Dirty Bosons.

I] General introduction to disorder:

$$H_{pure} + \int dx\, V(x)\,\rho(x) \qquad V(x) \text{ random variable.}$$

$$V(x) = \sum_i \delta(r - r_i)\, V_0$$

r_i random.

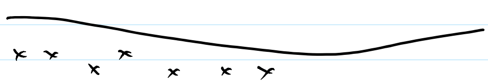
$\hookrightarrow$ 2 number. $\left\{ \begin{array}{ll} V_0 & \text{strength of disorder} \\ n & \text{average density of impurities} \end{array} \right.$

Poissonian disorder.

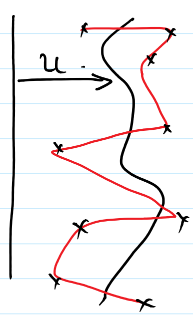
$\langle O \rangle \leftarrow$ quantum average. $H_{pure} + \int V\rho$

$$\overline{\langle O \rangle} = \frac{\int \mathcal{D}V\, P(V)\, \langle O \rangle_{H_{pure} + \int V\rho}}{\int \mathcal{D}V\, P(V)}$$

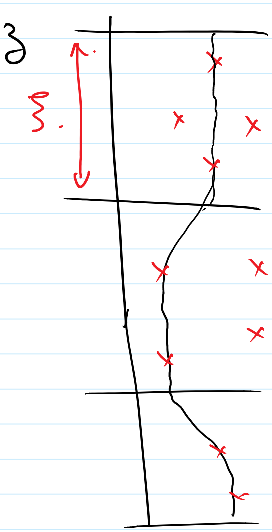
Disorder is weak.



$\rightarrow$ central limit theorem.



$$\frac{c}{2}\int (\nabla u)^2\, dz$$



$$V + \underbrace{\delta V(x,z)}_{\text{Gaussian fluctuations}}$$

$\delta V(r)$.

$$\frac{n_i V_0}{\overline{\delta V(r)\,\delta V(r')}}$$

$$\overline{\delta V(r)\,\delta V(r')} = \mathcal{D}\,\delta(r-r')$$
$$f(r-r').$$

$$\overline{\delta v\,\delta v\,\delta v\,\delta v} = \overline{\delta v\ \delta v}\ \ \overline{\delta v\,\delta v}.$$

$$\overline{\delta V(q)\,\delta V(q')} = \delta_{qq'}\,f(q) \longleftarrow \text{cst}\cdot\ \delta(r-r').$$

$$\mathcal{P}(\delta v) = e^{-\int dq\, f^{-1}(q)\,\delta v^*(q)\,\delta v(q)}$$

$$\overline{\langle \mathcal{O} \rangle} = \frac{1}{\left(\int \mathcal{D}v\, e^{-\delta v^2}\right)} \int \mathcal{D}\delta v\ \ e^{-\int \delta v^2} \left[\frac{\int \mathcal{D}\phi\ \mathcal{O}(\phi)\ e^{-S_{pure}+\int \delta v\,\rho}}{\int \mathcal{D}\phi\ \ e^{-S_{pure}+\int \delta v\,\rho}} \right]$$

$\delta v\ \ \delta v.$

$$\int \mathcal{D}\phi(x,\tau)$$

\# How to get rid of denominator:

→ Supersymetry.

⚠ non interacting particles

$$\int dx\ \ e^{-ax^2} \longrightarrow \frac{1}{a} \to \frac{1}{\det a}. \int d\eta_1 d\eta_1 \ldots\ e^{-\eta^+ A\eta} = \det.$$

Grassman variables $\eta_1\eta_2 = -\eta_2\eta_1.$

$$\left[\int \ldots\right]\frac{1}{[\quad]} \to [\quad]_{\text{commuting}} \left[\int d\eta d\eta \quad\right]_{\text{Grassman}}$$

→ Keldysh technique.

time - dependent actions.

