Quantum phases of ultracold atoms in optical lattices and magnetic microtraps

Eugene Demler

Physics Department, Harvard University

Collaborators

Ehud Altman (Harvard)
Ignacio Cirac (Garching)
Luming Duan (CalTech)
Walter Hofstetter (Harvard, ...)
Adilet Imanbekov (Harvard)
Misha Lukin (Harvard)
Ludwig Mathey (Harvard)
Anders Sorensen (Harvard)
Charles Wang (Harvard)
Fei Zhou (Utrecht, ...)
Peter Zoller (Innsbruck)
Bose-Einstein condensation of atomic gases

Anderson et al., Science (1995)

Ultralow density condensed matter system

\[ n \sim 10^{14} \text{ cm}^{-3} \]

\[ T_{\text{dec}} \sim 1 \mu \text{K} \]

Interactions are weak and can be described theoretically from first principles
Strongly interacting bosons in optical lattices

C. Orgel et al., Science (01); M. Greiner et al., Nature (02)

Standing wave laser fields produce a periodic potential for atoms

Bose Hubbard model

\[ \mathcal{H} = -t \sum \langle \psi_i | \psi_j \rangle + U \sum n_i^2 - \mu \sum n_i \]

\( \mu \)
\( \text{MI} \)
\( N = 2 \)
\( \text{MI} \)
\( N = 1 \)

\( N_t \gg U \) Superfluid phase
\( N_t \ll U \) Mott Insulator
Superfluid to Insulator Transition

Greiner et al., Nature (02) following Jackel et al. PRL (98)
Outline

Two component Bose mixtures in optical lattices
Questions: Competition of several ordered phases.
Fractionalized phases in $d>1$ without time reversal breaking.

Spin 1 bosons in optical lattices
Questions: Exotic spin order (nematic).
Pairing in systems with repulsive interactions.

Fermions in optical lattices
Questions: Pairing of fermions with repulsive interactions. High $T_c$ mechanism.

Boson-Fermion mixtures in 1d optical lattices
Questions: Competing orders. Polarons.

Atoms in magnetic microtraps

Fractional quantum Hall states of atoms in optical lattices
Questions: Charges and statistics of quasiparticles
Two component mixtures of bosonic atoms in optical lattices

Example: $^{87}$Rb

$|\uparrow\rangle = |F=1, m_F=-1\rangle$

$|\downarrow\rangle = |F=2, m_F=-2\rangle$

Mandal et al., Nature 2003

Two component Bose-Hubbard model

$\mathcal{H} = -t_\uparrow \sum_{\langle ij \rangle} a_{i\uparrow}^\dagger a_{j\uparrow} - t_\downarrow \sum a_{i\downarrow}^\dagger a_{j\downarrow}$

$+ U_{\uparrow\downarrow} \sum n_{i\uparrow} (n_{i\uparrow} - 1) + U_{\downarrow\downarrow} \sum n_{i\downarrow} (n_{i\downarrow} - 1) + U_{\uparrow\downarrow} \sum n_{i\uparrow} n_{i\downarrow}$

Nature of insulating phases?
Two component bosonic mixtures in optical lattices. Magnetic order in insulating phases

\[ H = - t_1 \sum_{i,j} a_{i\uparrow}^+ a_{j\uparrow} - t_2 \sum_{i,j} a_{i\downarrow}^+ a_{j\downarrow} \]

\[ + U_{1\uparrow} \sum_i n_{i\uparrow} (n_{i\uparrow} - 1) + U_{1\downarrow} \sum_i n_{i\downarrow} (n_{i\downarrow} - 1) + U_{1\uparrow \downarrow} \sum_i n_{i\uparrow} n_{i\downarrow} \]

Insulating phases with \( N = 1 \) atom per site. Average densities \( n_\uparrow = n_\downarrow = 1/2 \).

Easy plane ferromagnet

\[ |\Psi> = \prod (a_{i\uparrow}^+ + e^{i\phi} a_{i\downarrow}^+) |0> \]

Easy axis antiferromagnet

\[ |\Psi> = \prod_{i \in A} a_{i\uparrow}^+ \prod_{i \in B} a_{i\downarrow}^+ |0> \]
Quantum magnetism of bosons in optical lattices

XXZ magnetic systems with tunable interactions
Kuklov, Svistunov, PRL (03);
Duan, Demler, Lukin, PRL (03)

\[ \mathcal{H} = J_2 \sum_{ij} \hat{b}_i^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_i + J_1 \sum_{ij} (\hat{b}_i^\dagger \hat{b}_j^\dagger + \hat{b}_i^\dagger \hat{b}_j) \]

\[ J_2 = \frac{t^2 + t'^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \]

\[ J_1 = -\frac{t\uparrow t\downarrow}{U_{\uparrow\downarrow}} \]

