Swimming in Sand, part 3

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Lectures on the mechanics of interaction with granular media including biological & physics experiments, numerical, theoretical and physical robot models
Topics in the lectures

(revised)

• General principles in terrestrial locomotion
• Intro to granular media
• Drag, lift and flow fields during localized intrusion in granular media
• Modeling approaches: DEM & RFT
• Sandfish biological experiments
• Sandfish modeling: robot
• Sandfish modeling: DEM
• Biological tests of model predictions
• RFT modeling of sand-swimming
Swimming in Sand

Papers:
Maladen et al, Science, 2009
Maladen et al, Robotics: Science & Systems conference 2010 (Best paper award)
Maladen et al, J. Royal Society Interface, 2011
Maladen et al, ICRA, 2011

Pdfs and links to movies here:
http://crablab.gatech.edu/pages/publications/index.htm
The sandfish lizard

Sandfish (Scincus scincus)

- Native to Sahara desert
- Adaptations for living in sand: countersunk jaw, fringe toes, smooth scales, flattened sidewalls
- One of ~10 species classified *subarenaceous*: “swims” within sand

Mass ~ 16 grams
Swimming without use of limbs

Opaque markers

Nematode (*C. elegans*) in fluid
Hang Lu, Georgia Tech
Kinematics during steady swimming

Single period sinusoidal wave, traveling head to tail

$y = A \sin \left( \frac{2\pi}{\lambda} (x + v_w t) \right) \quad v_w = \lambda f$

$n=11 \text{ animals} \quad \text{mass}=16.2 \pm 4 \text{ g}$

$R^2 > 0.95 \text{ at all phases in cycle}$
Swimming kinematics

Travelling sinusoidal wave, kinematics independent of $\phi$

$A/\lambda = 0.20 \pm 0.04$ LP

$0.22 \pm 0.06$ CP

$n=11$ animals

mass=16.2 ± 4 g
Swimming speed vs frequency & wave efficiency

\[ \eta = \frac{v_x}{\lambda} = \frac{v_x}{f \lambda} = \frac{v_x}{\lambda} \]

Measures amount of “slip” relative to movement in a tube

- \( \eta = 0.53 \pm 0.12 \)
- \( \eta = 0.49 \pm 0.12 \)

Wave efficiency (\( \eta \approx 0.5 \)) is independent of \( \phi \)

\( n = 11 \) animals
\( \text{mass} = 16.2 \pm 4 \text{ g} \)
Swimming by the sandfish inspired robot

10 cm

Robot on the surface

Submerge robot to a depth of 4 cm in closely packed bed

$\xi=1,$
$A/\lambda=0.2$
$f=1 \text{ Hz}$

$\xi=1,$
$A/\lambda=0.2$
$f=0.25 \text{ Hz}$

Robot sub-surface   Real time

Ryan D. Maladen, Paul B. Umbanhowar, and Daniel L. Goldman, Georgia Tech
Integrating WM with DEM simulation

Particles above the robot rendered transparent

Box dimensions: 108cm x 40cm x 15cm
Number of particles: 3e5
Particle size : 0.6cm
Motors controlled to generate sandfish’s traveling sinusoidal wave *kinematics*.

\[
\beta (i, t) = \tan^{-1} \left[ \frac{2\pi A}{\lambda} \cos \left( \frac{2\pi}{l} x_{i+1} + 2\pi ft \right) \right]
\]

\[
- \tan^{-1} \left[ \frac{2\pi A}{5 \text{ cm}} \cos \left( \frac{2\pi}{P} x_i + 2\pi ft \right) \right]
\]

\[
\sim 10^5, \text{ 3 mm “glass” particles}
\]
Simulate granular medium: Discrete Element Method

Specify particle-particle/particle-intruder interaction rule

(e.g., see book by Rappaport)

Model validation: rod drag

3 mm diameter glass beads
3 cm long SS cylinder

elasticity  
\[ F_n = k \delta^{3/2} - G_n v_n \delta^{1/2} \]

\[ F_s = \mu F_n \]

dissipation

friction

\[ k = 2 \times 10^6 \text{ kg s}^{-2} \text{ m}^{-1/2} \]

\[ G_n = 15 \text{ kg s}^{-1} \text{ m}^{-1/2} \]

\[ \mu_{pp} = 0.1 \]

50:50 mix of 3.0, 3.4 mm “glass spheres”
Simulate granular medium: Discrete Element Method

Specify particle-particle/particle-intruder interaction rule

(e.g., see book by Rappaport)

Anesthetize animal, tilt platform until it slides down, obtain $\mu_{pb}$

An animal-particle friction = 0.27

50:50 mix of 3.0, 3.4 mm “glass spheres”

