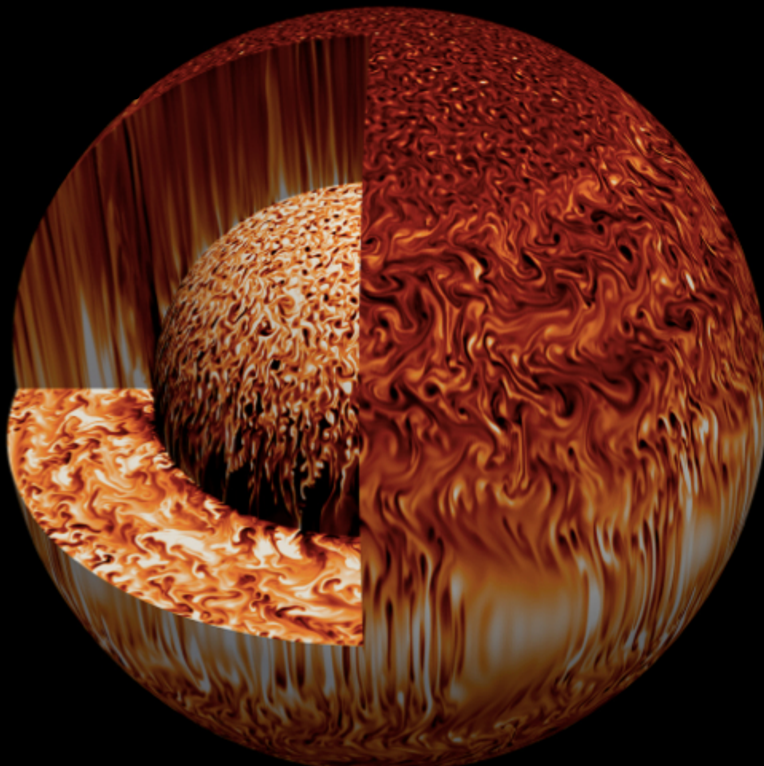
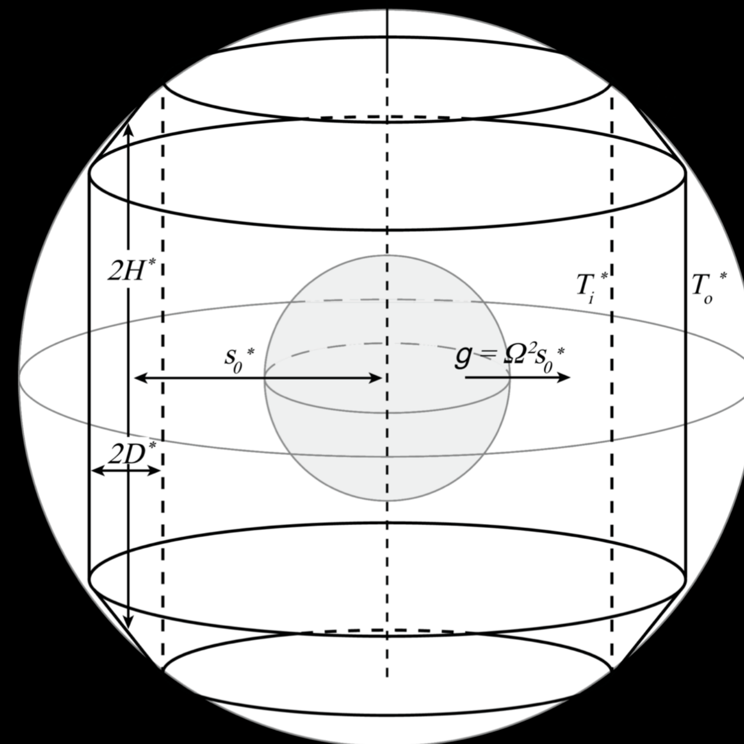
QL vs GQL for a model of Jupiter: “The Busse Annulus”

Courtesey T. Gastine





Busse (1976)

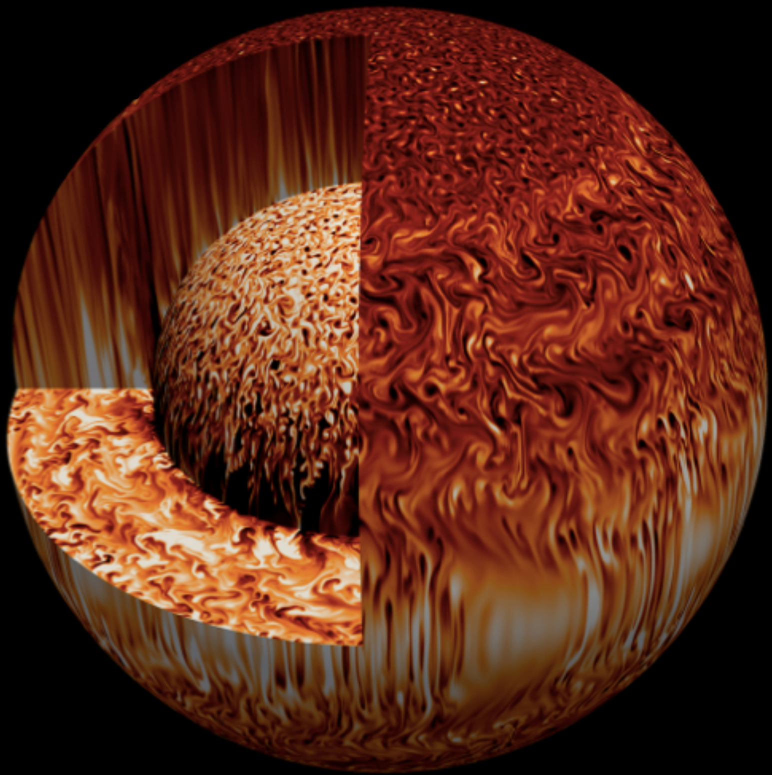
$$E\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) + 2\boldsymbol{e}_z \times \boldsymbol{u} = -\nabla P - Ra' T\,\boldsymbol{e}_y + E\nabla^2\boldsymbol{u},$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T = \frac{1}{Pr}\nabla^2 T,$$

