Dynamics of disordered systems

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Plan of Lectures

1. Introduction
2. Coarsening processes
3. Formalism
4. Dynamics of disordered spin models
First lecture
Plan of the 1st Lecture

Plan

1. Equilibrium vs. out of equilibrium classical systems.

2. How can a classical system stay far from equilibrium?
   From single-particle to many-body.
   Diffusion
   Phase-separation & domain growth
   Glasses
   Driven systems
   Active matter

3. Details on the non-equilibrium behaviour
Plan of the 1st Lecture

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1. **Equilibrium vs. out of equilibrium classical systems**

2. How can a classical system stay far from equilibrium?
   - From single-particle to many-body.
   - Diffusion
   - Phase-separation & domain growth
   - Glasses
   - Driven systems
   - Active matter

3. Details on the non-equilibrium behaviour
Introduction

General setting

- Closed & open systems
- Equilibrium & out of equilibrium
  - Long time scales
  - Forces & energy injection
- Individual & collective effects
Introduction

General setting

• Closed & open systems

• Equilibrium & out of equilibrium
  – Long time scales
  – Forces & energy injection

• Individual & collective effects
Isolated systems
Dynamics of a classical isolated system

Foundations of statistical physics.

**Question:** does the dynamics of a particular system reach a flat distribution over the constant energy surface in phase space?

Ergodic theory, ∈ mathematical physics at present.

Dynamics of a (quantum) isolated system: a problem of current interest, recently boosted by cold atom experiments.

**Question:** after a quench, i.e. a rapid variation of a parameter in the system, are at least some observables described by thermal ones?

When, how, which? some comments in 4th lecture
Introduction

General setting

- Closed & open systems
- Equilibrium & out of equilibrium
  - Long time scales
  - Forces & energy injection
- Individual & collective effects
Statistical mechanics

Equilibrium ensembles

- Isolated system $\Rightarrow$ conserved energy $\mathcal{E}$
- Ergodic hypothesis

$$S = k_B \ln \mathcal{N}$$

$$\beta \equiv \frac{1}{k_B T} = \frac{\partial S}{\partial \mathcal{E}} \bigg|_{\mathcal{E}}$$

Microcanonical definition

$$\mathcal{E} = \mathcal{E}_{\text{syst}} + \mathcal{E}_{\text{env}} + \mathcal{E}_{\text{int}}$$

Neglect $\mathcal{E}_{\text{int}}$ (short-range int.)

$$\mathcal{E}_{\text{syst}} \ll \mathcal{E}_{\text{env}}$$

$$p_{eq}(\mathcal{E}_{\text{syst}}) = g(\mathcal{E}_{\text{syst}}) e^{-\beta \mathcal{E}_{\text{syst}}}/Z$$

Canonical ensemble
Open systems

Aim

Our interest is to describe the statics and dynamics of a classical (or quantum) system coupled to a classical (or quantum) environment.

The Hamiltonian of the ensemble is

\[ H = H_{\text{syst}} + H_{\text{env}} + H_{\text{int}} \]

The dynamics of all variables are given by Newton (or Heisenberg) rules, depending on the variables being classical (or quantum).

The total energy is conserved, \( \mathcal{E} = ct \) but each contribution is not, in particular, \( \mathcal{E}_{\text{syst}} \neq ct \), and we’ll take \( e_0 \ll \mathcal{E}_{\text{syst}} \ll \mathcal{E}_{\text{env}} \).
Take a mechanical point of view and call $\{\vec{\zeta}_i\}(t)$ the variables, e.g. the particles’ coordinates $\{\vec{r}_i(t)\}$ and momenta $\{\vec{p}_i(t)\}$.

Choose an initial condition $\{\vec{\zeta}_i\}(0)$ and let the system evolve.

- For $t_w > t_{eq}$: $\{\vec{\zeta}_i\}(t)$ reach the equilibrium pdf and thermodynamics and statistical mechanics apply (e.g., temperature is a well-defined concept).
- For $t_w < t_{eq}$: the system remains out of equilibrium and thermodynamics and (Boltzmann) statistical mechanics do not apply.
Dynamics in equilibrium

Conditions

Take an open system coupled to an environment

Necessary:

— The bath should be in equilibrium
  same origin of noise and friction.

— Deterministic force
  conservative forces only, \( \vec{F} = -\nabla V \).

