ULTRACOLD COLLISIONS
- PETHICK AND SMITH
- METCHALF AND VAN DER STRATEN

Elastic - needed for evaporative cooling
Inelastic - limits density

**Potentials**

**Ground State**:
Induced dipole
(Van der Waals)

\[
U = -C_6/R^6
\]

\[
\Rightarrow U = -C_6/R^6
\]

**Excited States**: Resonant dipole interaction
Dipole moment between the \(s\) and \(p\) states

\[
U = \pm C_3/R^3 \quad \text{Long-range}
\]

For \(s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \Rightarrow S = 0, 1 \Rightarrow \text{singlet or triplet}

- These potentials generally support bound vibrational states
- \(H_{HF} \cdot s \Rightarrow [H_{HF}, S] \neq 0

**Individual Collision Channels** are generally not pure triplet or singlet.
Partial wave analysis of potential scattering

(Joachain, "Quantum Collision Theory")

Potential \( V \)

\( i k z \)

\( e \)

Assume that \( V \) is central (spherically symmetric):

\( \psi (r, \theta, \phi) = R(r) Y_{\ell m} (\theta, \phi) \)

For \( r \rightarrow 0 \), i.e. outside the range of \( V \):

\( \psi \rightarrow A e^{i k z} + A f(\theta, \phi) e^{i k r} \)

\( f(\theta, \phi) = f(\theta) \) for sph. symm is scattering amplitude

\( f(\theta) \) accounts for the effect of the potential

Also, for \( r \rightarrow \infty \):

\( R(r) \approx \frac{A e^{i (k r + \frac{\pi}{2})}}{r} - \frac{B e^{i (k r - \frac{\pi}{2})}}{r} \)

Incoming spherical wave

Outgoing

Incoming and outgoing fluxes are equal (elastic)

\( \Rightarrow R(r) \approx e^{i k r} \sin \left( \frac{k r - \frac{\pi}{2} + \delta_0}{k} \right) \)

\( \delta_0 = \text{phase shifts}; \) these contain the effect of \( V \).
EX \ SQUARE WELL

\[ V(x) = \begin{cases} \varepsilon & x < 0 \\ 0 & 0 \leq x < \infty \end{cases} \]

Assuming \( V = 0 \)

\[ \varepsilon / k > 0 \]

EX \ HARD SPHERE

\[ V(x) = \begin{cases} \varepsilon & x < 0 \\ 0 & 0 \leq x < \infty \end{cases} \]

\[ \varepsilon / k < 0 \]

IT TURNS OUT THAT

\[ F(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1) \ell^\ell \sin \theta \rho_\ell (\cos \theta) \]

AND

\[ \frac{d\sigma}{d\Omega} = |F(\theta)|^2 \]

Particles scattered into \( d\Omega \) per incident flux.

\[ \sigma = \int \frac{d\sigma}{d\Omega} \; d\Omega = \frac{4\pi}{k} \sum_{\ell=0}^{\infty} (2\ell+1) \sin^2 \theta \rho_\ell = \frac{2\pi}{k} \sum_{\ell=0}^{\infty} \rho_\ell \]

\[ = \frac{4\pi}{k} \text{Im} \; F(\theta) \text{ Optical Theorem} \]

The usefulness of partial wave analysis comes if only a few \( \rho_\ell (k) \neq 0 \)

Each \( \ell \) corresponds to a particular angular solution to the S.E. with angular momentum \( \ell \)
There is a maximum $l$ depending on the range $\rho_0$ of $V$ and incident energy $k\hbar = tK\rho_0 \Rightarrow l_{\text{max}} \approx K\rho_0$

Consider $N\alpha$ with $\rho_0 \approx 10\alpha_0$:

$$K\hbar T = \frac{k^2 K^2}{2m} \Rightarrow K\rho_0 \approx 1.6 \text{ at } 1\text{MeV} \Rightarrow l_{\text{max}} \approx 2$$

$K\rho_0 \approx 0.5 \text{ at } 1\text{MeV} \Rightarrow l_{\text{max}} \approx 0$ (s-wave)

Proton wave analysis is very useful for uncorrelated collisions.

