Course outline

1.

A reminder about concepts and an overview of experiments: how to entangle atoms and photons and realise quantum gates.

2.

Tests of complementarity and exploration of the quantum/classical boundary with coherent states of radiation
2. Tests of complementarity and exploration of the quantum/classical boundary with coherent states of radiation

Entangle a qubit with a mesoscopic system: how to encode information in a large object

When is a coherent field “quantum” or “classical”?

How to prepare large Schrödinger cats with a resonant atom/field interaction?
Outline of lecture

2.1. A complementarity experiment at the quantum/classical boundary
   Realization of a thought experiment based on Rabi oscillation and Ramsey interferometry

2.2. Single atom/mesoscopic field entanglement: how a coherent field evolves from quantum to classical.
   - An unexpected aspect of Rab Oscillation
   - A new tool to prepare and study Schrödinger cats
2.1. A complementarity experiment at the Quantum/Classical boundary
The “strangeness” of the quantum

- Feynman: Young’s slits experiment contains all the mysteries of the quantum
The “strangeness” of the quantum: a thought experiment about complementarity (Bohr-Einstein debate, Solvay 1927)

- Microscopic slit: set in motion when deflecting particle. Which path information and no fringes
- Macroscopic slit: insensitive to interfering particle. No which path information: fringes are observed
- Wave and particle are complementary aspects of the quantum object.
A “modern” version of Bohr’s proposal

- Mach Zehnder interferometer

- Interference between two well-separated paths.
  - Getting a which-path information?
A “modern” version of Bohr’s proposal

- Massive beam splitter: negligible motion, no which-path information, fringes
- Microscopic beam splitter: which path information and no fringes
Complementarity and entanglement

- A more general analysis of Bohr’s experiment
  - Initial beam-splitter state $|0\rangle$
  - Final state for path b $|\alpha\rangle$
  - Particle/beam-splitter state $\Psi = \Psi_a |0\rangle + \Psi_b |\alpha\rangle$
    - Particle/beam-splitter entanglement
      - (an EPR pair if states orthogonal)
  - Final fringes signal $\langle \Psi_a | \Psi_b \rangle \langle 0 | \alpha \rangle$
    - Small mass, large kick
      - NO FRINGES $\langle 0 | \alpha \rangle = 0$
    - Large mass, small kick
      - FRINGES $\langle 0 | \alpha \rangle = 1$
Entanglement and complementarity

Entanglement with another system destroys interference

- explicit detector (beam-splitter/ external)
- uncontrolled measurement by the environment (decoherence)

Complementarity, decoherence and entanglement intimately linked
A more realistic system: Ramsey interferometry

- Two resonant $\pi/2$ classical pulses on an atomic transition e/g

Which path information?
- Atom emits one photon in $R_1$ or $R_2$

Ordinary macroscopic fields
- (heavy beam-splitter)
  - Field state not appreciably affected. No "which path" information
  - FRINGES

Mesoscopic Ramsey field
- (light beam-splitter)
  - Addition of one photon changes the field. "which path" info
  - NO FRINGES
Coherent states of the field: a system evolving from quantum to classical

Field radiated by a classical source in the mode

$$| \alpha > = e^{-|\alpha|^2/2} \sum_{n} \frac{\alpha^n}{\sqrt{n!}} | n >$$

Poissonian distribution of the photon number

$$p(n) = e^{-|\alpha|^2} \frac{|\alpha|^n}{n!}$$

Representation in the complex plane

$$| \alpha | = \sqrt{n} = \Delta n \quad \rightarrow \quad \frac{\Delta n}{\bar{n}} = \frac{1}{|\alpha|}$$

"Quantum" field
Big fluctuations

$$| \alpha | \approx 1$$

"Classical" field
Small fluctuations

$$| \alpha | \gg 1$$

: a continuous parameter to explore the quantum classical boundary
Experimental requirements

• Ramsey interferometry
  – Long atomic lifetimes
  – Millimeter-wave transitions
    • Circular Rydberg atoms

\[ \pi/2 \text{ pulses in mesoscopic fields} \]
  – Very strong atom-field coupling
    • Circular Rydberg atoms

• Field coherent over atom/field interaction
  • Superconducting millimeter-wave cavities
General scheme of the experiments

Rev. Mod. Phys. 73, 565 (2001)
Resonant atom-cavity interaction: Rabi oscillation in vacuum

Initial state $|e,0>$

Vacuum Rabi frequency
$\Omega = 50 \text{ kHz}$

In a large coherent field, Rabi frequency becomes $\Omega \sqrt{n}$

Oscillatory Spontaneous emission and strong coupling regime.
Bohr’s experiment with a Ramsey interferometer

