Optimal information storage in noisy synapses under resource constraints

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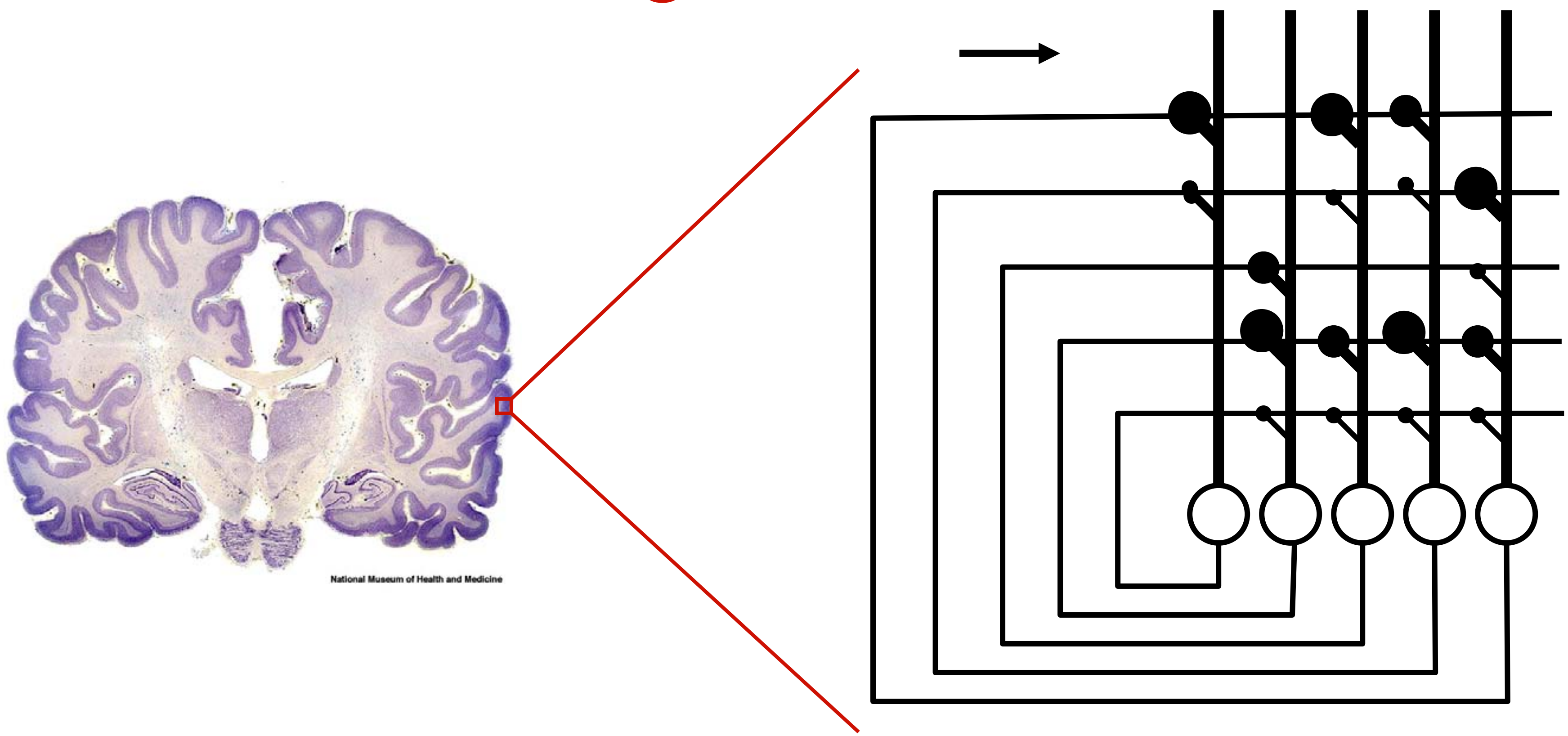
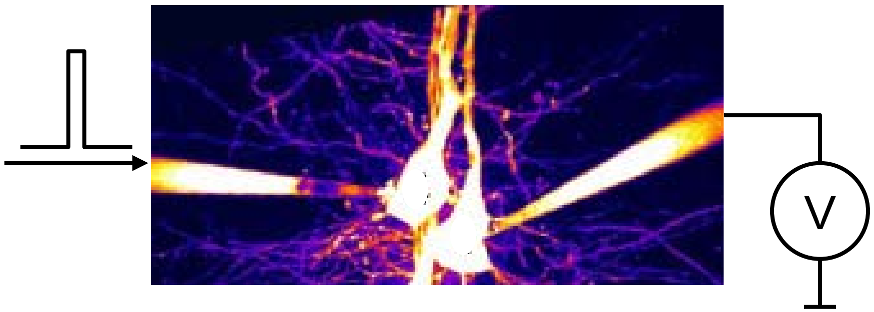
Modifications of synaptic connections store long-term memories



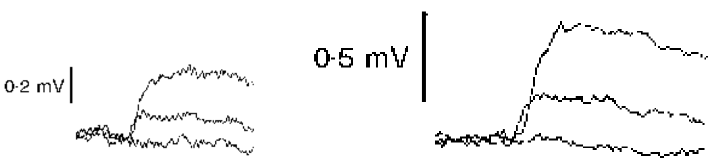
Chklovskii et al. (2004)

Can we understand properties of synapses by optimizing information storage in neuronal networks?

