Quantum Turbulence: Why fluid dynamicists should care

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Introductory comments

- Fluid dynamic phenomena range from angstroms to cosmic scales, but new phenomena appear when the length, time and temperature scales change by a factor of 10, say.
- Today, will discuss turbulent flows originating on the angstrom scale and at very low temperatures; tomorrow, some features on astrophysical scales at high temperatures.
- Both talks at an intermediate level of detail that focuses on concepts.

PNAS 118, e2018406118 (2021)

Phenomenology of quantum turbulence in superfluid helium

Ladislav Skrbek[a,1], David Schmoranzer[a], Šimon Midlik[a], and Katepalli R. Sreenivasan[b,1]

Edited by David A. Weitz, Harvard University, Cambridge, MA, and approved February 22, 2021 (received for review December 4, 2020)

Quantum turbulence—the stochastic motion of quantum fluids such as ^4He and ^3He-B, which display pure superfluidity at zero temperature and two-fluid behavior at finite but low temperatures—has been a subject of intense experimental, theoretical, and numerical studies over the last half a century. Yet, there does not exist a satisfactory phenomenological framework that captures the rich variety of experimental observations, physical properties, and characteristic features, at the same level of detail as incompressible turbulence in conventional viscous fluids. Here we present such a phenomenology that captures in simple terms many known features and regimes of quantum turbulence, in both the limit of zero temperature and the temperature range of two-fluid behavior.

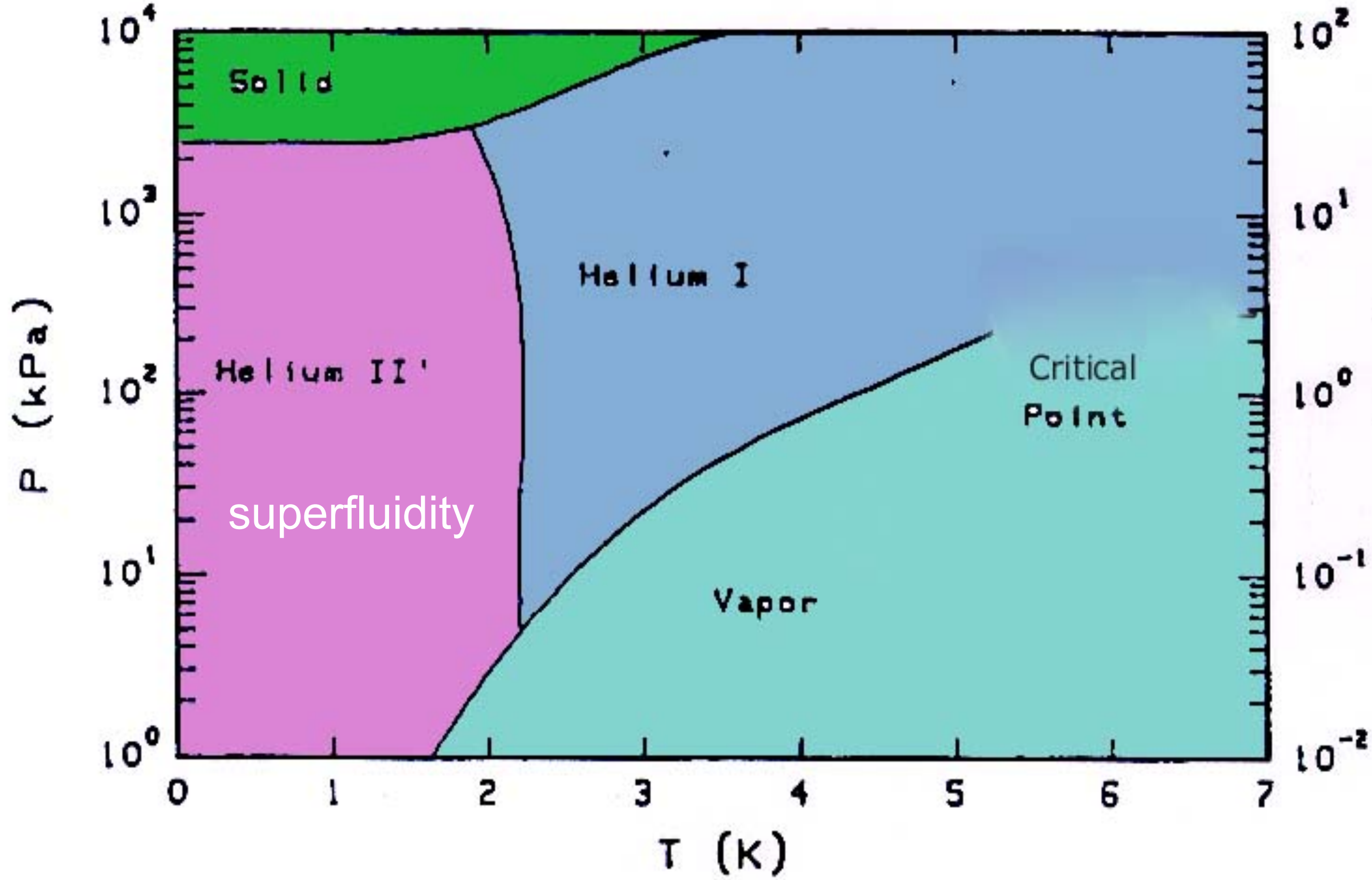
quantum turbulence | pure superfluid state | two-fluid state | Vinen and Kolmogorov turbulence

Turbulence is ubiquitous in Nature. Although it is an unfinished problem in science, incompressible turbulence in classical viscous fluids described by the Navier–Stokes equations, especially its decay without sustained production, as in the case of homogeneous and isotropic turbulence (HIT), is understood sufficiently well at the phenomenological level. Its properties can be described in surprisingly tangible detail (1). Quantum turbulence (QT) (2, 3) occurs in quantum fluids displaying superfluidity and two-fluid behavior at finite temperatures, such as the liquid phase of ^{4}He and the thermal excitations can be described hydrodynamically as a fluid with finite viscosity. It coexists with the inviscid superfluid component carrying no entropy. The total density ρ of the liquid is nearly temperature independent and satisfies $\rho = \rho_n + \rho_s$. In the $T \to 0$ limit, helium is entirely a superfluid ($\rho_s/\rho \to 1$ and $\rho_n/\rho \to 0$), while superfluidity vanishes at the high temperatures just stated ($\rho_s/\rho \to 0$ and $\rho_n/\rho \to 1$).

Under isothermal conditions, the two fluids move independently when flow velocities are small. When a certain critical velocity is exceeded, however, thin

Phase diagram of helium-4

(liquefaction ~1910: Kamerlingh Onnes, 1913 Nobel Prize)



The superfluid flows without friction (like a perfect fluid).

For helium-3, superfluidity sets in at millikelvin
for BEC at microkelvin (2005 Nobel Prize Cornell, Wieman)

Letters to the Editor

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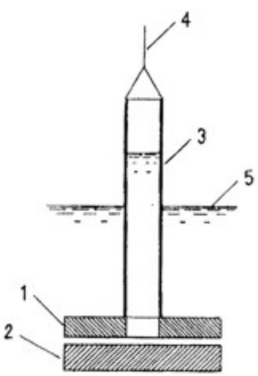
NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 83.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Viscosity of Liquid Helium below the λ-Point

THE abnormally high heat conductivity of helium II below the λ-point, as first observed by Keesom, suggested to me the possibility of an explanation in terms of convection currents. This explanation would require helium II to have an abnormally low viscosity; at present, the only viscosity measurements on liquid helium have been made in Toronto[1], and showed that there is a drop in viscosity below the λ-point by a factor of 3 compared with liquid helium at normal pressure, and by a factor of 8 compared with the value just above the λ-point. In these experiments, however, no check was made to ensure that the motion was laminar, and not turbulent.

The important fact that liquid helium has a specific density ρ of about 0·15, not very different from that of an ordinary fluid, while its viscosity μ is very small comparable to that of a gas, makes its kinematic viscosity $\nu = \mu/\rho$ extraordinary small. Consequently when the liquid is in motion in an ordinary viscosimeter, the Reynolds number may become very high, while in order to keep the motion laminar, especially in the method used in Toronto, namely, the damping of an oscillating cylinder, the Reynolds number must be kept very low. This requirement was not fulfilled in the Toronto experiments, and the deduced value of viscosity thus refers to turbulent motion, and consequently may be higher by any amount than the real value.



The very small kinematic viscosity of liquid helium II thus makes it difficult to measure the viscosity. In an attempt to get laminar motion the following method (shown diagramatically in the accompanying illustration) was devised. The viscosity was measured by the pressure drop when the liquid flows through the gap between the disks 1 and 2; these disks were of glass and were optically flat, the gap between them being adjustable by mica distance pieces. The upper disk, 1, was 3 cm. in diameter with a central hole of 1·5 cm. diameter, over which a glass tube (3) was fixed. Lowering and raising this plunger in the liquid helium by means of the thread (4), the level of the liquid column in the tube 3 could be set above or below the level (5) of the liquid in the surrounding Dewar flask. The amount of flow and the pressure were deduced from the difference of the two levels, which was measured by cathetometer.

The results of the measurements were rather striking. When there were no distance pieces between the disks, and the plates 1 and 2 were brought into contact (by observation of optical fringes, their separation was estimated to be about half a micron), the flow of liquid above the λ-point could be only just detected over several minutes, while below the λ-point the liquid helium flowed quite easily, and the level in the tube 3 settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I at normal pressure.

The experiments also showed that in the case of helium II, the pressure drop across the gap was proportional to the square of the velocity of flow, which means that the flow must have been turbulent. If, however, we calculate the viscosity, assuming the flow to have been laminar, we obtain a value of the order 10^{-9} C.G.S., which is evidently still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small gap, comes out higher than 50,000, a value for which turbulence might indeed be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present upper limit (namely, 10^{-9} C.G.S.) is already very striking, since it is more than 10^4 times smaller than that of hydrogen gas (previously thought to be the fluid of least viscosity). The present limit is perhaps sufficient to suggest, by analogy with supraconductors, that the helium below the λ-point enters a special state which might be called a 'superfluid'.

As we have already mentioned, an abnormally low viscosity such as indicated by our experiments might indeed provide an explanation for the high thermal conductivity, and for the other anomalous properties observed by Allen, Peierls, and Uddin[2]. It is evidently possible that the turbulent motion, inevitably set up in the technical manipulation required in working with the liquid helium II, might on account of the great fluidity, not die out, even in the small capillary tubes in which the thermal conductivity was measured; such turbulence would transport heat extremely efficiently by convection.

P. KAPITZA.

Institute for Physical Problems,
Academy of Sciences,
Moscow.
Dec. 3.

[1] Burton, NATURE, **135**, 265 (1935); Wilhelm, Misener and Clark, *Proc. Roy. Soc.*, A, **151**, 342 (1935).

[2] NATURE, **140**, 62 (1937).

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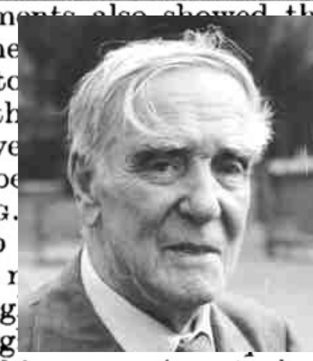


Nobel Prize for Kapitza in 1978

for his basic inventions and discoveries in the area of low-temperature physics

"The choice of the theme of my Nobel Lecture presents some difficulty for me."

