UCF and $1/f$ noise in Diffusive Metals

Collaborators:
B. Golding, W.H. Haemmerle (AT&T Bell Labs)
P. McConville, J.S. Moon, D. Hoadley (MSU)

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Outline

• 1/f noise in metals - background
• Low-temperature 1/f noise and UCF
• Symmetries and Random Matrix Theory
• 1/f noise vs. B in Bi, Li, and Ag
• Comparison of $L_\phi$ from WL and UCF
• Appendix: Tunneling systems in disordered solids
1/f noise in metals

Power spectral density: \[ S_V(f) \equiv 4 \int_0^\infty \langle V(t)V(0) \rangle \cos(2\pi ft) dt \]

Experimental observations: \[ S_V(f) = 4k_B T R + I^2 S_R(f) \]

with \[ S_R(f) \propto \frac{1}{f^\alpha} \] and \[ S_R(f) \propto \frac{1}{\text{volume}} \]
How to produce a 1/f spectrum

Single two-level fluctuator:

\[ S_R(\omega) \sim \frac{\tau}{1 + (\omega \tau)^2} \]

Many fluctuators, \( P(\log(\tau)) \sim \text{const.} \)
The Dutta-Horn model

Broad distribution of thermally activated processes produces nearly $1/f$ spectrum:

$$S_R(\omega) = \delta R^2 \int_0^\infty \frac{2\tau}{1 + \omega^2 \tau^2} D(\tau) d\tau$$

$\Delta E \gg kT \Rightarrow$ distribution is nearly flat on scale of $kT$
Mobile defects $\Rightarrow$ 1/f noise

Additional evidence:

- annealing reduces noise
- symmetry properties of noise
- correlates with low resistivity ratio

Limitations of Dutta-Horn model:

- assumes $T$-independent coupling of defect motion to resistance
- “local interference” model
1/f noise vs. T in Bi films


Bi low carrier density ⇒ high resistivity

1/f noise grows as T decreases!
Why does 1/f noise grow at low T?

density of mobile defects decreases as T drops
⇒ coupling of defect motion to resistance must increase

not local interference
Recall from previous lectures:
Universal conductance fluctuations (UCF) in wires

\[ G \text{ varies randomly with } B \text{ or } E_F \]

\[ \delta G \approx \frac{e^2}{h} \text{ for } L < L_\phi \]
UCF “magnetofingerprint” depends on exact positions of scatterers
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\[ \delta G \approx \frac{e^2}{h} \]

for phase-coherent sample with \( L < L_\phi \) (phase coherence length)

Altshuler & Spivak, JETP Lett. 42, 447 (1985)
UCF in macroscopic samples ($L > L_{\phi}$):

$$\left( \frac{\delta G}{G} \right)^2 = \frac{1}{N} \left( \frac{\delta G_{\text{box}}}{G_{\text{box}}} \right)^2$$

$N = \text{number of "coherence boxes"}$
Temperature dependence of “UCF noise”

(quasi-2D case)

1. Conductance change in “coherence volume”

\[
\delta G^2_1 \approx \frac{1}{\beta} \left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2} \alpha(k_F \delta r) \left(\frac{L}{l_e}\right)^{2-d} \frac{l_e}{t}
\]

motion of single defect: 

multiply by number of defects in box: \(n_s(T) \times L^2_\phi t\) \(\alpha(x) = 1 - \frac{\sin^2(x/2)}{(x/2)^2}\)

energy averaging: If \(k_B T > \frac{\hbar}{\tau_\phi}\), then \(\times \frac{\hbar \tau_\phi^{-1}}{k_B T} = \frac{L^2_T}{L^2_\phi}\) \(L_{\text{min}} \equiv \min\{L_\phi, L_T\}\)

Total conductance fluctuation in box: \(\delta G^2_{\text{box}} \approx \frac{1}{\beta} \left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2} \alpha(k_F \delta r) l_e L^2_{\text{min}} n_s(T)\)
Temperature dependence of “UCF noise”

2. Conductance fluctuation in entire sample

\[
\left( \frac{\delta G_{\text{total}}}{G_{\text{total}}} \right)^2 = \frac{1}{N_{\text{boxes}}} \left( \frac{\delta G_{\text{box}}}{G_{\text{box}}} \right)^2
\]

\[
N_{\text{boxes}} = \frac{L_{\phi}^2}{L_w}
\]

\[
G_{\text{total}} = \frac{w}{L} G_{\text{box}}
\]

\[
\delta G_{\text{total}}^2 \approx \frac{1}{\beta} \left( \frac{e^2}{h} \right)^2 \frac{1}{(k_F l_e)^2} \alpha(k_F \delta r) \frac{L_{\phi}^2 w}{L^3} l_e L_{\text{min}}^2 n_s(T)
\]
Temperature dependence of “UCF noise”

\[ \delta G_{total}^2 \approx \frac{1}{\beta} \left( \frac{e^2}{h} \right)^2 \frac{1}{(k_F \delta r)^2} \alpha(k_F \delta r) \frac{L_{\phi}^2 W}{L^3} l_e L_{\text{min}}^2 n_s(T) \]

