“Introduction to cold atoms and Bose-Einstein condensation (II)”

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7/7/04
Boulder Summer School
Bose-Einstein condensation  * 1925

Satyendra Nath Bose  Albert Einstein
History of Bose-Einstein condensation (mainly exp.)

- Theoretical prediction
  1924/25 Bose and Einstein

- Superfluidity in liquid helium
  1938 Fritz London
  1983 Reppy et al. (Cornell): BEC of helium in vycor

- Excitons (complicated interactions - no BEC observed)

- Dilute atomic gases
  Spin-polarized hydrogen:
  agenda & experimental techniques (since late ‘70s)
  MIT (Greytak, Kleppner) BEC ‘98
  Amsterdam (Silvera, Walraven)
  also: Harvard, BC, Turku, Cornell, Moscow

- Alkali atoms:
  Laser cooling (‘80s)
  Focused programs in Boulder and at MIT (since early ‘90s)
  June ‘95: Boulder (Cornell/Wieman)
  Sept. ‘95: MIT (W.K.)
  July ‘95 [indirect evidence]: Rice (Hulet)

now:
many experiments
3 particles, total energy = 3

Classical
3 particles, total energy = 3

Identical
3 particles, total energy = 3

Bosons
3 particles, total energy = 3

Bosons
3 particles, total energy = 3

3 | 0 0% 11% 10%
2 | 1 33% 11% 20%
1 | 1 33% 44% 30%
0 | 1 33% 33% 40%

Fermions
3 particles, total energy = 3

10 % probability for triple occupancy

30 % probability for double occupancy

Classical
3 particles, total energy = 3

Bosons are gregarious!
Fermions are loners!

33 % probability for triple occupancy

33 % probability for double occupancy
\[
\begin{align*}
n(\varepsilon) &= \begin{cases} 
\frac{1}{e^{(\varepsilon - \mu)/k_B T}} & \text{classical particles} \\
\frac{1}{e^{(\varepsilon - \mu)/k_B T} - 1} & \text{Bose-Einstein statistics} \\
\frac{1}{e^{(\varepsilon - \mu)/k_B T} + 1} & \text{Fermi-Dirac statistics}
\end{cases}
\end{align*}
\]
\[ n(\varepsilon) = \begin{cases} 
\frac{1}{e^{(\varepsilon - \mu)/k_B T} - 1} & \text{bosons Bose-Einstein statistics} \\
\frac{1}{e^{\varepsilon/k_B T} - 1} & \text{photons Planck’s blackbody spectrum} 
\end{cases} \]

\( \mu = 0 \) (photon number not conserved)
BEC is the most random state in nature

For a given number & energy and for bosons in 3D

Bose 1924
Einstein 1924, 1925

\[ P(N_0) \propto \# \text{ of possible configurations for } N-N_0 \text{ uncondensed atoms} \]

\[ \langle N_0 \rangle \]

\[ \# \text{ of condensed atoms } N_0 \]
Why do photons not Bose condense?

If we put in “extra” photons into the lowest mode they will be absorbed by the walls and thus increase the total entropy.

Bose-Einstein distribution with $\mu=0$

$N=N_c$

Condensate!

$N>N_c$
Are different particles absolutely identical?

Necessary assumption for indistinguishability

In quantum field theory they are excitations of the same field

Tests of the (anti-) symmetry of the state for bosons/fermions at the level of $10^{-9}$ and $10^{-26}$
Development of quantum statistics in three years

1924 Bose’s paper
1924/25 Three papers by Einstein

Einstein mentioned hydrogen, helium and the electron gas as possible candidates for BEC

1925 Pauli exclusion principle
1926 Fermi-Dirac statistics

Confusion about which statistics to apply

1927 Things were cleared up

Particles are no longer statistically independent!
If now we adopt the solution of the problem that involves symmetrical eigenfunctions, we should find that all values for the number of molecules associated with any wave have the same a priori probability, which gives just the Einstein-Bose statistical mechanics.* On the other hand, we should obtain a different statistical mechanics if we adopted the solution with antisymmetrical eigenfunctions, as we should then have either 0 or 1 molecule associated with each wave. The solution with symmetrical eigenfunctions must be the correct one when applied to light quanta, since it is known that the Einstein-Bose statistical mechanics leads to Planck's law of black-body radiation. The solution with antisymmetrical eigenfunctions, though, is probably the correct one for gas molecules, since it is known to be the correct one for electrons in an atom, and one would expect molecules to resemble electrons more closely than light-quanta.