From Landau (1941)

[Helium II] … possesses a number of peculiar properties, the most important of which is superfluidity **discovered by P.L. Kapitza**…

Physics World: volume 21, no.8, pp.27-30 (2008)

Superfluidity: three people, two papers, one prize

Most accounts of the controversial discovery of superfluid helium by Peter Kapitza, Jack Allen and Don Misener are often incomplete or simply wrong. **Allan Griffin** tries to set the record straight

University of Toronto

Misener family collection

Three of the best
Peter Kapitza (left) was awarded one half of the 1978 Nobel Prize for Physics for the discovery of superfluidity 40 years earlier. Jack Allen (middle) and Don Misener (right) discovered the phenomenon at the same time but did not get the same recognition.

The discovery of superfluidity in liquid helium-4 was announced to the scientific world on 8 January 1938, when two short papers were published back to back in *Nature*. One was by Peter Kapitza (*Nature* **141** 74), the director of the Institute for Physical Problems in Moscow, and the other was by two young Canadian physicists, Jack Allen and Don Misener (*Nature* **141** 75), both working at the Royal Society Mond Laboratory at the University of Cambridge in the UK. Both studies reported that liquid helium flowed with almost no measurable viscosity below the transition temperature of 2.18 K.

Very soon afterwards, theoretical work by Lev Landau, Fritz London and Laszlo Tisza showed that this

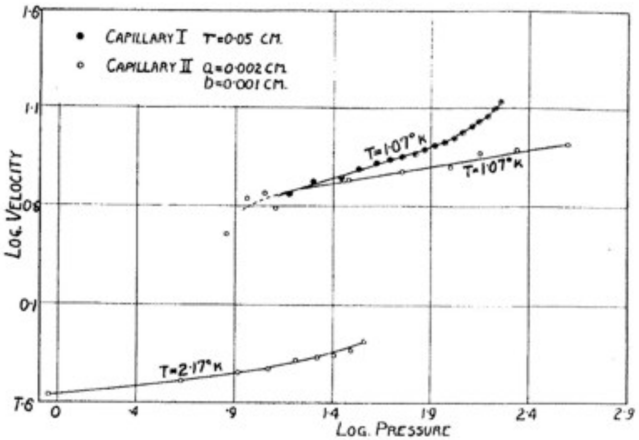
Kapitza, by then 84, was given half of that year's Nobel Prize for Physics with a somewhat vague citation reading "for his basic inventions and discoveries in the area of low-temperature physics". The other half did not go to Allen and Misener. Indeed, apart from a single mysterious sentence in the longer Nobel citation, the work of the two Canadians was completely ignored.

In his Nobel address Kapitza broke with tradition and said nothing about the work on superfluid helium for which he was being honoured. Instead, on the grounds that he had abandoned work on low-temperature physics decades earlier, he reviewed his most recent research on thermonuclear reactions. Today, science popularizers generally give sole credit for the discovery

Flow of Liquid Helium II

A SURVEY of the various properties of liquid helium II has prompted us to investigate its viscosity more carefully. One of us[1] had previously deduced an upper limit of 10^{-6} c.g.s. units for the viscosity of helium II by measuring the damping of an oscillating cylinder. We had reached the same conclusion as Kapitza in the letter above; namely, that due to the high Reynolds number involved, the measurements probably represent non-laminar flow.

The present data were obtained from observations on the flow of liquid helium II through long capillaries. Two capillaries were used; the first had a circular bore of radius 0·05 cm. and length 130 cm. and drained a reservoir of 5·0 cm. diameter; the second was a thermometer capillary 93·5 cm. long and of elliptical cross-section with semi-axes 0·001 cm. and 0·002 cm., which was attached to a reservoir of 0·1 cm. diameter. The measurements were made by raising or lowering the reservoir with attached capillary so that the level of liquid helium in the reservoir was a centimetre or so above or below that of the surrounding liquid helium bath. The rate of change of level in the reservoir was then determined from the cathetometer eye-piece scale and a stopwatch; measurements were made until the levels became coincident. The data showing velocities of flow through the capillary and the corresponding pressure difference at the ends of the capillary are given in the accompanying table and plotted on a logarithmic scale in the diagram.



Capillary I		Capillary II			
T = 1·07° K.		T = 1·07° K.		T = 2·17° K.	
Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)
13·9	183·5	8·35	402	0·837	36·6
11·5	154·5	6·92	218	0·757	31·3
10·3	127·7	6·88	143	0·715	26·1
9·0	105·0	6·30	101	0·685	21·1
8·2	83·5	6·05	56	0·655	16·4
7·5	65·7	5·55	30	0·609	12·1
6·9	49·3	4·70	11·3	0·570	8·3
6·1	34·1	4·39	9·2	0·525	4·3
5·2*	20·3	3·92	13·0	0·433	0·9
4·5*	15·2	2·88	7·2		

The following facts are evident:

(a) The velocity of flow, q, changes only slightly for large changes in pressure head, p. For the smaller capillary, the relation is approximately $p \propto q^4$, but at the lowest velocities an even higher power seems indicated.

(b) The velocity of flow, for given pressure head and temperature, changes only slightly with a change of cross-section area of the order of 10^3.

(c) The velocity of flow, for given pressure head and given cross-section, changes by about a factor of 10 with a change of temperature from 1·07° K. to 2·17° K.

(d) With the larger capillary and slightly higher velocities of flow, the pressure-velocity relation is approximately $p \propto q^3$, with the power of q decreasing as the velocity is increased.

If, for the purpose of calculating a possible upper limit to the viscosity, we assume the formula for laminar flow, that is, $p \propto q$, we obtain the value $\eta = 4 \times 10^{-9}$ c.g.s. units. This agrees with the upper limit given by Kapitza who, using velocities of flow considerably higher than ours, has obtained the relation $p \propto q^2$ and an upper limit to the viscosity of $\eta = 10^{-9}$ c.g.s. units.

The observed type of flow, however, in which the velocity becomes almost independent of pressure, most certainly cannot be treated as laminar or even as ordinary turbulent flow. Consequently any known formula cannot, from our data, give a value of the 'viscosity' which would have much meaning. It may be possible that the liquid helium II slips over the surface of the tube. In this case any flow method ... of

Submitted on
Dec 22, 1937
(19 days later)

thermal conduction capillary will not be likely to be greater than 50 cm./sec. It seems, therefore, that undamped turbulent motion cannot account for an appreciable part of the high thermal conductivity which has been observed for helium II.

J. F. ALLEN.
A. D. MISENER.

Royal Society Mond Laboratory,
Cambridge.

Dec. 22.

[1] Burton, E. F., NATURE, 135, 265 (1935).
[2] Allen, Peierls and Uddin, NATURE, 140, 62 (1937).

Some Experiments at Radio Frequencies on Supraconductors

MEASUREMENTS were made on an extruded tin wire carrying an alternating current of a frequency of about 200 kilocycles per second superposed upon a direct current. The resulting magnetic field at the surface of the wire was thus caused to pulsate cyclically.

"A marked change in the viscosity takes place at 2.19 K, the temperature of transition of helium I to helium II."

J.O. Wilhelm, A.D. Misener & A.R. Clark, *Proc. Roy. Soc.* **151**, 342-347 (1935)

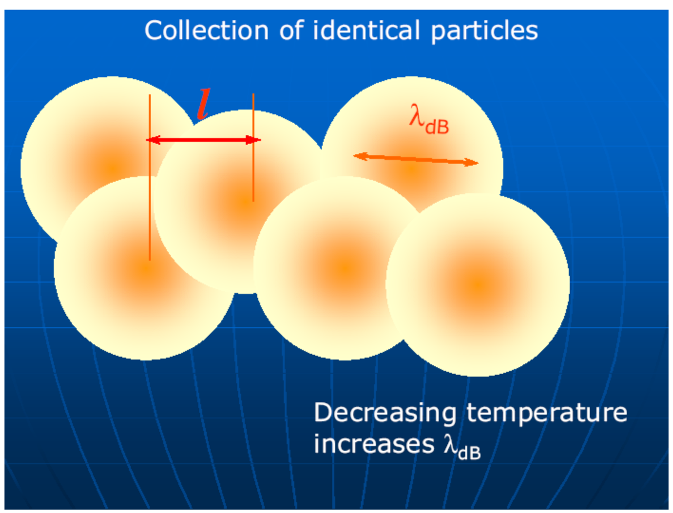
"It is not enough to make a discovery: one must also evaluate its significance for the development of science. But even that's not enough: a scientist must proceed from the essence of the discovery to produce others. It is only after this that he can consider that the discovery belongs to him."

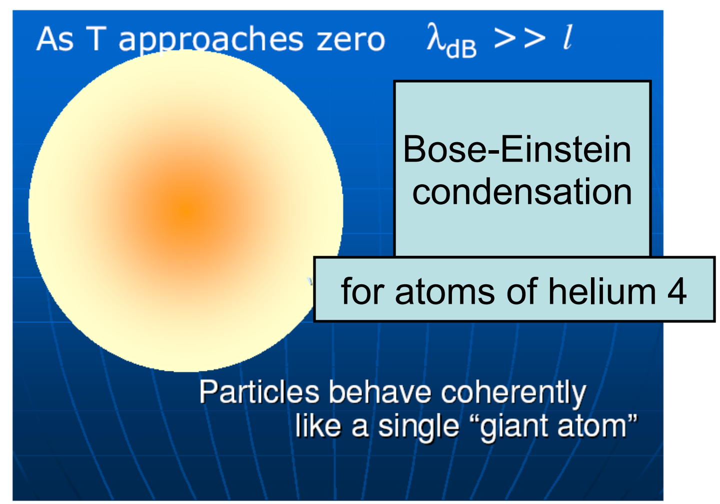
P.L. Kapitza, as quoted by Andronikashvili, in "Reflections on Helium", AIP Press (1980)

1. 1929: de Broglie wavelength, $\lambda_{dB} = h/(2\pi m k_B T)^{1/2}$, is a measure of the wavefunction spread.

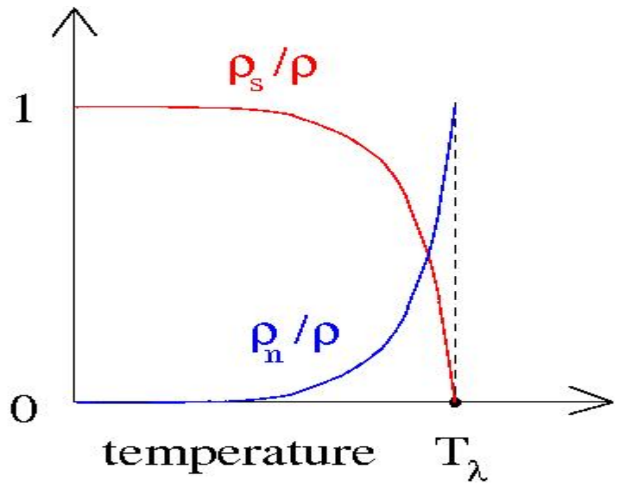
2. If $(V/N)^{1/3} >> \lambda_{dB}$, particles behave individually, and Maxwell-Boltzmann statistics apply.

3. If $(V/N)^{1/3} < \lambda_{dB}$, particles behave collectively, and either BE statistics or FD statistics apply.





Phenomenological model for He II



$T_\lambda \approx 2.17K$ (London: 3.2K)

Superfluid: density ρ_s, velocity v_s, no viscosity, no entropy

Normal fluid: density ρ_n, velocity v_n, carries viscosity and entropy

"coexisting but non-interacting and interpenetrating"

F. London 1900-1954

Nature **141**, 643 (1938)

L. Tisza 1907-2009

Nature **141**, 913 (1938)

For an ideal gas, S.N. Bose and A. Einstein had proposed that a large number of particles will occupy the lowest energy state ("condense") below certain temperature.

London's bold suggestion was that, even though liquid helium was far from an ideal gas, helium-4 atoms will condense into a ground state below a critical temperature.

Landau's two-fluid model for He II

Starts with the quantum ground state, with "elementary excitations" with particular spectra (phonons and rotons).