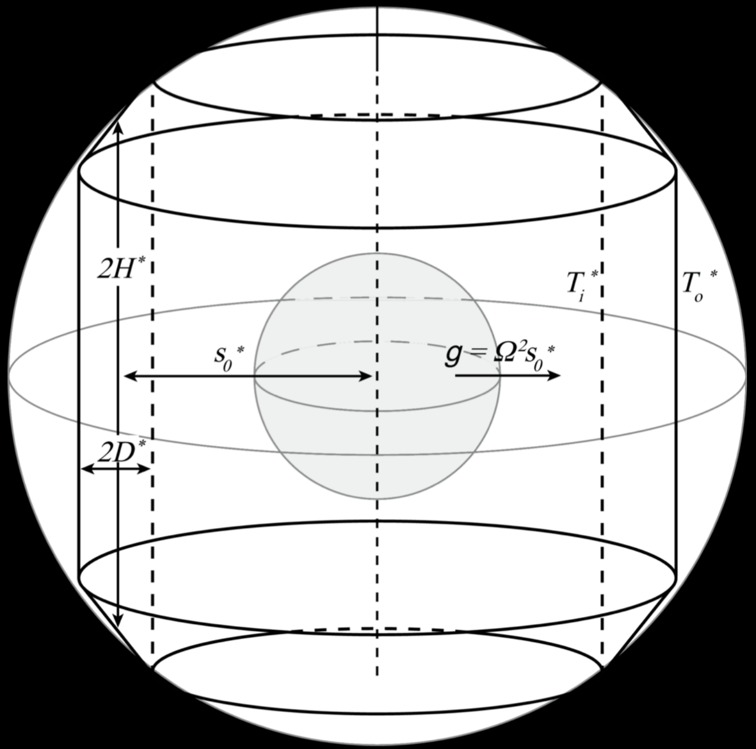
$$\nabla\cdot\boldsymbol{u} = 0.$$

$$Ra' = \frac{\alpha g \Delta T d}{\nu \Omega}, \quad E = \frac{\nu}{\Omega d^2}, \quad Pr = \frac{\nu}{\kappa}.$$

Deep Flows: 2D occurs due to strong rotation



Courtesey T . Gastine



Busse (1976)

$$\mathbf{u} = -\nabla \times (\psi(x,y)\mathbf{e}_z) + \mathbf{u}'(\mathbf{r}) \quad \hat{T} = \hat{T}(x,y), \; |\mathbf{u}'| \ll |\nabla \times (\psi(x,y)\mathbf{e}_z)|$$

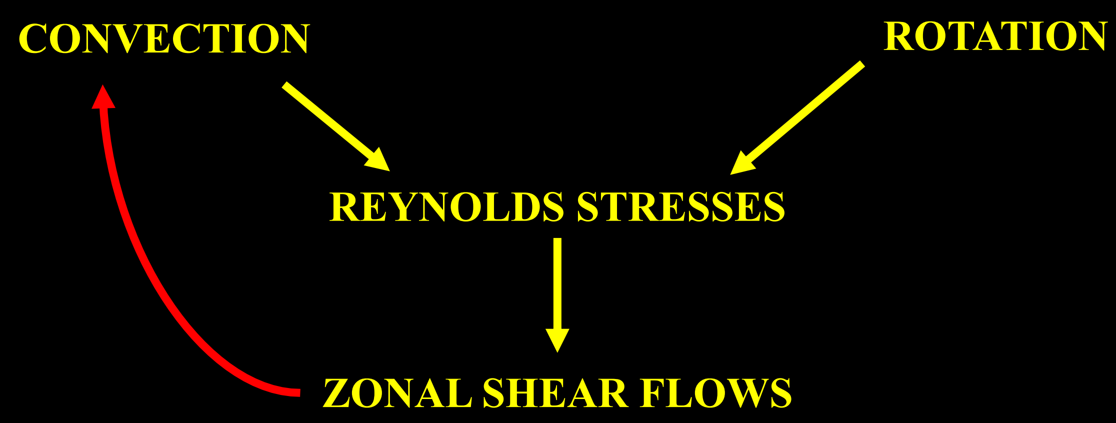
$$\frac{\partial \zeta}{\partial t} + J(\psi,\zeta) - \beta\frac{\partial \psi}{\partial x} = -\frac{Ra}{Pr}\frac{\partial \hat{T}}{\partial x} - C|\beta|^{1/2}\zeta + \nabla^2\zeta$$

$$\frac{\partial \hat{T}}{\partial t} + J(\psi,\hat{T}) = -\frac{\partial \psi}{\partial x} + \frac{1}{Pr}\nabla^2\hat{T} \quad J(A,B) = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial B}{\partial x}\frac{\partial A}{\partial y}$$

$$\beta = 2(\theta_T - \theta_B)\frac{d}{EL_z}$$

$$C = \left(\frac{1}{|\theta_T - \theta_B|}\frac{d}{L_z}\right)^{1/2}$$

Deep Models: jet formation. 2d-Dynamics



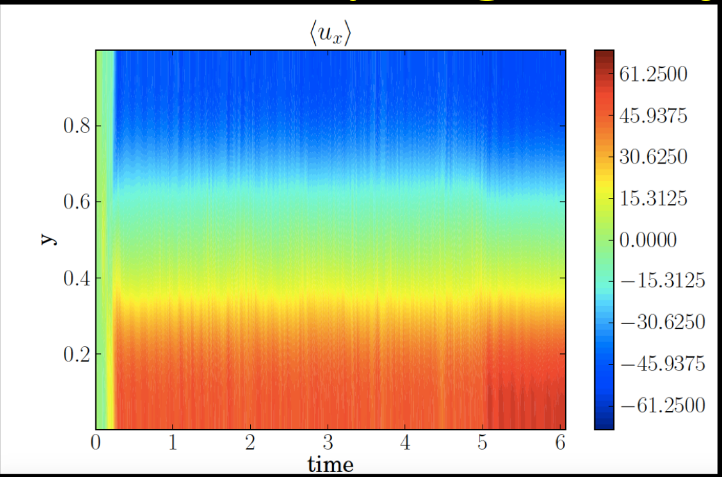
- As convective driving is increased, convection sets in for $Ra > Ra_c$
- Convective cells interact with rotation via beta-effect to drive zonal (shear) flows.
  - Thermally driven Rossby waves
  - Turbulent eddies with correlations → Reynolds stress
- Shear flow acts back to suppress the convection and lead to saturation
  - Shear inhibits convection
- Depending on parameters one can get a wide variety of behaviour.