By changing atomic and lattice properties we can manipulate

- sign of interactions
  - ferromagnetic \( U_{\uparrow\downarrow} \gg U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \)
  - antiferromagnetic \( U_{\uparrow\downarrow} \ll U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \)
- anisotropy
  - \[ \left| \frac{J_2}{J_1} \right| > 1 \] easy axis
  - \[ \left| \frac{J_2}{J_1} \right| < 1 \] easy plane
Two component mixture of bosonic atoms in optical lattice

Phase diagram

Altman, Hofstetter, Demler, Lukin

Antiferromagnetic case

\[ U_{\uparrow\uparrow} = U_{\downarrow\downarrow} = 2U_{\uparrow\downarrow} \]

\[ \frac{2zt_{\uparrow\downarrow}}{U_{\uparrow\uparrow}} \]
Second order coherence in the insulating state of bosons.
Hanbury-Brown-Twiss experiment for spinless bosons.
Altman, Demler, Lukin, c-m/0306226

Time of flight imaging

First order coherence, $n(r)$

Second order coherence $G(r,r') = \langle n(r) n(r') \rangle - \langle n(r) \rangle \langle n(r') \rangle$

$G(r,r') = A(t) \sum \delta(r-r'+ \frac{\hbar t}{m} \mathbf{G})$
Probing spin order of bosons

Antiferromagnetic insulating state of spin-$\frac{1}{2}$ bosons

$$|\psi\rangle = \prod_{i \in A} a_i^+ \prod_{j \in B} a_j^+ |0\rangle$$

$$G(r, r') = \langle n(r) n(r') \rangle - \langle n(r) \rangle \langle n(r') \rangle$$
Designing exotic phases

Optical lattice in 2 or 3 dimensions: polarization and frequencies may be different for different directions

Exactly solvable lattice model on a honeycomb lattice by Kitaev

\[ H = J_x \sum_{ij \in x} 6_i^x 6_j^x + J_y \sum_{ij \in y} 6_i^y 6_j^y + J_2 \sum_{ij \in 2} 6_i^2 6_j^2 \]

- Can be created with 3 sets of standing wave light beams
- Has non-trivial topological order, anyons, ...
Spin $F=1$ atoms in optical lattices

Hubbard Hamiltonian: Demler, Zhou, PRL (01)

$$\mathcal{H} = -t \sum_{ijm} a_{im}^+ a_{jm} + U_0 \sum_i n_i^2 + U_2 \sum_i \vec{S}_i^2 - \mu \sum_i n_i$$

Symmetry constraints: $N_i + S_i = \text{even}$ \hspace{1cm} $S_i \leq N_i$

Phase diagram for $d > 1$

Imambekov, Lukin, Demler, PRA (03)

$N = 3$ nematic

$N = 2$ singlet/nem

$N = 1$ nematic

Insulating phases

Nematic phase breaks spin rotational symmetry but not time reversal symmetry. $\langle \vec{S}^2 \rangle = 0$ \hspace{1cm} $\langle S_\alpha S_\beta \rangle \neq 0$

$|N\rangle = \prod_i (n_x a_{ix}^+ + n_y a_{iy}^+ + n_z a_{iz}^+)^N |0\rangle$

Spin singlet phase

$|S\rangle = \prod_i (a_{ix}^{+2} + a_{iy}^{+2} + a_{iz}^{+2})^{N/2} |0\rangle$
Nematic insulating phase for $N=1$

Effective $S=1$ spin model

\[ \chi = -J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - J_2 \sum_{\langle \langle ij \rangle \rangle} (\vec{S}_i \cdot \vec{S}_j)^2 \]

\[ J_1 = \frac{2t^2}{u_0 + u_2} \quad J_2 = \frac{2t^2}{3(u_0 + u_2)} + \frac{4t^2}{3(u_0 - 2u_2)} \]

Two site problem

<table>
<thead>
<tr>
<th>$S_{\text{tot}}$</th>
<th>$\vec{S}_i \cdot \vec{S}_j$</th>
<th>$(\vec{S}_i \cdot \vec{S}_j)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
<td>4</td>
</tr>
</tbody>
</table>

Singlet state is favored when $J_2 > J_1$

Can not have singlets on neighboring bonds

Classical nematic state is a superposition of $S_{\text{tot}} = 0$ and $S_{\text{tot}} = 2$ on each bond
Singlet and nematic insulating phases for $N = 2$

$S_i = 0$ and $S_i = 2$ states are allowed

$U_2 S_i^2$ favors $S_i = 0$

Spin exchange ($\sim t^2/U_0$) allows "scattering"

$S_i = 0$ $S_j = 0 \Rightarrow S_i = 2$ $S_j = 2$ $S_i + S_j = 0$

First order singlet–nematic transition $\frac{2t^2}{U_0 U_2} \sim \frac{1}{2}$

For even filling factors $S-N$ transition

$\frac{2N^2 t^2}{U_0 U_2} \approx 9$
$S=1$ bosons in optical lattice.