I am glad to … pay tribute to L. Tisza for introducing, as early as 1938, the conception of the macroscopical description of helium II by dividing its density into two parts and introducing, correspondingly, two velocity fields…. However, his entire quantitative theory (microscopic as well as thermodynamic-hydrodynamic) is in my opinion entirely wrong.

L.D. Landau, *Phys. Rev.* **75**, 884 (1949)

L.D. Landau 1908-1968

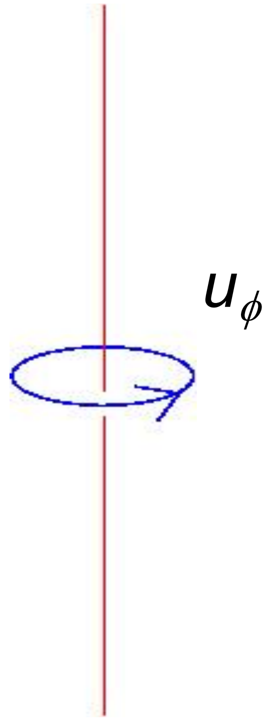
1962 Nobel Prize

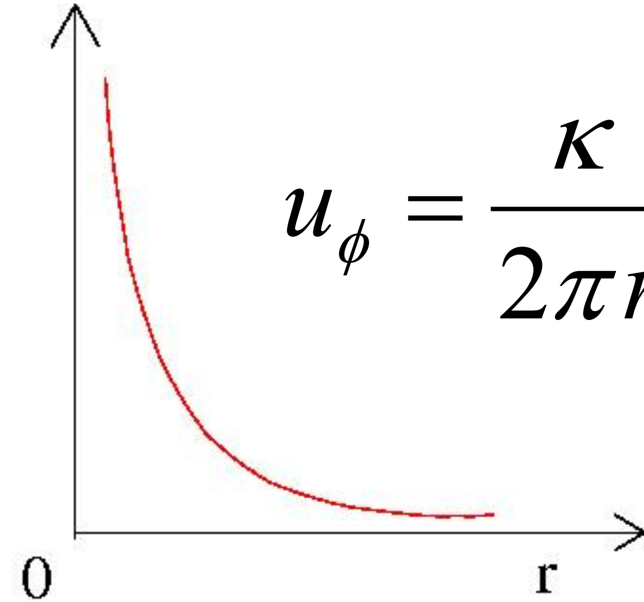
Landau's picture was incomplete as well, and has later been augmented by others. The present understanding is that the helium atoms indeed undergo Bose condensation and the superfluid velocity is the gradient of the phase of condensate wavefunction. But the condensate is not the superfluid. Only some 10% of the fluid is the condensate at 0 K, whereas all of it is superfluid.

N.N. Bogoliubov
J. Phys. USSR 11, **23** (1956)
Bose condensation and its role

P.C. Hohenberg & P.C. Martin
Annals of Physics, **34**, 291 (1965)
full critique and microscopic theory

quantized vortices in helium II





$$u_\phi = \frac{\kappa}{2\pi r}$$

Onsager
1903-1976

Wave function: $\psi = \psi_0 \exp(i\varphi(r))$, $\psi_0 \rightarrow 0$ as $r \rightarrow 0$ and $\rightarrow 1$ as $r \rightarrow \infty$

Velocity is the gradient of φ(r). The increment of its gradient over any closed path must be a multiple of 2π, for the wave function to remain single valued.

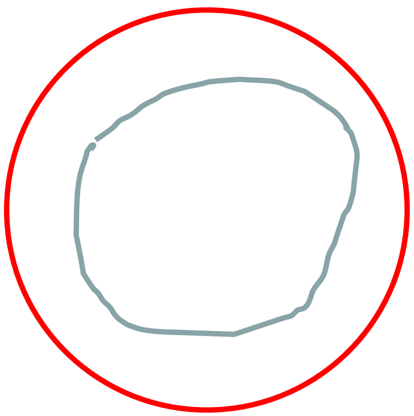
"Thus, the well-known invariant called hydrodynamic circulation is quantized; the quantum of circulation is h/m."

Onsager (1949)

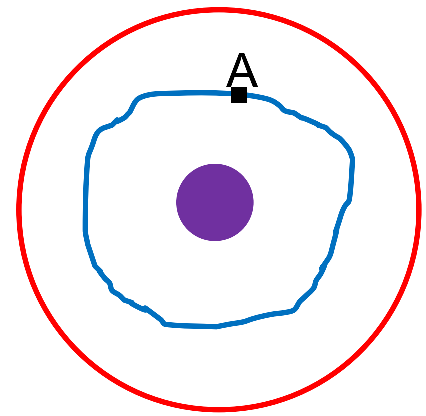
$$\psi(\mathbf{x},t) = \psi_0(\mathbf{x},t)\ \exp\,[i\phi(\mathbf{x},t)]$$

magnitude phase

$$\mathbf{u}_s\,(\mathbf{x},t) = (\hbar/m)\ \nabla\phi(\mathbf{x},t) \qquad (1)$$



$$\Gamma = \oint u_s\,.\,dr$$
$$= (\hbar/m) \oint \nabla\phi(\mathbf{x},t).d\mathbf{r}$$
$$= \nabla\phi^{tot}$$



simply connected

Can shrink the contour to zero without ever leaving the domain. Stokes theorem applies.

But ω = curl u_s = 0 from (1).

Thus Γ = 0 for all contours

doubly connected

Can't apply Stokes theorem

For the wavefunction to be same at A after n rotations, we should have, from (1), $\nabla\phi^{tot} = 2\pi n$. Thus,

$\Gamma = (h/m)\ n = \kappa n$; stable for n = 1.

Except for a few angstroms from the center of the core, the laws obeyed are those of classical hydrodynamics [e.g., Biot-Savart].

R.P. Feynman: 1918-1988

If … two oppositely directed sections of [vortex] line approach closely, … the lines (which are under tension) may snap together and join connections a new way …

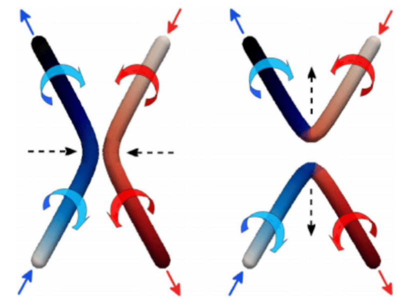
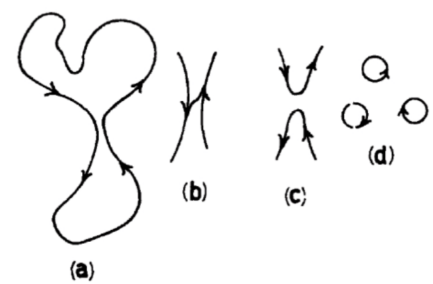
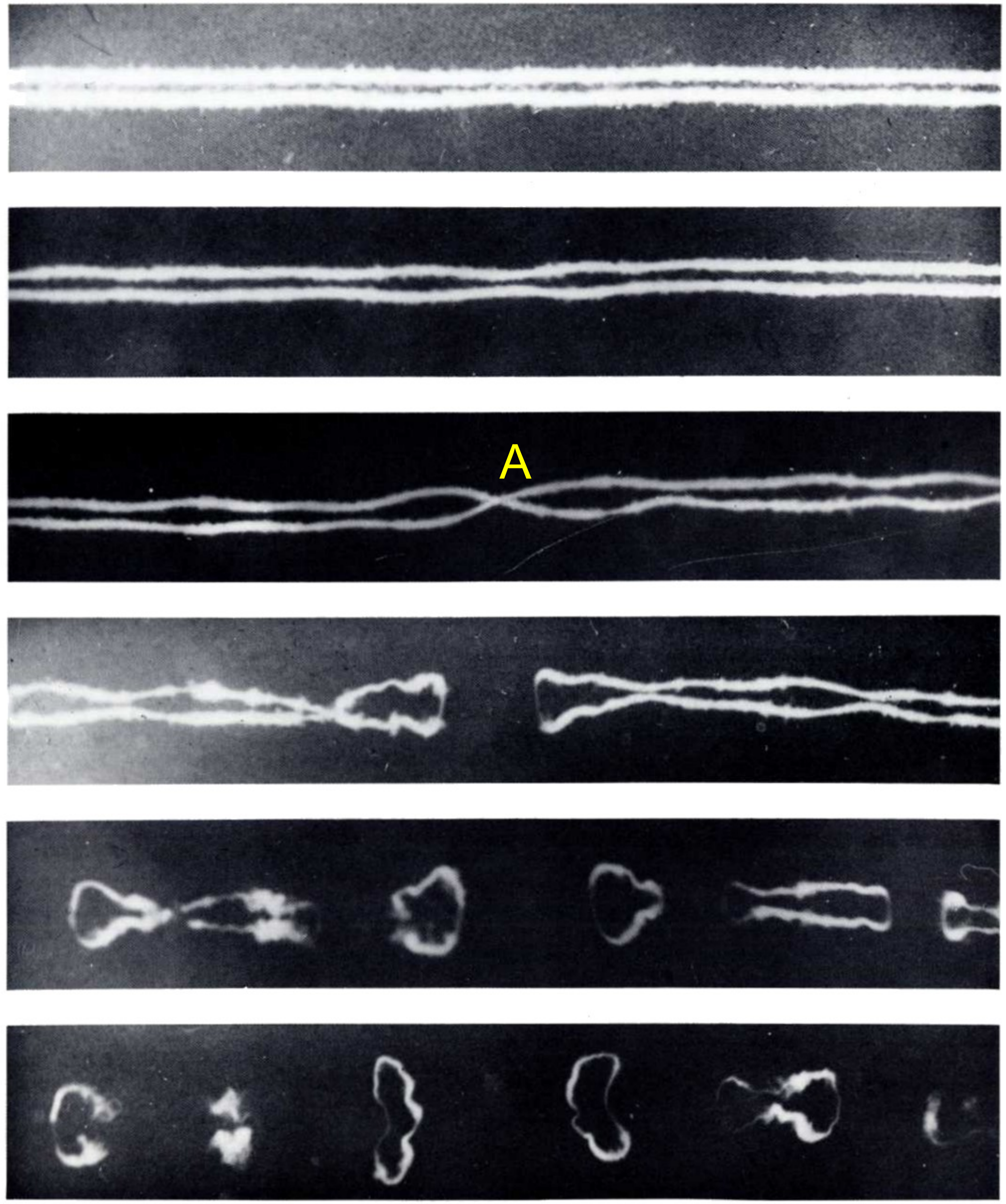


Figure 3.7 Schematics of vortex reconnection. Left: before reconnection; right: after reconnection.



Prog. Low Temp. Phys. 1, 17 (1955)

Crow, S. C. (1970). "Stability theory for a pair of trailing vortices". *AIAA J.* **8**: 2172.