History of BEC

W. Pauli, Z. Phys. 41, 81 (1927):

“We shall take the point of view also advocated by Dirac, that the Fermi, and not the Einstein-Bose, statistics applies to the material gas.”
A. Einstein (December 1924) about BEC:
“The theory is pretty, but is there also some truth to it?”

Fritz London

He realized in 1938 that BEC is an observable phenomenon
Thermal de Broglie wavelength ($\propto T^{-1/2}$) equals distance between atoms ($= n^{-1/3}$)

$n_{\text{crit}} \propto T^{3/2}$

"High" density: $n_{\text{water}}: T = 1 \text{ K}$

BUT: molecule/cluster formation, solidification $\Rightarrow$ no BEC 😞
(b) The densities are so high and the temperatures so low—those required to exhibit a noticeable departure—that the van der Waals corrections are bound to coalesce with the possible effects of degeneration, and there is little prospect of ever being able to separate the two kinds of effect.
BEC:
$\lambda_{dB} > n^{-1/3}$
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\[ \lambda_{dB} > n^{-1/3} \]
Thermal de Broglie wavelength ($\propto T^{-1/2}$) equals distance between atoms ($= n^{-1/3}$)

$n_{\text{crit}} \propto T^{3/2}$

“High” density: $n_{\text{water}}: T = 1 \text{ K}$

BUT: molecule/cluster formation, solidification

$\Rightarrow$ no BEC

“Low” density: $n_{\text{water}}/10^9: T = 100 \text{ nK} - 1 \text{ \(\mu\text{K}$

seconds to minutes lifetime of the atomic gas

$\Rightarrow$ BEC
What is Bose-Einstein condensation (BEC)?

High Temperature $T$:
- thermal velocity $v$
- density $d^{-3}$
- "Billiard balls"

Low Temperature $T$:
- De Broglie wavelength
  \[ \lambda_{dB} = \frac{h}{mv} \propto T^{-1/2} \]
- "Wave packets"

$T = T_{\text{crit}}$:
- Bose-Einstein Condensation
  \[ \lambda_{dB} \approx d \]
- "Matter wave overlap"

$T=0$:
- Pure Bose condensate
- "Giant matter wave"
Length Scales @BEC
Length Scales @BEC
### Length and energy scales in BEC

<table>
<thead>
<tr>
<th>Size of the atom</th>
<th>$a$</th>
<th>3 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation between atoms</td>
<td>$n^{-1/3}$</td>
<td>200 nm</td>
</tr>
<tr>
<td>Matter wavelength</td>
<td>$\lambda_{dB}$</td>
<td>1 $\mu$m</td>
</tr>
<tr>
<td>Healing length</td>
<td>$2\pi\xi$</td>
<td>2 $\mu$m</td>
</tr>
<tr>
<td>Size of confinement</td>
<td>$a_{osc}$</td>
<td>30 $\mu$m</td>
</tr>
</tbody>
</table>

- $a << n^{-1/3} \leq \lambda_{dB} < a_{osc}$
- $k_B T_{s-wave} >> k_B T_c$ (Gas!)
- $k_B T > U_{int} > \hbar \omega = (\hbar^2/m)na$ (BEC)
Cast of characters: nK tools

**Cooling**
- Laser cooling
- Evaporative cooling

**Atoms for BEC**

**Traps**
- Magnetic traps
- Optical traps

**How to observe BEC**
- Absorption imaging
- Dispersive imaging

**Manipulation of BEC**
- Magnetic fields
- Rf
- Optical dipole force
The cooling methods
Laser cooling = Precooling

atomic beam 600 K

Laser beams

cold atoms 10...100 μK

\[ \hbar W_{abs} < \hbar W_{em} \]