$k = 2 \times 10^6 \text{ kg s}^{-2} \text{ m}^{-1/2}$
$G_n = 15 \text{ kg s}^{-1} \text{ m}^{-1/2}$
$\mu_{pp} = 0.1$
$\mu_{pb} = 0.27$

Elasticity dissipation:

$$F_n = k \delta^{3/2} - G_n v_n \delta^{1/2}$$

Friction:

$$F_s = \mu F_n$$
Simulated sand-swimming

Particles above rendered transparent

$A/\lambda=0.25$, $\xi=1$, $f=1$ Hz
Trajectories of body markers
Speed vs frequency and $\eta$

Swimming in 3 mm glass particles, in experiment and simulation

$A/\lambda = 0.25, \xi = 1$
Variation of amplitude -> optimal swimming in sand

Hypothesis: animal utilizes swimming kinematics which maximize escape into the sand → a template!

Peak at $A/\lambda = 0.23 \pm 0.01$

Fixed $f=2$ Hz $\xi=1$
Localized fluid

Redder particles $\rightarrow$ higher speed

1 cm

Speed (cm/sec)

max body width

Distance (cm)

Calculate mean particle speed as a function of perpendicular distance from body, along body.
Resistive forces during swimming

\[ \frac{A}{\lambda} = 0.25, \quad \xi = 1 \]
Motor activation (torque) pattern

Torque (N-cm)

RMS of torque (N-cm)

A/λ=0.2
Torque is frequency independent --> Frictional fluid
Minimum mechanical cost of transport

\[ A/\lambda \approx 0.25 \]
Power

At f=2.5 Hz, total power developed in the 15 gram swimmer is ~1 W.

1W/0.015 kg = 60 W/kg

Vertebrate muscle is capable of ~100 W/kg:

--Swoap et al, JEB, 1993 measured 154 W/kg at ~40 C in hind limb of desert iguana

--Carroll & Wainwright, Comp. Bio & Phys, 2006, max of 330 W/kg in epaxial musculature in a bass

so simulation is reasonable in this regard

Top is 5 cm below surface
Power generation and dissipation on the body

Each bar represents $0.8 \times 1.6 \text{ cm}$ cross-sectional area along the body or on the head.

Top is 5 cm below surface

$P = F \cdot \nu$

$P = \tau \cdot \omega$

$\text{Blue} = A / \lambda = 0.2$, $\text{black} = A / \lambda = 0.06$

$f = 4 \text{ Hz}$
Internal actuation generates kinematics

Motor driven

Muscle driven

Can we use the model to predict how the sandfish “turns on” its muscles to move its body?
Trunk musculature in a lizard
Muscle activity recordings during subsurface swimming

Musculature

B

Longissimus dorsi
Iliocostalis

Rectus Abdominus
External oblique

C

26 vertebrae in trunk & ~13 in tail

Recording Technique

Bipolar Hook Electrode

Epaxial Musculature

50 μm diameter stainless steel wire

Implantation sites

Apparatus

Steinmetz, Goldman, In prep, 2011
Swimming Muscle Activation (EMG)

Steinmetz, Goldman, In prep, 2011

Intensity = EMG burst area / EMG duration

Mean Intensity (mV)

n=8

P<0.01

Control: Intensity is recorded when animal is not moving
Speed independence

Steinmetz, Goldman, In prep, 2011

Normalized EMG Intensity of 50% marker at burst 3

Mean segment velocity for half cycle of undulation

Biological support for frictional fluid picture

Segmental Velocity (bl/s)

n=6 animals

Triangle=LP
Circle=CP

P > 0.05

Steinmetz, Goldman, In prep, 2011
Numerical Simulation Predicts an Increase in Motor Torque with Depth
Intensity increases with depth
Activation timing of the wave

Slowed x5

Flexion at 50%

EMG Onset Relative to Flexion

Body Flexion at 50% SVL (Deg)

Time (s)

0 0.24 0.48

50% EMG 70% EMG 90% EMG 110% EMG 50% Flexion (deg.)

Concave Convex Straight
Emergent Activation Pattern with Simple Model

\[ \theta = \frac{\theta_{\text{seg}}}{\theta_{\text{max}}} \]

Motor Torque

Speed of traveling wave of motor torque is faster than speed of mechanical wave

\[ \frac{v_{\text{wave}}}{v_{\text{mechanical}}} = 2 \]

Activation Timing

Graph showing angle vs. cycle percentage with a waveform and markers indicating onset and offset.