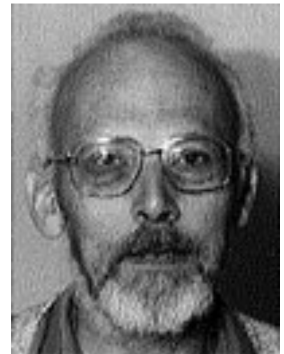
16. **Instability of a pair of trailing vortices.** The vortex trail of a B-47 aircraft was photographed directly overhead at intervals of 15 s after its passage. The vortex cores are made visible by condensation of moisture. They slowly recede and draw together in a symmetrical nearly sinusoidal pattern until they connect to form a train of vortex rings. The wake then quickly disintegrates. This is commonly called Crow instability after the researcher who explained its early stages analytically. *Crow 1970, courtesy of Meteorology Research Inc.*

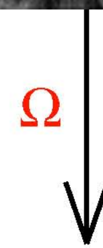
From Van Dyke's Album

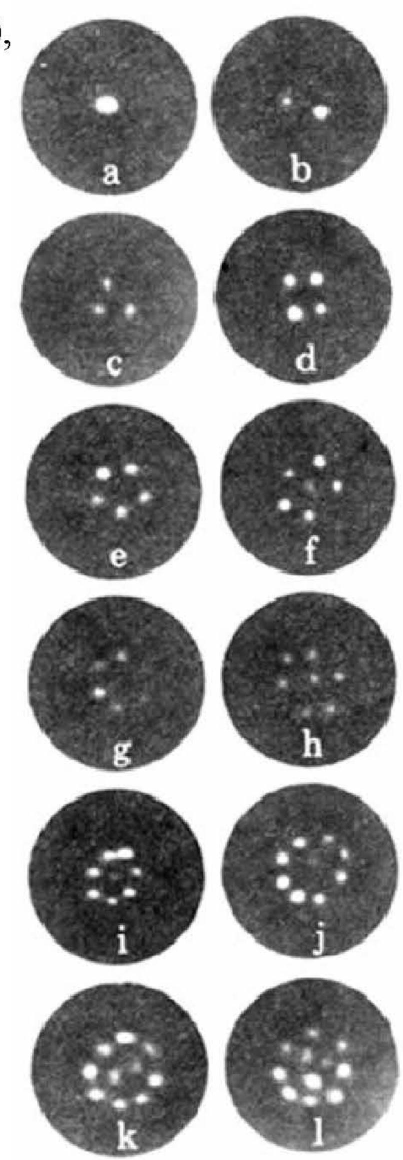
Previous observation of quantized vortices

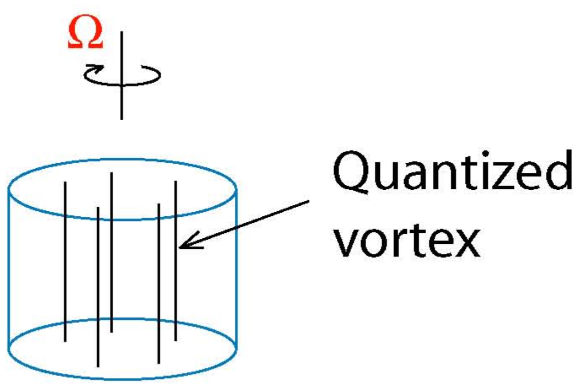
Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979), *Phys. Rev. Lett.* **43**, 214-217.

Quantized vortices were inferred and studied by H.E. Hall & W.F. Vinen (1956) *Proc. Roy. Soc. Lond.* **A238**, 204-214 and 215-234



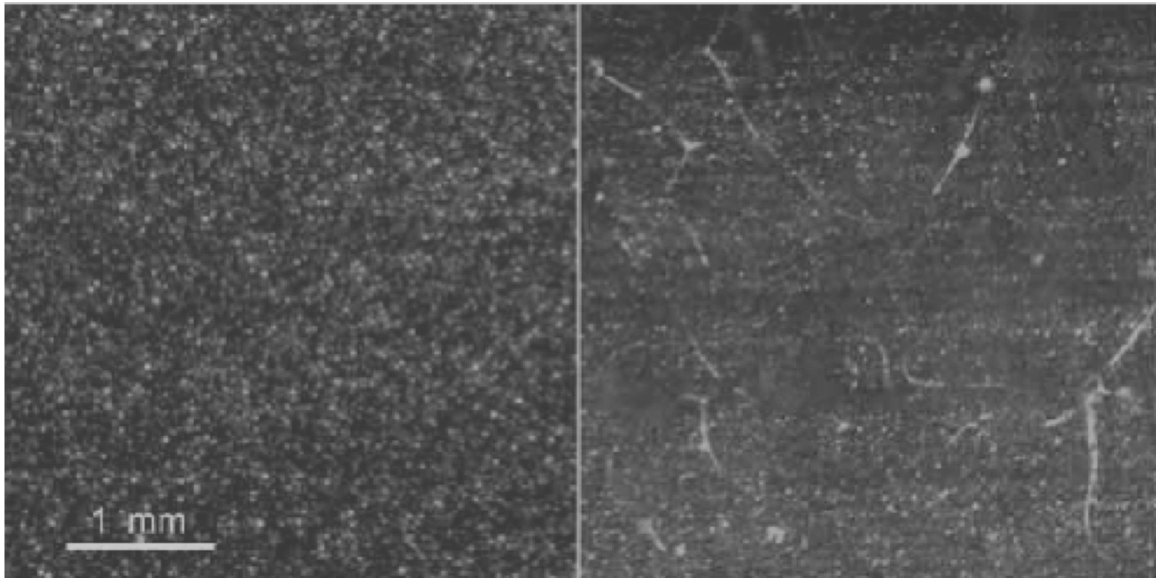






technique is not good for visualizing disordered vortices

50 years on…



~50 mK above T_λ **~50 mK below T_λ**

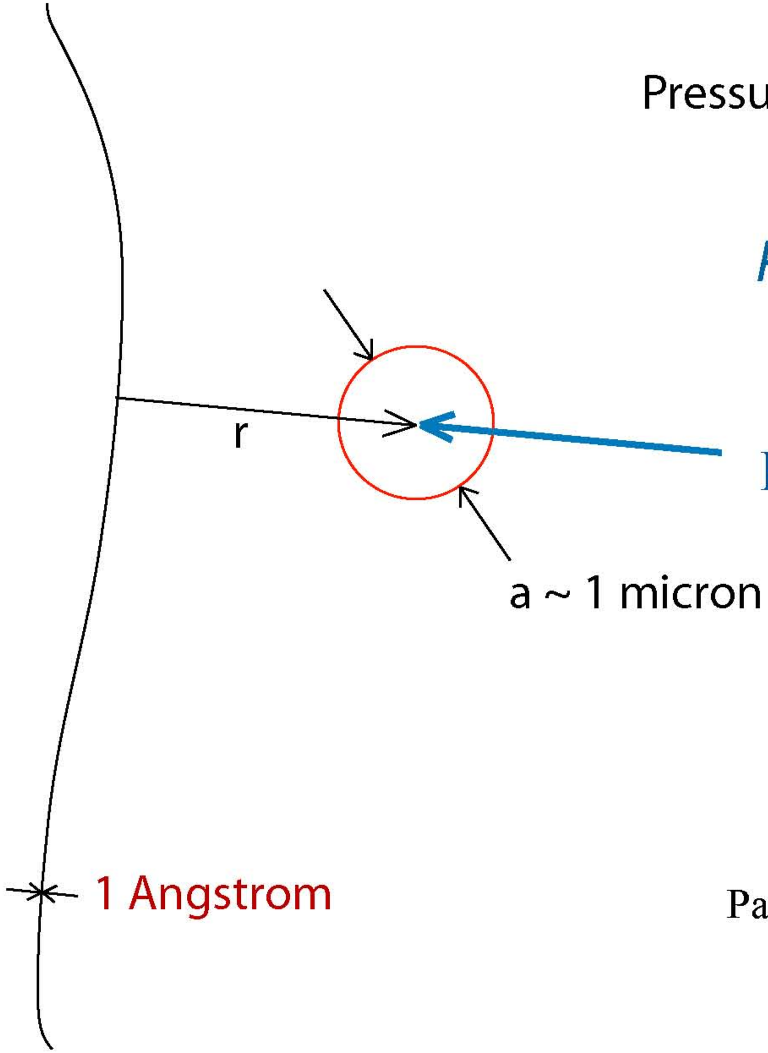
The left panel shows a suspension of hydrogen particles just above the transition temperature. The right panel shows the same particles after the fluid was cooled below the lambda point. Some particles have collected along filaments, while other are randomly distributed as before. Fewer free particles are apparent on the right only because the light intensity was reduced to highlight the brighter filaments in the image. Volume fraction $\cong 3X10^{-5}$.

G.P. Bewley, D.P. Lathrop & KRS, Nature 441, 558 (2006)

Particle Trapping

Pressure gradient:

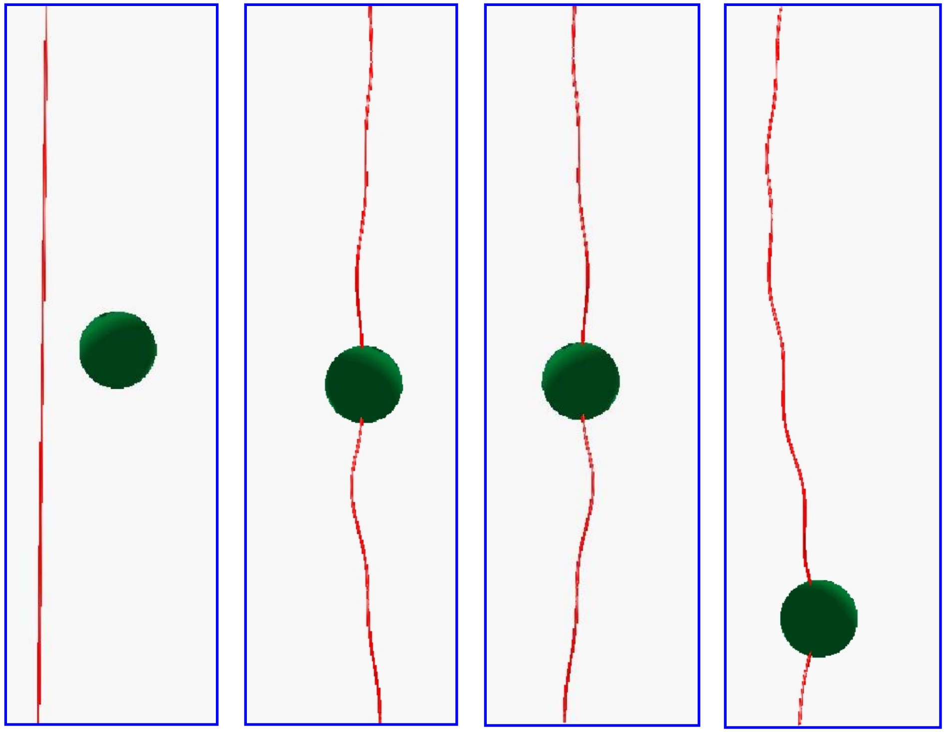
$$P = \frac{-\rho_s \kappa^2}{8\pi^2 r^2}$$