— Either the initial condition is taken from the equilibrium pdf, or the latter should be reached after an equilibration time \( t_{eq} \):

\[
P_{eq}(\vec{v}, \vec{r}) \propto e^{-\beta \left( \frac{mv^2}{2} + V(\vec{r}) \right)}
\]
Dynamics in equilibrium

Two properties

• One-time quantities reach their equilibrium values:

\[ \langle A(\{\vec{r}\}_\xi)(t) \rangle \to \langle A(\{\vec{r}\}_\xi) \rangle_{eq} \]

[the first average is over realizations of the thermal noise (and initial conditions) and the second average is taken with the equilibrium (Boltzmann) distribution]

• All time-dependent correlations are stationary

\[ \langle A_1(\{\vec{r}\}_\xi)(t_1)A_2(\{\vec{r}\}_\xi)(t_2) \cdots A_n(\{\vec{r}\}_\xi)(t_n) \rangle = \langle A_1(\{\vec{r}\}_\xi)(t_1 + \Delta)A_2(\{\vec{r}\}_\xi)(t_2 + \Delta) \cdots A_n(\{\vec{r}\}_\xi)(t_n + \Delta) \rangle \]

for any \( n \) and \( \Delta \). In particular, \( C(t, t_w) = C(t - t_w) \).

Proof: 3rd lecture
Plan of the 1st Lecture

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2. **How can a classical system stay far from equilibrium?**
   - From single-particle to many-body.
   - Diffusion
   - Phase-separation & domain growth
   - Glasses
   - Driven systems
   - Active matter

3. Details on the non-equilibrium behaviour
Too long equilibration time

Environment in but system away from equilibrium

- The equilibration time goes beyond the experimentally accessible times.
  \[ t_{\text{eq}} \gg t_{\text{exp}} \]

  **Microscopic system** with no confining potential, \( t_{\text{eq}x} = \infty \)
  e.g., **Diffusion processes**.

  **Macroscopic systems** in which the equilibration time grows with the system size,
  \[ \lim_{N \to 1} t_{\text{eq}}(N) \gg t \]
  e.g., **Critical dynamics**, **coarsening**, **glassy physics**.

- **Driven systems**
  \[ \vec{F} \neq -\nabla V(\vec{r}) \]
  e.g., **Sheared liquids**, **vibrated powders**, **active matter**.
Microscopic system

Brownian motion: diffusion

First example of dynamics of an open system

The system: the Brownian particle

The bath: the liquid

Interaction: collisional or potential

Canonical setting

A few Brownian particles or tracers • embedded in, say, a molecular liquid.

Late XIX, early XX (Brown, Einstein, Langevin)
Langevin approach

Stochastic Markov dynamics

From Newton’s equation $\vec{F} = m\vec{a} = m\vec{v}$ and $\vec{v} = \dot{\vec{x}}$ 

$$m\dot{v}_a = -\gamma_0 v_a + \xi_a$$

with $a = 1, \ldots, d$ (the dimension of space), $m$ the particle mass, $\gamma_0$ the friction coefficient, and $\xi$ the time-dependent thermal noise with Gaussian statistics, zero average $\langle \xi_a(t) \rangle = 0$ at all times $t$, and delta-correlations $\langle \xi_a(t)\xi_b(t') \rangle = 2\gamma_0 k_B T \delta_{ab} \delta(t - t')$.

Dissipation for $\gamma_0 > 0$ the averaged energy is not conserved, 

$$2\langle E_{syst}(t) \rangle = m\langle v^2(t) \rangle \neq 0.$$
Brownian motion

Normal diffusion

For simplicity, take a one dimensional system, $d = 1$.

The relation between friction coefficient $\gamma_0$ and amplitude of the noise correlation $2\gamma_0 k_B T$ ensures **equipartition** for the velocity variable

$$m\langle v^2(t) \rangle \to k_B T$$

for $t \gg t^v_r \equiv \frac{m}{\gamma_0}$ \text{Langevin 1908}

But the position variable $x$ **diffuses** since $e^{-\beta V}$ is not normalizable.

$$\langle x^2(t) \rangle \to 2D t \quad (t \gg t^v_r = m/\gamma_0)$$

$$D = \frac{k_B T}{\gamma_0}$$

diffusion constant.

The particle is out of equilibrium!
Brownian motion

Normal diffusion

For simplicity, take a one dimensional system, $d = 1$.