Insulating phases in magnetic field

A. Imambekov, M. Lukin, E. Demler, cond-mat/0401526

Even number of bosons per site

\[ \frac{N^2 t^2}{v_0 u_2} \]

\[ \text{SF} \]

\[ \text{CANTED NEMATIC} \]

\[ S=0 \quad S=2 \quad S=4 \quad S=6 \]

\[ H \]

Odd number of bosons per site

\[ \frac{N^2 t^2}{v_0 u_2} \]

\[ \text{SF} \]

\[ \text{CANTED NEMATIC} \]

\[ S=1 \quad S=3 \quad S=5 \quad S=7 \]

\[ H \]
$S=1$ Bosons in optical lattice
Magnetization plateaus

$\langle S_z \rangle$

$N$-even
$N$-odd

Magnetization plateaus in solid state systems

Shiramura et al.
67 : 1549 (1998)
$S=1$ atoms in optical lattice
Stern-Gerlach experiments

Apply magnetic field gradient

$\langle S_z \rangle$ per atom

Compare to formation of spin domains for $S=1$ Na atom in a single optical trap with magnetic field gradient. Stenger et al., Nature 1998
$S = 1$ atoms in optical lattice
Spin decoration experiment

\[ \{3 \} \{ 4 \} \{ 5 \} \{ N = 6 \} \{ 5 \} \{ 4 \} \{ 3 \} \]

$\langle S_z \rangle$ per site

1
Enhancing superfluidity of fermionic atoms using optical lattices
Hofstetter, Cirac, Zoller, Lukin, Dowler, PRL 89, 220707 (02)

Optical lattices enhance interactions and reduce kinetic energy of atoms. Both enhance superfluidity.

Effective description:
Hubbard model, $V < 0$

$$\mathcal{H} = -t \sum_{<i,j>} C_i^+ C_j + U \sum_i N_{ii} N_{ii}$$

Polarized bosonic atoms in optical lattices: superfluid - Mott insulator transition ($t \sim U \approx \text{kHz}$)

Theory: Jaksch et.al. PRL 81, 3108 (98)

Experiment: Greiner et.al. Nature 415, 39 (02)
Orgel et.al. Science 291, 2386 (01)
Enhancing superfluidity of fermionic atoms using optical lattices

\[ \mathcal{H} = -t \sum_{<i,j>} C_i^+ C_j + U \sum \delta n_i \delta n_i^+ \]

\( t \gg |U| \) BCS regime \( T_c \sim t e^{-\frac{7t}{U}} \)

\( t \ll |U| \) Condensation of composite bosons \( T_c \sim \frac{t^2}{U} \)

Highest transition temperature for \( t \sim U \)

\[ T_c^{\text{MAX}} \propto T_e^{\text{free}} \sqrt{n} |a_s| \]

In combination with effective atomic cooling due to turning on the optical lattice

\[ T_{\text{in}}^{\text{MAX}} \propto 0.1 T_{\text{f}}^{\text{free}} \]
Cold atom test of high-\(T_c\) mechanism in cuprates

Cold repulsive fermions in a lattice

Effective description: Hubbard model, \(U > 0\)

\[ \mathcal{H} = -t \sum_{\langle \sigma \rangle} \hat{c}_\sigma^\dagger \hat{c}_\sigma + U \sum_i \hat{n}_i \hat{n}_{i+1} \]

Antiferromagnetic order when lattice is completely filled (1 atom per site)

\(d\)-wave superfluidity at fractional fillings

\[ \begin{array}{c}
\uparrow \downarrow \\
\end{array} - \begin{array}{c}
\uparrow \downarrow \\
\end{array} \]
Second order interference from the BCS superfluid phase

\[ \Delta n(r, -r) | \Psi_{BCS} > = 0 \]

\[ \Delta n(r, r') = n(r) - n(r') \]

\[ \langle \Delta n(r, r')^2 \rangle \]

BCS

BEC

0

1

1.4

1.2

1.0

0.8

0.6

0.4

0.2

0

-1

-0.5

0

0.5

1
Boson-Fermion mixtures in 1d optical lattices

Mathey, Wang, Hofstetter, Lukin, Demler, quant-ph/0611074

Bosonic atoms are in the superfluid phase (high density). They mediate interactions between fermions and provide cooling.

- spin polarized fermions
- bosons

Boson-Fermion Hubbard Hamiltonian

\[ H = -t_b \sum_{ij} \epsilon_i \sigma_i \sigma_j - t_f \sum_{ij} \sigma_i \sigma_j \sigma_i \sigma_j - \mu_e \sum_i n_i - \mu_f \sum_i n_f,i \]

+ \[ U_e \sum_i n_i^2 + U_{ef} \sum_i n_i n_f,i \]

Instabilities

CDW: Periodic arrangement of fermions

Fermion pairing: Fermions bind into p-wave pairs
Interaction of fermions with the Bogoliubov mode of fermions is similar to electron-phonon coupling in solids.