$$F = \frac{4}{3}\pi a^3 \nabla P \sim \frac{1}{r^3}$$

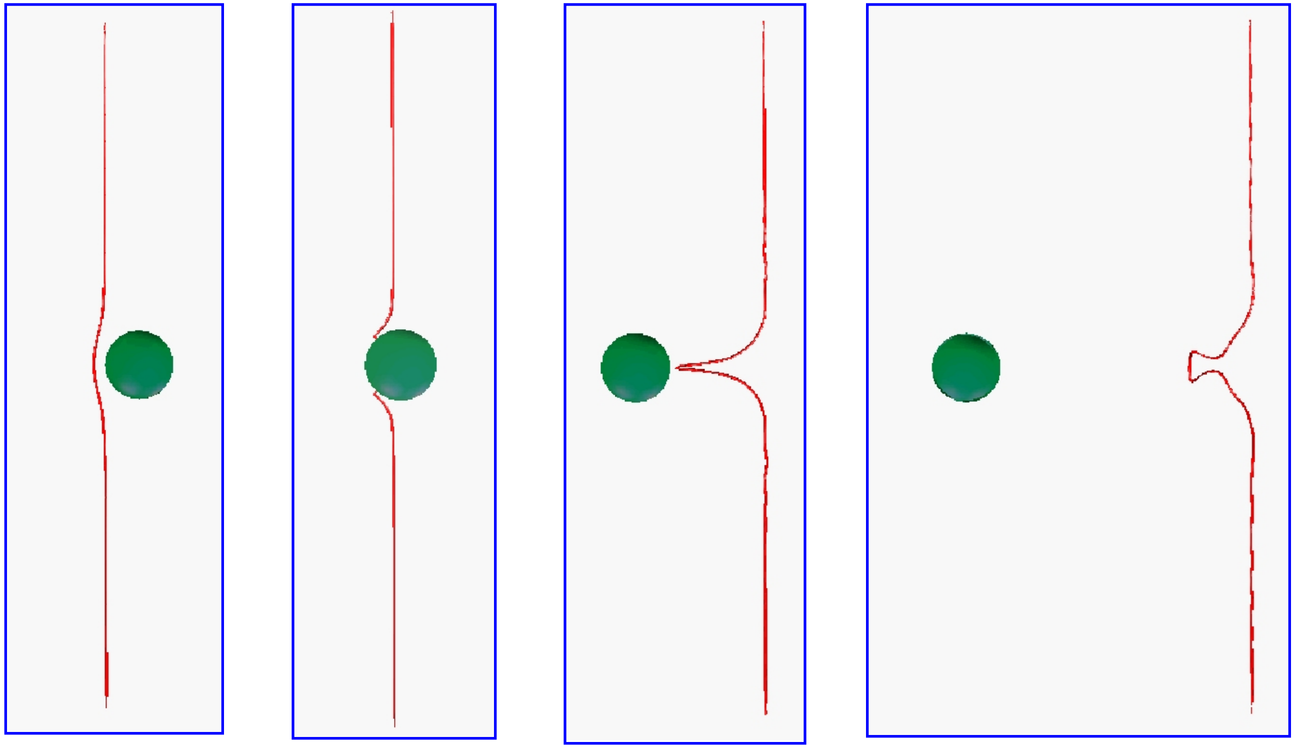
Parks, P.E. and Donnelly, R.J. (1966), *Phys. Rev. Lett.* **16**, 45–48.

sphere is trapped by vortex

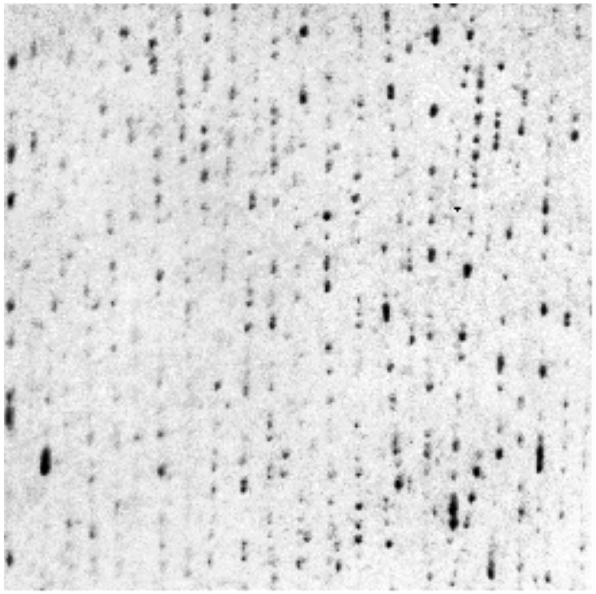


For a discussion of interaction between the fluid and particles in He II, see Sergeev, Barenghi & Kivotides, *Phys. Rev. B* **74**,184506 (2006); the simulations shown are by these authors.

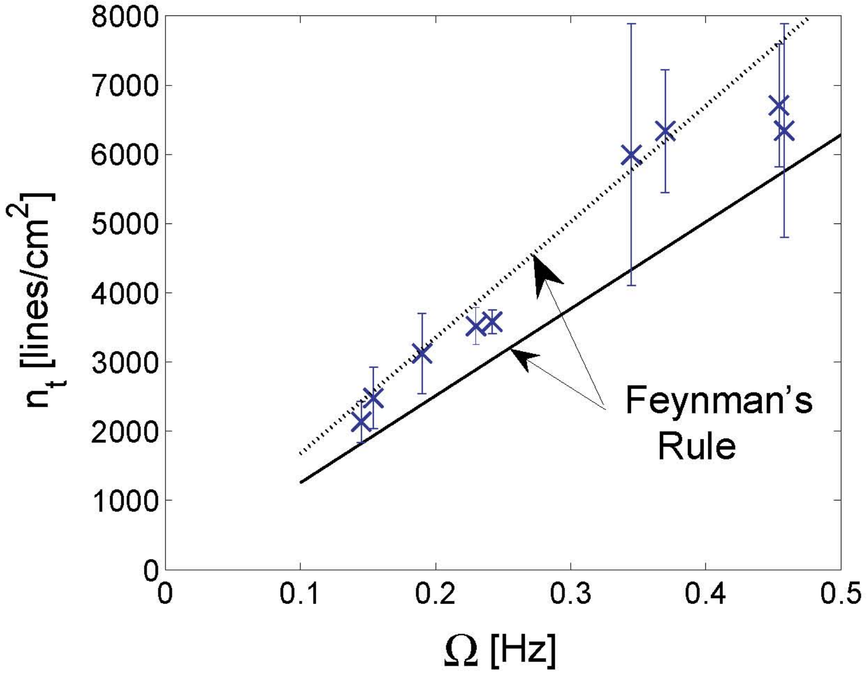
sphere escapes vortex



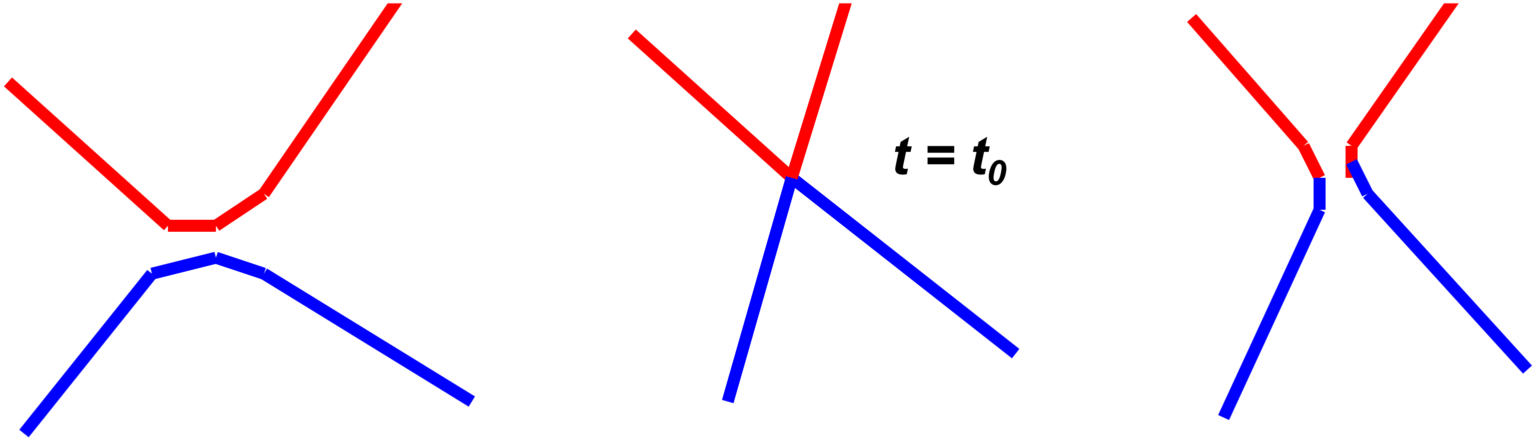
Number of vortices



c



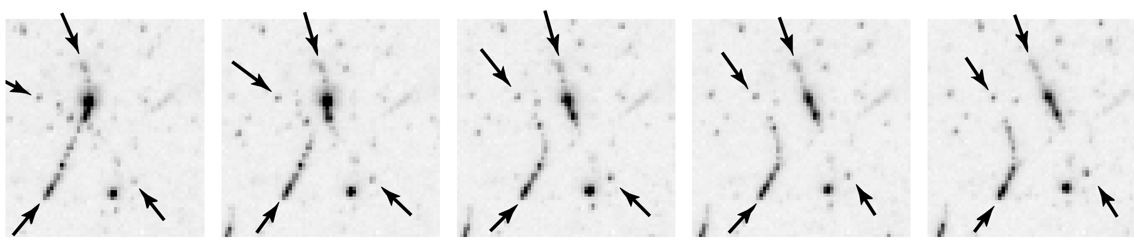
The left panel shows an example of particles arranged along vertical lines when the system is rotating steadily about the vertical axis. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. Their number follows Feynman's rule pretty well.



schematic

Two vortices of opposite signs, which are attracted to each other, collide, splice parts of one to parts of the other, and move away from each other in a different direction.

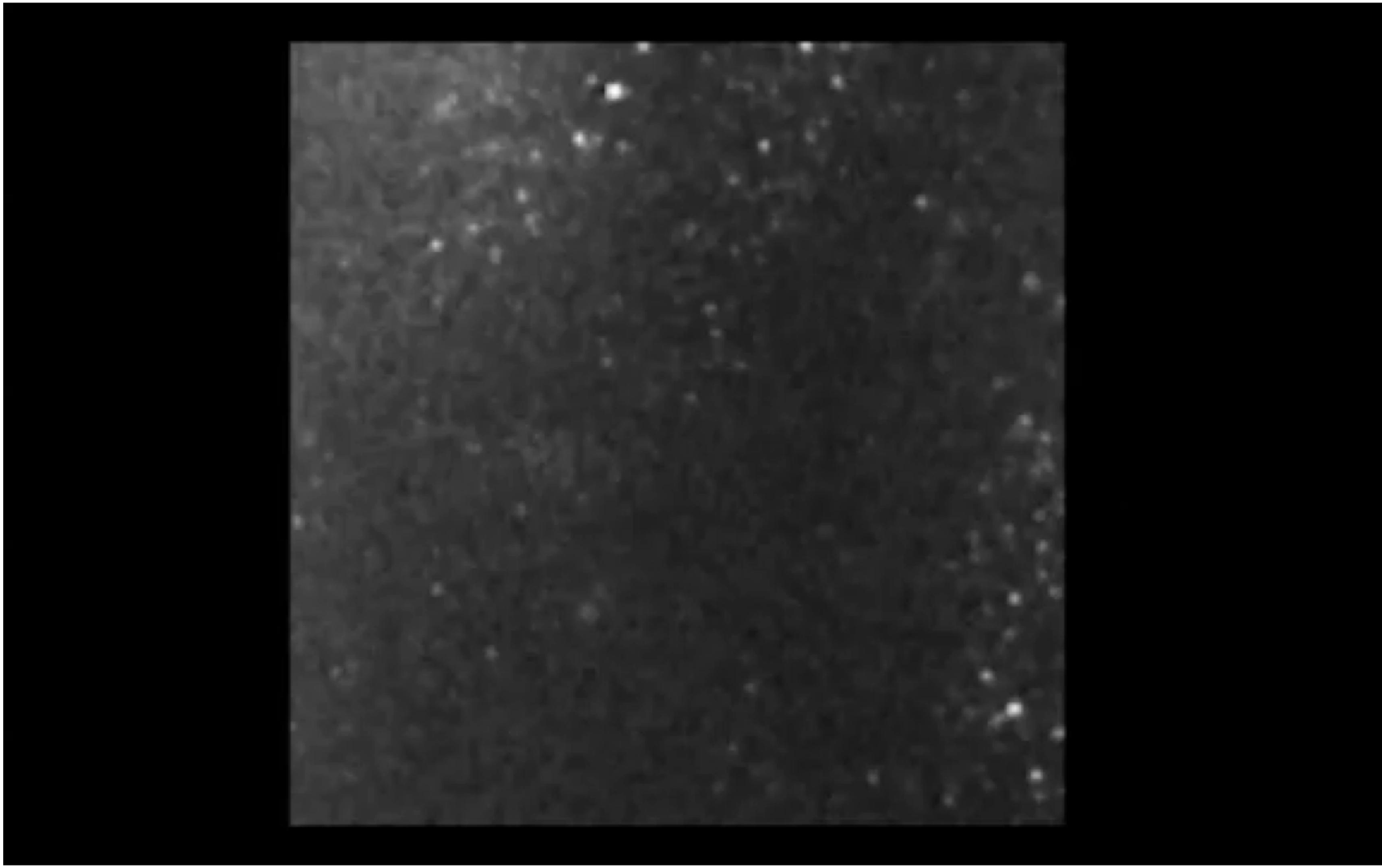
reconnection movie 2.avi



measurement

Images of hydrogen particles suspended in liquid helium, taken at 50 *ms* intervals, for $t > t_0$. Some particles are trapped on quantized vortex cores, while others are randomly distributed in the fluid. Before reconnection, particles drift collectively with the background flow. Subsequent frames show reconnection as the sudden motion of a group of particles.