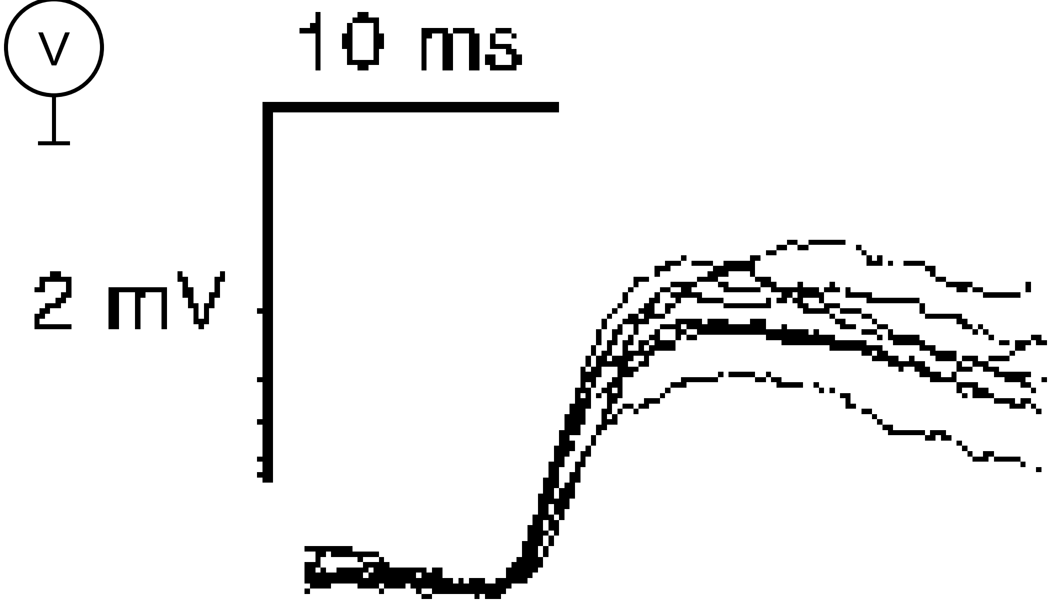
Synaptic properties (experiments)



1. Synaptic connections are sparse (~10%)



Hardingham & Larkman (1998)

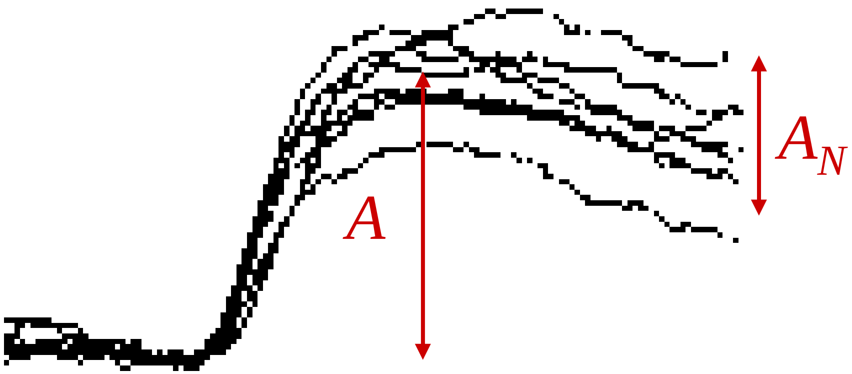


3. Some synapses are strong

2. Most synapses are weak & noisy

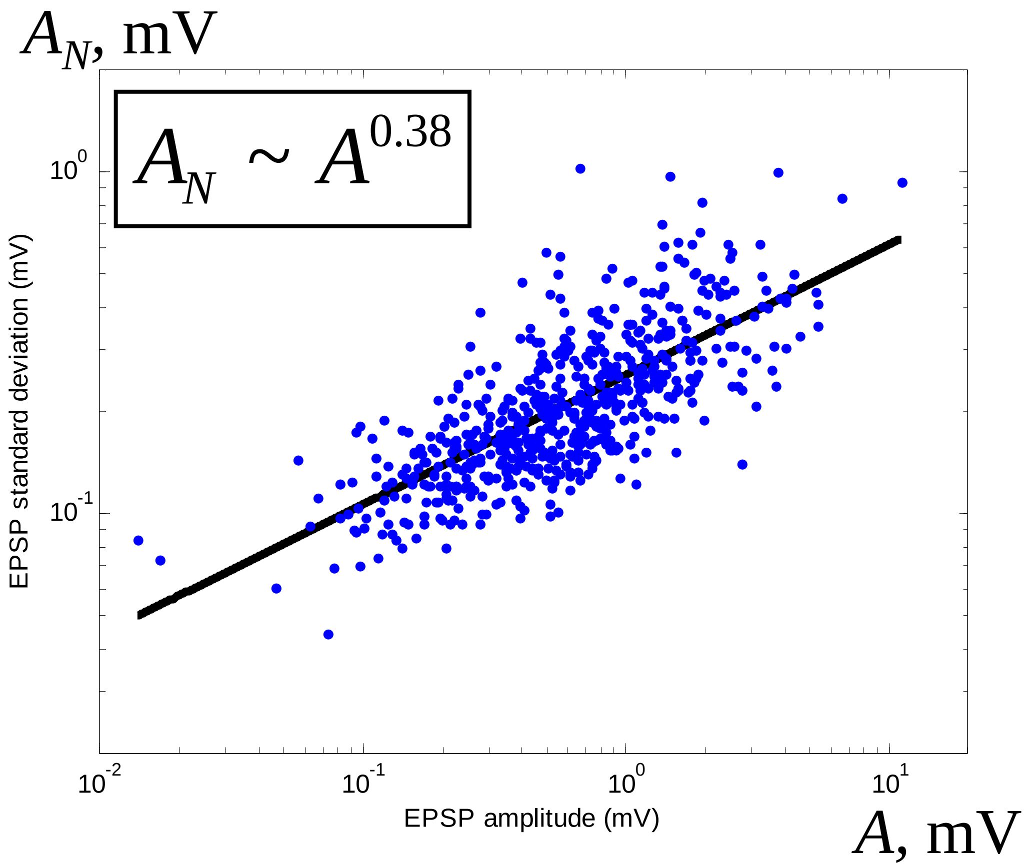
Can these properties follow from optimization?

Stronger synapses have greater SNR



A - mean EPSP amplitude (synaptic weight)

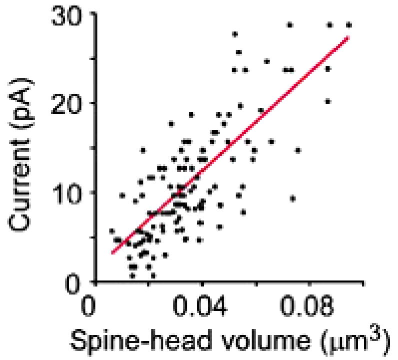
A_N – standard deviation of EPSP amplitude (synaptic noise)



Use stronger synapses for information storage?