Different order of limits $V_B \gg V_f$

Fermionic polaron:
Fermion plus a screening cloud of bosons

Pairing in BF mixtures is pairing of polarons
Boson–Fermion mixture in 1d optical lattice
Mathey, Wang, Hofstetter, Lukin, Demler
quant-ph/0401151

\( \frac{V_0}{E_R} \)

"slow bosons"

CDW

Paired fermions

"fast bosons"

Phase separation

\( a_{ef} \)
Boson-Fermion mixtures in 2d optical lattices

\[
\frac{U_{\text{eff}}}{E_R} \quad \text{SDW} \quad \text{d-wave SC} \quad \text{p-wave SC}
\]

\[
\text{CDW} \quad \text{s-wave SC}
\]

\[
n_f \quad 1 \quad 0.6
\]

40 K - $^{23}\text{Na}$ mixture

\[
\frac{T_c}{D} \quad \text{SDW}
\]

\[
n_f \quad 0.03 \quad 0.05
\]

\[
\text{Nd:YAG laser, } \lambda = 1.06 \mu m
\]

\[
N_0 = 1
\]

\[
a_{\text{eff}} = 174 a_0
\]

\[
a_{\text{ee}} = -129 a_0
\]

\[
a_{\text{sc}} = 52 a_0
\]

\[
D = 8t_f \quad \text{Bandwidth for fermions}
\]
Quantum Hall effect with ultracold atoms

Rotating condensates

N. Wilkin, J. Gunn, PRL 84, 6 (00)
N. Cooper, N. Wilkin, PRB 60, R16279 (99)
J. Reinders et.al., PRL 89, 120401 (02)

Creating effective magnetic field for neutral atoms in optical lattices

D. Jaksch, P. Zoller, New J. Phys. 5, 56 (03)
E. Mueller, cond-mat/0404306
A. Sorensen, E. Demler, M. Lukin, cond-mat/0405079

Rotating quadrupole potential + time dependent optical lattice

\[ V(x,y,t) = A \sin \omega t \cdot x \cdot y \]
\[ H_0 = -J_x(t) \sum_i a_i^\dagger a_{i,x} - J_y(t) \sum_i a_i^\dagger a_{i,y} \]

\[ J_x \neq 0 \quad J_y \neq 0 \]
Fractional quantum Hall effect with ultracold atoms in optical lattices

Expect fractional quantum Hall phases when

\[ \nu = \frac{\# \text{ atoms}}{\# \text{ fluxes}} = \frac{1}{2m} \]

Exact diagonalization for hard core bosons

We fix \( \nu = \sqrt{2} \), \( \lambda \) - flux density

\[ \Psi = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{1}{\lambda} \sum_{k} |z_k|^2} \]

(a) - overlap with the Laughlin wavefunction

(b) - energy gap to the lowest excited state
BEC in quasi one-dimensional magnetic microtrap

Condensate fragmentation in magnetic microtraps
Lehnardt et al., PRL 89, 040401 (2002)
Fortagh et al., PRA 66, 041604 (2002)
Correlated random potential in magnetic microtraps
D. W. Wang, M. Lukin, E. Demler, PRL 92, 76802 (04)

Random potential due to wire meandering

- Geometrical deformations at wavelengths smaller than ℏ average out
- Wire width fluctuations and long wavelength deformations are not important by the Biot-Savart law
- Lengthscale of the correlated random potential is set by the atom-wire separation, ℏ

J. Estève et al.
physics/0403020
Probing fragmented condensates

Shaking experiments

D - displacement of the confining potential

Dipole mode

Chaos

Self-trapping

? Bose glass?

Power spectra

\[ \log P(w) \]

Dipole

chao...

Self-trapping
Summary

Two component Bose mixtures in optical lattices can be used to design spin $\frac{1}{2}$ quantum systems. They can be used to study ferro and antiferromagnetic fractionalized spin states.

Spin $\frac{1}{2}$ bosons in optical lattices have a rich phase diagram with several insulating and superfluid phases.

Optical lattices are an efficient tool for reaching superfluidity of fermionic atoms. Fermions with repulsive interactions can provide important insights into the origin of high temperature superconductivity (cuprates).

Boson-Fermion mixtures can be used to study formation of polarons and competition between superfluidity (BCS) and charge density wave order in 1d systems.

Fragmented condensates in magnetic microtraps can elucidate the role of disorder for interacting systems.

A combination of oscillating quadrupole potential and time dependent optical lattices can be used to create fractional quantum Hall states of ultracold atoms.