- Illustrating complementarity: Store one Ramsey field in a cavity

  - Initial cavity state $|\alpha\rangle$
  - Intermediate atom-cavity state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|e,\alpha_e\rangle + |g,\alpha_g\rangle)$
  - Ramsey fringes contrast $|\langle\alpha_e|\alpha_g\rangle|$
    - Large field
      - $|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle$ FRINGES
    - Small field
      - $|\alpha_e\rangle = |0\rangle, |\alpha_g\rangle = |1\rangle$ NO FRINGE

Atom-cavity interaction time Tuned for $\pi/2$ pulse Possible even if C empty From quantum to classical
Quantum/classical limit for an interferometer

Fringes contrast versus photon number $N$ in first Ramsey field

Fringes vanish for quantum field

- photon number plays the role of the beam-splitter’s "mass"

Also an illustration of the $\Delta N \Delta \Phi$ uncertainty relation:

- Ramsey fringes reveal field pulses phase correlations.
- Small quantum field: large phase uncertainty and low fringe contrast

Nature, 411, 166 (2001)
An elementary quantum eraser

• Another thought experiment

Two interactions with the same beamsplitter assembly erase the which path information and restore the interference fringes
Ramsey “quantum eraser”

- A second interaction with the mode erases the atom-cavity entanglement

\[ \frac{1}{\sqrt{2}} (|e, 0\rangle + |g, 1\rangle) \]

Atom found in g: one photon in C whatever the path: no info and fringes

- Ramsey fringes without fields!
  - Quantum interference fringes without external field
  - A good tool for quantum manipulations
Entanglement between a mesoscopic coherent field and a single atom

The Ramsey interference experiment shows that, during a $\pi/2$ pulse, the atom and the field do not get entangled when $n >> 1$:

**NO ENTANGLEMENT** during time $t_{\pi/2} = \pi/2 \Omega \sqrt{n}$

Atom and field get however **ENTANGLED** if they are coupled for a longer time, of the order of $2\pi/\Omega$:

$$|e\rangle |\alpha\rangle \rightarrow |\Psi^+\rangle_{\text{atom}} |\alpha^+\rangle + |\Psi^-\rangle_{\text{atom}} |\alpha^-\rangle$$

Coherent field split into two components:
**Mesoscopic superposition of coherent states with opposite phases**

Rabi oscillation collapse and revivals revisited
To be classical a field in a cavity must be coherent and contain many photons on average.

Correspondance principle: a coherent field with many photons has small relative fluctuations and behaves asymptotically classically.

The interaction with an atom, which can emit or absorb at most one photon, is expected to leave a « large » field practically « unperturbed » and the « atom + field system » unentangled:

\[ |\alpha (0)> |\Psi_{\text{atom(0)}}> \rightarrow |\alpha (t)> |\Psi^{(\alpha)}_{\text{atom(t)}}> \]

How large must the photon number be for this classical limit to be valid?

It depends on how long the interaction lasts… A large field exhibits quantum features if the interaction with the atom has enough time to create entanglement…. and if there is no decoherence.

Mesoscopic physics in Quantum Optics
Single atom/mesoscopic field entanglement: how a coherent field evolves from quantum to classical
Classical Rabi oscillation

Two-level system \{|e>;|g>\} interacting with a resonant field

Rotating frame
Rotating wave approximation

Eigenstates of the hamiltonian

\[
\begin{align*}
|+\rangle &= \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \\
|\rangle &= \frac{1}{\sqrt{2}}(-|e\rangle + |g\rangle)
\end{align*}
\]

Rabi oscillation at frequency \(\Omega_{cl}\)
Classical Rabi oscillation: an interference effect

\[ \psi(t) = \frac{1}{\sqrt{2}} \left( e^{-i\Omega t/2} |+\rangle + e^{i\Omega t/2} |-\rangle \right) \]

Evolution: \[ |e\rangle \rightarrow |\psi(t)\rangle \]
Detection in \{|e\rangle, |g\rangle\} basis

Classical Rabi oscillation: a quantum beat between two indistinguishable paths
Rabi oscillation in a quantized field

Two-level system \( \{|e\rangle, |g\rangle\} \) interacting with a resonant quantized field \(|n\rangle\)

Jaynes-Cummings Hamiltonian

\[
H_{JC} = \frac{\hbar \Omega_0}{2} \left( a^+ |g\rangle \langle e| + a |e\rangle \langle g| \right)
\]

Exchange of a quantum of energy

Eigenstates of the Hamiltonian: "Dressed states"

\[
\begin{align*}
|+_n\rangle &= \frac{1}{\sqrt{2}} (|e,n\rangle + |g,n+1\rangle) \\
|_n\rangle &= \frac{1}{\sqrt{2}} (|e,n\rangle - |g,n+1\rangle)
\end{align*}
\]

Rabi oscillation between \(|e,n\rangle\) and \(|g,n+1\rangle\) at frequency \(\Omega_0 \sqrt{n+1}\).
The vacuum Rabi oscillation