But stronger synapses are costlier

Cost = Synaptic volume



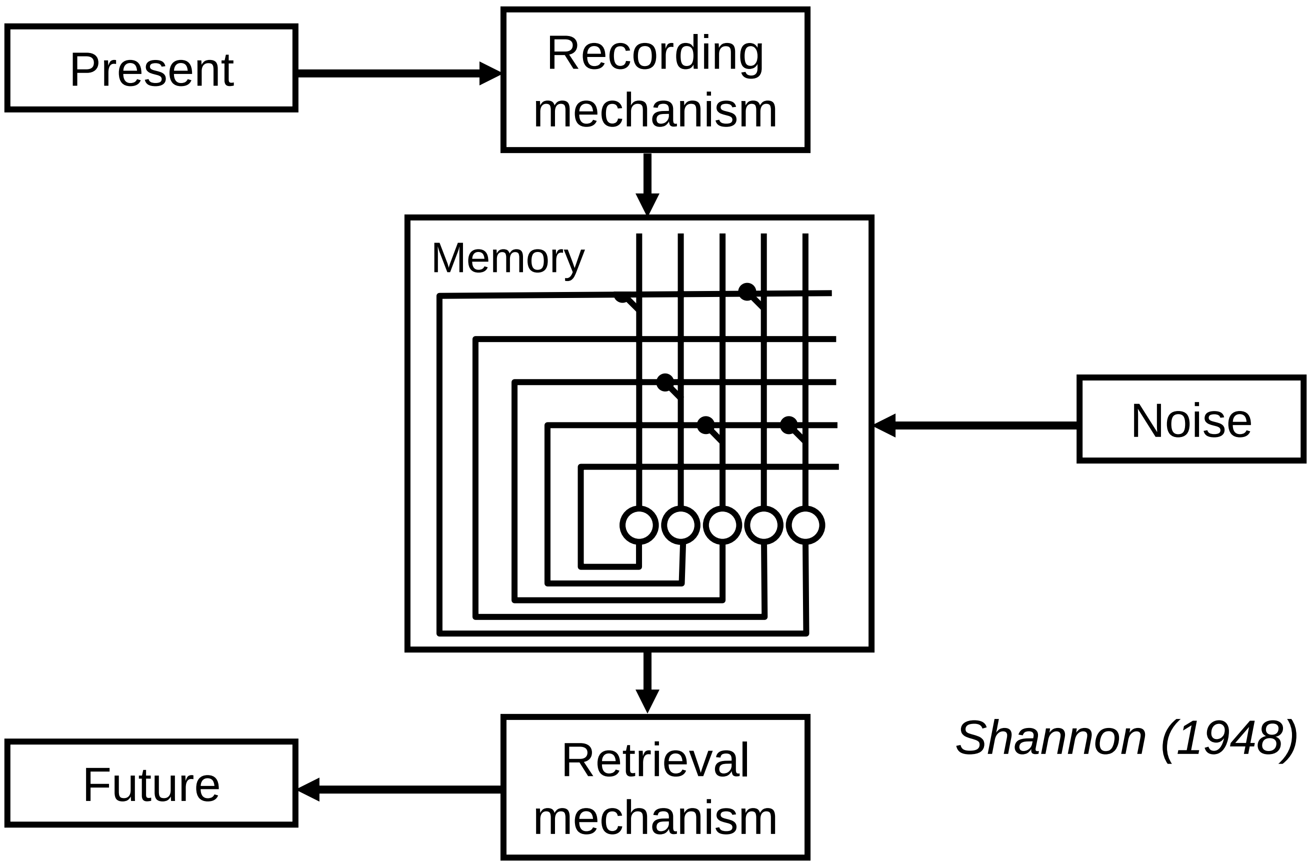
Matsuzaki M., Ellis-Davies G.R.C, Nemoto T., Miyashita Y., Iino M. & Kasai H. (2001)

Volume $$\frac{V}{V_N} = \left(\frac{A}{A_N}\right)^{\alpha}$$ SNR

V_N –volume of a synapse with unitary SNR

We balance competing requirements for greater information capacity and less cost

Memory as a communication channel from the present to the future



Shannon (1948)

We maximize "physical" information storage capacity per volume in the presence of noise

Information storage capacity of a Gaussian channel (α = 2)

Storage capacity per synapse: $I_{Synapse} = \frac{1}{2}\ln\left(1+\left\langle\frac{A^2}{A_N^2}\right\rangle\right)$

$\frac{V}{V_N} = \left(\frac{A}{A_N}\right)^2$

$\Bigg\} \quad I_{Synapse} = \frac{1}{2}\ln\left(1+\frac{\langle V\rangle}{V_N}\right)$

Storage capacity per unit volume: $I_{Volume} = I_{Synapse}/(\langle V\rangle + V_0) \quad = \frac{1}{2\left(\langle V\rangle + V_0\right)}\ln\left(1+\frac{\langle V\rangle}{V_N}\right)$