Brummell & Hart, Rotvig & Jones (2006), Tobias, Oishi & Marston (2017)

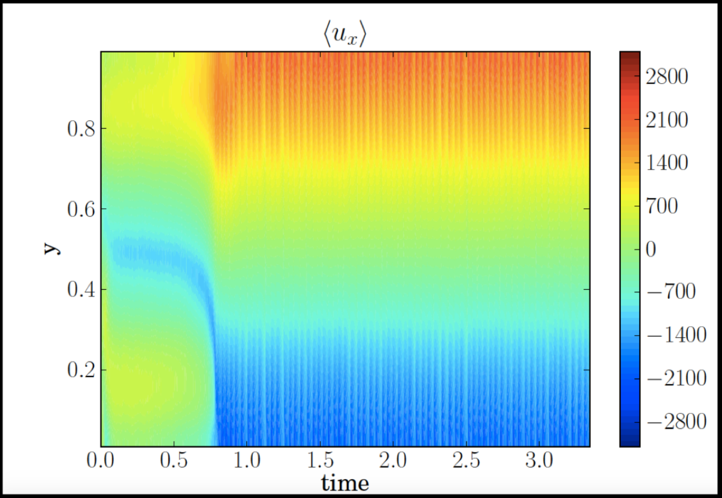
Deep Models: jet formation. 2d-Dynamics

Three types of behaviour

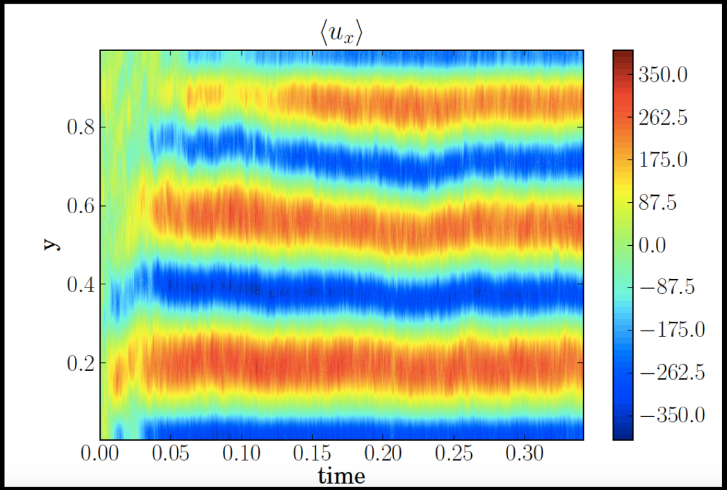
Quasi-steady large-scale jets



Brummell & Hart, Rotvig & Jones (2006), Tobias, Oishi & Marston (2017)



Quasi-steady multiple jets

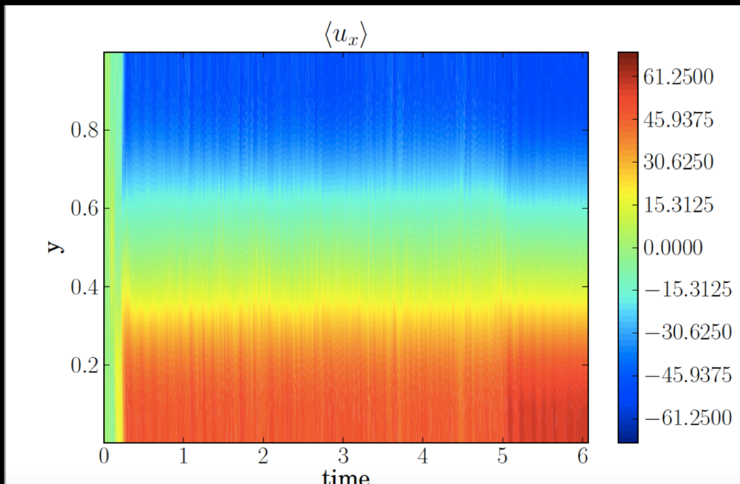


Bursting jets

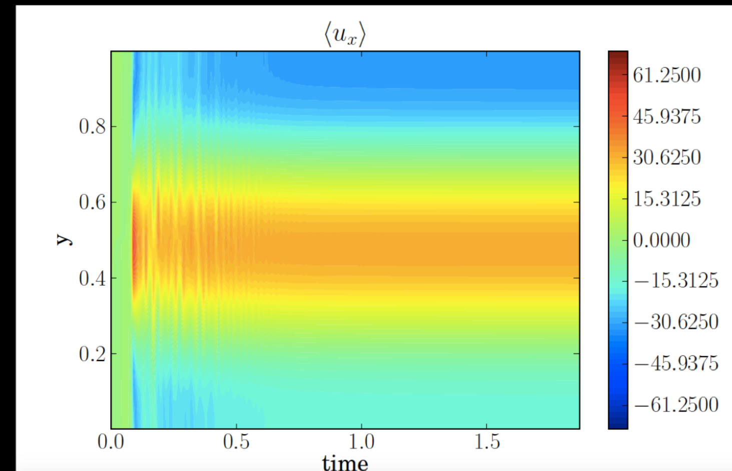
Predator-Prey Dynamics between zonal flows and convection

Reminiscent of zonal flow/turbulence interaction in pipe flow (Shih et al 2016)

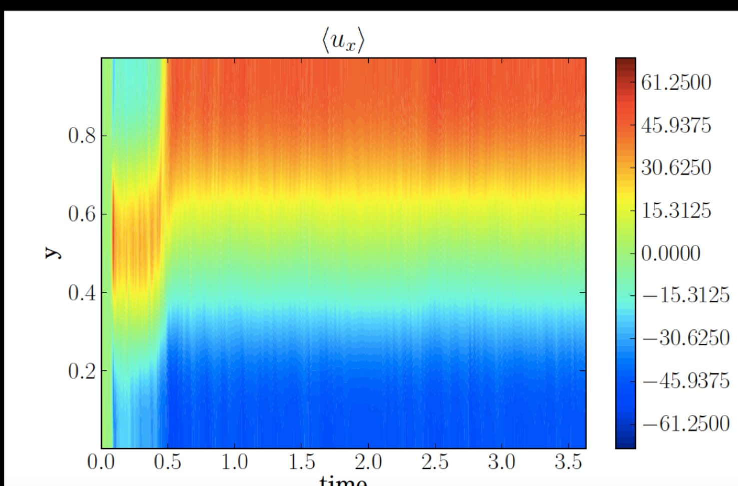
GQL vs QL?