The relation between friction coefficient $\gamma_0$ and amplitude of the noise correlation $2\gamma_0 k_B T$ ensures **equipartition** for the velocity variable

$$m\langle v^2(t) \rangle \rightarrow k_B T \quad \text{for} \quad t \gg t_r^v \equiv \frac{m}{\gamma_0} \quad \text{Langevin 1908}$$

But the position variable $x$ **diffuses** since $e^{-\beta V}$ is not normalizable.

$$\langle x^2(t) \rangle \rightarrow 2D t \quad (t \gg t_r^v = m/\gamma_0)$$

$$D = \frac{k_B T}{\gamma_0} \quad \text{diffusion constant.}$$

Coexistence of equilibrium ($v$) and out of equilibrium ($x$) variables
Macroscopic systems

Discussion of several macroscopic systems with slow dynamics due to

$$\lim_{N \to 1} t_{eq}(N) \gg t$$

Examples:

Ordering processes

Domain growth, phase separation

Systems with quenched disorder

Random ferromagnets, spin-glasses

Systems with frustrated interactions

Spin ices
Phase separation

Demixing transitions

Two species ● and ●, repulsive interactions between them.

Experimental phase diagram

Binary alloy, Hansen & Anderko, 54
Phase separation

Phase ordering kinetics

via spinodal process
with deep quench (unstable region)

via nucleation process
with shallow quench (metastable region)
Phase ordering kinetics

Are these quench dynamics fast processes? Can we simply forget what happens during the transient, $t_{eq}$, and focus on the subsequent *equilibrium* behaviour?

No!

It turns out that this is a very slow regime. Its duration grows with the size of the system and it diverges in the thermodynamic limit $N \to \infty$.

We understand the mechanisms for relaxation: interface local curvature driven dynamics and matter diffusion.

The domains get rounder
The regions get darker and lighter
Quenched disorder

*Quenched variables* are frozen during time-scales over which other variables fluctuate.

**Time scales**

$t_q$ could be the diffusion time-scale for magnetic impurities, the magnetic moments of which will fluctuate in a *magnetic system* or;

the flipping time of impurities that create random fields acting on other magnetic variables.

**Weak disorder** (modifies the critical properties but not the phases) vs.

**strong disorder** (modifies both).

*E.g.*, random ferromagnets $(J_{ij} > 0)$ vs. spin-glasses $(J_{ij} < 0)$. 
Spin-glasses

Magnetic impurities (spins) randomly placed in an inert host

Quenched random interactions

Interacting via the RKKY potential

\[ V(r) \propto \frac{\sin 2\pi k_F r}{r^3} \]

very rapid oscillations (change in sign) and slow power law decay

Standard lore: there is a 2nd order static phase transition at \( T_s \) separating a paramagnetic from a spin-glass phase.

No dynamic precursors above \( T_s \).

Glassy dynamics below \( T_s \) with aging, memory effects, etc.
What do glasses look like?

Simulation
Molecular (Sodium Silicate)

Confocal microscopy
Colloids (e.g. \( d \sim 162 \text{ nm in water} \))

Experiment
Granular matter

Simulation
Polymer melt
Structural glasses

Characteristics

- Selected *variables* (molecules, colloidal particles, vortices or polymers in the pictures) are coupled to their surroundings (other kinds of molecules, water, etc.) that act as *thermal baths in equilibrium*.

- There is *no quenched disorder*.

- The interactions each variable feels are still in competition, e.g. Lenard-Jones potential, *frustration*.

- Each variable feels a different set of forces, *time-dependent heterogeneity*.

Sometimes one talks about *self-generated disorder*. 
Structural Glasses

*e.g.*, colloidal ensembles

Micrometric spheres immersed in a fluid

In the glass: no obvious growth of order, slow dynamics with, however, scaling properties.

What drives the slowing down?
We do not know

**Long time-scales for relaxation**

Systems with *competing interactions* remain *out of equilibrium* and it is not clear

- whether there are phase transitions,
- which is the nature of the putative ordered phases,
- which is the dynamic mechanism.

Examples are:

- systems with quenched disorder,
- systems with geometric frustration,
- glasses of all kinds.

Static and dynamic mean-field theory has been developed – both classically and quantum mechanically – and they yield new concepts and predictions.

