Outline

- Introduction and motivation
- Critical states of matter
- Smectics
- Cholesterics
- Columnar phase
- Polymerized membranes
- Elastomers
Critical soft matter
Liquid crystal phases

- Crystal
- Smectic-C
- Smectic-A
- Nematic
- Isotropic

Cholesteric pitch
Smectic layer fluctuations
Vortex lines
Undulation instability on dilation

Strain-induced instability of monodomain smectic A and cholesteric liquid crystals

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A mechanism is proposed for the observed mechanical instability of monodomain smectic A and cholesteric liquid crystals subjected to uniaxial dilative stress. The threshold conditions for the instability are derived, and the possible roles of dislocations in controlling the instability and in producing large plastic distortions are discussed.

\[ \mathcal{H} \approx \frac{B}{2} \left[ \partial_z u + \frac{1}{2} (\nabla u)^2 \right]^2 + \frac{K}{2} (\nabla^2 u)^2 \]

FIG. 1. Periodic undulation of the layers of a dilated smectic A liquid crystal. Regions of maximum dilation are marked S.
Chiral liquid crystals: **cholesterics**

- Color selective Bragg reflection from cholesteric planes
- Temperature tunable pitch $\rightarrow$ wavelength
“Liquid crystals are beautiful and mysterious; I am fond of them for both reasons.” – P.-G. De Gennes
Bio-polymer liquid crystals: DNA

M. Nakata,
N. Clark, et al.
**Nematic Elastomer**

cross-linked polymer liquid crystal PLC

Terentjev
Finkelmann
Ratna

polymer

$n_i n_j$

nematic liquid-crystal

$Q_{ij}$

“We Solid” Liquid-Crystal

exhibits most conventional liquid-crystal phases (I, N, Sm-A, Sm-C, ...)

$\mathbf{u}_{ij}$
Nematic Elastomer

“Solid” Liquid-Crystal
Thermal response and stress-strain relation

Properties:
- spontaneous distortion (~ 400%) at $T_{IN}$, thermoelastic
- "soft" elasticity
- giant electrostriction

Applications:
- plastic displays
- switches
- actuators
- artificial muscle

Terentjev, et al
Nematic elastomer as heat engine

- monodomain nematic LCE
- 5cm x 5mm x 0.3mm
- lifts 30g wt. on heating, lowers it on cooling
- large strain (>400%)

\[ \eta \approx 10^5 \text{ Pa} \]

H. Finkelmann, Shahinpoor, et al
Visualization of soft deformation

a "liquid" solid $\iff$ a "solid"

liquid crystal

\[ \lambda_{zz} = \left( \frac{\varepsilon_{ii}}{\varepsilon_{\perp}} \right)^{\frac{1}{2}} \]

\[ \lambda_{xx} = \left( \frac{\varepsilon_{\perp}}{\varepsilon_{ii}} \right)^{\frac{1}{2}} \]

Warner, Terentjev ’90
Olmsted ‘94
**Elastic theory of NE**

- Construct rotationally invariant elastic theory of deformations about $\mathbf{u}_0$
- Study fluctuations and heterogeneities about $\mathbf{u}_0$

Must incorporate underlying rotational invariance of the nematic state

Some distortions cost no energy: **“soft” uniaxial solid**

$$f[\tilde{R}(\mathbf{x})] = f[O_T \tilde{R}(O_R \mathbf{x})]$$

$u' \approx \frac{(r-1)}{2\sqrt{r}} \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix} = (A_0^T)^{-1} \delta u A_0^{-1}$

- Vanishing energy cost for:
  $$\delta \mathbf{u} = \mathbf{O} \cdot \mathbf{u}_0 \cdot \mathbf{O}^T - \mathbf{u}_0$$

- Harmonic elasticity about nematic state:
  $$\mathbf{\varepsilon} = \mathbf{u} - \mathbf{u}_0$$
  $$\mathbf{H}_{\text{NE}}^0 = \mu_{zi} \varepsilon_{zi}^2 + B_z \varepsilon_{zz}^2 + \mu_\perp \varepsilon_{ij}^2 + \lambda \varepsilon_{ii}^2 + \lambda_{zi} \varepsilon_{zz} \varepsilon_{ii}$$

  0, required by rotational invariance

- Nonlinear elasticity about nematic state:
  $$\mathbf{H}_{\text{NE}} = B_z w_{zz}^2 + \mu_\perp w_{ij}^2 + \lambda w_{ii}^2 + \lambda_{zi} w_{zz} w_{ii}$$

  $$w_{zz} = \partial_z u_z + \frac{1}{2}(\nabla u_z)^2$$
  $$w_{ij} = \frac{1}{2}(\partial_i u_j) - \partial_i u_z \partial_j u_z$$
**Fluctuations and heterogeneity**

- Thermal fluctuations: \( \mathcal{Z} = \text{Trace}_u [e^{-\beta \mathcal{H}[u]}] \)

- Heterogeneity leads to strong qualitative effects of thermal fluctuations and network heterogeneity.

**Elastic “softness”**

\[ \mathcal{H}_{NE}^{\text{real}} = \mathcal{H}_{NE}[u] - u \cdot \sigma(r) - (\hat{n} \cdot \vec{g}(r))^2 \]

*encodes heterogeneity*
**Predictions**

- **Universal** elasticity: \[ \langle |\delta u(q)|^2 \rangle \sim q_{\perp}^{4+\eta} \text{, for } r_{\perp} > \xi_{\perp} \sim K^2 / \Delta \]

- **Non-Hookean elasticity:** \[ \sigma_{zz} \sim (u_{zz})^\delta \text{, } \delta > 1 \]
  (cf. non-Fermi liquid)

- **Length-scale dependent elastic moduli:**
  \[ K_{\text{eff}}(L) \sim L^\eta \text{, } \mu_{\text{eff}}(L) \sim L^{-\eta \mu} \text{, } B_{\text{eff}}(L) \sim B_0 \]

- **Macroscopically incompressible:** \[ \kappa_{\text{eff}} \sim \mu_{\text{eff}}(L) / B_{\text{eff}}(L) \to 0 \]

- **Universal Poisson ratios:**
  \[ u_{xx} > 0 \implies \begin{cases} u_{yy} = \frac{5}{7} u_{xx} \\ u_{zz} = -\frac{12}{7} u_{xx} \end{cases} \]
  \[ u_{zz} > 0 \implies u_{xx} = u_{yy} = -\frac{1}{2} u_{zz} \]