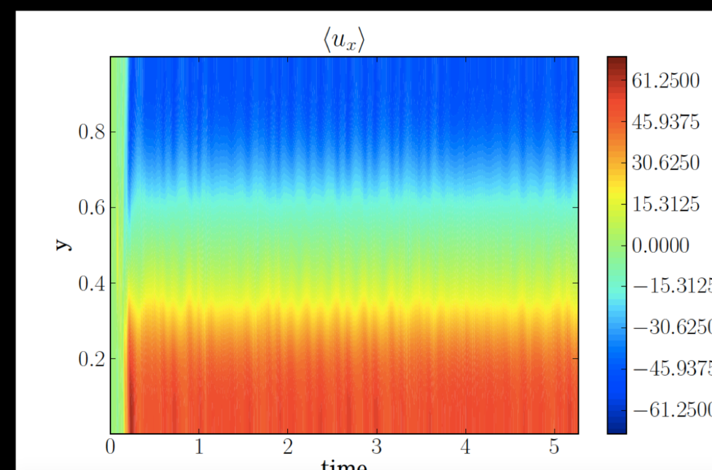
All DNS



0 QL

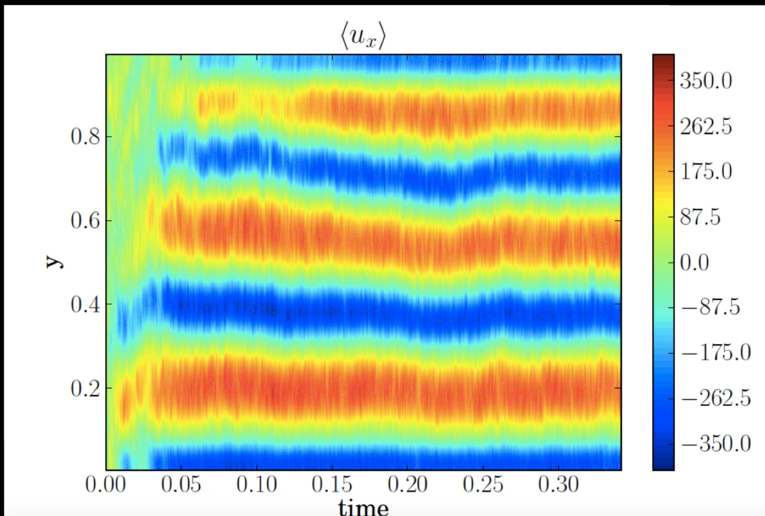


1 GQL



5 GQL

GQL vs QL?



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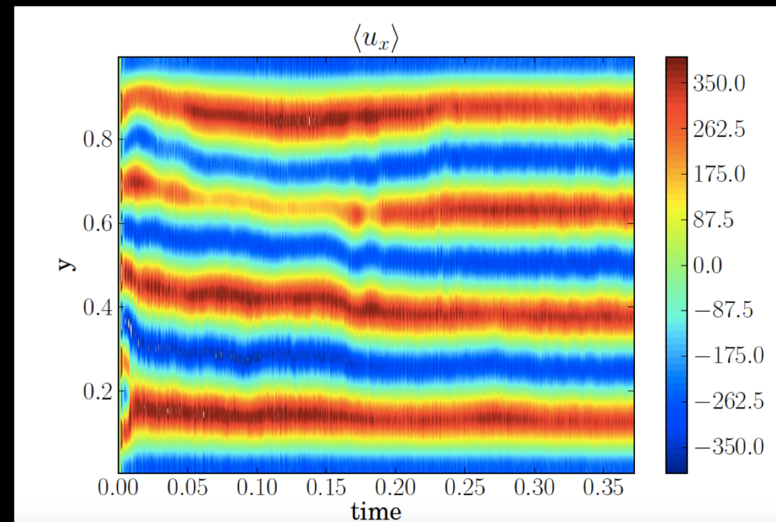
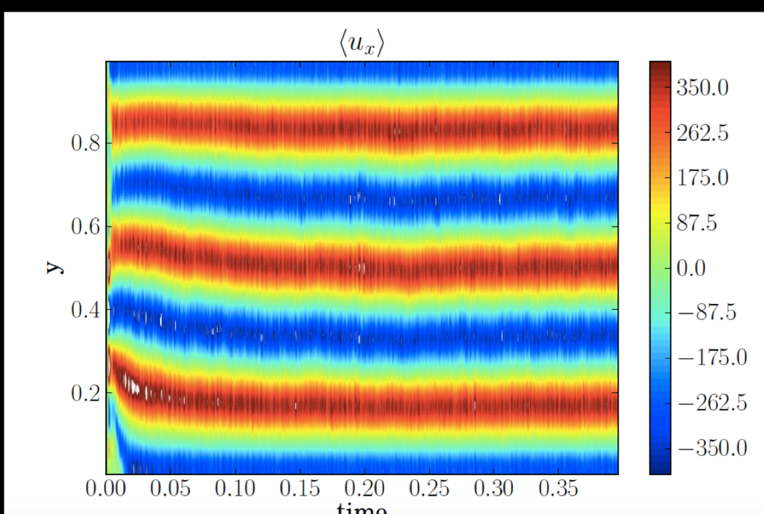
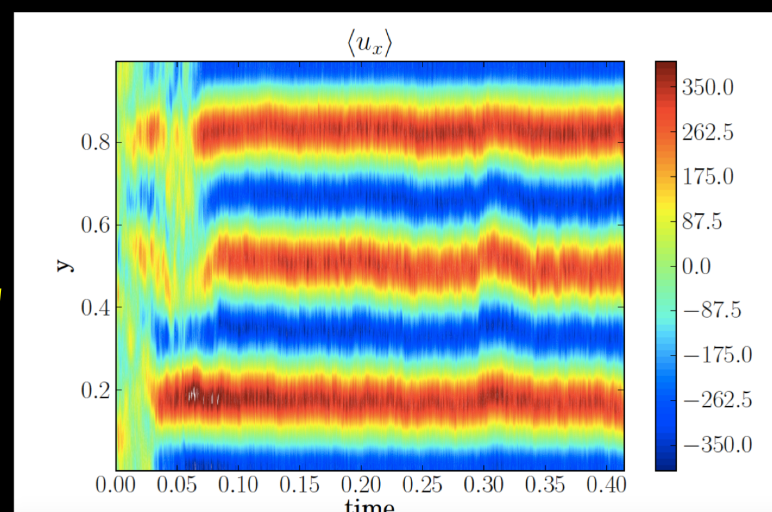