3. Tunneling model of disordered solids: \( n_s(T) \propto T \)

4. Dephasing length in quasi-2D: \( L_\phi \propto T^{-1/2} \)

Final answer: \( \delta G_{total}^2 \propto L_{\phi}^2 L_{\text{min}}^2 n_s(T) \propto \frac{1}{T} \)

Bi, t=11 nm
Symmetries and UCF: Random Matrix Theory

-- eigenvalues of random matrices do not obey Poisson statistics
-- eigenvalues exhibit level repulsion

Poisson: uncorrelated levels

Gaussian Orthogonal Ensemble

Wigner-Dyson; GOE
Poisson
Wigner-Dyson ensembles
(table courtesy of Boris Altshuler):

<table>
<thead>
<tr>
<th>Matrix elements</th>
<th>Ensemble</th>
<th>β</th>
<th>Realization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>Orthogonal</td>
<td>1</td>
<td>Time-reversal invariant</td>
</tr>
<tr>
<td>Complex</td>
<td>Unitary</td>
<td>2</td>
<td>Broken time-reversal invariance (e.g. by B field)</td>
</tr>
<tr>
<td>2x2 matrices??</td>
<td>Symplectic</td>
<td>4</td>
<td>T-invariant but with spin-orbit scattering</td>
</tr>
</tbody>
</table>
RMT and the UCF variance

\[ \delta G_1^2 \propto \left( \frac{e^2}{h} \right)^2 \frac{ks^2}{\beta} \]

\( k = \) number of independent eigenvalue sequences; \( s = \) level degeneracy

<table>
<thead>
<tr>
<th></th>
<th>B=0</th>
<th>B &gt; B_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low spin-orbit scattering</td>
<td>( \beta=1, k=1, s=2 ) (GOE)</td>
<td>( \beta=2, k=1, s=2 ) (GUE)</td>
</tr>
<tr>
<td>High spin-orbit scattering</td>
<td>( \beta=4, k=1, s=2 ) (GSE)</td>
<td>( \beta=2, k=1, s=1 ) (GUE + spin)</td>
</tr>
</tbody>
</table>
Noise vs. B in Bi


Noise drops by a factor of 2 in applied field $B > B_c = (h/e)/L$.

Confirms connection to UCF:

$$\delta G_1^2 \propto \left( \frac{e^2}{h} \right)^2 \frac{kS^2}{\beta}$$

Noise crossover function (Stone):

$$\frac{S_G(f, B)}{S_G(f, 0)} = \frac{-1}{2b^2} \Psi'' \left[ \frac{1}{2} + \frac{1}{b} \right]$$

$$b = \frac{8\pi B L^2_\Phi}{\Phi_0}$$
Other fun games with UCF and 1/f noise: Li

\[ \delta G_1^2 \propto \left( \frac{e^2}{h} \right)^2 \frac{k s^2}{\beta} \]

Li has low spin-orbit scattering \( \Rightarrow \beta = 1 \) at \( B=0 \)

\[ \text{GOE} \rightarrow \text{GUE} \]

\[ \beta = 1 \rightarrow 2 \]

Break spin-degeneracy when \( g\mu_B B > kT \)

\[ s = 2 \rightarrow 1 \]
\[ k = 1 \rightarrow 2 \]
\[ \beta = 2 \]
Can we obtain quantitative estimate of $L_\phi$ from $1/f$ noise vs. $B$?

Bi exhibits superconducting fluctuations at low $T$; try Ag

Noise vs. $T$ in Ag
Noise vs. B in Ag

McConville and Birge, PRB 47, 16667 (1993).

(crossover function with help from D. Stone)
Noise crossover function with spin & SO


UCF correlation function, T=0  \[ F_0(\Delta E, \Delta B, B) \equiv \langle \delta G(E_F, B) \delta G(E_F + \Delta E, B + \Delta B) \rangle \]

Response to small chance in impurity potential, T>0

\[ \delta G^2(B,T) = -\frac{4s^2}{\pi^2} \int \frac{d(\Delta E)}{2k_BT} K \left( \frac{\Delta E}{2k_BT} \right) \frac{d}{d\left(1/\tau_{\phi}\right)}(F_0(\Delta E, B)) \]

\[ K(x) = (x \coth x - 1)/\sinh^2 x \]

Incorporate spin effects: spin-orbit scattering, Zeeman splitting

\[ \delta G^2 = \left[ \frac{1}{4} \left( \delta G_s'(B) \right)^2 + \frac{3}{4} \left( \delta G_t'(B, L_{so}) \right)^2 \right]_{\text{Cooperon}} + \left[ \frac{1}{4} \left( \delta G_s' \right)^2 + \frac{1}{4} \sum_{M_z} \left( \delta G_t'(M_z g \mu_B B, L_{so}) \right)^2 \right]_{\text{Diffuson}} \]