Initial state $|e,0\rangle \rightarrow$ Rabi oscillation at $\Omega_0$

$|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0 t}{2}\right)|e,0\rangle - i \sin\left(\frac{\Omega_0 t}{2}\right)|g,1\rangle$

Maximal entanglement at $t = \frac{\pi}{\omega_0}$

Vacuum Rabi frequency

Atom-field entangled state

Formation of an EPR-pair

Rabi oscillation in a quantum field

Continuous evolution?
Rabi oscillation in a mesoscopic coherent field

\[ P_e(t) = \frac{1}{2} \sum_n p(n) \left( \cos(\Omega_n t) + 1 \right) \]

\[ \Omega_n = \Omega_0 \sqrt{n + 1} \]

Collapses and revivals

\[ \Omega_n \approx \Omega_0 \sqrt{n} + \Omega_0 \]

Collapse when the side components are phase-shifted by \( \pi \)

Revival when two successive components recover the same phase

\[ T_{\text{coll}} \sim \frac{\Omega_0}{\delta \Omega} \]

\[ T_{\text{rev}} \sim T_0 \frac{\Omega_0}{\sqrt{n} \Omega} \]
Collapse and revival in cavity QED

Also observed for $n \propto 1$ in
- closed cavities (Rempe, PRL 58, 353)
- ion traps (Meekhof, PRL 76, 1796)

Direct proof of field quantization (photon graininess)

What about larger $n$’s?
What about the field evolution in this complex Rabi oscillation process?
Classical limit

Spectrum of the Rabi frequencies

In a coherent field

\[ \Omega_n \sqrt{n} \to \infty \]

\[ \Omega_0 \to 0 \]

\[ \Omega_0 \sqrt{n} \to \Omega \]

In the classical limit

Atomic signal only depends on the energy of the field

Effect on the phase of the field?
Rabi oscillation in a mesoscopic field

Initial state: \[ |\psi_0\rangle = \frac{1}{\sqrt{2}} \left( \sum_n \alpha_n C_n |n\rangle + \sum_n \alpha^*_n C_n^* |n\rangle \right) \]

\[ C_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \]

\[ |\psi_+(t)\rangle > \quad |\psi_-(t)\rangle > \]

Mesoscopic field:

\[ |\phi_+(t)\rangle > = |\psi_+\rangle + |\alpha_+(t)\rangle \]

\[ |\phi_-(t)\rangle > = |\psi_-\rangle - |\alpha_-(t)\rangle \]

\[ |\alpha_+(t)\rangle > = e^{i\bar{n} \phi(t)} |\alpha e^{-i\phi(t)}\rangle \]

\[ \varphi(t) = \frac{\Omega_0 t}{4\sqrt{n}} \]

\[ \bar{n} \gg 1 \]

\[ \Omega_0 t < 16 \sqrt{n} \]

Atomic superposition of quantum phase \(-\varphi(t)\)

Coherent field of classical phase \(-\varphi(t)\)

Phase correlation
Rabi oscillation in a mesoscopic field

\[ | e, \alpha \rangle \rightarrow \frac{1}{\sqrt{2}} \left( |\alpha_+(t)\rangle |\psi^{a'}_+(t)\rangle + |\alpha_-(t)\rangle |\psi^{a'}_-(t)\rangle \right) \]

The atomic dipole and the field are phase-entangled
Generation of a Schrödinger-cat state

Classical limit
\[ \sqrt{n} \rightarrow \infty \quad \Omega_0 \rightarrow 0 \quad \Omega_0 \sqrt{n} \rightarrow \Omega \]

\[ \varphi = \frac{\Omega_0 t}{4\sqrt{n}} \rightarrow 0 \quad \overline{n} \varphi = \frac{\Omega_0 \sqrt{n} t}{4} \rightarrow \frac{\Omega t}{4} \]

\[ |e, \alpha \rangle \rightarrow \frac{1}{\sqrt{2}} \left( e^{-i\Omega t/2} |+\rangle + e^{i\Omega t/2} |-\rangle \right) |\alpha\rangle \]

No atom-field entanglement
Field « classical »

Classical Rabi oscillation
Field unchanged
Geometrical representation

Atomic state in the equatorial plane of the Bloch sphere

Coherent field in the Fresnel plane

Equatorial plane of the Bloch sphere

Phase correlation

Atomic dipole and field « aligned »
Evolution of the atom-field system

\[ |\text{Initial state}| = \frac{1}{\sqrt{2}} (|\alpha_+| + \alpha |\alpha_-|) \]

A microscopic object leaves its imprint on a mesoscopic one

Schrödinger-cat situation

"Size" of the cat = \( D \)

\[ D = 2\sqrt{n} \sin \left( \frac{\Omega_0 t}{4\sqrt{n}} \right) \]