\Rightarrow \text{cooling!}
Magnetic trapping - "thermos" for nanokelvin atoms

Phillips et al. (1985)
Pritchard et al. (1987)
Evaporative cooling using rf induced spinflips

Hess, Kleppner, Greytak et al. (1986/7)
Pritchard et al. (1989)
Ketterle et al. (1993/4)
Cornell et al. (1994)

$V = \mu B$

Position

⇒ controlling the trap depth
## Multi-stage cooling to BEC

<table>
<thead>
<tr>
<th></th>
<th>Temp. $T$</th>
<th>Density $n$ [cm$^{-3}$]</th>
<th>Phase space density $nT^{-3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oven</td>
<td>500 K</td>
<td>$10^{14}$</td>
<td>$10^{-13}$</td>
</tr>
<tr>
<td>Laser cooling</td>
<td>50 $\mu$K</td>
<td>$10^{11}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Evap. cooling</td>
<td>500 nK</td>
<td>$10^{14}$</td>
<td>2.6</td>
</tr>
<tr>
<td>BEC</td>
<td>(10 - 100 nK)</td>
<td>$3.10^{14}$</td>
<td>$10^{7}$</td>
</tr>
</tbody>
</table>
Sodium BEC experiment @ MIT
Laser cooling requires low density to avoid light absorption.
Limitations of Laser Cooling

\[ T \sim \mu K \]

Absorption excited state collisions \[ n \leq 10^{12} \text{cm}^{-3} \]
⇒ Phase space density \( 10^{-5} \)
Evaporative cooling is driven by elastic collisions

\[
\Gamma_{ec} = n \Gamma_{0} \rightleftarrows \Gamma_{loss}
\]

H. Hess (1986)
The problem ...

Absorption cross section for light:

$$\sigma_{opt} = \frac{3}{2\pi} \lambda^2 \approx 2 \cdot 10^{-9} \text{ cm}^2$$

Elastic collision cross section:

$$\sigma_{coll} = 8\pi a^2 \approx 2 \cdot 10^{-12} \text{ cm}^2$$
The solution …

- Dark light trap ("Dark SPOT MOT")
- Tight magnetic confinement
- ULTRA-high vacuum
- … a few years of engineering
Magnetic trapping
Magnetic trapping

\[ U_i(B) \]

**Internal energy**

\[ \nabla U_i(B(\vec{r})) = 0 \]

\[ \Delta U_i(B(\vec{r})) > 0 \]

Unless \[ \frac{d U_i(B)}{d B} = 0 \]

\[ \exists \text{ local extremum of } |\vec{B}| \]

Minimum \[ \rightarrow \text{ Wings' theorem} \]

\[ \rightarrow \text{ only weak-field seeking states} \]
Most common situation

\[ U_i(\tau) = \mu_i \cdot |\vec{B}(\vec{r})| \]

\[ \int \frac{dU_i(\theta)}{d\theta} \]

adiabatic condition:

Atom stays in HFS i.

classical analogon (cf. Levitron)

\[ U(\tau) = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta \]

= Const (precession)

choose \( \cos \theta < 0 \)

q.m., weak field:

\[ U(\tau) = \mu_B B \gamma \gamma m_\gamma \]
Adiabatic condition

\[
\begin{align*}
\vec{B}(+), \quad \vec{B}(+ + \Delta t) \\
\theta
\end{align*}
\]

\[
\omega_{\text{rot}} = \frac{d\theta}{dt} \ll \frac{U_i - U_B}{\hbar} = g \gamma m_B \frac{B}{\hbar} = \omega_{\text{Larmor}}
\]
Magnetic Traps

- Spherical Quadrupole

\[ U_{\text{Pot}} \propto |\vec{B}| \]

Linear potential

\[ \vec{B} = 0 \implies \text{Leaky!} \]

Solutions:
- rotating \( B \) Field
  - "TOP" trap
    - JILA
- Optical plug
  - MIT