Bewley, Poaletti, KRS & Lathrop, *PNAS* **105**, 13707 (2008)



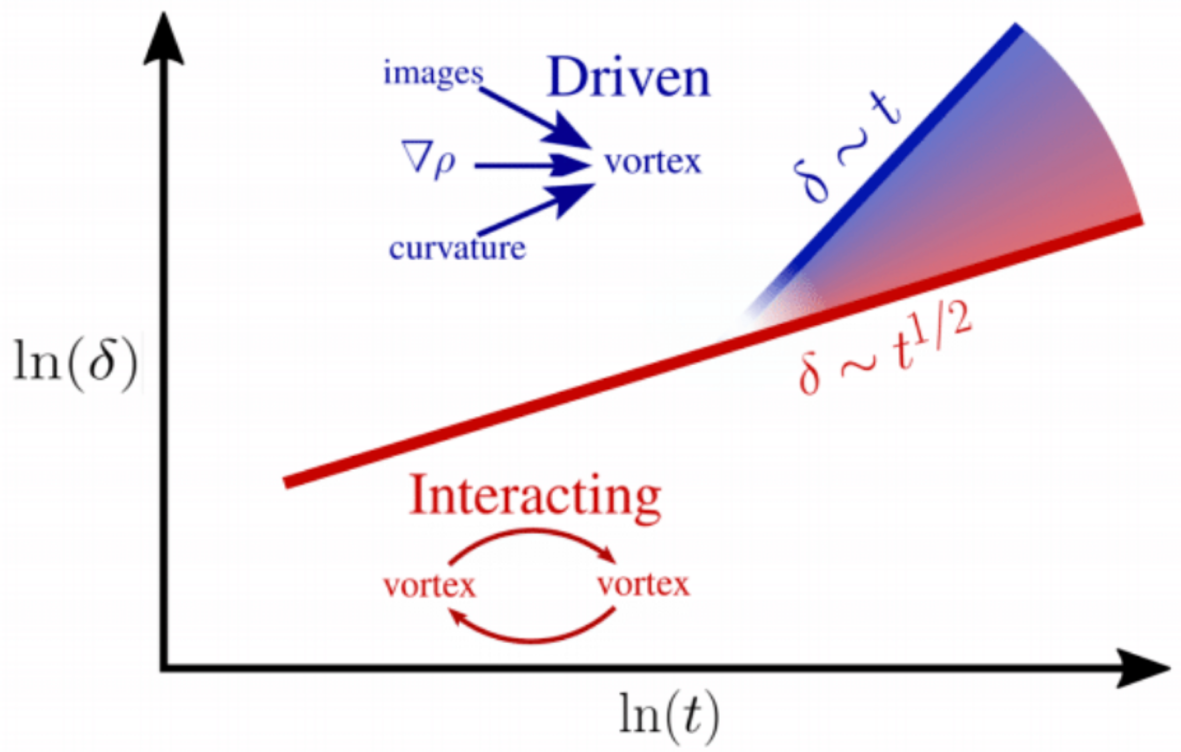
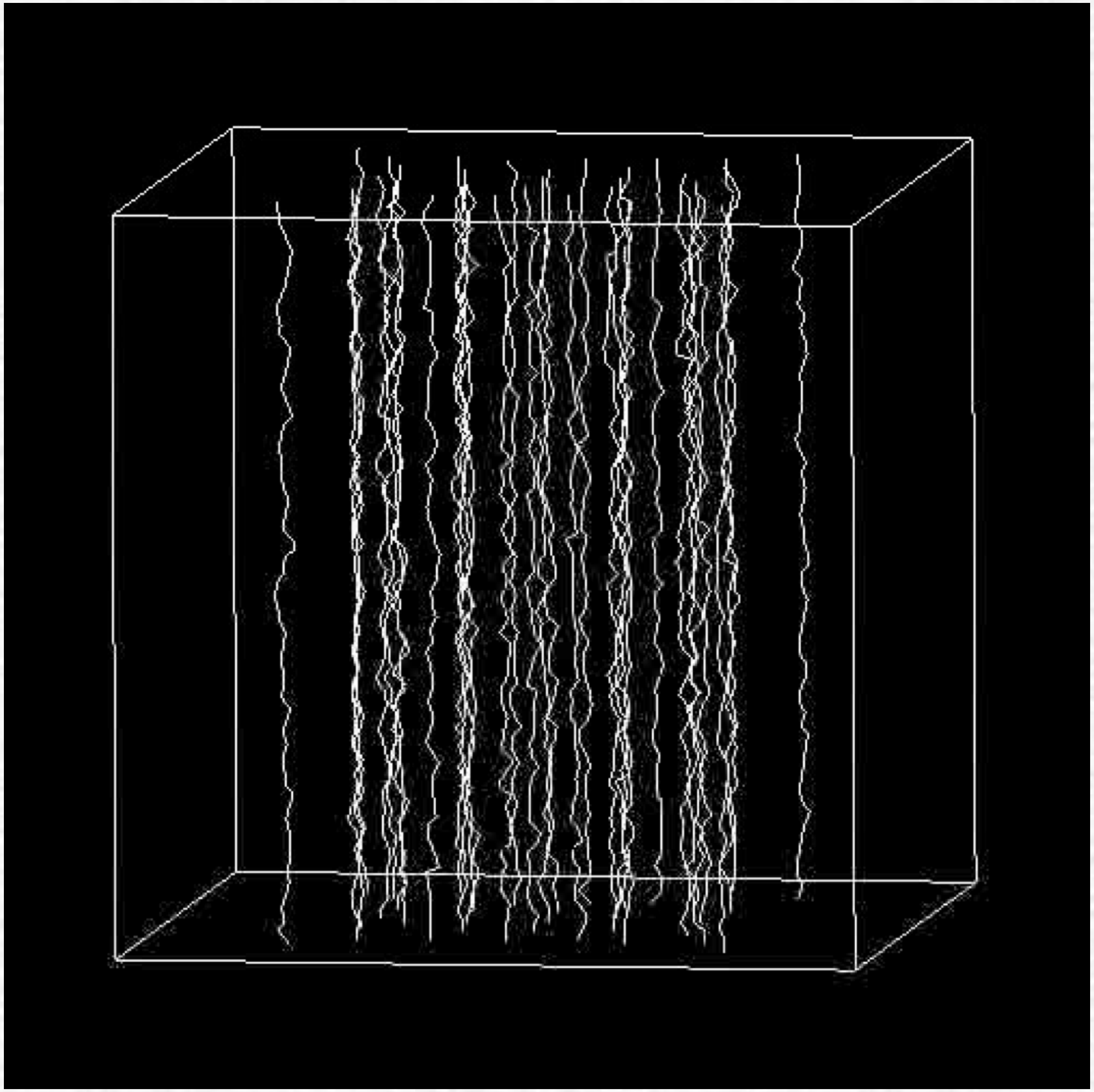


Figure 3.13 Routes to the reconnection of two vortex lines. According to Galantucci *et al.* (2019), two limiting scaling laws exist. In the first limiting behaviour,
$\delta(t) \sim |t - t_0|^{1/}$ … ines is de-
termined by th … nds. In the
second limitin … tex, hence
δ is set by the … ges, or by
the curvature … ssover oc-
curs between these two regimes. When δ becomes of the order of the vortex core radius or less, the scaling becomes $\delta(t) \sim |t - t_0|^{1/2}$, as predicted by Nazarenko & West (2003). Reproduced with permission from Galantucci *et al.* (2019).

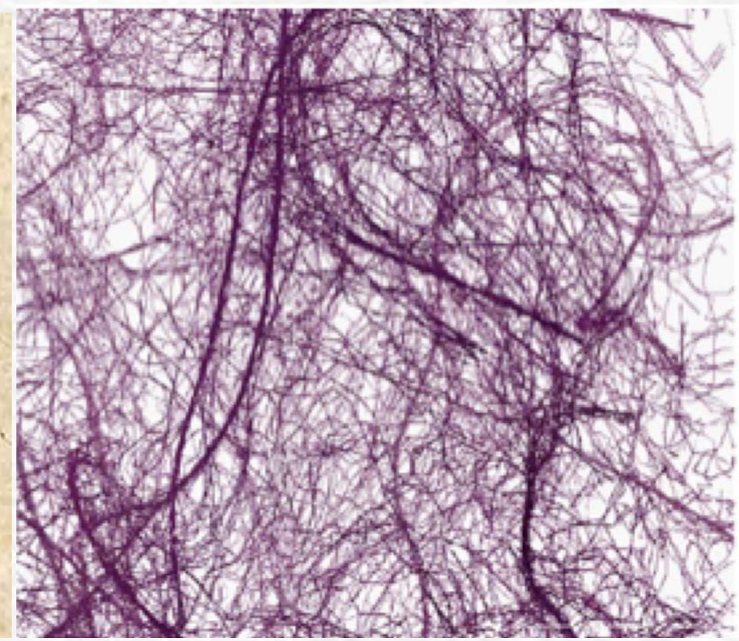
Taken from
C. Barenghi, L. Skrbek and KRS, PNAS (2022)

See also
E. Fonda, KRS and D.P. Lathrop, PNAS (2019)

Tsubota, Araki & Barenghi, *Phys. Rev. Lett.* **90,** 20530 (2003)



High-intensity vortex structures in homogeneous and isotropic turbulence [She, Jackson & Orszag, *Nature* **344** (1990)]



Vortex tan
turbulenc
(2006); pi
(1985), M.

Microscopi
J. Koplik a
(1993), by
which is a good model for the wavefunction in BEC.

S.Z. Alamri, A.J. Youd & C.F. Barenghi

PRL **101**, 215302 (2008) PHYSICAL REV

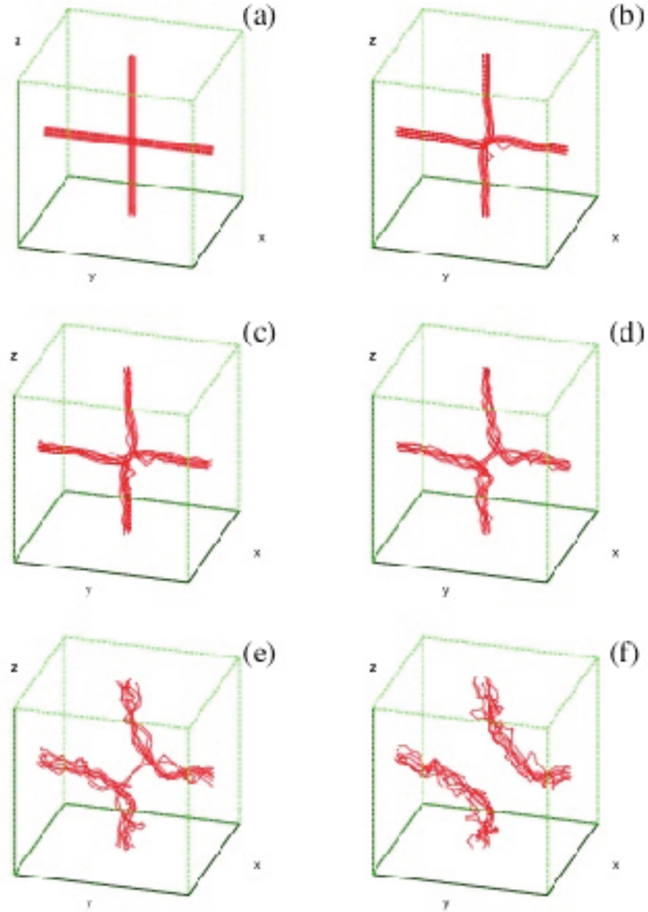


FIG. 1 (color online). Reconnection of two bundles of seven vortex strands each. (a) $t = 0$ s, (b) $t = 7.13$ s, (c) $t = 23.58$ s, (d) $t = 36.27$ s, (e) $t = 61.49$ s, (f) $t = 80.35$ s.