Legend:
- 30%
- 50%
- 70%
- 90%

Head and Tail markers with segmented angle representation.

Resting Length marker.

LP Sim.
Timing is similar between experiment and simulation.

\[
\frac{v_{\text{wave}}}{v_{\text{mechanical}}} = 1.5
\]
• **Goal**: gain analytic understanding using tools developed for small organisms swimming in fluids — *Resistive Force Theory*

• **Simplify**: no taper, flat head (in simulation $\eta=0.45$ for flat head, $\eta=0.57$ for tapered head, difference of $\sim20\%$)
Resistive force modeling

(after Gray and Hancock, 1954, Taylor 1952)

- **Assume** square cross-section swimming at constant speed at fixed depth with waveform:

\[ y = A \sin \left( \frac{2\pi}{\lambda} (x + v_{w}t) \right) \]
\[ \tan \theta = \frac{dy}{dx} = \frac{2A\pi}{\lambda} \cos \left( \frac{2\pi}{\lambda} (x + v_{w}t) \right) \]
\[ v_{y} = \frac{dy}{dt} = \frac{2A\pi v_{w}}{\lambda} \cos \left( \frac{2\pi}{\lambda} (x + v_{w}t) \right) \]
\[ \psi = \tan^{-1} \left( \frac{v_{y}}{v_{x}} \right) - \theta. \]

- Non-inertial movement (net thrust=net drag)

- Head drag = flat plate (or for taper use 30% flat plate, Schiffer, 2001)

- Insert force laws to solve for \( \eta = \frac{v_{x}}{v_{w}} \) for given \( A, \lambda \) and obtain \( v_{x} = \eta v_{w} = \eta \lambda f \)

\[ \delta F_{x} = F_{\perp}(\psi) \sin \theta - F_{\parallel}(\psi) \cos \theta \]
\[ \int_{0}^{\lambda} (\frac{F_{\perp}(\psi)}{\text{area}} \sin \theta - \frac{F_{\parallel}(\psi)}{\text{area}} \cos \theta) \sqrt{1 + \tan^{2} \theta} dx + F_{\text{head}} = 0 \]
Resistive force modeling

(after Gray and Hancock, 1954)

In low Re fluids, for long narrow element

\[ F_{\perp} \approx C_{\perp} v \sin \psi \]

\[ F_{\parallel} \approx C_{\parallel} v \cos \psi \]

\[ C_{\perp} : C_{\parallel} \approx 2:1 \]
Granular resistive forces

Obtain empirical drag laws for $F_\perp$ and $F_{\parallel}$

- Drag rod in simulation of 3 mm “glass” particles while varying $\phi$
- Use simulation to resolve forces on all surfaces
- Average in space and time during steady state, divide by area to find surface stresses
Granular resistive forces

Empirical granular resistive force laws

\[ F_\perp = C_S \sin \beta_0 \]
\[ F_\parallel = [C_F \cos \psi + C_L(1 - \sin \psi)] \]
\[ \tan \beta_0 = \gamma \sin \psi \]

Independent of speed

Dashed: Stokes drag on narrow/long ellipsoids in low Re fluid

(Forces shown for LP)
Resistive forces in DEM and RFT

Green=RFT (using steady state drag)
Black=DEM (measured instantaneously)

Square body, no taper, 3 mm particles
Resistive force modeling

(after Gray and Hancock, 1954)

- **Assume** square cross-section swimming at constant speed at fixed depth with waveform:
  
  \[ y = A \sin \frac{2\pi}{\lambda} (x + v_w t) \]
  \[ \tan \theta = \frac{dy}{dx} = \frac{2A\pi}{\lambda} \cos \frac{2\pi}{\lambda} (x + v_w t) \]
  \[ v_y = \frac{dy}{dt} = \frac{2A\pi v_w}{\lambda} \cos \frac{2\pi}{\lambda} (x + v_w t) \]
  \[ \psi = \tan^{-1} \left( \frac{v_y}{v_x} \right) - \theta. \]

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\[
\delta F_x = F_\perp (\psi) \sin \theta - F_\parallel (\psi) \cos \theta
\]

\[
\int_0^L \left( \frac{F_\perp (\psi)}{\text{area}} \sin \theta - \frac{F_\parallel (\psi)}{\text{area}} \cos \theta \right) \sqrt{1 + \tan^2 \theta} \, dx + F_{\text{head}} = 0
\]
RFT solution

Range=from 30% flat plate drag on head to flat plate head
Granular resistive forces

Empirical granular resistive force laws

\[ F_\perp = C_S \sin \beta_0 \]
\[ F_\parallel = [C_F \cos \psi + C_L (1 - \sin \psi)] \]
\[ \tan \beta_0 = \gamma \sin \psi \]