A – synaptic weight

A_N – noise amplitude

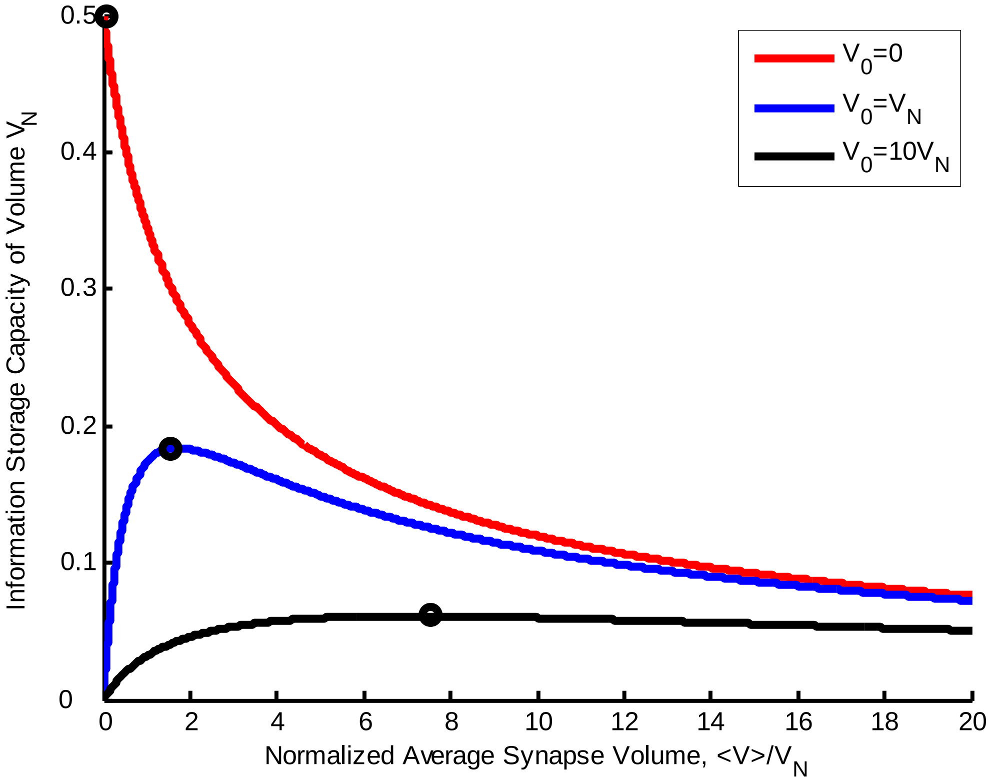
V – synapse volume

V_N – volume of synapse with unitary SNR

V_0 – accessory (wire) volume per synapse

Storage capacity per unit volume

$$I_{Volume} = \frac{1}{2\left(\langle V \rangle + V_0\right)} \ln\left(1 + \frac{\langle V \rangle}{V_N}\right)$$



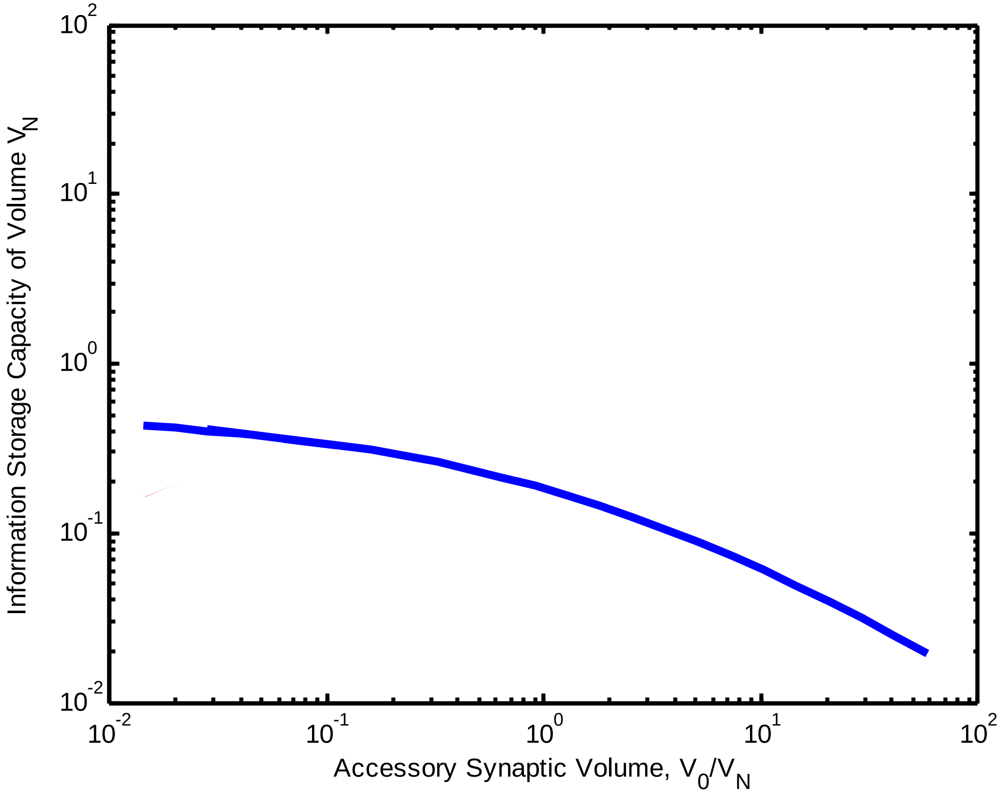
$<V>$ – average synapse volume

V_N – volume of synapse with unitary SNR

V_0 – accessory (wire) volume per synapse

Small synapses maximize information storage capacity provided accessory volume is not big

