Swimming in Sand, part 2

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Lectures on the mechanics of interaction with granular media including biological & physics experiments, numerical, theoretical and physical robot models
Topics in the lectures

(revised)

• General principles in terrestrial locomotion
• Intro to granular media
• Drag, lift and flow fields during localized intrusion in granular media
• Modeling approaches: DEM & RFT
• Sandfish biological experiments
• Sandfish modeling: robot
• Sandfish modeling: DEM
• Biological tests of model predictions
• RFT modeling of sand-swimming
Drag Induced Lift in Granular Media

Yang Ding, Nick Gravish, and Daniel I. Goldman

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Received 31 August 2010; published 13 January 2011)

Laboratory experiments and numerical simulation reveal that a submerged intruder dragged horizontally at a constant velocity within a granular medium experiences a lift force whose sign and magnitude depend on the intruder shape. Comparing the stress on a flat plate at varied inclination angle with the local surface stress on the intruders at regions with the same orientation demonstrates that intruder lift forces are well approximated as the sum of contributions from flat-plate elements. The plate stress is deduced from the force balance on the flowing media near the plate.

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PACS numbers: 45.70. Mg, 47.50. −d, 83.10. Rs

Objects moved through media experience drag forces opposite to the direction of motion and lift forces perpendicular to the direction of motion. The principles that govern how object shape and orientation affect these forces are well understood in fluids like air and water. These principles explain how wings enable flight through air and fins generate thrust in water [1].

Lift and drag forces are also generated by movement within dry granular media—collections of discrete particles that interact through dissipative contact forces. Generation and control of these forces while moving within granular media is biologically relevant to many animal locomotion systems. Following the method of [6], forces on the connecting rod between the intruder and the main body were determined in separate measurements and subtracted from $F_x$ and $F_y$. The grain bed was 75 PD wide by 53 PD deep by 75 PD long. The initial packing state of the grains was prepared by shaking the container moderately in the horizontal direction before each run. The volume fraction was determined through measurements of $\rho$, total grain mass ($M$), and occupied volume ($V$) to be $\frac{M}{\rho V} = 0.62 \pm 0.01$.

The simulation employed the soft-sphere discrete element method (DEM) [10] in which particle-particle and
Features of granular drag

* Insensitive to speed
* Increases with depth
* Insensitive to shape

What about the lift force?

Wieghardt (1975)

Lift in fluids

Air foil
Measure lift force on simple shapes
Experiment

$\phi = 0.62$

$0.32 \pm 0.02$ cm diameter glass particles

rod = 10 cm long

Note: larger particles (10x) than in previous drag experiments
Net forces on intruders

Net force in vertical plane

Positive lift force  Small lift force  Negative lift force

2.5 cm
Velocity field (in co-moving frame)

(in 0.3 mm diameter glas particles)
Discrete Element Method (DEM) simulation

Books:
• Rapaport, *The art of molecular dynamics simulation*, 2004
• Pöschel, *Computational granular dynamics : models and algorithms*, 2005
Flow field and streamlines in co-moving frame

3D simulation of 350,000 3 mm “glass” spheres (cross-section shown). Rod dragged at 10 cm/sec
Net forces on intruders

Net force in vertical plane

Positive lift force

Small lift force

Negative lift force

3 mm glass spheres, \( \phi = 0.62 \)

Red = simulation

Black = experiment
Particle interaction force Model

- Force is contact only, repulsive, non-conservative.
- **Spherical Particles.**
- Deformation treated as small overlap
- Normal force is a function of overlap and velocity
- Friction for tangential direction
Force Model (details)

- \( \vec{F}_{ij} = \vec{F}_{ij}^n + \vec{F}_{ij}^t \)
- \( \vec{F}_{ij}^n = (k_n \delta^\alpha + G_n \dot{\delta} \delta^\beta) \hat{n}_{ij} \)
- \( \alpha = 3/2 \) and \( \beta = 1/2 \), Hertz model*. \( G_n \) is a constant for nearly monodisperse particles.

- \( \vec{F}_{ij}^t = -\min(k_t |\vec{\xi}_{ij}|^{\dagger}, \mu_s |\vec{F}_{ij}^n|) \vec{V}_{ij} \)

- Slip term depends on past history:
  \( \vec{\xi}_{ij}(t) = \int_{t_0}^t \vec{V}_{ij}(t') dt' \)

Computation Process

- Contact force model
- Integration method: Explicit Euler
- Set boundary conditions: hardwall, soft wall, periodic, etc.

Many open source solvers and standard techniques to make run $N \log N$...

