$\text{AB}_2\text{X}_4$ spinels

- One of the most common mineral structures
- Common valence:
  - $A^{2+}, B^{3+}, X^{2-}$
  - $X=\text{O, S, Se}$

cubic Fd$\bar{3}$m
Deconstructing the spinel

- A atoms: diamond lattice
- Bipartite: not geometrically frustrated
Deconstructing the spinel

B atoms: pyrochlore

decorate the plaquettes of the diamond lattice
$\text{ACr}_2\text{O}_4$ spinels

- pyrochlore lattice
- $S=3/2$ Isotropic moment
- $X=O$ spinels: B-B distance close enough for direct overlap
- dominant AF nearest-neighbor exchange
H=0 Susceptibility

Frustration:

<table>
<thead>
<tr>
<th></th>
<th>Zn</th>
<th>Cd</th>
<th>Hg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_{CW}$ (K)</td>
<td>-390</td>
<td>-70</td>
<td>-32</td>
</tr>
<tr>
<td>$T_N$ (K)</td>
<td>12</td>
<td>7.8</td>
<td>5.8</td>
</tr>
<tr>
<td>$f$</td>
<td>33</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>
Degeneracy

Heisenberg model

\[ H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j = \frac{1}{2} \sum_t \left( \sum_{i \in t} \vec{S}_i \right)^2 + \text{const.} \]

Ground state constraint: total spin 0 per tetrahedron

Quantum mechanically: not possible
Classical spin liquid

- No LRO (Reimers)
Classical spin liquid

- No LRO (Reimers)
- Dipolar correlations

\[ S^\mu_i = b^\mu_{ab} \]
Classical spin liquid

- No LRO (Reimers)
- Dipolar correlations

\[ S_i^\mu = b_{ab}^\mu \]
Classical spin liquid

Unusual “ring” correlations seen in CdCr$_2$O$_4$ related

Y$_2$Ru$_2$O$_7$: J. van Duijn et al, 2007
Ordering

Many perturbations important for ordering:

- Spin-lattice coupling
- Further exchange
- Spin-orbit effects
- Quantum corrections

ZnCr$_2$O$_4$

CdCr$_2$O$_4$

HgCr$_2$O$_4$

S.H. Lee + many others
Magnetization Plateaus

Classically: \( M = M_s \frac{H}{H_s} \)

Plateau indicates 3:1 structure

H. Ueda at al, 2005/6
Magnetization Plateaus

- Plateau mechanism:
  - spin-lattice coupling favors collinearity
- Order on plateau may be selected by
  - spin-lattice
  - quantum effects

“R” state observed in neutrons

Matsuda et al
A-site spinels

Spectrum of materials

CoRh$_2$O$_4$  Co$_3$O$_4$  MnSc$_2$S$_4$  FeSc$_2$S$_4$

V. Fritsch et al. PRL 92, 116401 (2004); N. Tristan et al. PRB 72, 174404 (2005); T. Suzuki et al. (2006)

Naively unfrustrated
Why frustration?

- Roth, 1964: 2nd and 3rd neighbor exchange not necessarily small
- Exchange paths: A-X-B-X-B comparable
- Minimal model
- $J_1$-$J_2$ exchange
Ground state evolution

- Coplanar spirals
- Spiral surfaces:

$J_2 / J_1$: 0.2, 0.4, 0.85, 20
Monte Carlo

MnSc$_2$S$_4$

\( f = 11 \) at \( J_2/J_1 = 0.85 \)
Entropy and $J_3$ compete to determine ordered state.

Spiral spin liquid regime has intensity over entire spiral surface.
Comparison to Expt.

- Diffuse scattering
- Ordered state
- (qq0) spiral
- Specific heat?

Expt. agrees with theory for FM $J_1$
$\text{Cs}_2\text{CuCl}_4$

- Spatially anisotropic triangular lattice
- Cu$^{2+}$ spin-1/2 spins

$$H = \frac{1}{2} \sum_{i,j} \left[ J_{ij} \vec{S}_i \cdot \vec{S}_j - \vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j) \right]$$

- couplings:
  - $J=0.37\text{meV}$
  - $J'=0.3J$
  - $D=0.05J$
Neutron scattering

Coldea et al, 2001/03: a 2d spin liquid?

Very broad spectrum similar to 1d (in some directions of k space).
Roughly fits power law.

Fit of “peak” dispersion to spin wave theory requires adjustment of J,J’ by 40% - in opposite directions!
Dimensional reduction?

- Frustration of interchain coupling makes it less “relevant”
- First order energy correction vanishes
- Leading effects are in fact $O[(J')^4/J^3]$!
Dimensional reduction?

- Frustration of interchain coupling makes it less “relevant”
- First order energy correction vanishes.
- Numerics: $J'/J < 0.7$ is “weak”

Very different from spin wave theory

Very weak inter-chain correlations
Excitations

- Build 2d excitations from 1d spinons

- Exchange: \( \frac{J'}{2} (S_i^+ S_j^- + S_i^- S_j^+) \)

- Expect spinon binding to lower inter-chain kinetic energy

- Use 2-spinon Schroedinger equation
Broad lineshape: “free spinons”

- “Power law” fits well to free spinon result
- Fit determines normalization

\[ J'(k) = 0 \text{ here} \]
Bound state

Compare spectra at $J'(k)<0$ and $J'(k)>0$:

- Curves: 2-spinon theory with experimental resolution
- Curves: 4-spinon RPA with experimental resolution
Transverse dispersion

Bound state and resonance

Solid symbols: experiment
Note peak (blue diamonds) coincides with bottom edge only for $J'(k)<0$
Spectral asymmetry

Vertical lines: $J'(k) = 0$. 

$\omega [\text{meV}]$ vs $k'/\pi$ for $k'_y = 0$, $k'_y = 2\pi$, and $k'_y = 3\pi$. The figure shows the spectral asymmetry with vertical lines indicating the condition $J'(k) = 0$. 