base repulsion is stronger than
bose-bose repulsion. In 3D
Cooper pairing is a Fermi surface nesting instability.

For \( k_{F\uparrow} = k_{F\downarrow} \) (\( P = 0 \)), the \( \uparrow + \downarrow \) Fermi surfaces match spin-singlet pair \((k_{F\uparrow}, \uparrow)\) with \((-k_{F\downarrow}, \downarrow)\) to make pairs with momentum \( \mathbf{Q} = 0 \) coherently at all points on Fermi Surface, gapping entire Fermi Surface.

For \( P > 0 \) nesting is no longer perfect:

FFLO: pair these to make condensate of Cooper pairs with momentum \( \mathbf{Q} \)

But it only works over a "patch" of the Fermi surfaces.

To enhance FFLO superfluid pairing need to enhance the Fermi surface nesting of \( \uparrow \) and \( \downarrow \) Fermi surfaces.

Put fermions in an optical lattice to make Fermi surfaces less "rounded", flatter.
Back towards 1D: Put optical lattice along \(x, y\), not along \(z\)

\[
V(x, y, z) = -V_0 \left( \cos^2 \left( \frac{2\pi x}{\lambda} \right) + \cos^2 \left( \frac{2\pi y}{\lambda} \right) \right)
\]

\(\lambda = \text{wavelength of light making lattice}\)

Now single-particle dispersion of fermions is free along \(z\), but high effective mass for moving in bands along \(x, y\), direction (tunneling in optical lattice)

First Brillouin zone.

\[
\begin{array}{c}
\uparrow k_x, k_y \\
\downarrow -k_x, -k_y \\
\end{array}
\]

\[
\begin{array}{c}
k_F & \quad -2\pi/\lambda \\
-2\pi/\lambda & \quad 2\pi/\lambda
\end{array}
\]

Makes Fermi surfaces more neglected, (can also do this with other types of lattices)

enhances FFLO pairing, etc.

pairs with total momentum \(\vec{Q}\), \(-\vec{Q}\)

two condensates of Cooper pairs:

\[
\langle C_x(r) C_y(r) \rangle \approx \Delta \cos(Qz) + \text{higher harmonics.}
\]

But how to demonstrate this type of pairing is present? Not clean.
Transport in unitary Fermi gas, with $T \approx T_F$

Transport of: energy (thermal conductivity) \[ \leftarrow \] easiest to measure

spin

momentum (viscosity)

Carriers are atoms moving with speed $v \approx \frac{k_F}{m} = v_F$

strong scattering at unitarity: mean-free path $l \approx \frac{1}{v} \approx \text{interatomic spacing}$

So diffusivity $D \approx v_F l \approx \frac{k}{m} \approx 10^{4} (\mu m)^2 / s$

Go to higher $T$ $V \approx v_F \sqrt{\frac{T}{T_F}}$ mean-free path $l \approx \frac{1}{T \sigma} \sim \frac{1}{k_F \left( \frac{T}{T_F} \right)}$

so $D \approx \frac{k}{m} \left( \frac{T}{T_F} \right)^{3/2}$ for $T \gg T_F$

at low $T$ for $n_F = n_r \Rightarrow$ superfluid: spin is transported by dilute quasiparticles $l \to \infty$ as $T \to 0$

$D \to \infty$

Zwecklein (MIT) is measuring this by setting up spin-density gradient + watching it relax by diffusion.