For $s$-wave scattering:

$$f(\theta) = \frac{1}{K} e^{i\frac{\pi}{4} \sin^2 \beta_0} \Rightarrow \text{isotropic}$$

$$\beta = \frac{4\pi}{K^2} \sin^2 \beta_0 \approx \frac{4\pi}{K^2}$$

The $s$-wave scattering length is more useful than $\beta_0$:

$$\alpha = -\frac{\tan \beta_0}{\lim K \rightarrow 0 K} \Rightarrow \beta_0 \approx -K\alpha$$

As $K \rightarrow 0$

$$f(\theta) \approx -\alpha \text{ and } \beta \approx 4\pi \alpha^2$$

$\alpha > 0 \Rightarrow \beta < 0 \Rightarrow V\text{ repulsive (eg hard sphere)}$

$\alpha < 0 \Rightarrow \beta > 0 \Rightarrow V\text{ attractive (eg square well)}$
Note that $\alpha \to +\infty$ for $S_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\Rightarrow$ "Zero-energy resonances."

Consider square well

What happens to $\alpha$ as $V_0$ is increased?

$\Rightarrow$ 1 node appears in well

$\Rightarrow$ Bound state

$0 < S < \frac{\pi}{2}$  0 Bound states $\Rightarrow$ Attractive

$\frac{\pi}{2} < S < \pi$  1 " " Repulsive

$\pi < S < \frac{3\pi}{2}$  1 " " Attractive

$\frac{3\pi}{2} < S < 2\pi$  2 " " Repulsive

What does the scattered wave look like?

$\Psi_{5c}(r) \sim \frac{\sin(kr + S_0)}{r}$ for $S$-wave scattering

$\Rightarrow k \to 0$

But $S_0 = -k\alpha$

$\Rightarrow \Psi_{5c}(r) \sim \sin(k(r - \alpha)) \sim k(1 - \alpha/r)$
$\alpha$ is very important in quantum gases because it determines whether the interactions in the gas are attractive or repulsive.

For Bose-Einstein condensates ($\alpha > 0$):
- Stable BEC
- $\alpha < 0$: unstable to collapse

For Fermions ($\alpha > 0$):
- Weakly-bound bosonic molecules
- $\alpha < 0$: Cooper pairing

Mean-field energy $U = \frac{4\pi \hbar^2 a}{m}$
Photoassociation - best way to measure α

- Absorption of a laser photon tuned to a molecular resonance causes a spontaneous emission

- S.E. leads to loss of atoms by decaying to bound molecules or by heating atoms → signal is loss of atoms.

- Linewidth given by energy of exciting atoms
  \[1 \text{ kHz} \Rightarrow 21 \text{ MHz}\]
  → high precision

- Resonance frequencies of molecular transitions gives new information on long-range part of the excited state potential

\[\text{eg } C_3 \propto 1 \langle s1d1p \rangle^2 \propto \alpha\]

→ Spectral measurement of atomic lifetimes!
  Lifetime precision of \(10^{-4}\) is most precise measurement ever.
1. Photon PA also gives information about G.S. potential

\[ \text{Signal} \propto |<\psi|\psi'>|^2 \]

Free atoms vibration level of excited state

\[ \propto \int \psi_r \phi_v d^3v \text{ overing integral} \]

5/6 goes away new node's in \( \psi_r \)

![Graph showing PA signal and \( \psi_r \)]

2. Photon P.A. measures G.S. levels directly

\[ E_B = h(V_{BB} - V_{PA}) \]

\( \alpha \) depends sensitively on \( E_B \) of (rest bound) state

We have measured \( E_B \) in Li \((s=1)\) to 1 kHz \( \Rightarrow 10^{-7} \) precision.

\( \Rightarrow \alpha \) known to \( \sim 1\% \)
Feshbach Resonance - "Control Knob" for $A$

$s=0 \rightarrow s=1$ Potentials Tweak Difficulty with $B$-Field

$e^g L^3: F=2$

$F=1 \rightarrow \mu = -\mu_B$

Atoms in $F=1, m=1$ States are Spin-Polarized

At Sufficiently High Field $\Rightarrow s=1$

Atoms Coupled by $H_{\text{HF}}$

Molecules Unbound Molecules $\alpha < 0$ $s=0$ Molecular State

Molecules Weakly Bound Molecules $\alpha > 0$

$B_0 \rightarrow B$

Pair of Free Atoms are Resonant with a Molecular Level at $B_0$

$\alpha = \alpha_B \left(1 - \frac{AB}{B-B_0}\right)$ \Rightarrow Controllable Interactions

Adiabatically Ramp Through Resonance \Rightarrow Conversion

of Atomic Gas to a Molecular Gas - M. Bec.