0 QL

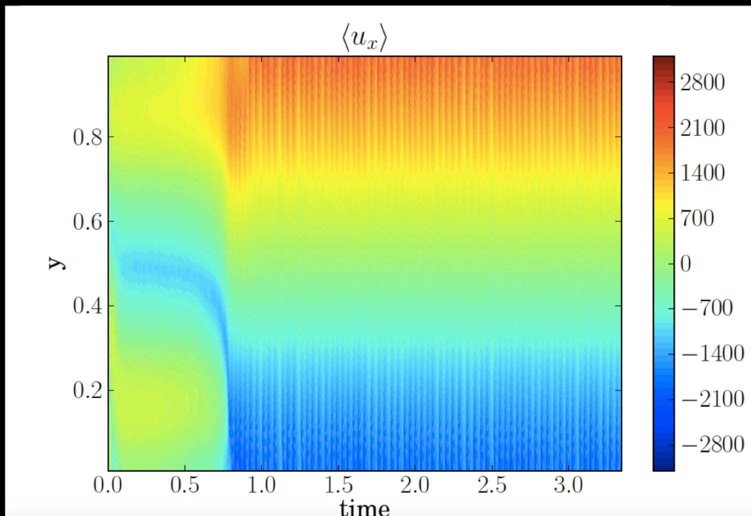


1 GQL

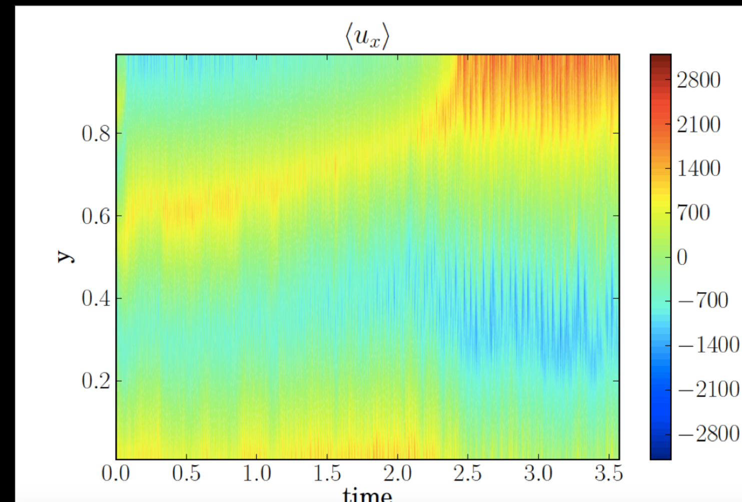


5 GQL

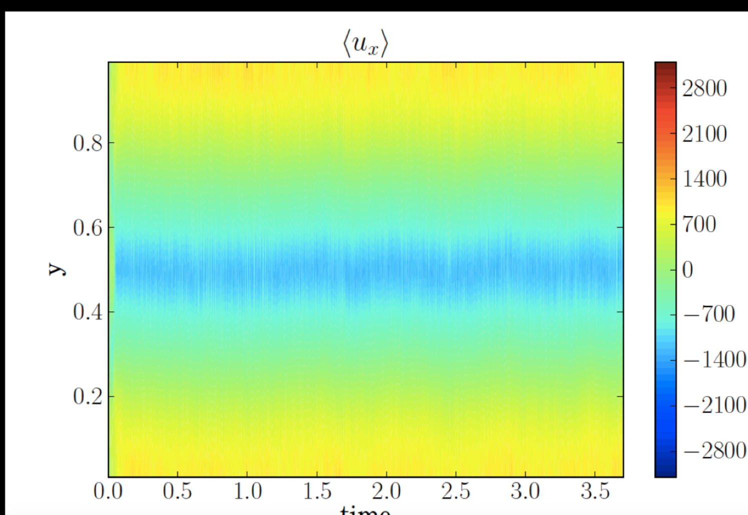
GQL vs QL?



