Heisenberg Magnets

In many cases, with half-filled shells especially, one obtains isotropic spins with Heisenberg interaction

$$H = \frac{1}{2} \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Even classically, frustration is rather different than in Ising systems.

$$\sum \vec{S}_i = 0$$

Can be achieved on a triangle

c.f. non-collinear

of course $\vec{S}_i \cdot \vec{S}_j \neq -1$

Still, one can find significant ground state degeneracy in some cases.

**Triangle AF $\rightarrow$ “No” 6-fold degeneracy**

Order at $T=0$. No order $T>0$ due to Meissner-Wegner

**Kagome**

Still have choices!

Turns out to have extensive entropy.

However $\rightarrow$ orders as $T \rightarrow 0$ due to “order by disorder” phenomena.
A nice method which is well "Luttinger-Tin" method.

Idea: Minimize $H$ relaxing constant $1s_1 = S$ instead just force $\sum |s|^2 = N S^2$.

Guaranteed to get energy $E_0 \leq E_0$ if $|s_1| = S$.

If you can find such a sol. which then satisfies $|s_1| = S$ you are done.

\[
M \equiv \sum_j \chi_j \ immature - \frac{q}{2} (1S_1^2 - S^2)
\]

Quadratic: $\chi_j \tilde{s}_j = \lambda \tilde{s}_j$ eigenvalue problem.

Energy: $\sum_j S^2 \rightarrow min \ \text{at} \ \text{min. eigenvalue.}$

1/ Translational invariance, eigenvectors satisfy block form

\[
\tilde{S}_j = e^{i\mathcal{Q} \cdot \mathcal{R}} u_j \ \text{in arbitrary}
\]

$u_j$ depends only upon basis site (i.e. periodic)

energies: $\lambda \rightarrow \varepsilon_n(\mathcal{Q})$ bands.

$\rightarrow$ Look for band minima.

* In general for Bovic lattice, can always satisfy $1S_1^2 = S^2$ constant by

\[
\tilde{S}_j = S (e^{i\mathcal{Q} \cdot \mathcal{R}} e^{i\mathcal{Q} \cdot \mathcal{R}} e^{i\mathcal{Q} \cdot \mathcal{R}}) = S \text{Re}(e^{i\tilde{\mathcal{Q}}})
\]

$\tilde{\mathcal{Q}} = \mathcal{Q} + \mathcal{R}$.
There are coplanar spins. \( \rightarrow \) another reason for ferroelectricity?

In some other circumstances, can make this work for non-Brooks lattices. Some examples of bond

<table>
<thead>
<tr>
<th>Kagome</th>
<th>1 flat bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>pyrochlore</td>
<td>2 ( \approx ) 4s</td>
</tr>
<tr>
<td>FCC</td>
<td>minimum decay lines</td>
</tr>
<tr>
<td>diamond ( J_2J_1/8 )</td>
<td>( \approx ) along surface</td>
</tr>
</tbody>
</table>

OK, but how good is this classical picture?

Simplest gauge \& spin-wave theory (1/5 expansion)

e.g. triangle lattice: ordered moment \( (T=0) \)

\[
\mathbf{M} = |\langle \hat{S} \rangle| = S - 0.2613 + 0.00555 \chi
\]

\[
= \begin{cases} 
0.742 & \text{for } S=1 \\
0.2447 & \text{for } S=1/2 
\end{cases} \quad (\approx 50\%)
\]

Evidently for \( S > 1 \) fairly clearly good, and perhaps not so bad for smaller \( S \)?

\[\text{In fact, strong evidence for Exact Diagonalization.}\]

Other methods?

\[\text{**Numerics} - \text{Exact Diagonalization (see LST)}\]

DMRG (mostly 1d...)

Series expansion (based)

QMC (S. Prell)

Recent DMRG, Series, \( \text{M}_{\text{S=1/2}} = 0.205 \pm 0.015 \)
Shaggy suggests SW good for Isotropic triangular AF

Other approaches? Talk more 3rd lecture
- Sku-Particles = MFT + Various WFs
- Exact Solns, (1d = special cases)

Then now to survey of experiments, I find interesting, Start to better understand some to less understood.

For $T > 0$ properties? How good is classical?

e.g. How does $C_n$ decrease? Where is specific heat peak?

![Graph showing heating curve]

$$T = \frac{TP}{|C_n|} \quad (S \to \infty) \approx 0.17$$

Classical MC

Q: Is $\tau$ large or small for small $S$?

(Answer for $S = \frac{1}{2}$ known for semi-emp.) $\tau_{i.e.} = 0.27$