Parameters

- Experimental hardness (k) is calculated using Hertz model* for 3mm glass beads using Young & Poisson modulii for glass. Simulated hardness is much smaller † but δ is always <1% radius.
- Restitution is measured by dropping one particle on another at 0.5 m/sec.
- Friction coefficients (µ) are measured by sliding block (with particles glued) on a slope with glued particles.
- Time step is set to be 1/20 collision time* and reducing it by a factor of 2 does not change measured force significantly.

\[
F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}
\]
\[
F_s = \mu F_n
\]

3 mm glass particles:

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (k)</td>
<td>(5.7 \times 10^9) kg s(^{-2}) m(^{-1/2})</td>
<td>(2 \times 10^6) kg s(^{-2}) m(^{-1/2})</td>
</tr>
<tr>
<td>Restitution coefficient</td>
<td>0.92 ± 0.03</td>
<td>0.88</td>
</tr>
<tr>
<td>(G_n)</td>
<td>(15 \times 10^2) kg m(^{-1/2}) s(^{-1})</td>
<td>15 kg m(^{-1/2}) s(^{-1})</td>
</tr>
<tr>
<td>(\mu_{\text{particle–particle}})</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>(\mu_{\text{particle–body}})</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Density</td>
<td>2.47 g cm(^{-3})</td>
<td>2.47 g cm(^{-3})</td>
</tr>
<tr>
<td>Diameter</td>
<td>3.2 ± 0.2 mm</td>
<td>3.0 mm (50%) and 3.4 mm (50%)</td>
</tr>
</tbody>
</table>

Validation: rod drag

3 mm diameter glass beads

3 cm long SS cylinder

Simulation: 50:50 mix of 3.0, 3.4 mm “glass spheres”
Simulation results

Forces

Depth dependence

Stress on the surface
Plate as a differential element

\[ \alpha = 180^\circ + \tau \]
Plate drag

Depth (at plate center) = 3.75 cm

10 cm long (into page), 0.03 cm thick, 2.54 cm wide
Flow field snapshot vs plate angle (in co-moving frame)
Local stresses are well approximated by plate elements.
Drag and lift on a plate

![Graph showing stress vs. angle for drag and lift forces.]

- Stress (N/cm²) vs. angle (°)
- Two lines represent different forces: $f_x$ and $f_z$
- Granular effect indicated
Integrate the force on the plates

\[ F_z = \int f_z(\alpha)(z/d)\,dA \]
Drag and lift on a plate

- granular
- fluid
- $f_x$
- $f_z$

stress (N/cm$^2$)

$\alpha$ (degree)
Coulomb's method

(after Wieghardt, 1975)

1. Find the slip plane which separate flowing region and non-flow region

2. Analyze force balance on the wedge-shaped region with the plate as a boundary

From Nedderman, *Statics and kinematics of granular materials*, 1992
Examine flowing material near plate
Characterize the flow field

Direction of the flow

Area of the upward flowing region

\[ \bar{\psi} = 44^\circ \]
Apply Coulomb's method

\[ \sigma(\alpha) = \frac{W}{lw} \frac{\cos \beta \sin (\tilde{\psi} + \gamma)}{\sin (\alpha - \beta - \tilde{\psi} - \gamma)} \]

\[ \beta(\alpha) = \tan^{-1} \mu_{\text{eff}} \]

\[ \alpha_{vp} = 97^\circ \]

\[ \beta' = \beta(\alpha_{vp}) \]
Model result

σ (N/cm²)

(VP)  (P)

0  0.6

Simulation  Model

σ

180°  0°

Model  Simulation

F (N)

180°  90°  0°
Summary

• Drag force is insensitive to shape, lift force depends on shape and increase with depth

• DEM can quantitatively model granular flows

• Drag induced lift on nonplanar intruders can be computed as the sum of lift forces from independent planar (plate) elements which each experience a lift force resulting from the pushing of material up a slip plane.

• “Wedge” model gives reasonable estimate based on flowing region near plate
Swimming in Sand

Papers:

Maladen et al, Science, 2009
Maladen et al, Robotics: Science & Systems conference 2010 (Best paper award)
Maladen et al, J. Royal Society Interface, 2011
Maladen et al, ICRA, 2011

Pdfs and links to movies here:
http://crablab.gatech.edu/pages/publications/index.htm
The sandfish lizard

Sandfish (*Scincus scincus*)

- Native to Sahara desert
- Adaptations for living in sand: countersunk jaw, fringe toes, smooth scales, flattened sidewalls
- One of ~10 species classified *subarenaceous*: “swims” within sand

mass ~ 16 grams

1 cm
Taken by Sarah Steinmetz at GT micro-CT facility, with Prof. Bob Guldberg,
X-ray imaging to see within sand