→ Replica method.

$$\frac{1}{\int \mathcal{D}\phi\ \ e^{-S[v]}} = \left[\int \mathcal{D}\phi\ e^{-S[v]}\right]^{n-1}$$

$n-1$

$$\overline{\int \mathcal{D}\phi \; e^{-S[V]}}$$

$$\frac{\int \mathcal{D}\phi \, \mathcal{O}(\phi) \, e^{-S[V]}}{\int \mathcal{D}\phi \; e^{-S[V]}} = \left[\int \mathcal{D}\phi \, \mathcal{O}(\phi) \, e^{-S[V]}\right] \overbrace{\left[\int \mathcal{D}\phi \, e^{-S[V]}\right] \left[\cdots\right]}^{n-1}$$

$$= \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \mathcal{D}\phi_3 \ldots \mathcal{D}\phi_n \; \mathcal{O}(\phi_1) \; e^{-\sum_{i=0}^{n} S[\phi_i, V]}$$

$$S_0[\phi] + \int dx \; V(x) \, \rho_\phi(x)$$

$$\int \mathcal{D}V \; e^{-\int V^2} \int \mathcal{D}\phi_1 \ldots \mathcal{D}\phi_n \; \mathcal{O}(\phi_1) \, e^{-\sum_{i=1}^{n} S_0[\phi_i] - \sum_{i=1}^{n} V \rho(\phi_i)}$$

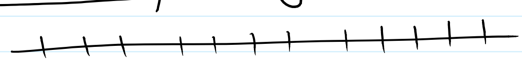
$$\frac{\int \mathcal{D}\phi_1 \ldots \mathcal{D}\phi_n \, \mathcal{O}(\phi_1) \, e^{-\sum_{i=1}^{n} S_0[\phi_i]} \; e^{-\mathcal{D}\left(\sum_{i=1}^{n} \rho(\phi_i)\right)^2}}{\int \mathcal{D}\phi_1 \ldots \mathcal{D}\phi_n \; e^{-\sum_{i=1}^{n} \cdots}} = \overline{\langle \mathcal{O}(\phi) \rangle}$$

II] 1D bosons + disorder:

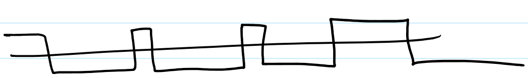
$$\underbrace{H_{Kin} + H_{int}}_{\text{pure 1D bosons}} + \int dx \; V(x) \, \rho(x)$$

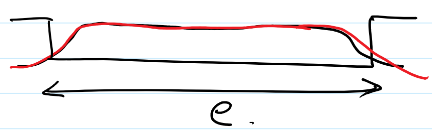
\# for bosons $U = 0$ is pathological.

$$V(x) = \begin{cases} +V_0 \\ -V_0 \end{cases}$$



pure all in $k=0$ state

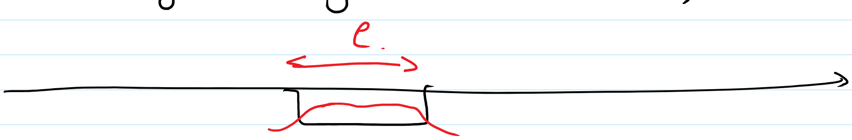




$$P = \left(\frac{1}{2}\right)^{\ell} \sim e^{-\ell \log 2}$$

$$P \approx \frac{\hbar}{\ell} \qquad E_{kin} \simeq \frac{1}{2m}\left(\frac{\hbar}{\ell}\right)^2 \qquad E_{tot} = \frac{1}{2m}\left(\frac{\hbar}{\ell}\right)^2 - V_0$$

ℓ large enough. ($E_{tot} < 0$)



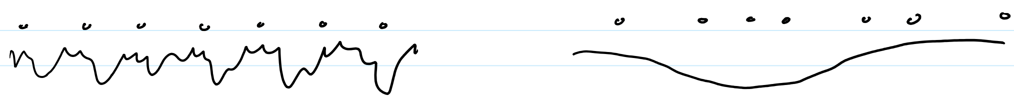
$\rho = \infty$

• int boson $\longrightarrow$ $H = \frac{1}{2\pi}\int dx \left[uK \cdot (\pi \Pi_\phi)^2 + \frac{u}{K} (\nabla\phi)^2 \right]$

$$\int dx \; V(x) \, \rho(x)$$

$$\left[\rho_0 - \frac{1}{\pi} \nabla\phi(x) + \rho_0 \, e^{i(2\pi\rho_0 x - 2\phi(x))} + h.c. \right]$$

TG + H.J Schulz EPL $\underline{3}$ 1287 (87); PRB $\underline{37}$ 325 (88).