Extensions of the RG have been proposed and are currently being explored.
Energy injection

Traditional: from the borders (outside)

Rheology

Transport
Rheology of complex fluids

Shear thinning $\tau_{\text{relax}}$ decreases, e.g. paints

Shear thickening $\tau_{\text{relax}}$ increases, e.g. cornstarch & water mix

e.g. review Brader 10
Drive & transport

Driven interface over a disordered background

A line Depinning & creep avalanches

e.g. review Giamarchi et al 05, connections to earthquakes Landes 16
Active matter

Definition

Active matter is composed of large numbers of active "agents", each of which consumes energy in order to move or to exert mechanical forces.

Due to the energy consumption, these systems are intrinsically out of thermal equilibrium.

Energy injection is done “uniformly” within the samples (and not from the borders).

Coupling to the environment (bath) allows for the dissipation of the injected energy.
Natural systems

Birds flocking
Natural systems

Bacteria

Escherichia coli - Pictures borrowed from the internet.
Artificial systems

Janus particles

Particles with two faces (Janus God)

e.g. Bocquet group ENS Lyon-Paris, di Leonardo group Roma
In these lectures:
Focus on relaxational cases (no weird forces)
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   Glasses
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   Active matter

3. More details on the non-equilibrium behaviour
   Dynamic classes
Dynamical classes

Distinction

- Let us review the phenomenology of each kind of glass in more detail.
- $T_g$ in structural glasses looks like a mixed 1st-2nd order transition.
- Spin-glass transition is seen as a conventional 2nd order transition.
- A family of models that capture this phenomenology.
  
  Kirkpatrick, Thirumalai & Wolynes, late 80s

- Static and dynamic analytic methods to study them.
- Results & open questions.
Glassy dynamic arrest

Observable (viscosity) vs control parameter (temperature)
Characteristic temperatures

Fragile glasses

\[ T > T_m \] (equilibrium) liquid

At \( T_m \), melting transition (avoided)

\[ T_g < T < T_m \] (metastable) supercooled liquid

At \( T_g \), \( \tau_{relax} = 100 \text{ sec} \) Dynamic crossover

\[ T < T_g \] Glassy non-equilibrium dynamics

At \( T_0 \), \( \tau_{relax} \rightarrow \infty \) ? Phase transition to ideal glass ?
Cooling rate effects

\[ r \equiv \frac{\Delta T}{\Delta t} \quad r_1 > r_2 > r_3 \]

There is no single value of \( V \) at fixed \( T < T_g(r) \).

The system is \textit{out of equilibrium} below \( T_g(r) \).
Observables

Positional order

The (fluctuating) local particle number density

\[ \rho(\vec{r}_0) = \sum_{i=1}^{N} \delta(\vec{r}_0 - \vec{r}_i) \]

with normalisation \( \int d^d \vec{r}_0 \rho(\vec{r}_0) = N \).

The density-density correlation function \( C(\vec{r} + \vec{r}_0, \vec{r}_0) = \langle \rho(\vec{r} + \vec{r}_0)\rho(\vec{r}_0) \rangle \)

that, for homogeneous (independence of \( \vec{r}_0 \)) and isotropic (\( \vec{r} \mapsto |\vec{r}| = r \)) cases, is simply \( C(\vec{r} + \vec{r}_0, \vec{r}_0) = C(r) \).

The double sum in \( C(\vec{r} + \vec{r}_0, \vec{r}_0) = \langle \sum_{ij} \delta(\vec{r} + \vec{r}_0 - \vec{r}_i)\delta(\vec{r}_0 - \vec{r}_j) \rangle \) has contributions from \( i = j \) and \( i \neq j \): \( C_{\text{self}} + C_{\text{diff}} \)
Observables

Positional order

The density-density correlation function

\[ C(\vec{r} + \vec{r}_0, \vec{r}_0) = \langle \rho(\vec{r} + \vec{r}_0) \rho(\vec{r}_0) \rangle = \sum_{ij} \langle \delta(\vec{r} + \vec{r}_0 - \vec{r}_i) \delta(\vec{r}_0 - \vec{r}_i) \rangle \]

is linked to the structure factor

\[ S(\vec{q}) \equiv N^{-1} \langle \tilde{\rho}(\vec{q}) \tilde{\rho}(-\vec{q}) \rangle = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} e^{-i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} \]

by

\[ N S(\vec{q}) = \int d^d r_1 \int d^d r_2 \ C(\vec{r}_1, \vec{r}_2) \ e^{-i\vec{q} \cdot (\vec{r}_1 - \vec{r}_2)} \]
Observables