Noise crossover function:

\[ \nu(B,T) = \frac{\delta G^2(B,T)}{\delta G^2(B=0,T)} \]
Weak Localization magnetoresistance

McConville and Birge, PRB 47, 16667 (1993).

\[ \sigma(B) = -\frac{e^2}{2\pi^2\hbar} \left[ \Psi\left(\frac{1}{2} + \frac{B_1}{B}\right) + \frac{1}{2} \Psi\left(\frac{1}{2} + \frac{B_2}{B}\right) - \frac{3}{2} \Psi\left(\frac{1}{2} + \frac{B_3}{B}\right) \right] \]

\[ B_1 = B_0 + B_{SO} \]
\[ B_2 = B_\phi \]
\[ B_3 = B_\phi + \frac{4}{3} B_{SO} \]

\[ B_0 = \frac{3\hbar}{4eD\tau_e} \]
\[ B_x = \frac{\hbar}{4eD\tau_x} = \frac{h/e}{8\pi L_x^2} \]
Comparison of $L_\phi$ from WL and UCF

Hoadley, McConville and Birge, PRB 60, 5617 (1999).

McConville and Birge, PRB 47, 16667 (1993).

Hoadley, McConville and Birge, PRB 60, 5617 (1999).
Epilogue
Trionfi, Lee, and Natelson, PRB 72, 035407 (2005).

AuPd alloy

High-purity Ag
Why don’t $L_{\phi}^{WL}$ and $L_{\phi}^{UCF}$ agree at low T in Ag?

- Crossover to strong spin-orbit scattering
  - But AuPd data is even stronger!
- Noise measurements are out of equilibrium?
  - Noise vs. B unchanged with drive current
A proposal to explain the discrepancy in Ag
Trionfi, Lee, and Natelson, PRB 72, 035407 (2005).
Does the 1/f noise saturate UCF?

\[ g(\ln(t)) = \text{const. for } \tau_{\text{min}} < \tau < \tau_{\text{max}} \implies \delta G^2 \approx \omega S_G(\omega) \ln \left( \frac{\tau_{\text{max}}}{\tau_{\text{min}}} \right) \]

It would take \( \sim 200 \) decades of 1/f noise at the level measured to saturate the UCF!
Summary

• 1/f noise in metals comes from defect motion
• 1/f noise is enhanced at low T due to long-range quantum interference (UCF)
• Noise vs. B reveals RMT crossovers (GOE $\rightarrow$ GUE, Zeeman splitting, etc.)
• Discrepancy in $L_\phi$ determined from WL and UCF
  – Maybe due to crossover from unsaturated to saturated UCF
Appendix: Tunneling systems in disordered (insulating) solids

I. Crystalline Solids

"Every atom knows its place"

Point defects are not mobile at low temperature (1eV $\approx$ 10,000K)
II. Disordered Solids

More disorder $\Rightarrow$ more low barriers
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More disorder $\Rightarrow$ more low barriers
Dynamics in a double-well potential

High T: over the barrier
(thermal activation)

Low T: through the barrier
(tunneling)

Asymmetry energy: $\varepsilon$

Tunneling energy: $\Delta = \hbar \omega_0 e^{-\lambda} \approx \hbar \omega_0 e^{-\frac{\sqrt{2mV}}{\hbar}d}$
\( \varepsilon, \Delta, k_B T \ll \omega_0 < V \)

\( \Rightarrow \) two-level tunneling system (TLS)

Hamiltonian in left-right basis

\[
H = \frac{1}{2} \begin{pmatrix} \varepsilon & -\Delta \\ -\Delta & -\varepsilon \end{pmatrix}
\]
\( \Delta = \hbar \omega_0 e^{-\lambda} \approx \hbar \omega_0 e^{-\sqrt{2mV/d}} \)

Eigenvalues:

\[
E_{0,1} = \pm \frac{1}{2} \sqrt{\varepsilon^2 + \Delta^2}
\]

Eigenstates:

\[
|\Psi_0\rangle = \sin \frac{\theta}{2} |L\rangle + \cos \frac{\theta}{2} |R\rangle
\]

\[
|\Psi_1\rangle = \cos \frac{\theta}{2} |L\rangle - \sin \frac{\theta}{2} |R\rangle
\]

where \( \tan \theta = \frac{\Delta}{\varepsilon} \)
The standard “tunneling model”
Anderson, Halperin and Varma; Phillips (1972)

Hypotheses: 1) $P(\varepsilon, \lambda) = P_0 \Rightarrow c(T) \sim T$
2) TLS scatter phonons $\Rightarrow \kappa(T) \sim T^2$

Experimental Data:
Zeller and Pohl (1971)