Lectures on Transition to Turbulence

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Overview

The purpose of these lectures is to present the development in our understanding of the emergence of turbulence in pipe and other shear flows where the laminar-turbulence transition is sub-critical. Scientific study of this problem goes back to Osborne Reynolds in 1883, but only in the last 15 years or so has there been significant progress. Today we can say with confidence that, at least in quasi-one-dimensional flows, the laminar-turbulent transition is actually a non-equilibrium phase transition. We can predict its universality class (directed percolation), and we can perform experiments on real fluids and do essentially exact computations based on the Navier-Stokes equations that confirm the theoretical predictions.

How does it come to be that a problem in fluid mechanics ends up being resolved by statistical mechanics and not partial differential equation theory? This may seem like an interesting example of a transdisciplinary success story. But it is even weirder than that. As I describe the discoveries and analyses that contributed to our current understanding, we will at times invoke concepts, results and techniques from fields such as: neuroscience, population ecology, high-energy hadron scattering and the spread of genes. Underlying everything, of course, is the renormalization group. It's completely mind-boggling that a partial differential equation, solved digitally in a computer or analoguely in a pipe filled with water, behaves in a small range of Reynolds number like water dripping through your coffee percolator!

Outline

Part 1. Why are we interested in transitional turbulence?

- i. Turbulent drag and its reduction
- ii. Propagation of turbulent fronts at high Reynolds number
- iii. Supercritical transitions to turbulence
- iv. Subcritical transitions to turbulence

Part 2. What we think we understand: Puffs

- i. Reynolds experiment – puffs (flashes) and turbulence exist
- ii. Hof experiment (2006) – turbulence doesn't exist
- iii. Hof experiment (2008) – accurate measurement of lifetime of puffs
- iv. Superexponential scaling and extreme value statistics
- v. Hof experiment (2011) – Puff splitting and the laminar-turbulence transition defined
- vi. Analogy with nerve fiber signal propagation
- vii. DNS on single puffs

viii. Activator-inhibitor/Predator-prey dynamics and stochasticity
ix. Numerical simulations of predator-prey model and puff decay and splitting
x. Directed percolation and the universality class of the single puff transition
xi. Experimental evidence for predator-prey dynamics
xii. Experimental evidence for directed percolation
xiii. Puff interactions

Part 3. What we think we understand: Slugs

i. Phenomenology of slugs
ii. Streamwise shear interactions in the predator-prey model
iii. Numerical simulations of extended predator-prey model and slugs phenomenology
iv. Propagation of turbulent fronts at low Re in the extended predator-prey model

Part 4. What we do not understand (yet)

i. Laminar-turbulent transition in 2D – a problem in pattern formation
ii. Connection of statistical mechanics framework with Navier-Stokes equations