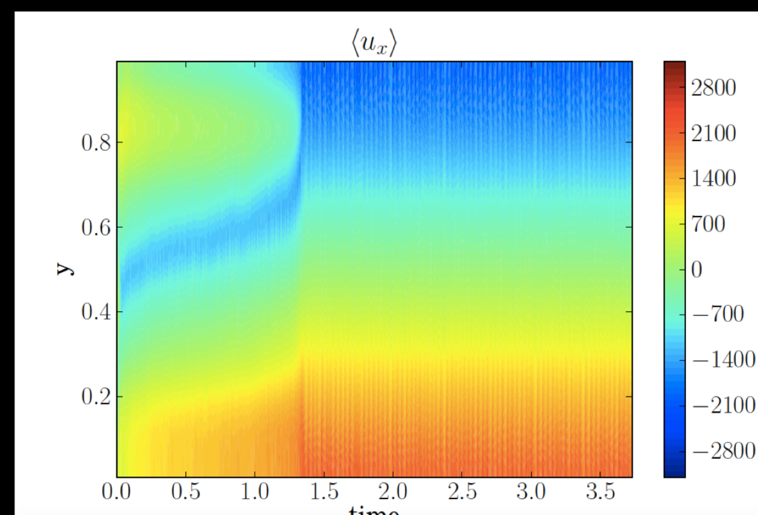
All DNS



0 QL

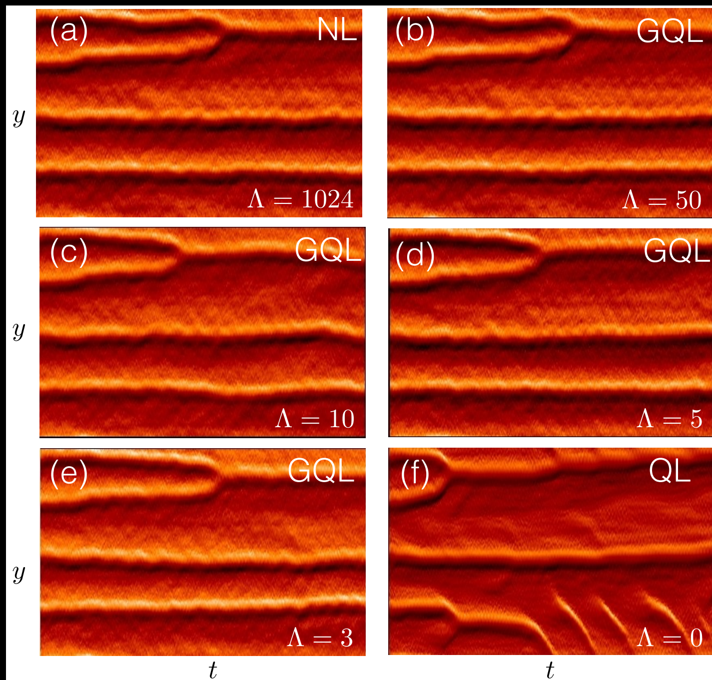


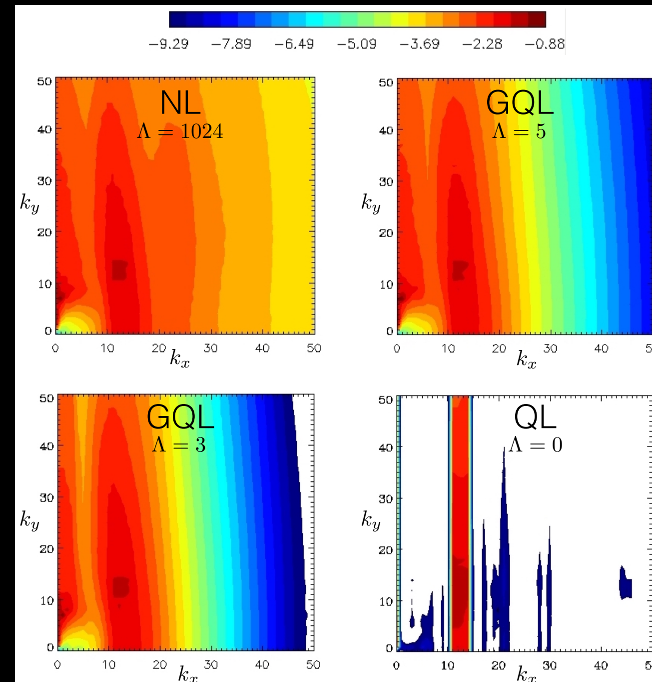
1 GQL



5 GQL

Barotropic Jet. QL vs GQL (Beta-plane)





Marston, Chini & Tobias, PRL (2016)

- **On a beta-plane even QL with a small cut-off Λ=3 can reproduce merging of jets.**
- **QL struggles to get number of jets correct.**
- **Formal derivation of GQL can be achieved via asymptotic expansion**
  - **Small parameter is related to degree of lack of statistical equilibrium**
  - **In this case, the small parameter can be related to ratio of dissipation to forcing.**

Low modes

$$\partial_T\bar{\zeta} + J(\bar{\psi},\bar{\zeta}) + \bar{\beta}\partial_X\bar{\psi} = -\partial_y(\overline{\zeta'\partial_\chi\psi'}) - \bar{\kappa}\,\bar{\zeta} + \bar{\mathcal{D}},$$

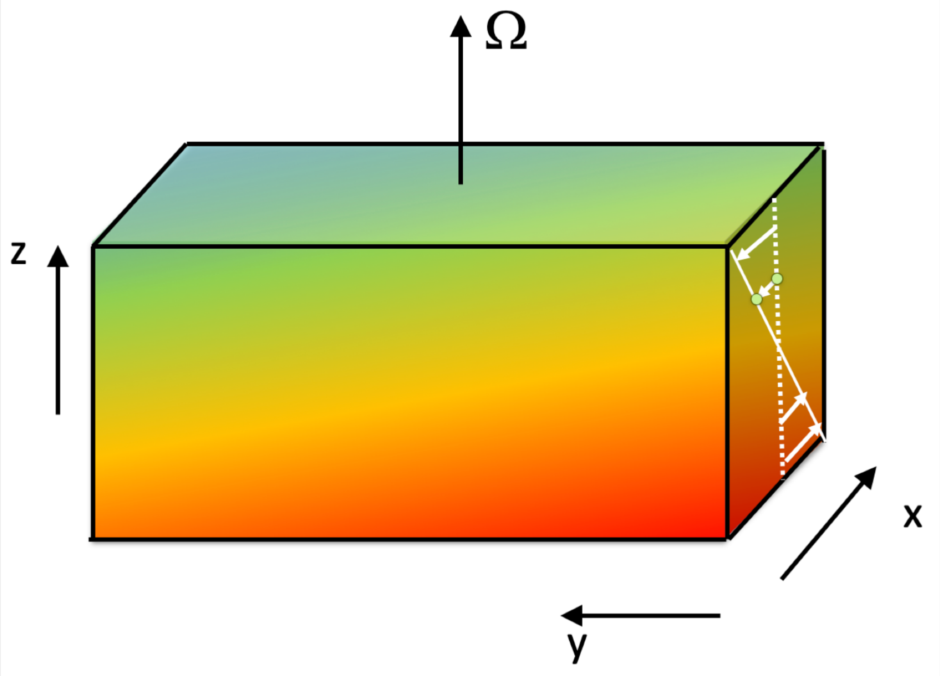
$$\partial_y^2\bar{\psi} = \bar{\zeta},$$

High modes

$$\partial_\tau\zeta' - \partial_y\bar{\psi}\partial_\chi\zeta' + \partial_\chi\psi'\partial_y\bar{\zeta} + \bar{\beta}\partial_\chi\psi' = \eta'(\tau),$$

$$(\partial_\chi^2 + \partial_y^2)\psi' = \zeta',$$

Another Example: 3d Rotating Convection with a thermal wind



- Another simple model of the interaction of convection with shear (relevant to stars/planets)
- Hathaway et al 1980
- Currie 2014
- Currie & Tobias 2019
- ϕ is latitude (box as shown is at the pole)
- T_y negative temperature increases southwards

$$T_0 = T_c + T_y y - z,$$

$$U_0 = -\frac{T_y Ra}{Ta^{1/2} \sin\phi}\left(z - \frac{1}{2}\right)$$

$$Ta = 4\Omega^2 d^4/\nu^2$$

$$Ra = \alpha g d^4 \partial T/\partial z/\kappa\nu$$

$$Pr = \nu/\kappa$$

- Saxton et al (2021)