- Ioffe-Pritchard Trap

\[ B \]

harmonic potential

\[ B_0 \]

0
Majorana Flops

Atom

Spherical Quadrupole Field

Spin does not flip $\Rightarrow$ Trap Loss

Spin flips

Potential Energy

Landau-Zener crossing

Time
Landau-Zener trap loss probability

\[ e^{\left[ -\frac{2\pi |V_{12}|^2}{\nu \times 1_{2}} \right]} \approx e^{-\frac{2\pi \mu B' x^2}{\kappa \nu}} \]

\[ = e^{-2\pi \left( \frac{x}{x_0} \right)^2} \]

@ \( x = x_0 \):

Larmor Freq. \( \frac{\mu B' x}{\kappa} \)

= Orbital Freq. \( \frac{d\theta}{dt} = \frac{\nu}{x} \)

\[ \Rightarrow \text{Trapping Time } \propto (\text{cloud diameter})^2 \]
**Top Trap** (JILA '94)

**\( \vec{B}_{\text{stat}} = B' \left( \frac{x}{y^2} \right) \)** Quadrupole Field

**\( \vec{B}_{\text{rot}} = B_0 \left( \cos \omega t \right) \)** Rotating bias field

**Time-averaged potential**

\[ U_{\text{Top}} = \frac{1}{2} \left( B'' r^2 + B'' z^2 \right) \]

\[ \frac{B''^2}{2 B_0} \quad \frac{4 B''^2}{B_0} \]

**Circle of death** \( \tau_D = B_0 / B' \)

\[ U_{\text{Top}} (\tau_D) = \frac{\mu B_0}{4} \]
Ioffe - Pritchard Trap

2 "Pinch" Coils
⇒ \( B_0, B_{\text{axial}} \)

4 "Ioffe" bars
⇒ \( B'_{\text{radial}} \)
$B_0 \neq 0$ traps

Ioffe-Pritchard configuration

$$\vec{B} = B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + B' \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} + B'' \begin{pmatrix} -\frac{y^2}{2} - \frac{1}{2}(x^2+y^2) \\ -\frac{y^2}{2} - \frac{1}{2}(x^2+y^2) \\ 2^2 \end{pmatrix}$$

$$B(x,y,z) = \left\{ \left[ B_0 + \frac{1}{2} B'' z^2 - \frac{B''}{4} (x^2+y^2) \right]^2 + \left( B' - B'' z/2 \right)^2 x^2 + \left( B' + B'' z/2 \right)^2 y^2 \right\}^{1/2}$$

$$\approx \frac{1}{2} \left[ \left( \frac{B'}{B_0} \right)^2 r^2 + B'' z^2 \right]$$

Large $B_0, B'$

$B'' z^2 = B_{\text{radial}}$

WARNING: Green terms limit trapping volume! (important for hot clouds)
IP - Trappology

in order of appearance

Permanent magnets      Rice
Cloverleaf trap        MIT
Baseball trap          Boulder
Four-dee               Rowland
Ioffe bars             Konstanz
3 coils, no extra bias  Munich
3 coils + extra bias   Paris
Ioffe bars, superconducting MIT
Pole piece             Orsay
BEC in a "cloverleaf" magnetic trap

MIT, March '96 [M.-O. Mewes et al., PRL 77, 416 (1996)]
Bias field adjustment is critical

\[ B_0 = 500 \, \text{G} \]
\[ B_0 = -10 \, \text{G} \]
\[ B_0 = 1 \, \text{G} \]
Gravito-Magnetic Trap

- Single coil carrying current $I_S$ levitates atoms against gravity with magnetic field gradient $\approx 8 \text{ G/cm}$. 
Gravito-Magnetic Trap

- Single coil carrying current $I_S$ levitates atoms against gravity with magnetic field gradient $\sim 8$ G/cm.
Gravito-Magnetic Trap

- Single coil carrying current $I_s$ levitates atoms against gravity with magnetic field gradient $\sim 8$ G/cm.
- Stable vertical confinement for $|z| > R/2$ above coil.