Information storage capacity as a function of accessory volume

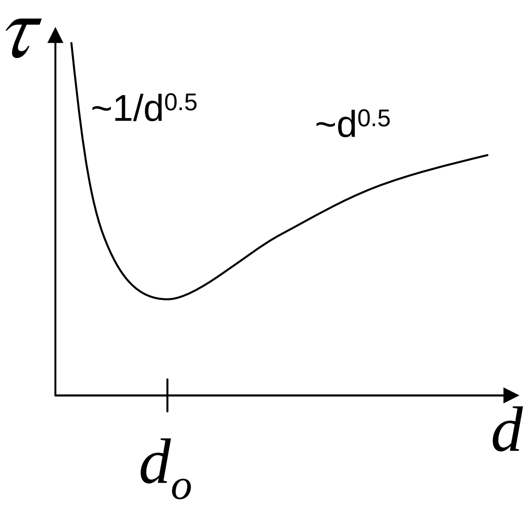


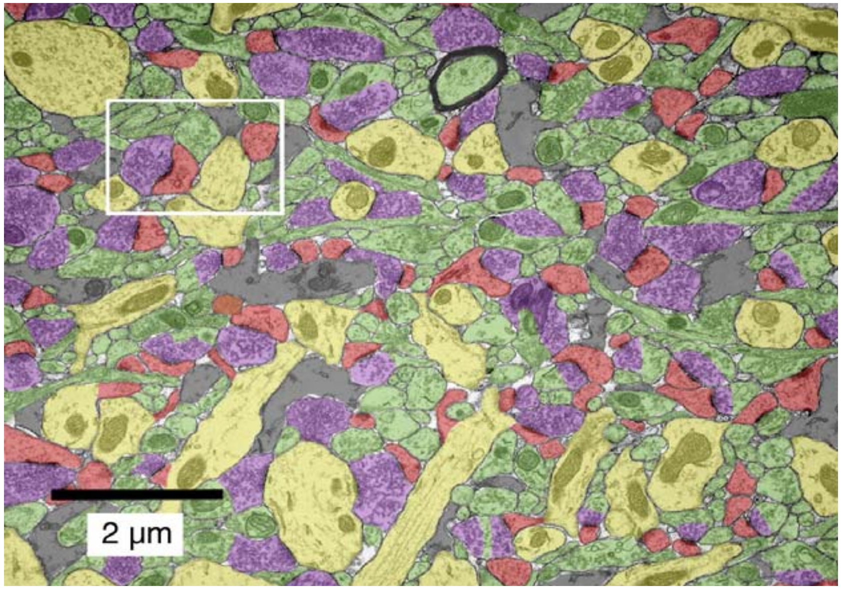
Small accessory volume maximizes information storage capacity

Why cannot wire volume be infinitesimally small?

Conduction delay in very thin (and very thick) wires is prohibitively long:

$$\tau = R/s = R/\beta d^{0.5}$$

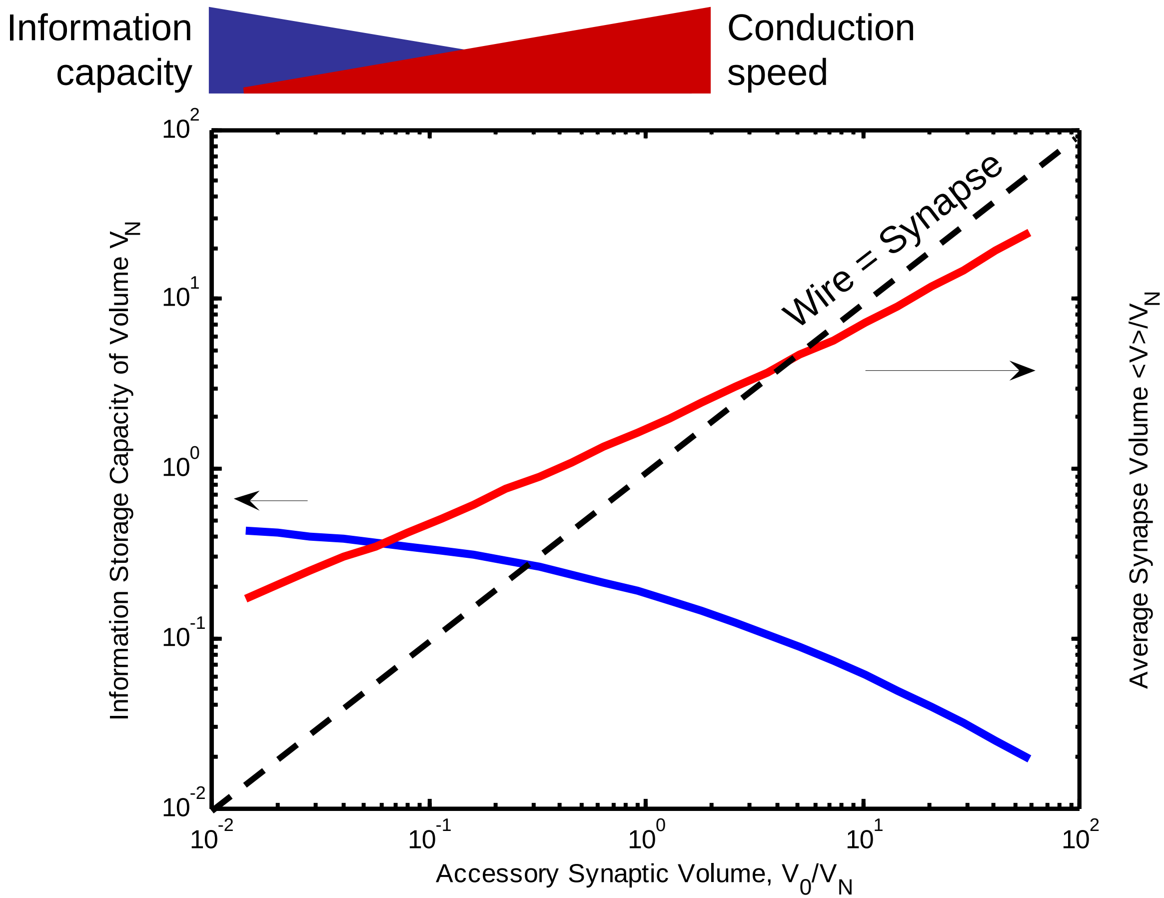




Wires and synapses occupy comparable volume

Chklovskii, Schikorski & Stevens (2002)