$\int V(x) \nabla\phi(x)$ $\leftarrow$ forward scattering.

$$\int (\pi\Pi)^2 (uK) + \frac{u}{K} (\nabla\phi)^2 - V(x) \nabla\phi .$$

$\eta(x) = \int_{-\infty}^{x} dy \; V(y)$. $\qquad \phi(x) \to \tilde{\phi}(x) + \eta(x) \frac{K}{2u}$.

$$\frac{u}{K} \left[\nabla\phi - \frac{K}{2u} V(x) \right]^2 \equiv \frac{u}{K} \left[\nabla\tilde{\phi} \right]^2$$

$$\left[\tilde{\phi}(x), \Pi(x') \right] = i \delta(x - x')$$

$$\int uK (\pi\Pi)^2 + \frac{u}{K} (\nabla\tilde{\phi})^2 + cst$$

$$\left\langle e^{i\theta(x,\tau)} e^{-i\theta(0,0)} \right\rangle_{H+V} = \left\langle e^{i\theta} e^{-i\theta} \right\rangle_{\tilde{H}} \equiv \left(\frac{1}{r} \right)^{\frac{1}{2K}}.$$

$\sigma = \frac{i}{\omega} \langle j ; j \rangle_{ret}$. $\qquad j = \partial_\tau \phi \leftrightarrow \nabla_x \theta$

$$j = \partial_\tau \left[\tilde{\phi} - \eta(x) \frac{K}{2u} \right] = \partial_\tau \tilde{\phi}$$

σ $\underline{\underline{\text{not}}}$ affected by the forward scattering.

$\rho \simeq e^{i 2\pi\rho_0 x - 2\phi(x)} \qquad \left\langle e^{i2\phi(x,\tau)} e^{-i2\phi(0,0)} \right\rangle$

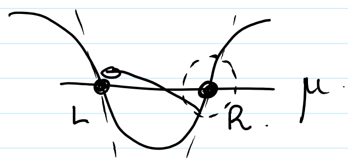
pure $\langle \quad \rangle = \left(\frac{\alpha}{r} \right)^{2K}$.

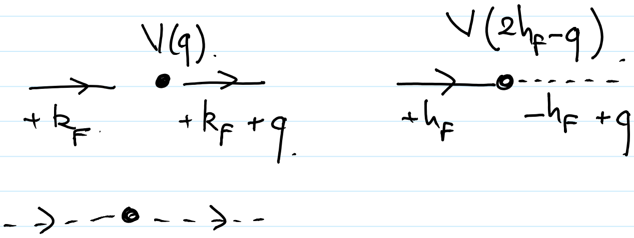
$$\langle \quad \rangle = \left\langle e^{i2\tilde{\phi}(x,\tau)} e^{-i2\tilde{\phi}(0,0)} \right\rangle e^{i\frac{K}{u}\int_{-\infty}^{x} dy \, V(y)} \; e^{-i\frac{K}{u}\int_{-\infty}^{0} dy \, V(y}$$

$$\langle \quad \rangle_{H_0+V} = \left\langle e^{i2\tilde\phi(x,\tau)}\, e^{-i2\tilde\phi(0,0)} \right\rangle e^{\frac{K}{iu}\int_{-\infty}^{x} dy\, V(y)}\, e^{-\frac{K}{iu}\int_{-\infty} dy\, V(y)}$$

$$= \left(\frac{\alpha}{r}\right)^{2K} e^{i\int_0^x dy\, V(y)}$$

$$\overline{\langle \quad \rangle} = \underbrace{\left(\frac{\alpha}{r}\right)^{2K}}_{\text{pure}} e^{-D|x|\cdot(\;)} \qquad e^{-|x|/\xi_F} \qquad \xi_F \simeq \frac{1}{\mathcal{D}}$$