Another Example: 3d Rotating Convection with a thermal wind

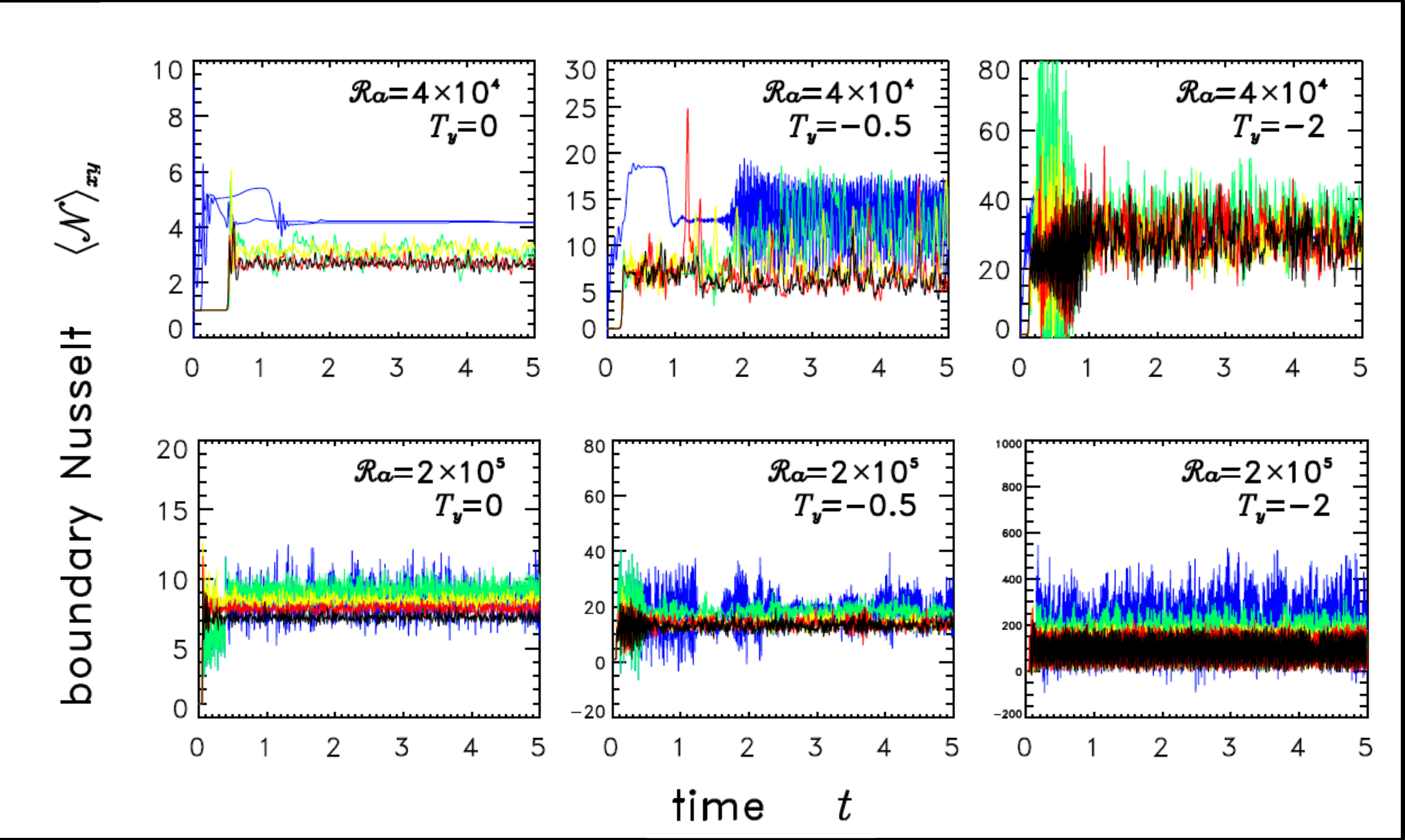
$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + U_0 \frac{\partial \boldsymbol{u}}{\partial x} + w \frac{\mathrm{d} U_0}{\mathrm{d} z} - Pr \nabla^2 \boldsymbol{u} + Ta^{1/2} Pr \boldsymbol{\Omega} \times \boldsymbol{u} + Pr \boldsymbol{\nabla} p - Ra Pr \theta \hat{\boldsymbol{e}}_z = -(\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u},$$

$$\frac{\partial \theta}{\partial t} + U_0 \frac{\partial \theta}{\partial x} - \nabla^2 \theta - w + T_y v = -(\boldsymbol{u} \cdot \boldsymbol{\nabla}) \theta,$$

- Turbulence can extract energy from both the temperature gradient directly or from the shear (if T_y non-zero)
- How well do QL/GQL do as T_y changes and shear increases