High Speed Camera (visible light)
Image Intensifier
Fluidized Bed
X-ray Source
1000 fps camera

Ryan Maladen
Sarah Steinmetz
Experimental apparatus

- X-ray source (80-160 kV)
- Image intensifier
- High speed (1000 fps) camera
- Fluidized bed of granular media (0.3-3 mm glass beads)
- Air flow pulses

- Animal is placed in holding pen
- Air pulses to the fluidized bed sets initial volume fraction $0.58<\phi<0.63$
- Gait is pulled up
- Animal moves onto sand, dives within
- Motion is recorded with high speed visible and x-ray imagers
Probing granular media

Robotic arm

Force Torque Sensor

Granular media

Fluidized bed

Drag Force

v

depth

Robot arm with 6 axis force/torque sensor

Maladen, Ding, Li, Goldman, *Science*, 2009
Gravish, Umbanhowar, Goldman, *PRL*, 2010
Granular media, a “frictional fluid”

Drag forces:

1. increase with depth
2. independent of speed
3. increase with increasing compaction (volume fraction $\phi$)

Drag experiments in 0.3 mm glass beads

Maladen, Ding, Li, Goldman, *Science*, 2009
0.25±0.04 mm diameter glass beads, particle density = 2.5 g/cm³, bed depth=15 cm
0.25±0.04 mm diameter glass beads, particle density = 2.5 g/cm³, bed depth=15 cm
Swimming without use of limbs

Opaque markers

Nematode (C. elegans) in fluid
Hang Lu, Georgia Tech
Side view

10 cm

Slowed 10x
Swimming kinematics (sagittal plane)

- n=11 animals
- mass=16.2 ± 4 g

C: θ (deg.)
- CP: 20
- LP: 30
- P<0.05

D: Depth (cm)
- CP: 2
- LP: 4
- P<0.05
Swimming kinematics (horizontal plane)
Traveling wave, head to tail
Kinematics during steady swimming

\[ y = A \sin \left( \frac{2\pi}{\lambda} (x + v_w t) \right) \quad v_w = \lambda f \]

Single period sinusoidal wave, traveling head to tail

R^2 > 0.95 at all phases in cycle

n=11 animals
mass=16.2 \pm 4 \text{ g}
Swimming kinematics

Travelling sinusoidal wave, kinematics independent of $\phi$

$A/\lambda = 0.20 \pm 0.04$ LP
$0.22 \pm 0.06$ CP

$\phi = 0.58$, LP
$\phi = 0.62$, CP

$n=11$ animals
mass $= 16.2 \pm 4$ g

$L =$ snout-vent length (SVL)

$P > 0.05$
Swimming speed vs frequency & wave efficiency

Wave efficiency ($\eta \sim 0.5$) is independent of $\phi$.

$$\eta = \frac{v_x}{v_w} = \frac{v_x}{f \lambda} = \frac{v_x}{\lambda}$$

Measures amount of "slip" relative to movement in a tube.

- $\phi = 0.58$, LP
- $\phi = 0.62$, CP

$n=11$ animals
Mass = 16.2 $\pm$ 4 g
Wave efficiencies of undulatory swimmers

(see Alexander, Vogel, Gray & Hancock, Lighthill, etc.)

Sarah Steinmetz

\[ \eta \]

$\eta$ vs. Reynolds Number:

- Non-Inertial
  - Terrestrial substrate
  - Water
  - Body Length (cm): 14, 36, 0.1
  - Reynolds Number:
    - $< 1$
    - $< 0.1$
    - $0.1 - 0.005$
    - $0.1 - 8$
    - $0.005 - 36$
    - $36 - 60$
- Inertial
  - Reynolds Number:
    - $\approx 10^3$
    - $\approx 10^5$
    - $\approx 10^9$

Granular media, Rough surface, Agar, Grains, Water

Particle size has little effect on swimming

\[ \eta \]

Glass beads with ±15% polydispersity

\[ A/\lambda \approx 0.2, \text{ independent of particle size too...} \]

...a template? (Full & Koditschek, JEB, 1999)
Sand swimming physical model design

HSR 5980SG
Digital standard servo

7 segment, 6 motor robot

5.87 ± 0.06 mm diameter plastic spheres, particle density = 1 g/cm³

Andrew Masse

Maladen et al, J. Royal Society Interface, 2011
Maladen et al., Int. Journal of Robotics Research (in press)
Maladen et al, Proc. of Robotics Science and Systems (2010); Best Paper Award
Sand swimming robot design

- Outer Lycra sleeve
- Inner latex sleeve

Masts for tracking position

Outer Lycra sleeve
Limbless robots

Applications of these robots
Kuka snake arm
Surgery robot, JHU
SINTEF, Norway
Choset et al.