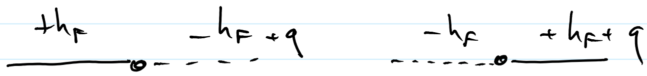


$$i v_F\left[\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L\right]$$
$$+ V(x)\left[\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L\right]$$

backward scattering

$$\int dx \left(V(x)\, e^{i[2\pi\rho_0 x - 2\phi(x)]} + h.c.\right)$$

$$\xi(x) = V(x)\, e^{i 2\pi\rho_0 x} \qquad \overline{\xi(x)\,\xi(x')} = e^{i2\pi\rho_0(x+x')}\,\delta(x-x')$$
$$\overline{\xi(x)\,\xi^*(x')} = e^{i2\pi\rho_0(x-x')}\,\delta(x-x')$$



$$H = \int dx\left[\xi(x)\, e^{-i2\phi(x)} + h.c.\right]$$

$$S = \sum_{j=1}^{n} \frac{1}{2\pi K}\int dx\, d\tau\left[(\partial_\tau \phi_j)^2 + (\partial_x \phi_j)^2\right]$$
$$-\cdot\mathcal{D}\int_0^\beta d\tau \int_0^\beta d\tau' \sum_{j,k}^{n} \int dx \cos\left(2\phi_j(x,\tau) - 2\phi_k(x,\tau')\right)$$

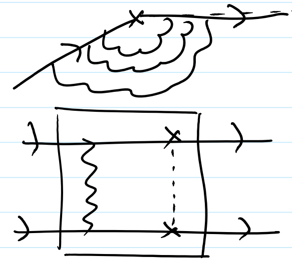
$$\left(\frac{1}{r}\right)^{2K}$$

RG approach: TG + HJ. Schulz

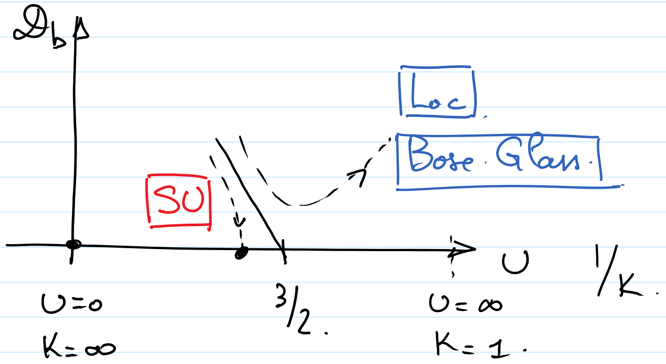
$$\frac{\partial \mathcal{D}_b}{\partial \ell} = (3-2K)\mathcal{D}_b \qquad \mathcal{D} = \mathcal{D}_0\, e^{(3-2K)\ell} \qquad \alpha(\ell) = \alpha_0\, e^{\ell}$$
$$\Lambda(\ell) = \Lambda_0\, e^{-\ell}$$

L^{3-2K}.

$$\begin{cases} \dfrac{dK}{dl} = -\dfrac{K^2}{2}\mathcal{D}_b \\[2mm] \dfrac{d\mathcal{D}_b}{dl} = \mathcal{D}_b(3-2K) \end{cases}$$

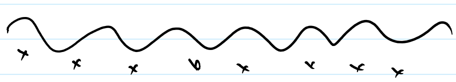


TG + E. Orignac / 0005220



[Extension of the BG phase to d>1 by scaling args : M.P.A. Fisher et al. PRB 40 546 (89).]

SU <--> CDW $K \simeq 1/2$



pinning of a CDW.

Variational method:

TG + P. Le Doussal PRB 53 15206 (2006)

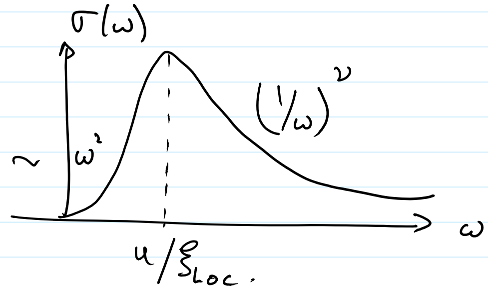
$$S = \frac{1}{K}\int\sum_j (\partial_\tau\phi)_j^2 + (\partial_x\phi)_j^2 \quad - \sum_{j}\int \cos(\phi_j - \phi_k)\, d\tau d\tau' dx$$