Independent of speed

(Forces shown for LP)

Dashed: Stokes drag on narrow/long ellipsoids in low Re fluid
Wave efficiencies of undulatory swimmers

(see Alexander, Vogel, Gray & Hancock, Lighthill, etc.)

\[ \eta \]

Sarah Steinmetz

RFT captures form of $\eta$ vs $A/\lambda$

Gray=Analytic solutions (head drag neglected)

- Dark blue: RFT solution for un-tapered body
- Light blue: Scaled RFT
- Dashed: Simulation

$\eta$ vs $A/\lambda$ graph showing the relationship between $\eta$ and $A/\lambda$ with different line styles and colors representing the various solutions and simulations.
Competition of effects leads to maximum

\[ \frac{v_x}{fL} = \frac{\eta \lambda f}{fL} = \eta \times \frac{\lambda}{L} \]

Body lengths/cycle =

MEDIUM EFFECT

GEOMETRY EFFECT

Go faster with increasing A

Go slower with increasing A
RFT captures functional form & location of optimum

Sandfish simulation in loose packed 3 mm glass beads

Speed (bl/cycle)

\[ A/\lambda \approx 0.2 \]
RFT force approximation is good at intermediate \( A/\lambda \) but not good instantaneously at small \( A/\lambda \)

\[ A/\lambda = 0.06 \quad \text{and} \quad A/\lambda = 0.22 \]

Green=RFT (using steady state drag)
Black=DEM (measured instantaneously)
Why thrust is over-estimated in RFT

Examine *transient* response in rod drag

10 cm long rod, 4 cm deep
Force buildup occurs over a characteristic length.
Analytic approximations

Gray=Analytic solutions (head drag neglected)
Direction of motion of segments

Blue arrows are velocity of each element
Small $A$

$$y = A \sin \frac{2\pi}{\lambda} (x + v_w t)$$

$$\tan \theta = \frac{dy}{dx} = \frac{2A \pi}{\lambda} \cos \frac{2\pi}{\lambda} (x + v_w t)$$

$$\psi = \tan^{-1} \left( \frac{v_y}{v_x} \right) - \theta.$$
\[ \delta F_x = F_\perp(\psi) \sin \theta - F_\parallel(\psi) \cos \theta \]

\[ P_\perp = \frac{F_\perp}{bd} = C_{S\perp}^1, \]

\[ P_\parallel = \frac{F_\parallel}{bd} = C_{||}^1 (\pi/2 - \psi) \]

\[ \eta = 2\pi^2 \left( \frac{C_{S\perp}^1}{C_{||}^1} - 1 \right) \left( \frac{A}{\lambda} \right)^2 \]
\[ \delta F_x = F_\perp(\psi) \sin \theta - F_\parallel(\psi) \cos \theta \]
\[ \delta F_x = F_{\perp}(\psi) \sin \theta - F_{\parallel}(\psi) \cos \theta \]

\[ P_{\perp} = \frac{F_{\perp}}{b d} = C_{\perp}^3 \psi, \]

\[ P_{\parallel} = \frac{F_{\parallel}}{b d} = C_{S\parallel}^3 + C_{\parallel}^3 \psi, \]

\[ \eta = 1 - \frac{4C_{S\parallel}^3}{4C_{\perp}^3 - C_{\parallel}^3 (A/\lambda)^{-1}} \]
Near exact scaling!

Why is $\eta$ independent of $\phi$?

Force laws for 0.3 mm particles

$F_{\perp}(N)$

$F_{\parallel}(N)$
Localized fluid achieves same state

Initial low $\phi$ state

Initial high $\phi$ state

"wake" achieves similar $\phi$
RFT over-estimates $\eta$

Hypothesis: Scale thrust (but not drag) by 50%
Summary

• Yielding terrestrial substrates---solid and fluid-like response to stress
  – many open locomotion questions

• Volume fraction qualitatively affects drag force: LP $\rightarrow$ fluid-like, CP $\rightarrow$ fracturing solid

• Granular lift forces are sensitive to shape dependent and can be approximated by summing plate elements

• Sandfish lizard swims within granular media ("frictional fluid") of different preparations using similar body undulation kinematics
  – Template for swimming in sand?

• DEM, robot and RFT models capture mechanics of sand-swimming:
  – $v_x$ vs $f$, $\eta \approx 0.5$, optimality condition $A/\lambda = 0.2$

• RFT systematically deviates from DEM model
  – Ding et al, in prep, will show that instantaneous force=average drag force is not a good approximation