Gavin Miller
Hirose et al.
Choset et al.

Surgery robot, JHU
Choset et al.
SINTEF, Norway
Applications of granular swimmers

- Search and rescue
- Lunar surface
- Martian sand
- Desert IED detection
- Exploration
- Rubble - earthquake
- Rubble - earthquake
Control of the motors

Angle between adjacent segments modulated using:

Angular approximation of a sinusoidal traveling wave

$$\beta_i = \beta_0 \xi \sin\left(\frac{2\pi \xi i}{6} - 2\pi f t\right)$$

$$\beta(i,t) - \text{motor angle of the } i^{th} \text{ motor at time } t, \ (i=1-6)$$

$$\beta_o - \text{maximum angular amplitude, determines } A/\lambda$$

$$\xi - \text{number of wavelengths along the body (period)}$$

$$f = \text{undulation frequency}$$
Swimming by the sandfish inspired robot

Robot on the surface

Robot sub-surface

Real time

Submerge robot to a depth of 4 cm in closely packed bed

\(\xi = 1,\frac{A}{\lambda} = 0.2, f = 1\ \text{Hz}\)

\(\xi = 1,\frac{A}{\lambda} = 0.2, f = 0.25\ \text{Hz}\)
Robot swimming subsurface: x-ray video

Buried 4 cm deep.

$\xi=1,$
$A/\lambda=0.2$
$f=0.25 \text{ Hz}$
Comparison of robot model and sandfish

Set $A/\lambda = 0.2$, $\xi=1$ (from animal experiment)

- $v_x$ increases linearly with $f$ (like sandfish)

$$\eta = \frac{v_x}{v_w} = \frac{v_x}{f\lambda} = \frac{v_x}{f}$$

- $\eta = 0.33 \pm 0.03$ (unlike sandfish $\eta \approx 0.5$)
Why is the performance different?

Some potential reasons:
Scaling, smoothness, friction, body morphology, GM properties…

Need insight into locomotor-medium interaction at particle level and a tool that we can vary the above
Sand swimming robot simulation

Box dimensions: 108cm x 40cm x 15cm
Number of particles: 3 x 10^5
Particle size: 0.6cm

Maladen et al., J. Roy Soc. Interface 2011
Maladen et al., Int. Journal of Robotics Research
Part I: Simulating and validating media

Discrete Element Method (DEM) simulation

3 parameter collision contact model:

**normal**: elastic & dissipative

+ **tangential**: friction

\[
F_n = k \delta^{3/2} - G_n v_n \delta^{1/2}
\]

\[
F_s = \mu F_n
\]

\( k = 2 \times 10^5 \, \text{kg s}^{-2} \, \text{m}^{-1/2} \)

\( G_n = 5 \, \text{kg s}^{-1} \, \text{m}^{-1/2} \)

\( \mu_{pp} = 0.1 \)

50:50 mix of 5.81, 5.93 mm “plastic” spheres, particle density = 1 g/cm³
DEM simulation has predictive power

\[ a_{\text{peak}} \]

R=2 cm Al monodisp

Fit a(t) profile at this velocity

Blue=experiment
Black=simulation
Parameters

6 mm plastic particles:

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<tr>
<th>Parameters</th>
<th>Experiment</th>
<th>Simulation</th>
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</thead>
<tbody>
<tr>
<td>Hardness (k)</td>
<td>$1.7 \times 10^8 \text{ kg s}^{-2} \text{ m}^{-1/2}$</td>
<td>$2 \times 10^5 \text{ kg s}^{-2} \text{ m}^{-1/2}$</td>
</tr>
<tr>
<td>Restitution coefficient</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td>$G_n$</td>
<td>$1 \times 10^2 \text{ kg m}^{-1/2} \text{ s}^{-1}$</td>
<td>5 kg m$^{-1/2}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\mu_{\text{particle–particle}}$ ($\mu_{\text{pp}}$)</td>
<td>0.073</td>
<td>0.080</td>
</tr>
<tr>
<td>$\mu_{\text{body–particle}}$ ($\mu_{\text{bp}}$)</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Density</td>
<td>$1.03 \pm 0.04 \text{ g cm}^{-3}$</td>
<td>1.06 g cm$^{-3}$</td>
</tr>
<tr>
<td>Diameter</td>
<td>$5.87 \pm 0.06 \text{ mm}$</td>
<td>5.81 mm (50%) and 5.93 mm (50%)</td>
</tr>
<tr>
<td>Granular volume</td>
<td>188 PD $\times$ 62 PD $\times$ 35 PD</td>
<td>188 PD $\times$ 62 PD $\times$ 24 PD</td>
</tr>
</tbody>
</table>