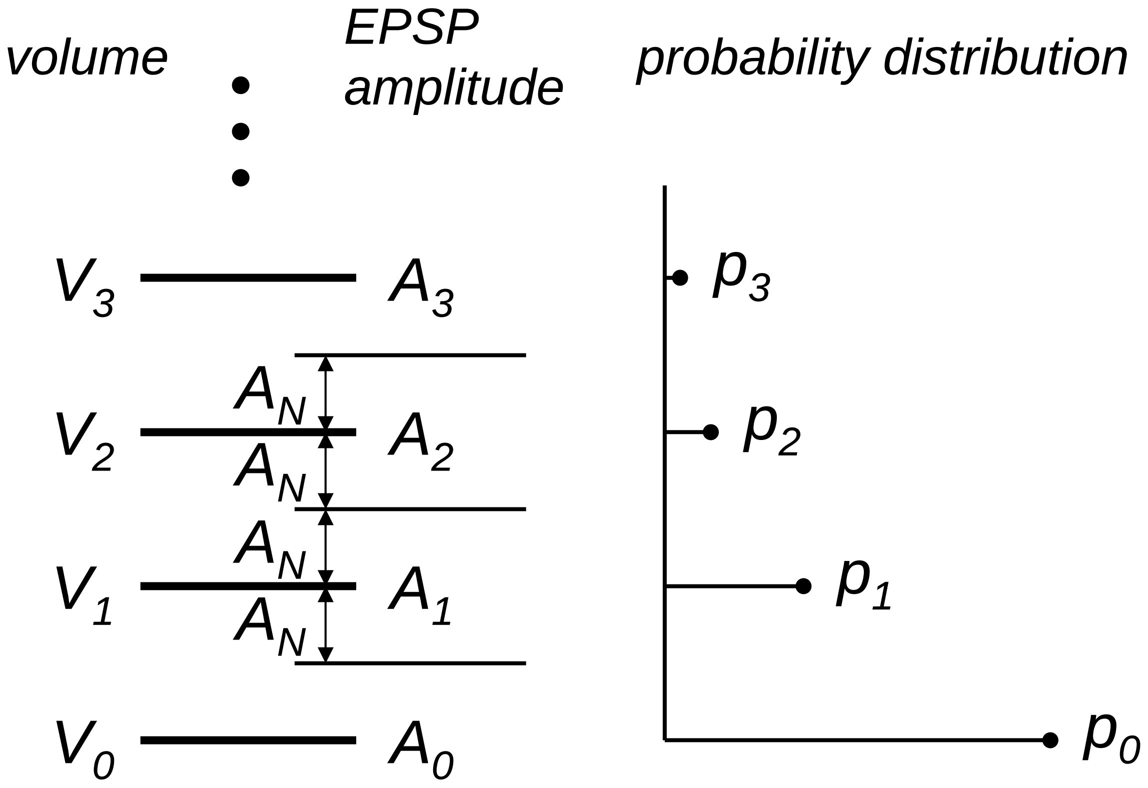
Maximum information storage capacity without compromising time delays



Small and, hence, noisy synapses maximize information storage capacity

What is the optimal distribution of synaptic weights?

Approximate treatment of noise: discrete synaptic states



Optimal distribution over discrete states: Boltzmann exponent

Maximize $I_{synapse} = -\sum_i p_i \ln p_i$ provided $\bar{V} = \sum_i p_i V_i$

$$I_{synapse} = -\sum_i p_i \ln p_i - \beta\left(\sum_i p_i V_i - \bar{V}\right) - \lambda\left(\sum_i p_i - 1\right)$$

$$p_i = \frac{1}{Z}\exp(-\beta V_i)$$

$I_{volume} = \dfrac{I_{synapse}}{\bar{V}}$ is maximized when Z=1

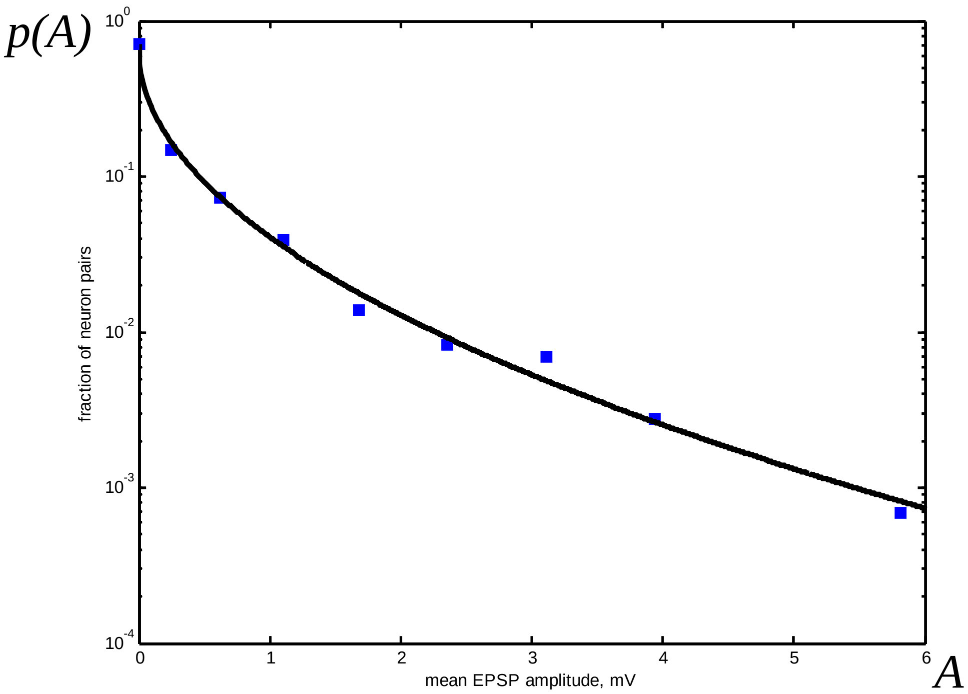
$$p_i = \exp(-\beta V_i) \qquad \sum_i \exp(-\beta V_i) = 1$$

Balasubramanian, Kimber & Berry (2001)

de Polavieja (2002,2004)

Distribution of synaptic weights is a stretched exponential

$$\left.\begin{array}{l} p_i = \exp(-\beta V_i) \\ \dfrac{V}{V_N} = \left(\dfrac{A}{A_N}\right)^{\alpha} \end{array}\right\} \Rightarrow \quad p_i = \exp\left(-\beta\left(V_0 + V_N\left(\frac{A_i}{A_N}\right)^{\alpha}\right)\right)$$



■ Experimental data (thousands of neuronal pairs)

