Exotic Quantum Phase (I) [Senthil]

- Central Ideas in "conventional" condensed matter:
  1. $e^-$ retains its integrity as quasiparticles
  2. Notion of order parameter/spontaneous symmetry breaking
     ▶ These ideas have been challenged by experimental findings such as fractional quantum Hall and high-$T_c$ superconductivity.

- Some conceptual questions:
  ▶ Does every quantum phase have elementary excitation?
  ▶ Can interacting bosons have metallic (i.e. $f$ finite) phase?
  ▶ Is order in phase necessarily described by Landau order parameter?
  ▶ Does $e^-$ have to survive as quasiparticle in phases?
  ▶ Does clean metal always have sharp Fermi surface?
  ▶ Can solid with odd $e^-$ per cell have non-symmetry breaking ground state?

- In general, interacting system has Hamiltonian of form:
  \[ H = T + V \]
  e.g. Hubbard model: \[ H = -t \sum \langle \sigma \alpha \rangle \sum \sigma \uparrow \downarrow \]
  ▶ The two extreme limits ($T \gg V$ or $V \gg T$) is easy.
     e$^-$ wave-like, e$^-$ particle-like
  ▶ Interesting physics occurs in intermediate regime $T \sim V$.

- Consider delocalized limit:
  ▶ Bose fluid: \( V \propto \prod_{i<j} f(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{T} \sum_{i<j} W(\vec{r}_i - \vec{r}_j)} \)
    For ideal gas, \( f = \text{const.} \); \( T \to 0 \) \( f \to 0 \)
    For hard core boson, \( f \to \begin{cases} \text{const.} & \text{as} \ T \to 0 \\ \text{const.} & \text{as} \ T \to \infty \end{cases} \)
    \[ Z = |<b>|^2 \leq 1 \text{ interaction} \]
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\[ \tau \propto e^{-\frac{1}{2} \sum_{ij} u(R_i - R_j)^2} \phi_{\text{slate}}(\{R_i,\sigma_i\}) \]

**Fermi fluid:**

- Special case: Gutzwiller wavefunction
  \[ \phi_{\text{Gutz}} = [\prod_i (1 - (1-g) n_i n_i)] \phi_{\text{slate}} = g^{\sum_i n_i} \phi_{\text{slate}} \]
- Note that in general we can write down a Fermionic wavefunction
  by
  \[ \phi_F = \phi_B \cdot \phi_{\text{slate}} \]
- Thus, we can think \( \phi_B \) as a spinless "slave" of the
  Fermion. This motivates "slave boson" mean-field theory.

**Slave Boson Mean-field Theory**

- Write \( C_{\alpha} = b_i f_{i\alpha} \). Replace microscopic \( H \) by approximate
  \( H_{\text{MF}} \) in which holons & spinons are non-interacting (but which
  \( H_{\text{MF}} \) is determined self-consistently):
  \[ H_{\text{MF}} = H[b] + H[f] \]
  \[ H[b] = \sum_{ij} t_{ij} (b_i^\dagger b_j + h.c.) + V_{\text{int}} [b_i^\dagger b_j] \]
  \[ H[f] = -\sum_{ij} t_{ij}^* (f_{i\alpha}^\dagger f_{j\alpha} + h.c.) \]
- The metallic phase given by condensing \( b_i \):
  \[ C_{\alpha} = \langle b_i \rangle f_{i\alpha} \]
  \[ \rightarrow \langle c_\alpha \rangle = |\langle b_i \rangle|^2 \langle f_{i\alpha}^\dagger f_{i\alpha} \rangle \]
  \[ \rightarrow \text{quasiparticle residue} \ \Xi = |\langle b_i \rangle|^2 \]
- If instead we're interested in correlated superconductor:
  \[ \phi_{\text{sc}} = \phi_B \cdot \phi_{\text{BCS}} \]
  or
  \[ H[f] = \Delta f_i f_{i\alpha} + \Delta^* f_{i\alpha}^\dagger f_{i\dagger}^\dagger \]
- These capture physics of heavy electron metal &
  phenomenology of cuprates.
Next consider the localized limit.

\( H = - \sum t_{ij} b_i^\dagger b_j + h.c. + \frac{1}{2} \sum n_i(n_i - 1) \)

These often end up as Neel state or quantum paramagnet.

- e.g. spin ladder
- e.g. Valence bond solid: bond pattern that spontaneously breaks lattice translation symmetry 
  (can be caused by both magnetic or phonon coupling)

Quantum Spin Liquid

- Mott insulator whose ground state is not smoothly connected to band insulator. [technical defn]
- OR: spin-1/2 quantum system whose ground state does not break any underlying symmetries [rough defn]
- From theoretical considerations, quantum spin liquid can exists in \( d > 1 \).

- Experimentally, it seems that quantum spin liquid do exists in \( d > 1 \) (e.g. organic, kagome, hyper-kagome)

Interesting properties of quantum spin liquid

1. exotic excitations (with fractional spin, non-local interactions described through gauge fields, etc.)
2. ordering not captured by broken symmetry (e.g. "topological order" that capture global property of wavefunction)
3. platform for onset of many unusual phenomena (e.g. superconductivity from spin liquid state? )
4. great experimental setting for violating "conventional" condensed matter.
Possible systems where quantum spin liquid can be found:
- Geometrically frustrated magnets (e.g., Kagome)
- Intermediate correlation regime (e.g., organics)

Exotic Mott Insulator at intermediate correlation:
- Recall that \( H_{\text{eff}} = \sum J_{ij} \vec{S}_i \cdot \vec{S}_j + K \Sigma_{\vec{r}} (P_{234} + P_{345}) \)
- Ring exchange tends to favor spin liquid more than nearest neighbor
- Returning to wavefunction formulation: \( \Psi = \Psi_b \cdot \Psi_{\text{spin}} \)
- Mott insulator + freezing of charge motion
  \( \Rightarrow \) use \( \Psi_b \) of localized Bose solid \( \Psi_b \)
- But then spin correlation would be roughly the same as in ordinary metal.
  \( \leftarrow \) coming from \( \Psi_{\text{spin}} \)
- A competing state would be \( \Psi = \Psi_b \cdot \Psi_{\text{spin}} \), with \( \Psi_{\text{spin}} \) again a localized Bose solid \( \Psi_b \). The resulting state is still a Mott insulator, but may be a spin singlet.