$$\left[1 - \frac{1}{2}(\phi_j - \phi_j)^2\right]$$

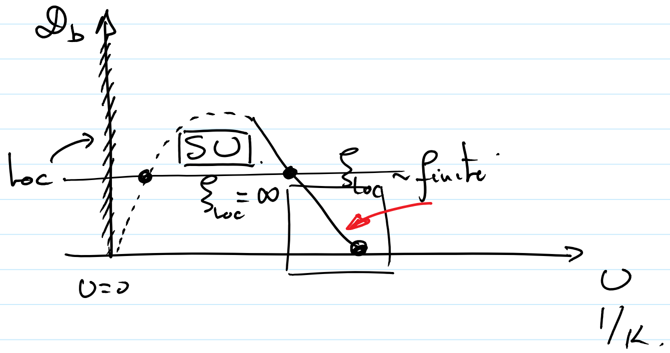
$$S_0 = \sum_{q\omega_n}^{n-1}\sum_{jk}\int \phi^{*j}_{q\omega_n}\, G_0^{-1\,jk}(q,\omega_n)\, \phi^k_{q\omega_n}$$

$$F \text{ "}\leq\text{" } F_0 + \langle S - S_0\rangle_S = F_{\text{trial}}.$$

$$\frac{\partial F_{trial}}{\partial G^{dk}(q,\omega_n)} = 0$$



$$\xi_{Loc} \simeq \left(\frac{1}{\mathcal{D}_b}\right)^{\nu(K)}$$



SU:

$\sigma \equiv \infty \quad (T=0)$

Loc

$\sigma = 0 \quad (T=0)$

$$\frac{\partial \mathcal{D}}{\partial \ell} = \mathcal{D}(3-2K) \qquad \mathcal{D}(\ell) = \mathcal{D}_0\, e^{(3-2K)\ell}$$

$$\mathcal{D}(\ell^*) \simeq \mathcal{O}(1) \qquad \xi^{loc}_{\ell^*} \simeq 1 \qquad \xi^{loc}_{\ell^*} = \xi^{loc}_0\, e^{-\ell^*}$$

$$\mathcal{D}_0\, e^{(3-2K)\ell^*} = 1 \qquad e^{\ell^*} = \left(\frac{1}{\mathcal{D}_0}\right)^{\frac{1}{3-2K}}$$

$$\xi^{loc}_0 \simeq \mathcal{O}(1)\left(\frac{1}{\mathcal{D}_0}\right)^{\frac{1}{3-2K}}$$

$$\langle \psi(x)\psi^\dagger(0)\rangle \sim e^{-|x|/\xi_{Loc}}$$

$$G(L) \sim e^{-L/\xi_{Loc}}$$

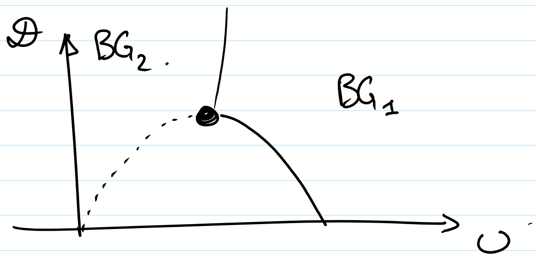
$$\langle \psi(x)\psi^\dagger(0)\rangle \sim \left(\frac{1}{x}\right)^{\frac{1}{2K}} \qquad K = 3/2$$

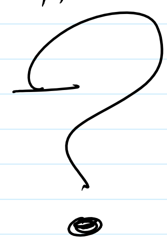
Universal exponent at the SU - BG transition

1/3 [See also V. Krashurnikov et al PRB 53 13091 (96)]

Check universality from RG → 2nd order RG
general argument

Z. Ristivojevic, A. Petkovic, P. Le Doussal, TG.
PRL 109. 026402 (2012).





F. Hrahsheh, T. Vojta PRL 109 265303 (2012)

L. Pollet, N. Prokofev, B. Svistunov.
PRB 87 144203 (2013)

finite T ? $\Longrightarrow$ Many body Localization.

T. Nattermann. et al. PRL 91 056603 (2003)

$\hookrightarrow$ bath

No bath. ? (see Lect. D. Huse)