The field acts as a Which-Path detector

Contrast of the Rabi oscillation \( C(t) = |\langle \alpha_+(t) | \alpha_-(t) \rangle| = e^{-D^2(t)} \)
New insights on collapse and revival

\[ |e, \alpha > \rightarrow \frac{1}{\sqrt{2}} \left( |\alpha_+(t) > |\psi_{at}^+(t) > + |\alpha_-(t) > |\psi_{at}^-(t) > \right) \]

\[ C(t) = |< \alpha_+(t) | \alpha_-(t) >| = e^{-D(t)^2} \]

Collapse as soon as the two components are well separated

Field states merge again into a superposition state

Revival of the Rabi oscillation

\[ \psi_{field}^+ \sim \sqrt{\frac{\Omega}{4\sqrt{n}}} \left( |i\alpha > + (-1)^{n+1} \left| \frac{-i\alpha}{\Omega_0} \right> \right) \]

- Unconditional mesoscopic states superposition
- The field has a defined parity
- Size of the cat-distance D

\[ \phi = \frac{\Omega_0 T_R}{\sqrt{n}} \approx \pi \]

\[ T_R \approx 2T_0 \sqrt{n} \]

Maximal entanglement: "Schrödinger cat state"
An exact calculation

Rabi oscillation in 20 photons

Atom initially in $|g\rangle$

Q function evolution in 20 photons
Field phase distribution measurement

How to measure a coherent field phase-shift?

Homodyne method

Injection of a coherent field $|\alpha >$

Second injection $| -\alpha e^{i\phi} >$

Resulting field $| \alpha (1 - e^{i\phi}) >$

Back to the vacuum state $\phi_S = 0$

A probe atom is sent in $| g >$

- Field in the vacuum state $P_g \approx 1$
- Field in an excited state $P_g \approx 1/2$

$P_g (\phi_S) = a$ signal to measure the field phase distribution

Field phase-shifted by $\Delta \phi$ $\rightarrow$ Maximum displaced by $\Delta \phi$
Experimental field phase distribution

Maximum<1 (thermal field)

Width of the peak

$\propto \frac{1}{\sqrt{n}}$
Phase splitting in quantum Rabi oscillation: timing of the experiment

Injection of a coherent field $|\alpha>$

A first atom is sent and interacts resonantly with the field

Detection of the atom
Field projected on

$|\psi_{field}> = \frac{1}{\sqrt{2}} (|\alpha_+> + |\alpha_->)$

Injection of $|-\alpha e^{i\phi}>$

A probe atom is sent in $|g>$

$P_g(\phi)$: two peaks corresponding to the vanishing of each component

$\phi_s = \phi$
Vanishing of $|\alpha_+>$

$\phi_s = -\phi$
Vanishing of $|\alpha_->$
Evidence of the phase splitting

\[ v = 335 \text{m/s} \]

\[ T_{\text{int}} = 32 \mu s \approx 1.5T_0 \]

\[ \bar{n} = 36 \]

Measured phase \[ \varphi = 23^\circ \]

Expected value \[ \varphi = \frac{\Omega_0 t_{\text{int}}}{4\sqrt{n}} = 23^\circ \]

Experiment and theory in very good agreement
Evolution of the phase distribution

\[ \bar{n} = 30 \]
\[ \varphi = \frac{\Omega_0 t_{\text{int}}}{4\sqrt{\bar{n}}} \]

2 velocities
Various number of photons

\[ v_a = 335 \text{ m/s}, \ t_{\text{int}} \approx 1.5 T_0 \]
\[ \varphi_{\text{exp}} = 19^\circ \]

\[ v_b = 200 \text{ m/s}, \ t_{\text{int}} \approx 2.5 T_0 \]
\[ \varphi_{\text{exp}} = 37^\circ \]
Experiment vs theory

Measured phase vs theoretical phase \( \theta = \frac{\Omega_0 t_{\text{int}}}{4\sqrt{n}} \)

theory (slope 1)

numerical simulations
- second mode
- thermal field
- relaxation

experimental points

Experiment and simulations in very good agreement

Experiment vs theory
Test of coherence: induced quantum revivals

Initial Rabi rotation
Stark pulse (duration short compared to phase rotation).
Collapse
Reverse phase rotation
Equivalent to a Z rotation by $\pi$

And slow phase rotation

A spin echo experiment

Separation and recombination of field components by Stark switching

Rabi oscillation revivals
Conclusions and perspectives

Larger and longer lived cats (n in the hundreds) with better cavities

Prepare and detect \( |\alpha, 0\rangle + |0, \alpha\rangle \)
(similar to \( |n, 0\rangle + |0, n\rangle \) « high noon states »)

Non local field states in two cavities

Wigner function measurements and decoherence studies of cat states
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