Positional order

In isotropic cases, i.e. liquid phases, the pair correlation function

$$\frac{N}{V} g(r) = \text{average number of particles at distance } r$$

from a tagged particle at \( \vec{r}_0 \)

is linked to the structure factor

$$S(\vec{q}) = 1 + \frac{N}{V} \langle \sum_{i=1}^{N} \sum_{j=1}^{N} e^{-i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} \rangle$$

by

$$S(\vec{q}) = 1 + \frac{N}{V} \int d^d r \ g(r) e^{i\vec{q} \cdot \vec{r}}$$

Peaks in \( g(r) \) are related to peaks in \( S(q) \). The first peak in \( S(q) \) is at \( q_0 = \frac{2\pi}{\Delta r} \) where \( \Delta r \) is the distance between peaks in \( g(r) \) (that is close to the inter particle distance as well).
Observables

Liquid

“Introduction to Modern Statistical Mechanics”, Chandler (OUP)
Observables

Experiments & simulations of liquids

Inter-peak distance between the peaks in $g(r)$ is $\Delta r \simeq \sigma \simeq 3\text{Å}$

Position of the first peak in $S(q)$ is at $q_0 \simeq \frac{2\pi}{\Delta r} \simeq 2\text{ Å}^{-1}$

“Structure Factor and Radial Distribution Function for Liquid Argon at 85K”,

"RMC Analyses Solve High-Speed Phase-Change Mechanism"

Matsunaga, Kojima, Yamada, Kohara, Takata (2006)
Two-time observables

Correlations

\[ C_{AB}(t, t_w) \equiv \langle A[\{\vec{r}^i(t)\}] B[\{\vec{r}^i(t_w)\}] \rangle \]

average over realizations of the dynamics (initial conditions, random numbers in a MC simulation, thermal noise in Langevin dynamics, etc.)
Correlation functions

One can define a two-time dependent density-density correlation

\[ \langle \delta \rho(\vec{x}, t) \delta \rho(\vec{y}, t_w) \rangle \]

The angular brackets indicate a “thermal” average; i.e. over different dynamical histories (runs of simulation/experiment)

Upon averaging one expects:

- isotropy (all directions are equivalent)
- invariance under translations of the reference point \( \vec{x} \).

Thus, \( \langle \delta \rho(\vec{x}, t) \delta \rho(\vec{y}, t_w) \rangle = g(r; t, t_w) \), with \( r = |\vec{x} - \vec{y}| \). Its Fourier transform is \( F(q; t, t_w) \) and it has a self part \( F_s(q; t, t_w) \).
Super-cooled liquids

Equilibrium decay above $T_g$

The intermediate or self correlation

$$F_s(q; t, t_w) = N^{-1} \sum_{i=1}^{N} \langle e^{i \vec{q}(\vec{r}_i(t) - \vec{r}_i(t_w))} \rangle$$

MD simulations of silica
Experiments in glycerol

Note the plateau
The relaxation time $\tau_\alpha$ increases by 5 orders of magnitude
Super-cooled liquids

but the structure is always the one of a liquid!

No important change in structure in the full range of temperatures in which the relaxation time, $\tau_\alpha$ varies by 5-10 orders of magnitude.
The plateau

here, in a binary Lennard-Jones mixture

First stationary relaxation towards the plateau: ‘cages’

Is the plateau an order parameter?

Figure from J-L Barrat & Kob 99
First stationary relaxation towards the plateau: ‘cages’

Colloids \textit{Weeks et al. 02}

Powders \textit{Pouliquen et al. 03}

The particles’ displacement is much smaller than the particle radius.

Second non-stationary relaxation below the plateau: ‘structural’
Response to perturbations

The perturbation couples linearly to the observable $B[\{\vec{r}_i\}]$

$$E \rightarrow E - hB[\{\vec{r}_i\}]$$

The linear instantaneous response of another observable $A(\{\vec{r}_i\})$ is

$$R_{AB}(t, t_w) \equiv \left\langle \frac{\delta A[\{\vec{r}_i\]](t)}{\delta h(t_w)} \right|_{h=0} \right.$$  