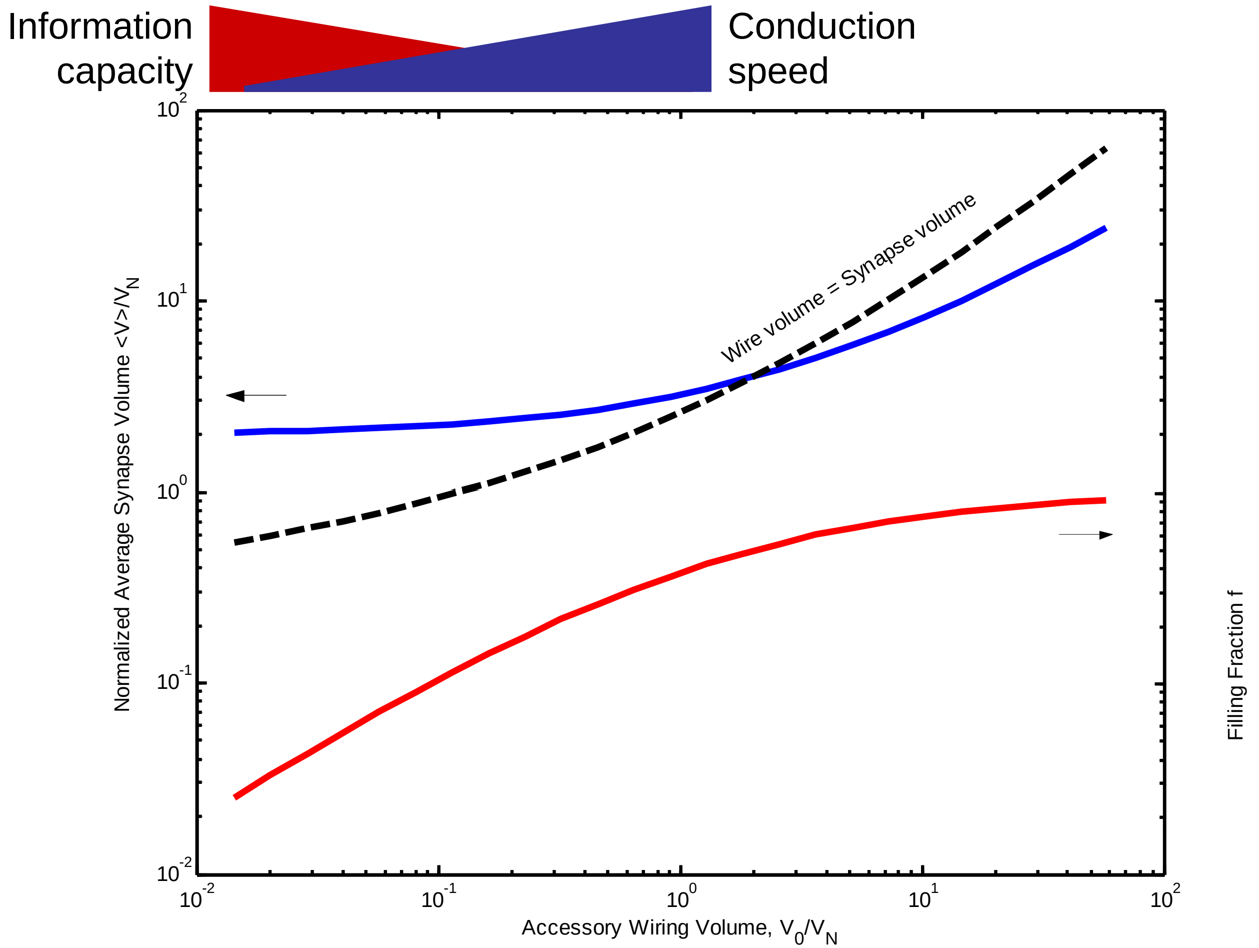
— Theoretical fit:

$p = \exp(-A^{0.49})$

or using $A_N \sim A^{0.38}$

$p = \exp(-(A/A_N)^{0.79})$

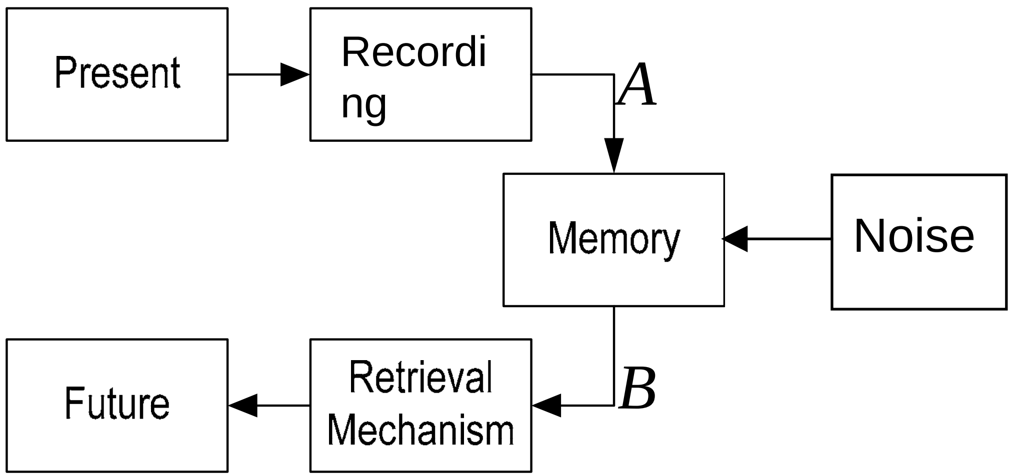
Results for equidistant synaptic states



Sparse connections with typically small but occasionally big weights maximize information storage capacity

Can we solve the optimization problem without making restrictive assumptions?

Reverse problem: Given noise model and signal distribution, for what cost function is this distribution optimal?