Return to the original question:

What quantitative relationship does

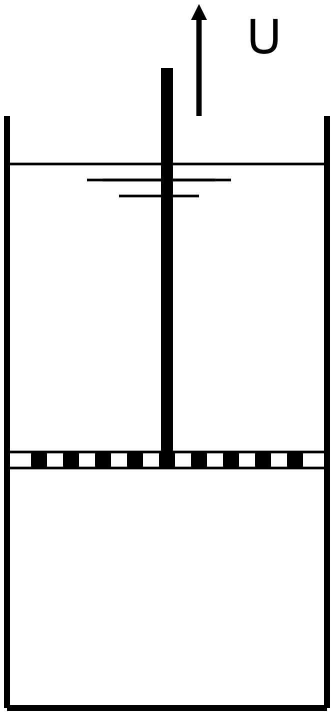
quantum turbulence

bear to

classical turbulence?

Different types of turbulence are possible but consider only that in which the two turbulence fields are tightly coupled by mutual friction.

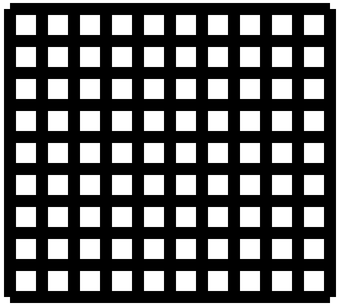
Classical turbulence behind pull-through grid



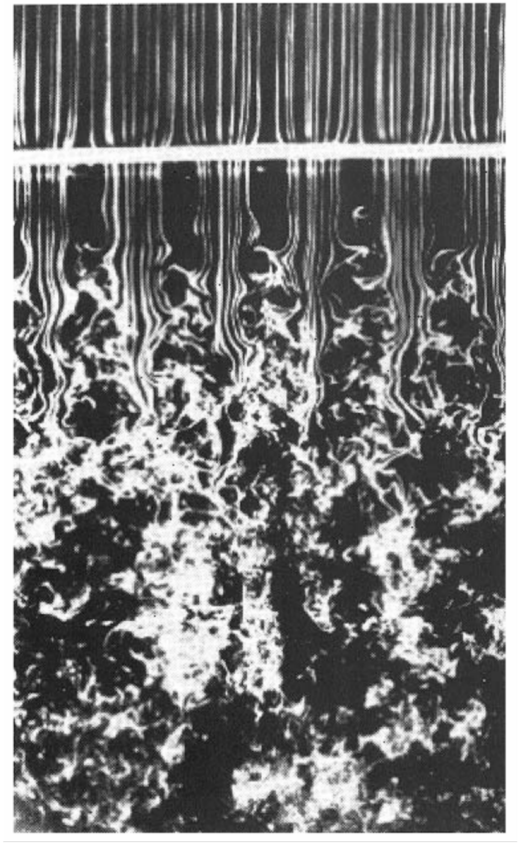
tank of water

$$<\omega^2>^{1/2} \sim t^{-1.5}$$

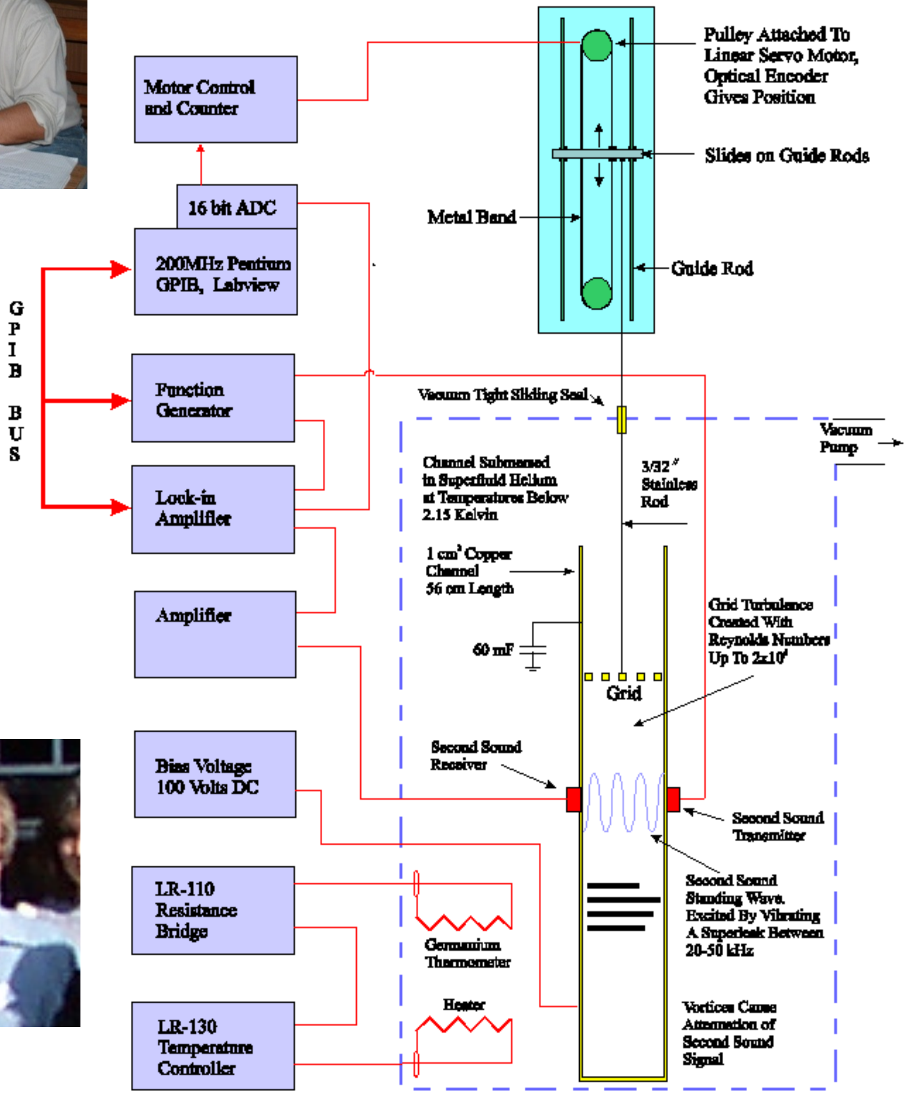
nearly isotropic turbulence is generated.