The linear integrated response or dc susceptibility is

$$\chi_{AB}(t, t_w) \equiv \int_{t_w}^{t} dt' R_{AB}(t, t')$$
ac response to perturbations

\[
\chi(\omega, t_w) = \int_0^{t_w} dt' R(t_w, t') h(\omega, t') = \int_0^{t_w} dt' R(t_w, t') h e^{i\omega t'}
\]

\[
\chi'(\omega, t_w) = \text{Re} \chi(\omega, t_w) \quad \text{(in phase)}
\]

\[
\chi''(\omega, t_w) = \text{Im} \chi(\omega, t_w) \quad \text{(out of phase)}
\]

are related by Kramers-Kröning

\[
\chi''(\omega, t_w) = -\pi^{-1} P \int d\omega' \frac{\chi'(\omega, t_w)}{\omega' - \omega}
\]

In equilibrium \( \chi(\omega, t_w) \to \chi(\omega) \)
Dynamics in super-cooled liquid

Propylene carbonate $T_{VF} = 132 \text{ K}, T_g = 159 \text{ K}, T_m = 218 \text{ K}$

Fig. 2, U. Schneider et al.

Low frequency peak $\approx$ Cole-Davison function or FT of stretched exp.

Schneider, Lunkenheimer, Brand, & Loidl 98
Still lower temperature

Out of equilibrium relaxation

L-J mixture J-L Barrat & Kob 99

Colloids Viasnoff & Lequeux 03

The equilibration time goes beyond the experimentally accessible times

The same is observed in all other glasses.
Still lower temperature

Ageing effects

L-J mixture J-L Barrat & Kob 99
Colloids Viasnoff & Lequeux 03

$|g_1(t_w, t)|^2$

t - $t_w$ (sec)

$F_s(q; t, t_w)$

Ageing the relaxation is slower for older systems
The plateau

Binary Lennard-Jones mixture

First stationary relaxation towards the plateau: ‘cages’

Note that the structural relaxation is stationary at $T > T_g$ (left) and non-stationary $T < T_g$ (right)
Real space view

First stationary relaxation towards the plateau: ‘cages’

Colloids *Weeks et al. 02*

The particles’ displacement is much smaller than particle radius.

Second non-stationary relaxation below the plateau: ‘structural’

The one that demonstrates the out of equilibrium physics

Powders *Pouliquen et al. 03*
An insulating spin-glass

Thiospinel

Self-correlation
spontaneous

Thermo-remanent magnetisation
induced (a response)

Herisson & Ocio 01
An insulating spin-glass

Thiospinel

There seems to be a plateau (maybe inclined) separating a stationary from a non-stationary regime

Herisson & Ocío 01
Ferromagnet vs glass

Not so different as long as correlations are concerned

2d Ising model - spin-spin

Sicilia et al. 07

Lennard-Jones - density-density

Kob & Barrat 99
Different two-time regimes

Interpretation

• In phase ordering kinetics, thermal fluctuations within domains vs. domain wall motion.
• In particle systems, rattling within cages vs. structural relaxation.

Cages in colloidal suspensions

Domain growth in the 2d Ising model.
Response functions

Glasses and spin-glasses

Lennard-Jones mixture
Kob & J-L Barrat 98

A metallic spin-glass
Vincent et al. 96
Summary

Spin-glasses & coarsening systems

2nd order phase transition at $T_c$

<table>
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<tr>
<th>Paramagnet</th>
<th>Critical slowing down</th>
<th>Spin-glass/ferromagnet</th>
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<tbody>
<tr>
<td>Exponential relax</td>
<td>Non-exponential relax</td>
<td></td>
</tr>
<tr>
<td>Equilibrium</td>
<td>Long-relaxation to equilibrium</td>
<td>Non-equilibrium</td>
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</tbody>
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Separation of time-scales

Stationary

Aging

Aging means that correlations and responses depend on $t$ and $t_w$

ac susceptibilities depend on $\omega$ and $t_w$
Fragile glasses

Time-scales from calorimetric measurement of entropy

What is making the relaxation so slow?
Is there growth of static order?
Which one?
Phase space picture?
Summary

**Structural glasses**

**Crytallization** at $T_m$ is avoided by cooling fast enough.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Supercooled liquid</th>
<th>Glass</th>
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<tbody>
<tr>
<td>Exponential relax</td>
<td>Non-exponential relax</td>
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<tr>
<th>Equilibrium</th>
<th>Metastable equilibrium</th>
<th>Non-equilibrium</th>
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<tbody>
<tr>
<td>Separation of time-scales &amp;</td>
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<tr>
<td>An exponential number of metastable states !</td>
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<td></td>
</tr>
<tr>
<td>Stationary</td>
<td>Aging</td>
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</tbody>
</table>

**Aging** means that correlations and responses depend on $t$ and $t_w$.

ac susceptibilities depend on $\omega$ and $t_w$.