Another Example: 3d Rotating Convection with a thermal wind



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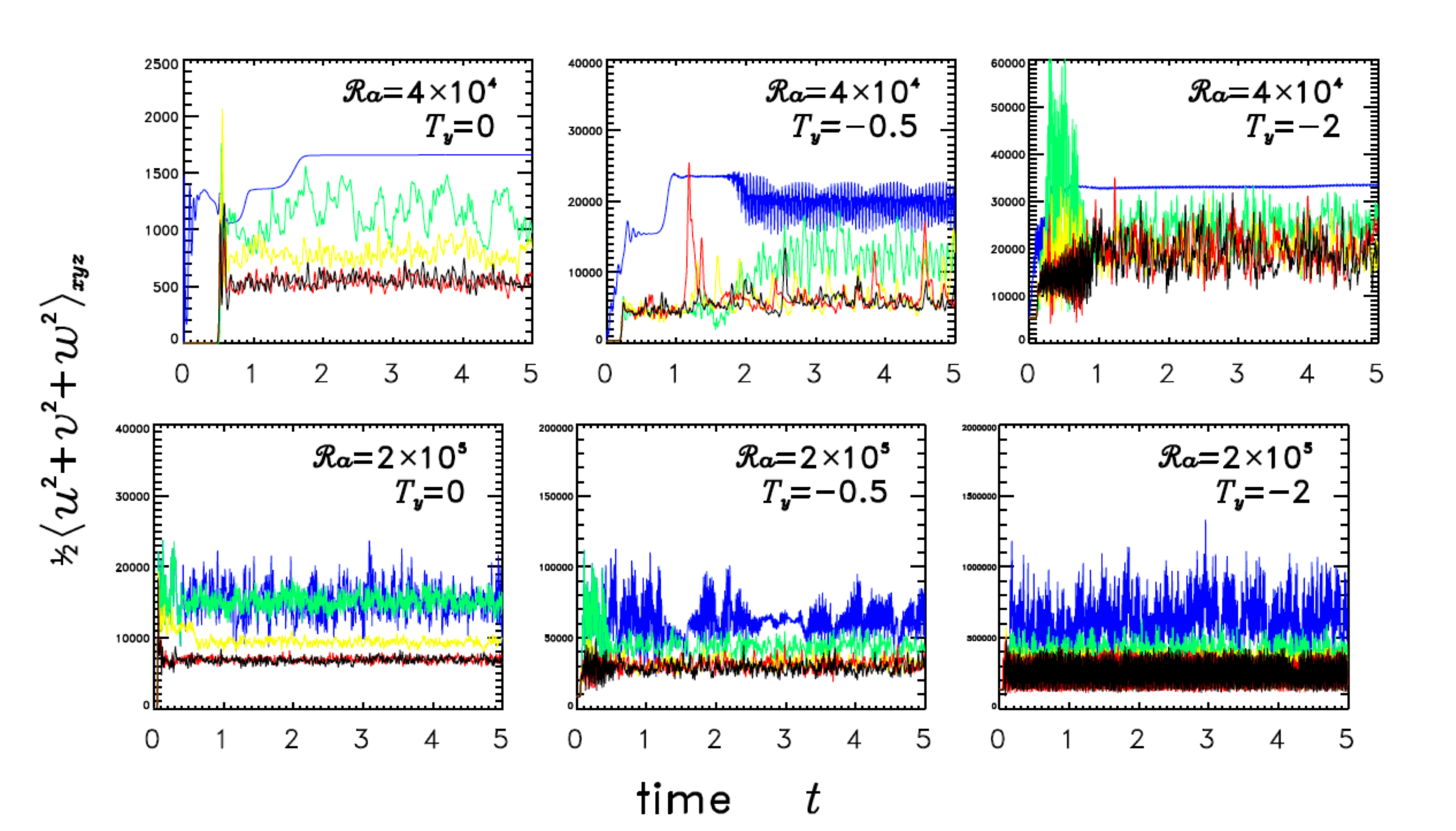
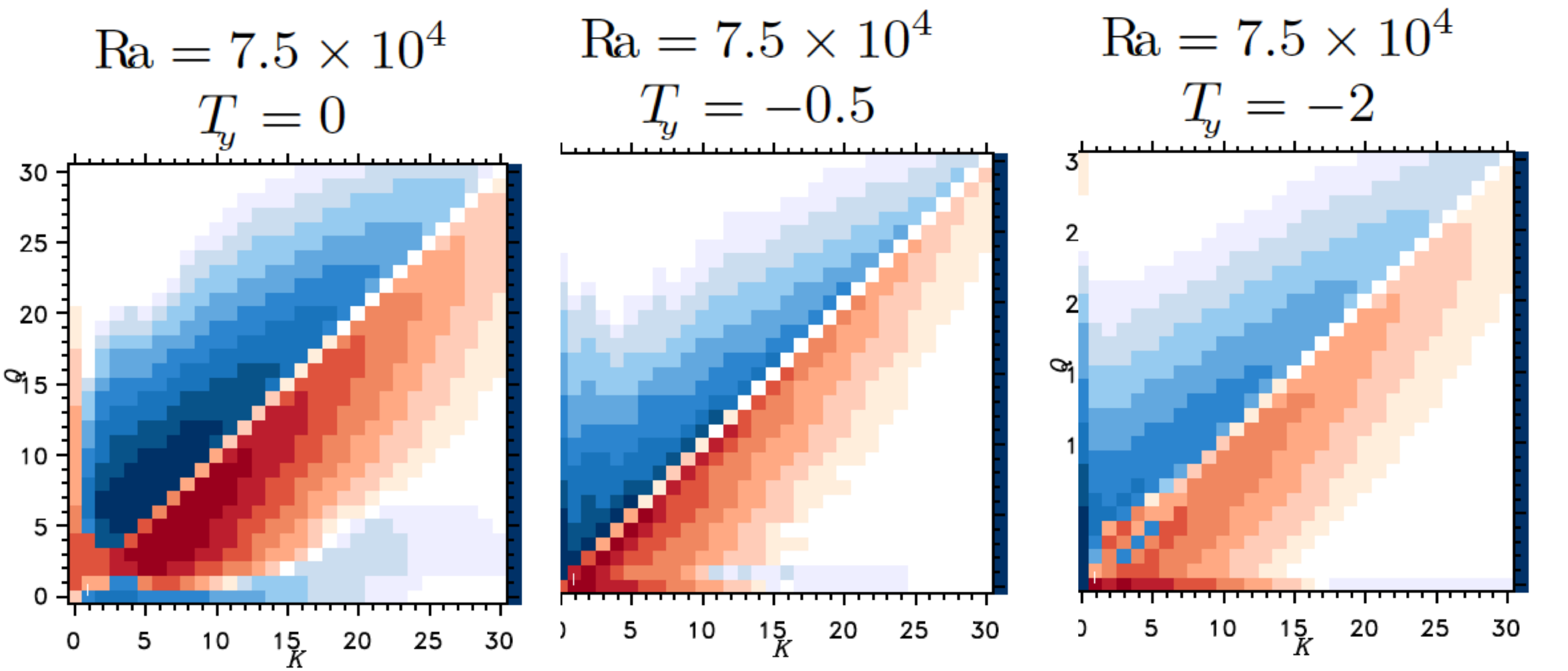


Another Example: 3d Rotating Convection with a thermal wind

Transfer Functions



Non-local inverse cascade (Reynolds Stress)

Extract Energy from Large-scale shear