Kicked rotor and Anderson localization

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Classical anisotropic diffusion

Diffusion tensor diagonal in the (1,2,3) axes: $D_{11} \neq D_{22} = D_{33}$

Approximate expressions:

$$D_{11} = \frac{K^2}{4} \left( 1 + \frac{\varepsilon^2}{4} \right)$$

$$D_{22} = D_{33} = \frac{K^2}{16}$$
Schematic view of the experiment

Kicks (amplitude quasi-periodically modulated with time)

\[ H = \frac{p^2}{2} + k \cos \theta \left( 1 + \epsilon \cos \omega_2 t \ \cos \omega_3 t \right) \sum_n \delta(t - nT) \]
Numerical results for the three-color kicked rotor

Momentum distribution

$K$ (kick strength)
Numerical results for the three-color kicked rotor

Momentum distribution

Localized regime

$K$ (kick strength)

Diffusive regime
How to identify unambiguously the transition?

$|\psi(p)|^2$

$\langle p^2(t) \rangle$

3 increasing $K$ values

Time (number of kicks)
How to identify unambiguously the transition?

At criticality, one expects an anomalous diffusion with

$$\langle p^2(t) \rangle \sim t^\gamma$$

with

$$\gamma = \frac{2}{3}$$

3 increasing $K$ values
Phase diagram of the Anderson transition

\[ \alpha = \frac{d \log \langle p^2(t) \rangle}{d \log t} \]  
(from numerics)

\[ \alpha = 0 \quad \text{Localized} \]
\[ \alpha = \frac{2}{3} \quad \text{Critical} \]
\[ \alpha = 1 \quad \text{Diffusive} \]

1000 kicks

\[ H = \frac{p^2}{2} + K \cos \theta \left( 1 + \varepsilon \cos \omega_2 t \cos \omega_3 t \right) \sum_n \delta(t - n) \]
From localization to diffusive regime: experimental results

$\langle p^2(t) \rangle \sim t$

$\langle p^2(t) \rangle \sim t^{2/3}$

Localized (K=4.51)
Critical (K=6.04)
Diffusive (K=7.47)
Fit $A + Bt^{2/3}$
Experimental momentum distributions

momentum (in units of 2 recoil momenta)

localized

critical distribution

diffusive

$|\Psi|^2$
Experimental momentum distributions

- localized (exponential)
- diffusive (Gaussian)

Population vs. momentum (in units of 2 recoil momenta)
From localized to diffusive regime

Experimental results

Numerical results

\[ \frac{1}{\Pi_0^2(t)} \propto \langle p^2(t) \rangle \]

log scale

time (number of kicks, log scale)

localized regime (slope 0)
diffusive regime (slope 1)
critical regime (slope 2/3)
localized regime (slope 0)
Critical regime of the quasi-periodic kicked rotor

Power law fit $<p^2(t)> \sim t^\alpha$

with $\alpha = 0.669 \pm 0.002$

Excellent agreement with the one-parameter scaling law over 7.5 decades
Rescaled dynamics at various times (numerics)

\[ \ln \Lambda(K, t) \]

\[ K \] (kick strength)

Critical point

increasing time
Finite time scaling

$$\Lambda(t) = \frac{\langle p^2(t) \rangle}{t^{2/3}} = \mathcal{F} \left( \frac{\xi(K)}{t^{1/3}} \right)$$

The "displacement" is proportional to $$\xi(K)$$.
Finite time scaling analysis of numerical results

Scaling function \( \Lambda(t) = \frac{1}{\Pi_0^2(t) t^{2/3}} \)

Localization length

Critical point \( K_c = 6.6 \)

Critical exponent \( \xi \sim |K - K_c|^{-\nu} \)


Numerical data up to \( 10^6 \) kicks, latest result: \( \nu = 1.58 \pm 0.02 \)
The critical regime is the horizontal line.

Problem: it requires very long times to accurately measure the position of the transition as well as the critical exponent.

\[
\Lambda(K, t) = \frac{\langle p^2(t) \rangle}{t^{2/3}} \approx \frac{1}{\Pi_0^2(t) t^{2/3}}
\]

\( \Pi_0(t) \): Population in the zero-velocity class

- The critical regime is the horizontal line.
- Problem: it requires very long times to accurately measure the position of the transition as well as the critical exponent.
Experimental measurement of the critical exponent

Scaling function:
\[ \Lambda = \frac{\langle p^2(t) \rangle}{t^{2/3}} = F \left( \frac{\xi(K)}{t^{1/3}} \right) \]

Fit using:
\[ \frac{1}{\xi(K)} = \alpha(K - K_c)^\nu + \beta \]

\( \beta \) : cut-off taking into account experimental imperfections

\( \nu = 1.64 \pm 0.08 \)

Universality of the critical exponent: experimental test

The critical exponent is universal

Weighted average:
\( \nu = 1.63 \pm 0.05 \)

Table of data sets

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \frac{\omega_p}{2\pi} )</th>
<th>( \frac{\omega_s}{2\pi} )</th>
<th>Path in ((K, \varepsilon))</th>
<th>( K_c )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.89</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>4.01 → 8.08</td>
<td>6.67</td>
</tr>
<tr>
<td>B</td>
<td>2.89</td>
<td>( \sqrt{7} )</td>
<td>( \sqrt{17} )</td>
<td>4.01 → 8.08</td>
<td>6.68</td>
</tr>
<tr>
<td>C</td>
<td>2.89</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>3.0435 → 10.0435</td>
<td>5.91</td>
</tr>
<tr>
<td>D</td>
<td>2.89</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>7.50 → 7.50, 0.73</td>
<td>( \varepsilon_c = 0.448 )</td>
</tr>
<tr>
<td>E</td>
<td>2.00</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>3.01 → 5.7, 0.73</td>
<td>4.69</td>
</tr>
<tr>
<td>F</td>
<td>2.31</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>4.01 → 9.08</td>
<td>6.07</td>
</tr>
<tr>
<td>G</td>
<td>2.47</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>4.01 → 9.08</td>
<td>5.61</td>
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<tr>
<td>H</td>
<td>3.46</td>
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<td>( \sqrt{13} )</td>
<td>4.01 → 9.08</td>
<td>6.86</td>
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<tr>
<td>I</td>
<td>3.46</td>
<td>( \sqrt{5} )</td>
<td>( \sqrt{13} )</td>
<td>4.01 → 9.08</td>
<td>7.06</td>
</tr>
</tbody>
</table>


\( \nu = 1.58 \)
Prediction of the self-consistent theory of localization

Very good agreement for the position of the critical point $K_c$

Only fair agreement for the critical "conductance" $\Lambda_c$

M. Lopez et al, NJP 15, 065013 (June 2013) arXiv:1301.1615
Momentum distribution at the critical point

- Very localized initial state $\Rightarrow \langle |\psi(p, t)|^2 \rangle$ is a direct measure of the average intensity Green function $G(0, p; t)$
- Numerical experiment at the critical point:
  - Time invariant shape (neither Gaussian, nor exponential)
Momentum distributions at criticality

Distributions at various times

Distributions at various times rescaled by the critical $t^{1/3}$ law
Experimental measurements in the critical regime

- Characterized by a specific scaling: $p \propto t^{1/3}$

Raw experimental data

Rescaled data

Experimentally measured critical Green function

Analytical prediction (Airy function)

Experimental points (with error bars)

Residual /Airy function

Residual /Gaussian

Residual /Exponential

Rescaled momentum $p/t^{1/3}$

Lemarié et al, PRL 105, 090601 (2010)