There might be an **equilibrium transition** to an **ideal glass** at $T_s$. 
Challenges in classical non-equilibrium macroscopic systems

- **Coarsening**
  The systems are taken across *usual phase transitions*.
  The *dynamic mechanisms* are well-understood:
  competition between equilibrium phases & topological defect annihilation.
  The difficulty lies in the calculation of observables in a time-dependent non-linear field theory.

- **Glasses**
  Are there phase transitions?
  The dynamic mechanisms are not well understood.
  The difficulty is conceptual (also computational).

- **General question**
  Do these, as well as sheared liquids or active matter, enjoy some kind of thermodynamic properties?
Methods

Many body systems

• **Coarsening phenomena**

  Identify the order parameter $\phi(\vec{x}, t)$ (a field). Write Langevin or Fokker-Planck equations for it and analyse them. A difficult problem. Non-linear equations. Neither perturbation theory nor RG methods are OK. Self-consistent resummations tried.

• **Glassy systems**

  The "order parameter" is a composite object depending on two-times. Spin models with quenched randomness yield a mean-field description of the dynamics observed. Classes of systems (ferromagnets, spin-glass and fragile glasses) captured.
Disordered spin systems

Classical $p$-spin model

$$H_{syst} = - \sum_{i_1 < \ldots < i_p}^{N} J_{i_1 i_2 \ldots i_p} s_{i_1} s_{i_2} \ldots s_{i_p}$$

Ising, $s_i = \pm 1$, or spherical, $\sum_{i=1}^{N} s_i^2 = N$, spins.

Sum over all $p$-uplets on a complete graph: fully-connected model.

Random exchanges $P(J_{i_1 i_2 \ldots i_p}) = e^{-p! J_{i_1 i_2 \ldots i_p}^2 / (2N^{p-1} J^2)}$

Extensions to random graphs possible: dilute models.

$p = 2$ Ising: Sherrington-Kirkpatrick model for spin-glasses

$p = 2$ spherical $\approx$ mean-field ferromagnet

$p \geq 3$ Ising or spherical: models for fragile glasses
Disordered spin systems

Random $K$-sat problem

A clause is the ‘logical or’ between $K$ requirements imposed on Boolean variables $x_i$ chosen randomly from a pool of $N$ of them.

A formula is the ‘logical and’ between $M$ such clauses, $F = \bigwedge_{\ell=1}^{M} \bigvee_{i=1}^{K} x_i^{(\ell)}$. It is satisfied when all $M$ clauses are.

The search for a solution can be set as the search for the spin configuration(s) with vanishing energy

$$H_{\text{syst}} = \alpha 2^{-K} N + \sum_{R=1}^{K} (-1)^R \sum_{i_1 < \ldots < i_R} J_{i_1 i_2 \ldots i_R} s_{i_1} s_{i_2} \ldots s_{i_R}$$

with $\alpha = M/N$, Ising spins, $s_i = \pm 1$, and interactions

$$J_{i_1 \ldots i_R} = 2^{-K} \sum_{\ell=1}^{M} C_{\ell, i_1} \ldots C_{\ell, i_R}$$

with $C_{\ell, i_k} = +, -$ for the condition $x_{i_k}^{(\ell)} = \text{T,F}$ and $C_{\ell, i_k} = 0$ otherwise.

Sum of classical dilute $p \leq K$-spin models
Methods

for classical and quantum disordered systems

Statics

TAP Thouless-Anderson-Palmer \[\\text{fully-connected (complete graph)}\]

Replica theory \[\\text{Gaussian approx. to field-theories}\]

Cavity or Peierls approx. \[\\text{dilute (random graph)}\]

Bubbles & droplet arguments \[\\text{finite dimensions}\]

functional RG

Dynamics

Generating functional for classical field theories (MSRJD).

Schwinger-Keldysh closed-time path-integral for quantum dissipative models (the previous is recovered in the \(\hbar \to 0\) limit).

Perturbation theory, renormalization group techniques, self-consistent approx.
Methods

for classical and quantum disordered systems

Statics

TAP Thouless-Anderson-Palmer \{ fully-connected (complete graph) \\
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Dynamics

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Perturbation theory, renormalization group techniques, self-consistent approximations
Some references

Spin-glasses


Theory and methods


and unpublished notes (see webpage & www.lpthe.jussieu.fr/~leticia)

Growing lengths