\[
F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}
\]

\[
F_s = \mu F_n
\]
Integrated numerical simulation

Maladen, Ding, Kamor, Umbanhowar, Goldman, in prep, 2010

Multi-body solver + DEM simulation

DEM code computes forces from segment collisions with grains and grain/grain collision

“Motors” are controlled to drive travelling wave

5 cm
Part II: Simulating the robot

Multi-body simulator Working model (WM) 2D

Angular approximation of sinusoidal traveling wave

\[ \beta_i = \beta_0 \xi \sin\left(\frac{2\pi \xi i}{6} - 2\pi ft\right) \]

(like in experiment)

Lycra skin – particle friction estimated experimentally \( \mu_{\text{particle–robot}} : 0.27 \)

Maladen et al., J. Roy Soc. Interface, 2011
Integrating WM with DEM simulation

Particles above the robot rendered transparent

Box dimensions: 108cm x 40cm x 15cm
Number of particles: 3e5
Particle size: 0.6cm
Simulated robot vs. physical robot

Simulated and physical robot swimming speeds agree!

vs. sandfish

\[ \eta = 0.36 \]

\[ \eta = 0.34 \]

\[ \eta = 0.5 \]
Changing smoothness of wave
activate different numbers of motors

\[ \beta_i = \beta_0 \xi \sin\left(\frac{2\pi \xi i}{N} + 2\pi ft\right) \]

Maladen et al., J. Roy Soc. Interface, 2011
Wave efficiency vs # of segments

Resistive force theory prediction

η = 0.5

7 segment, 6 motor robot

Sandfish!
Changing friction

Body-particle friction dominates

7 segment, 6 motor robot

Fixed body particle friction
vary particle-particle friction

Fixed particle-particle friction
vary particle-body friction

Body-particle friction dominates
Motor torque vs. time

![Graph showing motor torque over time with bar chart indicating torque amplitude for different motors.](Image)
Motor torque vs. frequency

Swims in a “frictional fluid”—friction dominates all forces

7 segment, 6 motor robot
Use physical model to test for template

**Hypothesis:** Sandfish kinematics are adapted to rapidly swim within sand $\rightarrow$

sinusoidal wave of $A/\lambda \approx 0.2$ is a template for this behavior

$A/\lambda \approx 0.2$, single period

Test effect of $A/\lambda$ on performance
Vary sand swimming kinematics

Vary $A/\lambda$ for a single period wave

![Graph showing the relationship between $\eta$ and $A/\lambda$. The graph includes data points for robot experiments and robot simulations.](image)
Vary sand swimming kinematics

$A/\lambda = 0.05$
$\lambda$ – High / $\eta$ – Low

$A/\lambda = 0.55$
$\lambda$ – low / $\eta$ – High

Highest performance gait → robot advances most body-lengths per cycle

10 cm
Maximum performance of the physical model

\[ A/\lambda \approx 0.2 \]

Robot simulation

Robot experiment

SINGLE PERIOD WAVE

Sandfish
Competition of effects leads to maximum

Body lengths/cycle = \[ \frac{v_x}{fL} = \frac{\eta \lambda f}{fL} = \eta \times \frac{\lambda}{L} \]

MEDIUM EFFECT

Go faster with increasing A

GEOMETRY EFFECT

Go slower with increasing A
Sandfish kinematics maximize robot speed
Vertical control surface?
Robot with tiltable head and masts for subsurface tracking

Andrew Masse
Active head to control vertical position

- Pitch control of wedge-shaped head (-30° to 30°) using a single servo-motor
- Embedded tilt sensor: accelerometer & gyro
Drag and lift on wedge-like shapes

6mm plastic particles

\[ \alpha = 140^\circ \]

\[ \alpha = 40^\circ \]

\[ \alpha = 90^\circ \]

\[ \alpha = 140^\circ \]

Diagram showing the forces acting on a wedge-like shape with different wedge angles. The force diagram illustrates the lift and drag forces as a function of the wedge angle. The plot shows a decrease in both lift and drag forces as the wedge angle increases from 40° to 140°.
Sensitive dependence of lift force on tilt angle
Head movement?

10 cm

Slowed 10x
END