Cost function is given by the KL divergence up to two arbitrary constants $v > 0$ and v_0

$$V(A) = v \int p(B|A) \ln \frac{p(B|A)}{p(B)} dB + v_0$$

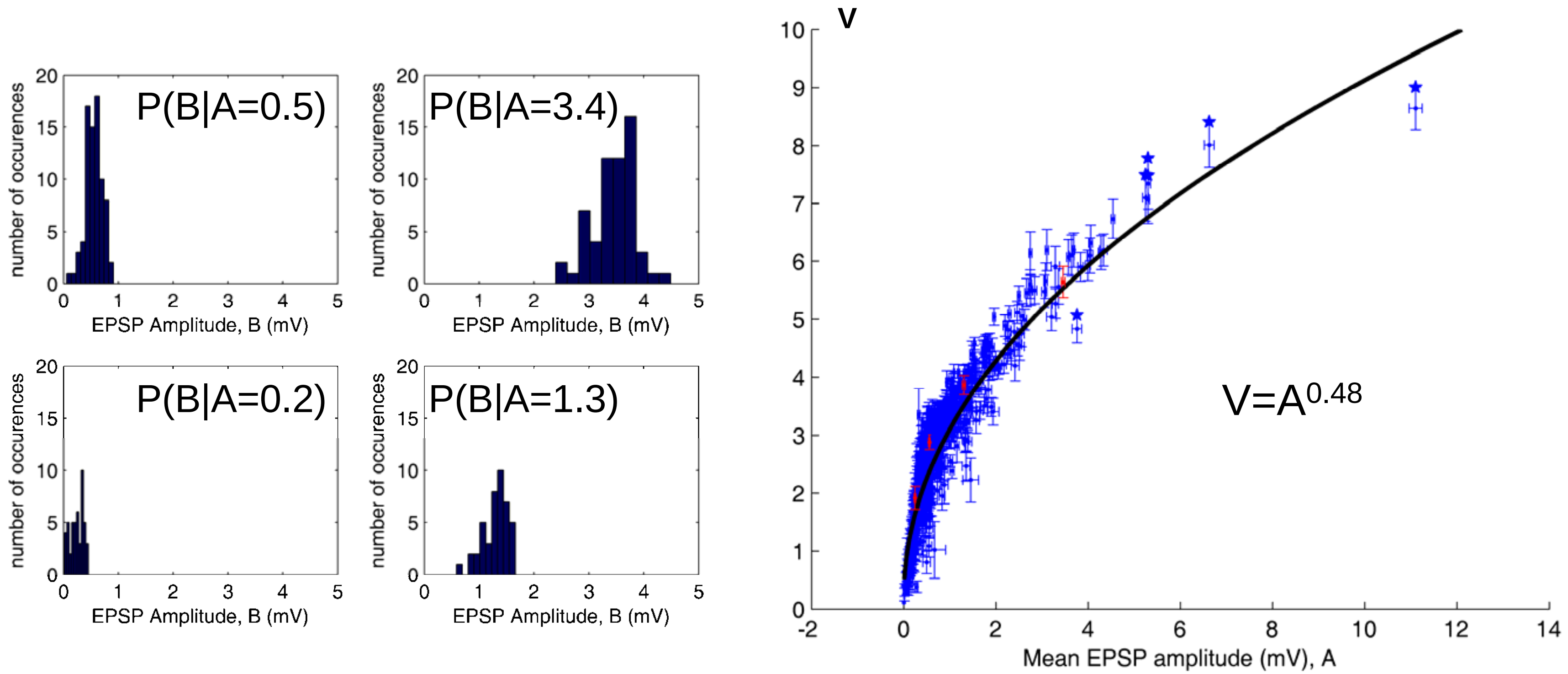
Gastpar et al. (2003)

The Gaussian channel and the discrete-states model are special cases of this relationship

$$V(A) \sim \int p(B\,|\,A) \ln \frac{p(B\,|\,A)}{p(B)}\, dB \sim \int B^2 \exp\left(-(B-A)^2\right) dB \sim A^2$$

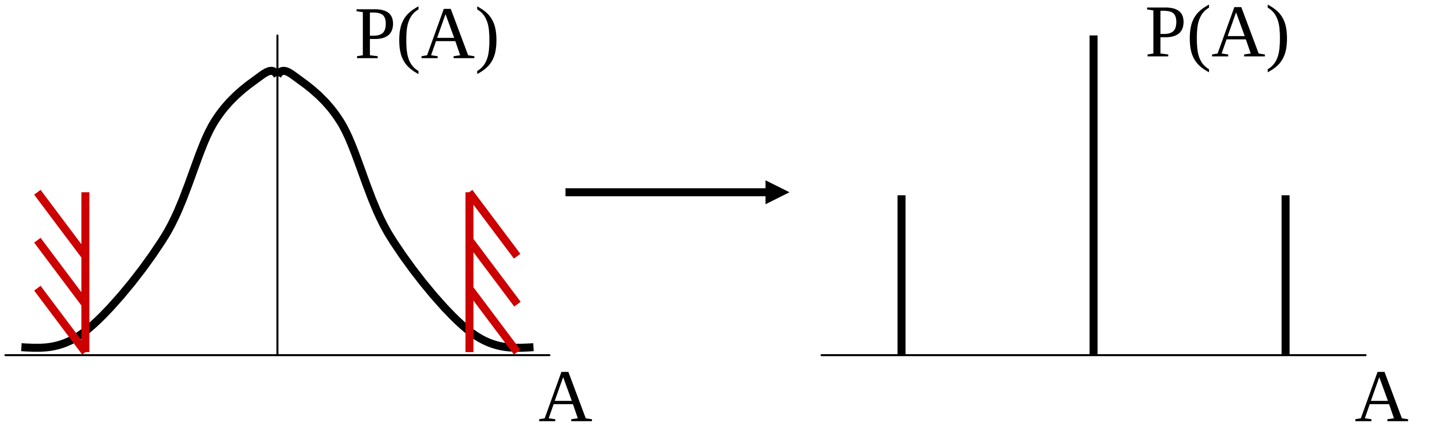
$$V\left(A_j\right) \sim \sum_i p\left(B = A_i \,|\, A_j\right) \ln \frac{p\left(B = A_i \,|\, A_j\right)}{p\left(B = A_i\right)} \sim -\ln\left(p\left(A_j\right)\right)$$

Calculation of synaptic cost function from the experimentally measured signal distribution



Consistent with the exponent 0.49 obtained in the discrete states model

Are optimal synapses discrete or continuous?



Capacity achieving distribution for many reasonable channels is discrete

Summary

Maximization of information storage capacity per unit volume explains:

- Sparseness of synaptic connections
- Noisiness and weakness of typical synapses
- Wide distribution of synaptic weights

and does not preclude the possibility of discrete synaptic states

Acknowledgments

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Lav Varshney (MIT)