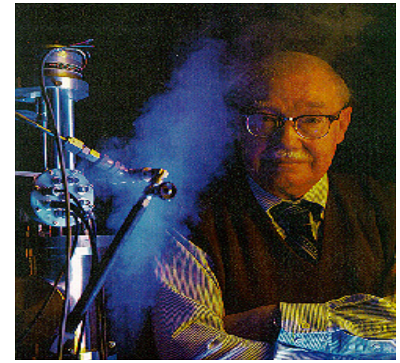


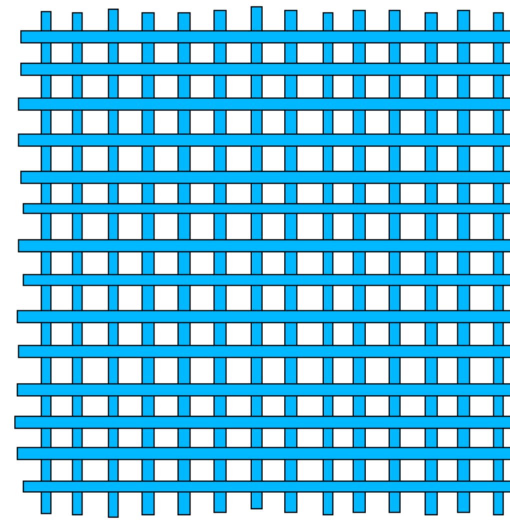
square grid of bars



grid turbulence in air: reoriented; Corke & Nagib

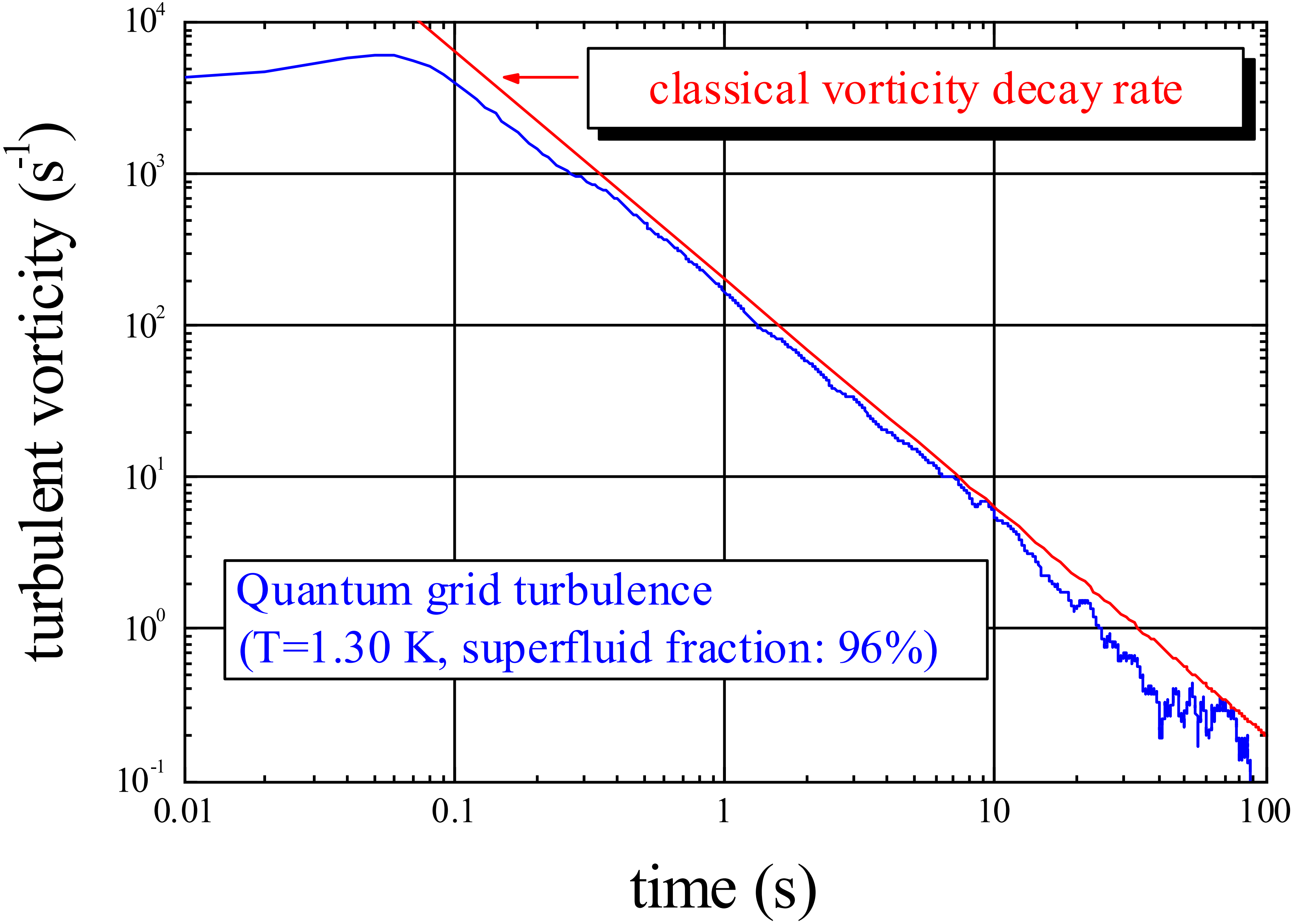




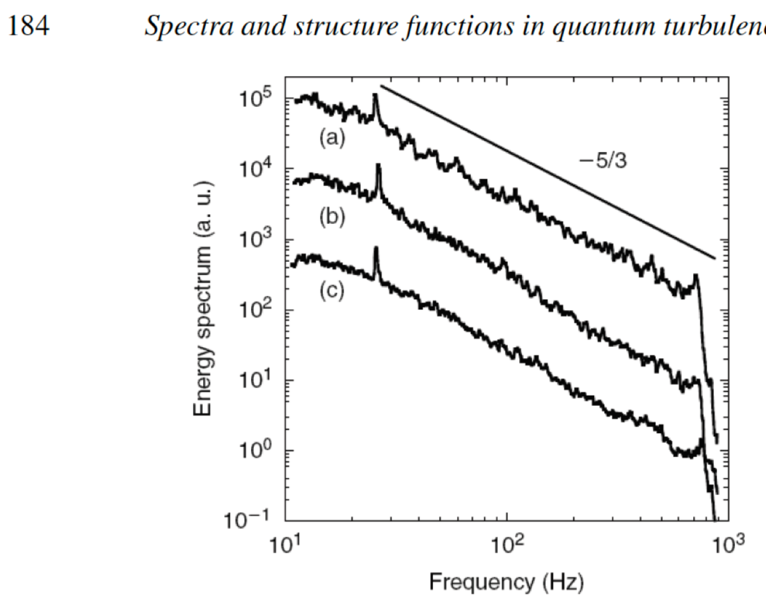


turbulence-generating grid
(as in Comte-Bellot & Corrsin)

Stalp, Niemela, Vinen, Donnelly, Skrbek, etc



classical vorticity decay rate
Quantum grid turbulence
(T=1.30 K, superfluid fraction: 96%)
turbulent vorticity (s^{-1})
time (s)
10^4
10^3
10^2
10^1
10^0
10^{-1}
0.01
0.1
1
10
100



Superfluid turbulence in Kármán flow:
J. Maurer & P. Tabeling, *Europhys. Lett.* **43**, 29 (1998)

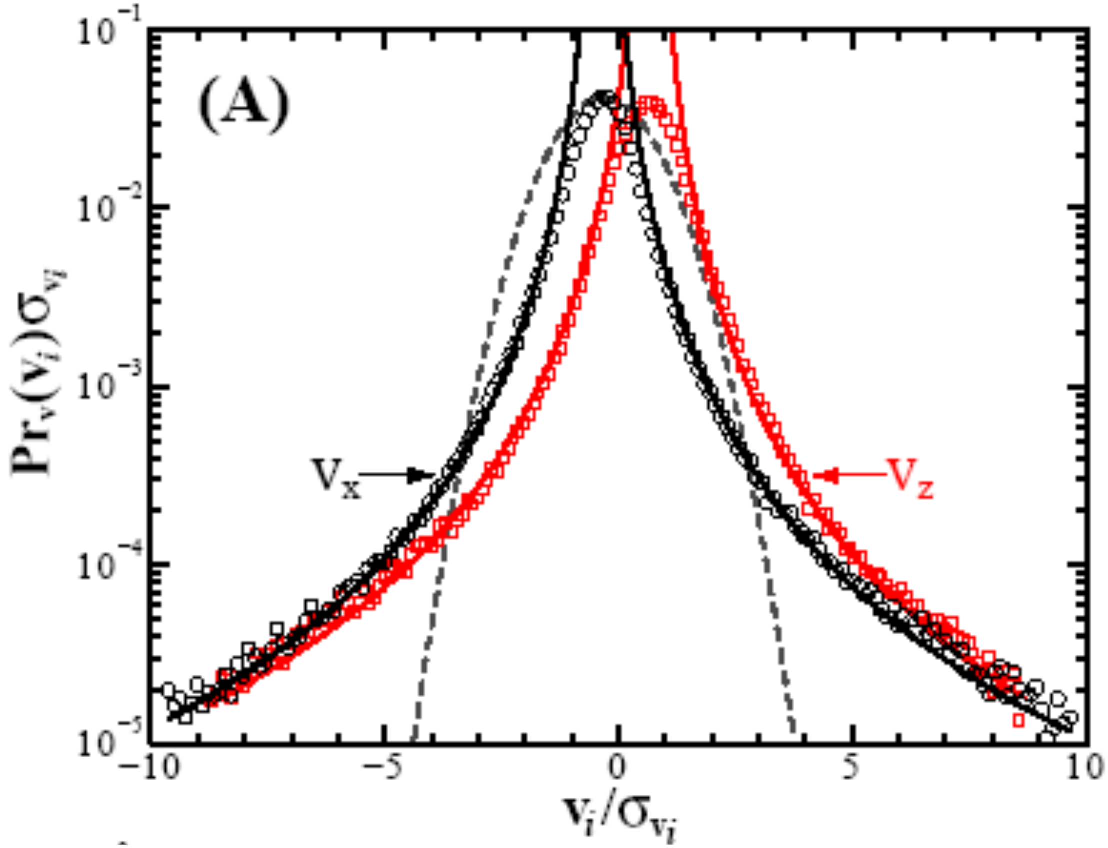
Obvious? Surprising?

A.N. Kolmogorov 1903-1987

In simulations:

- C. Nore, M. Abid & M.E. Brachet, *Phys. Rev. Lett.* **78**, 3896 (1997)
- T. Araki, M. Tsubota & S.K. Nemirovskii, *Phys. Rev. Lett.* **89**, 145301 (2002)
- M. Kobayashi & M. Tsubota, *Phys. Rev. Lett.* **94**, 065302 (2005)
- P.E. Roche *et al. Europhys. Lett.* **77**, 66002 (2007)
- Now many others

Nearly homogeneous turbulence following a counterflow



Pr(v) ~ $|v|^{-3}$
due to quantized vortices

No instances (away from solid boundary) where power-law tails exist for velocity distributions in classical turbulence.

Paoletti, Fisher, KRS & Lathrop, *Phys. Rev. Lett.* (2008)

Comparison of classical and quantum turbulence

- **–5/3 slope in the spectral form is common**
- **Decay law is the same as in classical turbulence**

Quantum turbulence (helium II)

- Velocity PDF follows a power law
- “Dissipation mechanism has to be quite different---but what is it?

Classical turbulence (3D)

- Velocity distribution is nearly normal
- Energy dissipation occurs because of fluid viscosity; may be related to the appearance of certain types of Hölder singularities of the weak solutions of the Euler equation (the so-called Onsager “conjecture”)

While both the large and small scales are different in the two cases, the energy dynamics in a wide range of scales seem to be locked in step. These aspects are reasonably well understood now.

Remarks on "dissipation mechanism" in quantum turbulence

(introduction to energy cascades, quantum length, Kelvin waves, and Kelvin cascade)

Kolmogorov cascade: Input at L, cascades downscale without dissipation, truncated at $\eta = (\nu^3/\varepsilon)^{1/4}$

Define quantum length scale, $l = (\kappa^3/\varepsilon)^{1/4}$

(not too much smaller than η)

Kolmogorov cascade cannot proceed beyond l: because the granularity of vortex lines will then become visible

Energy propagates as Kelvin waves on individual vortices (Fonda et al. PNAS 111, 4707, 2014)

Kelvin cascade by nonlinearities (V.B. Eltsov & V.S. L'vov, JETP Lett. 111, 389, 2020)

Eventually it gets radiated, probably gets dissipated in boundary layers of the container.

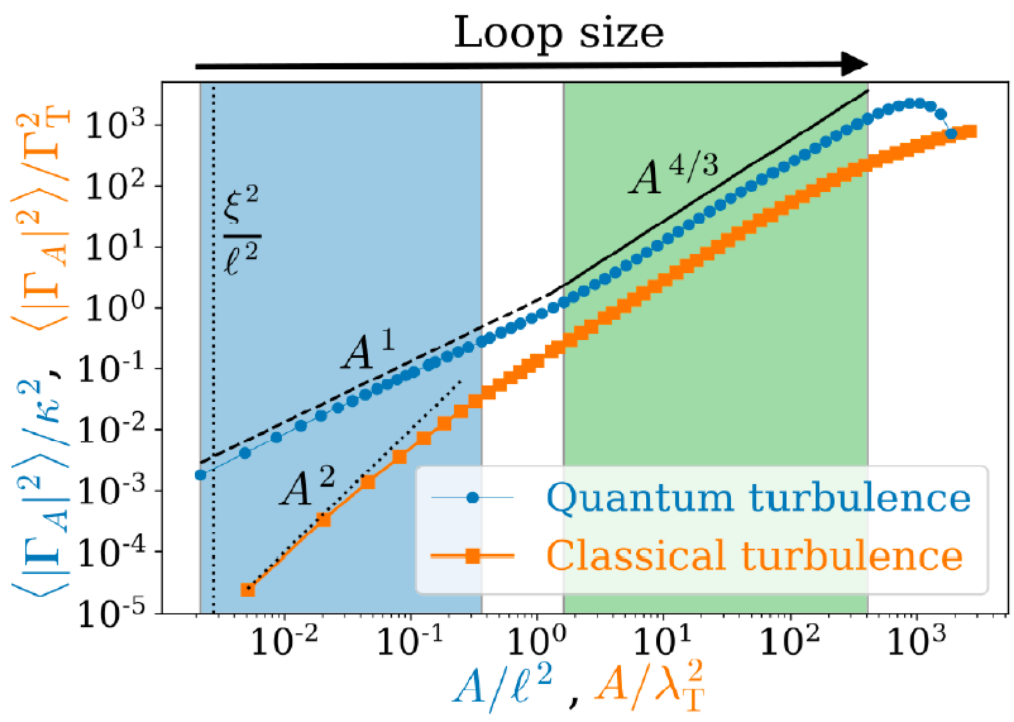


Figure 9.14 Scaling of the circulation in classical and quantum turbulence. The quantity $\langle|\Gamma_A|^2\rangle$ (in units of the square of the quantum of circulation κ) is plotted vs the area A enclosed by the loop C (in units of ℓ^2 for quantum turbulence and in units of the square of the Taylor microscale λ_T for classical turbulence). Top (blue) curve: numerical simulation of the Gross-Pitaevskii equation; the initial condition is an ABC flow. Bottom (orange) curve: numerical simulation of the Navier-Stokes equation. The left blue region and the right green region represent the quantum length scales and the classical length scales, respectively. Reproduced with permission from Müller *et al.* (2021).

V.L. Ginzburg (2003 Nobel Prize in Physics)

***Physics Today*, May 1990, page 9; *Uspekhi* 42 (4), 353, 1999**

Classified Physics into

Microphysics, Astrophysics and Macrophysics

(the small, the large and the complex)

One of the 11 items of Macro- or complex physics:

"Strongly Nonlinear Phenomena: Turbulence"

What is the broad inference?

The underlying allure in the study of complex system is the notion of 'universality'; that is, "microscopic details could be different, but classes of macroscopic phenomena are quantitatively alike".

This is true at some coarse level but appears to need further caveats when explored more deeply. That is, details do matter in principle.